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A Comment on String Solitons†

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We derive an exact string-like soliton solution of $D = 10$ heterotic string theory. The solution possesses $SU(2) \times SU(2)$ instanton structure in the eight-dimensional space transverse to the worldsheet of the soliton.

4/93

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In [1], an exact multi-fivebrane soliton solution of heterotic string theory was presented. This solution represented an exact extension of the tree-level supersymmetric multi-fivebrane solutions of [2,3]. For this class of fivebrane solutions, the generalized curvature incorporating the axionic field strength possesses a (anti) self-dual structure [4,5] and is referred to as an "axionic instanton" (see [6] and references therein). Exactness is shown for the heterotic solution based on algebraic effective action arguments and $(4,4)$ worldsheet supersymmetry [1]. The gauge sector of the heterotic solution possesses $SU(2)$ instanton structure in the four-dimensional space transverse to the fivebrane. In more recent work, Kounnas [7] described a method of obtaining string solutions with non-trivial backgrounds by using $N=4$ superconformal building blocks with $\hat{c}=4$. In particular, he proposed the existence of an exact solution with $SU(2)\times SU(2)$ instanton structure.

In this paper we obtain an explicit space-time background corresponding to Kounnas' conformal field theory by constructing an exact string-like solution of $D=10$ heterotic string theory from a modification of the fivebrane ansatz. In the eight-dimensional space transverse to the string, the solution contains two independent $SU(2)$ instantons each embedded in a separate $SO(4)$ subgroup of the gauge group. The arguments demonstrating exactness of this solution follow those of [1].

The tree-level supersymmetric vacuum equations for the heterotic string are given by

$$\begin{aligned}
\delta\psi_M &= \left(\nabla_M - \tfrac{1}{4}H_{MAB}\Gamma^{AB}\right)\epsilon = 0,\\
\delta\lambda &= \left(\Gamma^A\partial_A\phi - \tfrac{1}{6}H_{ABC}\Gamma^{ABC}\right)\epsilon = 0,\\
\delta\chi &= F_{AB}\Gamma^{AB}\epsilon = 0,
\end{aligned} \tag{1}$$

where $\psi_M,\ \lambda$ and χ are the gravitino, dilatino and gaugino fields. The Bianchi identity is given by

$$dH = \frac{\alpha'}{4}\left(\mathrm{tr}R\wedge R - \mathrm{tr}F\wedge F\right). \tag{2}$$

The (9+1)-dimensional Majorana-Weyl fermions decompose down to chiral spinors according to $SO(9,1)\supset SO(1,1)\otimes SO(4)\otimes SO(4)$ for the $M^{9,1}\to M^{1,1}\times M^4\times M^4$ decomposition. The ansatz

$$\begin{aligned}
\phi &= \phi_1 + \phi_2,\\
g_{\mu\nu} &= e^{2\phi_1}\delta_{\mu\nu} \qquad \mu,\nu = 2,3,4,5,\\
g_{mn} &= e^{2\phi_2}\delta_{mn} \qquad m,n = 6,7,8,9,\\
g_{ab} &= \eta_{ab} \qquad a,b = 0,1,\\
H_{\mu\nu\lambda} &= \pm 2\epsilon_{\mu\nu\lambda\sigma}\partial^\sigma\phi \qquad \mu,\nu,\lambda,\sigma = 2,3,4,5,\\
H_{mnp} &= \pm 2\epsilon_{mnpk}\partial^k\phi \qquad m,n,p,k = 6,7,8,9
\end{aligned} \tag{3}$$

with constant chiral spinors $\epsilon_\pm = \epsilon_2 \otimes \eta_4 \otimes \eta_4'$ solves the supersymmetry equations with zero background fermi fields provided the YM gauge field satisfies the instanton (anti) self-duality condition

$$F_{\mu\nu} = \pm\frac{1}{2}\epsilon_{\mu\nu}{}^{\lambda\sigma}F_{\lambda\sigma}, \qquad \mu,\nu,\lambda,\sigma = 2,3,4,5$$
$$F_{mn} = \pm\frac{1}{2}\epsilon_{mn}{}^{pk}F_{pk}, \qquad m,n,p,k = 6,7,8,9. \tag{4}$$

