Metastable Flux Configurations and de Sitter Spaces

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Abstract

We derive stability conditions for the critical points of the no-scale scalar potential governing the dynamics of the complex structure moduli and the axio-dilaton in compactifications of type IIB string theory on Calabi-Yau three-folds. We discuss a concrete example of a T^6 orientifold. We then consider the four-dimensional theory obtained from compactifications of type IIB string theory on non-geometric backgrounds which are mirror to rigid Calabi-Yau manifolds and show that the complex structure moduli fields can be stabilized in terms of H_{RR} only, *i.e.* with no need of orientifold projection. The stabilization of all the fields at weak coupling, including the axio-dilaton, may require to break supersymmetry in the presence of H_{NS} flux or corrections to the scalar potential.

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1 Introduction

There are many ways to obtain models of particle phenomenonlogy using string theory. A good starting point is to construct a model with $\mathcal{N} = 1$ supersymmetry in four dimensions. One can obtain such models, for example, by compactifying M-theory on G_2-holonomy manifolds, F-theory on Calabi-Yau four-folds or type II theories on Calabi-Yau orientifolds. It is a beautiful fact that these models have a moduli space of vacuum states. However, concrete predictions can only be made if the mechanism which picks the vacuum state of string theory can be identified. By including fluxes as background fields the continuous ambiguity associated with the vacuum expectation values of the moduli fields is replaced by a discrete freedom associated with the choice of flux numbers. However, the number of possible vacuum states is still enormous and it has been argued to built a whole landscape of solutions. However, most of these string theory backgrounds have flat directions and the number of solutions with all moduli stabilized is very limited.

Stabilizing all the scalar fields associated with a Calabi-Yau compactification of string theory at weak coupling is a particularly hard problem. In the context of compactifications of type IIB string theory on a Calabi-Yau orientifold, for example, one of the fields which is conventionally stabilized using fluxes is the axio-dilaton. This removes the arbitrariness associated with the vacuum expectation value of this field. However at the same time this means that the string coupling constant is no longer a parameter whose value we can freely choose but it is determined in terms of fluxes. Weak coupling can then only be achieved if some flux numbers can be chosen to be large. But taking such a limit and making the moduli fields heavy is difficult, and it has been conjectured in ref. [1] that it is actually impossible. This situation may change once supersymmetry is broken and as a result it is very important to determine the properties of flux configurations leading to stable critical points of the scalar potential while breaking supersymmetry. This is the aim of the present paper.

Lets illustrate this idea in the interesting example of the KKLT model [2]. Here complex structure moduli and the axio-dilaton acquire an expectation value due to perturbative fluxes while preserving an $\mathcal{N} = 1$ supersymmetry. Non-perturbative corrections to the superpotential cause the radial modulus ρ to become heavy compared to the AdS cosmological constant while the masses of the complex structure moduli will generically be of the order to the inverse AdS length and may not be heavy enough to be considered stabilized [1]. This situation changes once these vacua are lifted to dS spaces. According to ref. [2] this can be achieved by assuming the presence of an anti-D3 brane which contributes a factor

$$\Delta V \sim \frac{1}{(\mathrm{Im}\rho)^3}, \tag{1.1}$$

to the scalar potential. Once this contribution is taken into account the potential for the radial modulus displays a metastable minimum at which the scalar potential takes a positive value. Moreover, the masses of the moduli are of the order to the AdS scale which can be much larger than the dS scale and as a result after supersymmetry is broken all the moduli fields can turn out to be heavy enough.

Adding anti-D3 branes is one way to uplift the potential to positive values. Following the argument of ref. [4], it should also be possible to obtain a potential contribution resembling the one resulting from anti-D3 branes by considering flux configurations for which $\mathcal{D}_I W \neq 0$ for some I. From the no-scale form of the potential it follows that such a contribution is positive and it's dependence on ρ is precisely equal to the one originating from anti-D3 branes. This makes it a natural alternative to the KKLT model. Since $\mathcal{D}_I W \neq 0$ the flux can no longer be imaginary self-dual (ISD) but will acquire an imaginary anti-self dual (IASD) component. Requiring that the scalar potential is critical in the complex structure and axio-dilaton directions imposes conditions on the fluxes which we will derive in the present paper while the radial modulus is not stabilized.

We then consider the four-dimensional theory obtained from compactifications of type IIB string theory on backgrounds which are mirror to rigid Calabi-Yau manifolds, *i.e.* non-geometric backgrounds with no Kähler structure. In this case case the flux induced superpotential does depend explicitly on all scalar fields, *i.e.* the complex structure moduli and the axio-dilaton. Mirror symmetry implies that on the type IIB side the Kähler potential for the axio-dilaton differs from the conventional one obtained from dimensional reduction [1]. This fact enables us to find a scalar potential which stabilizes all the complex structure moduli in terms of RR fluxes only while requiring no orientifold charge. However the axio-dilaton is not fixed and slides off to weak coupling. The axio-dilaton could be stabilized if H_{NS} is taken into account and supersymmetry is broken to render the scalar fields heavy enough. Another possibility is to take perturbative corrections to the Kähler potential and non-perturbative corrections to the superpotential into account [1].

The organization of this paper is as follows. In section 2 we consider geometric compactifications of type IIB string theory on Calabi-Yau three-folds. We derive the conditions imposed on the flux configurations to lead to stable critical points of the scalar potential in the complex structure and axio-dilaton directions. We explicitly show that the critical points do correspond to minima of the potential by computing the Hessian matrix. We illustrate the idea in the example of a torus orientifold. In section 3 we consider the four-dimensional theory obtained from compactifications of type IIB strings on mirrors of rigid Calabi-Yau manifolds. We find a scalar potential which stabilizes all the complex structure moduli in terms of RR fluxes only while requiring no orientifold charge. We discuss several possibilities to stabilize the axio-dilaton at weak coupling. We then end with some conclusions and speculations.

2 Type IIB string theory compactified on Calabi-Yau three-folds

In this section we discuss geometric compactifications of type IIB strings and analyze the critical points of the scalar potential. To set up the notation we start in subsection 2.1 deriving the form of the scalar potential following closely ref. [5]. In subsection 2.2 we derive the conditions to obtain a critical point of the potential. In subsection 2.3 we explicitly check that the critical points correspond to minima by computing the Hessian matrix. In subsection 2.4 we present a concrete example.

2.1 The scalar potential

Our starting point is the low-energy effective action of type IIB strings in the ten-dimensional Einstein frame

$$\begin{aligned} S_{IIB} &= \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-g}\left[R-\frac{\partial_M\tau\partial^M\bar{\tau}}{2(\mathrm{Im}\tau)^2}-\frac{G\cdot\overline{G}}{12\mathrm{Im}\tau}-\frac{\tilde{F}_{(5)}^2}{4\cdot 5!}\right] \\ &\quad -\frac{1}{8i\kappa_{10}^2}\int\frac{C_{(4)}\wedge G\wedge\overline{G}}{\mathrm{Im}\tau}+S_{loc}. \end{aligned} \tag{2.1}$$

Here the axio-dilaton τ is written in terms of the RR scalar $C_{(0)}$ and the dilaton ϕ according to

$$\tau = C_{(0)} + ie^{-\phi}, \tag{2.2}$$

and

$$\tilde{F}_{(5)} = F_{(5)} - \frac{1}{2}C_{(2)} \wedge H_{NS} + \frac{1}{2}B_{(2)} \wedge H_{RR}, \tag{2.3}$$

where H_{RR} and H_{NS} are the two three-forms with potentials $C_{(2)}$ and $B_{(2)}$ respectively and $G = H_{RR} - \tau H_{NS}$. The condition that $\tilde{F}_{(5)}$ is self-dual should be imposed by hand. The Bianchi identity for the five-form field is

$$d\tilde{F}_{(5)} = H_{NS} \wedge H_{RR} + 2\kappa_{10}^2 T_3 \rho_3^{loc}. \tag{2.4}$$

By integrating the Bianchi identity over the internal manifold $\mathcal{M}_6$, we get

$$\frac{1}{(2\pi)^4\alpha'^2}\int_{\mathcal{M}_6} H_{NS} \wedge H_{RR} + Q_3^{loc} = 0, \tag{2.5}$$

where we have used the relation $2\kappa_{10}^2 T_3 = (2\pi)^4\alpha'^2$. This identity means the sum of the D3 charges from background fields and localized sources vanishes. From Eq. (2.1) one obtain the four-dimensional scalar potential by dimensional reduction

$$V = \frac{1}{24\kappa_{10}^2(\mathrm{Im}\rho)^3}\int_{\mathcal{M}_6} d^6y\sqrt{g}\frac{G\cdot\overline{G}}{\mathrm{Im}\tau} - \frac{i}{4\kappa_{10}^2(\mathrm{Im}\rho)^3}\int_{\mathcal{M}_6}\frac{G\wedge\overline{G}}{\mathrm{Im}\tau} \tag{2.6}$$