The chiralities of the spinors ϵ_2, η_4 and η_4' are correlated by

$$(1\mp\gamma_3)\epsilon_2 = (1\mp\gamma_5)\eta_4 = (1\mp\gamma_5)\eta_4' = 0, \tag{5}$$

so that three-quarters of the spacetime supersymmetries are broken. An exact solution is obtained as follows. Define a generalized connection by

$$\Omega^{AB}_{\pm M} = \omega^{AB}_M \pm H^{AB}_M \tag{6}$$

in an $SU(2)\times SU(2)$ subgroup of the gauge group, and equate it to the gauge connection A_M [8] for $M = 2,3,4,5,6,7,8,9$ so that $dH = 0$ and the corresponding curvature $R(\Omega_\pm)$ cancels against the Yang-Mills field strength F in both subspaces (2345) and (6789). For $e^{-2\phi_1}\Box\, e^{2\phi_1} = e^{-2\phi_2}\Box\, e^{2\phi_2} = 0$, the curvature of the generalized connection can be written in covariant form [4,5]

$$\hat{R}^\alpha{}_{\beta\gamma\lambda} = \delta_{\alpha\lambda}\nabla_\gamma\nabla_\beta\phi_1 - \delta_{\alpha\gamma}\nabla_\lambda\nabla_\beta\phi_1 + \delta_{\beta\gamma}\nabla_\lambda\nabla_\alpha\phi_1 - \delta_{\beta\lambda}\nabla_\gamma\nabla_\alpha\phi_1$$
$$\pm\epsilon_{\alpha\beta\gamma\mu}\nabla_\lambda\nabla_\mu\phi_1 \mp \epsilon_{\alpha\beta\lambda\mu}\nabla_\gamma\nabla_\mu\phi_1, \tag{7}$$

where $\alpha,\beta,\gamma,\lambda,\mu = 2,3,4,5$ and

$$\hat{R}^i{}_{jkl} = \delta_{il}\nabla_k\nabla_j\phi_2 - \delta_{ik}\nabla_l\nabla_j\phi_2 + \delta_{jk}\nabla_l\nabla_i\phi_2 - \delta_{jl}\nabla_k\nabla_i\phi_2$$
$$\pm\epsilon_{ijkm}\nabla_l\nabla_m\phi_2 \mp \epsilon_{ijlm}\nabla_k\nabla_m\phi_2, \tag{8}$$

where $i,j,k,l,m = 6,7,8,9$. It easily follows that

$$\hat{R}^\alpha{}_{\beta\gamma\lambda} = \mp\frac{1}{2}\epsilon_{\gamma\lambda}{}^{\mu\nu}\hat{R}^\alpha_{\beta\mu\nu}, \tag{9}$$

and

$$\hat{R}^i{}_{jkl} = \mp\frac{1}{2}\epsilon_{kl}{}^{mn}\hat{R}^i_{jmn}, \tag{10}$$

from which it follows that both F and R are (anti) self-dual in both four-dimensional subspaces. This solution becomes exact since $A_M = \Omega_{\pm M}$ implies that all the higher order corrections vanish. Both the algebraic effective action arguments and the $(4,4)$ worldsheet supersymmetry arguments of [1] can be used in essentially the same manner to demonstrate exactness of the string solution. The explicit solution for ϕ_1 and ϕ_2 in (3) is given by

$$\begin{aligned} e^{2\phi_1} &= e^{2\phi_{1_0}} \left(1 + \sum_{i=1}^{N} \frac{\rho_i^2}{|\vec{x} - \vec{a}_i|^2}\right), \\ e^{2\phi_2} &= e^{2\phi_{2_0}} \left(1 + \sum_{j=1}^{M} \frac{\lambda_j^2}{|\vec{y} - \vec{b}_j|^2}\right), \end{aligned} \tag{11}$$

where $\vec{x}$ and $\vec{a}_i$ are four-vectors and ρ_i instanton scale sizes in the space (2345), and $\vec{y}$ and $\vec{b}_j$ are four-vectors and λ_j instanton scale sizes in the space (6789). Axion charge quantization then requires that $\rho_i^2 = e^{-2\phi_{1_0}} n_i \alpha'$ and $\lambda_j^2 = e^{-2\phi_{2_0}} m_j \alpha'$, where n_i and m_j are integers. Note that for $N = 0$ or $M = 0$ we recover the solution of [1]. It is interesting to note that both the charge $Q_2 = -1/2 \int_{S^7} {}^*H$ and the mass per unit length $\mathcal{M}_2$ of the infinite string diverge. By contrast, all classes of fivebrane solutions have finite charge and mass per unit length as a result of the preservation of half the spacetime supersymmetries and the saturation of a Bogmol'nyi bound. The fact that three-quarters of the spacetime supersymmetries are broken for this solution means that the saturation of the Bogomol'nyi bound is no longer guaranteed, but it is unclear as to whether this would necessarily imply infinite mass per unit length for the string. It would be interesting to see whether any finite mass per unit length analogs of this solution exist, especially in the context of the conjectured dual theory of fundamental fivebranes [9]. Another interesting point is that the $D = 8$ instanton number N_8 for this string solution is in general nonzero for gauge group $E_8 \times E_8$ ($N_8 = NM$, where N and M are the $D = 4$ instanton numbers in the (2345) and (6789) spaces respectively), since in this case $(\mathrm{TrF}^2)^2$ is nonvanishing. This is to be contrasted with the zero $D = 8$ instanton number found for the string soliton solution of Duff and Lu [10]. †

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