By using the flux induced superpotential [3] which is explicitly given by

$$W = \int_{\mathcal{M}_6} G \wedge \Omega, \tag{2.7}$$

and the Kähler potential

$$\mathcal{K} = -3\log[-i(\rho-\bar{\rho})] - \log[-i(\tau-\bar{\tau})] - \log[-i\int_{\mathcal{M}_6}\Omega\wedge\overline{\Omega}], \tag{2.8}$$

where ρ is the radial modulus, the scalar potential (2.6) can be transformed into the standard $\mathcal{N}=1$ supergravity form

$$V = e^{\mathcal{K}}\left(g^{a\bar{b}}\mathcal{D}_aW\mathcal{D}_{\bar{b}}\overline{W} - 3|W|^2\right) \tag{2.9}$$

where a and b label all moduli and the axio-dilaton. Because the superpotential is independent of ρ the scalar potential takes the no-scale form

$$V = e^{\mathcal{K}}F_I\bar{F}^I, \tag{2.10}$$

where I and J label the complex structure moduli and the axio-dilaton. Here and in the following we will be using the notation of [6]

$$F_I = \mathcal{D}_IW, \qquad Z_{IJ} = \mathcal{D}_I\mathcal{D}_JW, \qquad U_{IJK} = \mathcal{D}_I\mathcal{D}_J\mathcal{D}_KW, \tag{2.11}$$

and indices are raised using the inverse of the Kähler metric $g_{I\bar{J}} = \partial_I\partial_{\bar{J}}\mathcal{K}$.

2.2 Critical points of the scalar potential

The scalar potential will be critical in the complex structure and axio-dilaton directions if the first derivatives vanish, *i.e.* if

$$\partial_I V = e^{\mathcal{K}} \left(Z_{IJ} \bar{F}^J + F_I \overline{W} \right) = 0. \tag{2.12}$$

One, but not the most general, solution of this condition is given by flux configurations satisfying $F_I = 0$. Using the explicit expression for the superpotential we have

$$F_i = \int_{\mathcal{M}_6} G \wedge \chi_i \qquad \text{and} \qquad F_\tau = -\frac{1}{\tau - \bar{\tau}} \int_{\mathcal{M}_6} \overline{G} \wedge \Omega, \tag{2.13}$$

where χ_i is the basis of harmonic $(2,1)$ forms and with lower case indices i, j we label the complex structure moduli only. This implies that the non-vanishing components of G can lie in the $(0,3)$ or $(2,1)$ directions. In other words, G is ISD, $\star G = iG$. Moreover, this critical point is stable because the scalar potential (2.10) is positive semi-definite and at the critical points the potential vanishes.

In the following we would like to find the most general solution of Eq. (2.12). We start by rewriting Eq. (2.12) in the form

$$\begin{aligned} &Z_{\tau\tau} \bar{F}^\tau + Z_{\tau j} \bar{F}^j + F_\tau \overline{W} = 0, \\ &Z_{i\tau} \bar{F}^\tau + Z_{ij} \bar{F}^j + F_i \overline{W} = 0. \end{aligned} \tag{2.14}$$

Note that

$$Z_{ij} = \kappa_{ijk} \frac{\int_{\mathcal{M}_6} G \wedge \overline{\chi}^k}{\int_{\mathcal{M}_6} \Omega \wedge \overline{\Omega}}, \quad Z_{\tau i} = -\frac{1}{\tau - \bar{\tau}} \int_{\mathcal{M}_6} \bar{G} \wedge \chi_i, \quad Z_{\tau\tau} = 0. \tag{2.15}$$

A simple computation (we include the details in an appendix) shows that the first condition in Eq. (2.14) is equivalent to

$$\int_{\mathcal{M}_6} G \wedge \star G = 0, \tag{2.16}$$

while the second condition leads to

$$(B\bar{B}_k + A\bar{A}_k) \int_{\mathcal{M}_6} \Omega \wedge \overline{\Omega} + \kappa_{ijk} A^i B^j = 0. \tag{2.17}$$

Here we introduced the Hodge decomposition

$$G = A\Omega + A^i \chi_i + \bar{B}^{\bar{i}} \bar{\chi}_{\bar{i}} + \bar{B}\overline{\Omega} \tag{2.18}$$

and κ_{ijk} are the Yukawa couplings. From here we conclude that the potential for the scalars (2.10) does not have a critical point for an arbitrary choice of flux. Only if Eq. (2.16) and Eq. (2.17) are satisfied can we find a critical point in all directions except the size. This is not always possible. If $H_{NS} = 0$, for example, then the dilaton cannot be stabilized since the only non-vanishing contribution to the dilaton potential comes from the overall factor $e^{\mathcal{K}}$. As a result no critical point exists since Eq. (2.16) is violated.

It is not difficult to see that all flux combinations can lead to critical points of the potential except if G is given by a combination of the following components

$$G_{(3,0)} + G_{(0,3)}, \qquad G_{(3,0)} + G_{(2,1)}, \qquad G_{(3,0)} + G_{(0,3)} + G_{(2,1)}, \tag{2.19}$$

or their complex conjugates. A flux of the form $G_{(3,0)} + G_{(0,3)}$, for example, is easily seen to violate the condition (2.16).

Among the possible flux combinations leading to critical points of the scalar potential only a flux lying in the $(2,1)$ or $(1,2)$ directions preserves supersymmetry. The $(2,1)$ component obviously preserves supersymmetry, as it satisfies

$$\mathcal{D}_I W = \mathcal{D}_\rho W = 0. \tag{2.20}$$

However a flux in the $(1,2)$ direction also preserves supersymmetry if accompanied by a change in the sign of the tadpole due to fluxes. The reason for this is that type IIB supergravity in ten dimensions is invariant under the change of sign of all RR fluxes. Changing the signs of RR fields replaces G by $-\bar{G}$ and as a result a flux lying in the (2,1) direction should lead to the same physics as a flux in the (1,2) direction. The (1,2) component does satisfy the conventional supergravity constraint $\mathcal{D}_I \widetilde{W} = \mathcal{D}_\rho \widetilde{W} = 0$, but with a superpotential given by

$$\widetilde{W} = \int_{M_6} \overline{G} \wedge \Omega. \tag{2.21}$$

The derivation of this superpotential will be discussed in appendix B. The two superpotentials W and $\widetilde{W}$ are related to each other by a CPT transformation. Any other flux components satisfying Eq. (2.16) and (2.17) will not preserve supersymmetry and lead to a positive cosmological constant or vanishing cosmological constant if only a $(3,0)$ (or $(0,3)$) component is turned on. On the other hand, due to the no-scale structure of the potential the radial modulus cannot be stabilized.

2.3 The Hessian matrix

The no-scale potential is positive definite. As a result solutions which lead to a vanishing potential at the critical point $V_\star$ are necessarily stable. However, we are interested in solutions for which $V_\star > 0$ and as a result we have to check the stability of the solutions. In order to determine if the critical points are stable we compute the Hessian matrix H. It turns out that it only has positive eigenvalues which means that the critical points are minima in the complex structure and axio-dilaton directions. Indeed, the second derivatives of the scalar potential are given by

$$\begin{aligned}
\partial_I \partial_J V &= e^{\mathcal{K}} \left(U_{IJK} \bar{F}^K + 2 Z_{IJ} \bar{W} \right) \\
\partial_I \partial_{\bar{J}} V &= e^{\mathcal{K}} (g_{I\bar{J}} F_K \bar{F}^K - R_{I\bar{J}K}{}^L F_L \bar{F}^K + 2 F_I \bar{F}_{\bar{J}} + Z_{IL} \bar{Z}_{\bar{J}\bar{K}} g^{L\bar{K}} + g_{I\bar{J}} |W|^2)
\end{aligned} \tag{2.22}$$

The critical points will be stable if

$$d\Sigma^2 = H_{\alpha\beta} dw^\alpha dw^\beta \geq 0, \tag{2.23}$$

where w^α labels all coordinates, *i.e.* α and β label the axio-dilaton, complex structure moduli and their complex conjugates. Using formulas which are explicitly presented in appendix A we obtain

$$d\Sigma^2 = e^{\mathcal{K}} g^{\gamma\sigma} \left(Z_{\alpha\gamma} \overline{Z}_{\beta\sigma} dw^\alpha dw^\beta + g^{\tau\bar{\tau}} U_{\alpha\gamma\tau} \overline{U}_{\beta\sigma\bar{\tau}} dw^\alpha dw^\beta \right) \tag{2.24}$$

where $U_{\alpha\gamma\sigma} = \mathcal{D}_\alpha \mathcal{D}_\gamma \mathcal{D}_\sigma W$ and $Z_{\alpha\gamma} = \mathcal{D}_\alpha \mathcal{D}_\gamma W$ are the generalization of U_{IJK} and Z_{IJ}. As a result the hessian matrix is positive semi-definite and the critical points correspond to minima.

2.4 An example

In this section we describe a concrete example in terms of a type IIB orientifold compactification. This example is closely related to examples discussed in refs. [4] and [7]. We will be following their notation. Let x^i and y^i, for $i = 1, 2, 3$ be the six real coordinates on T^6. These coordinates are subjected to the periodic identifications $x^i \equiv x^i + 1$ and $y^i \equiv y^i + 1$. The complex structure is parameterized by complex parameters τ^{ij}, and

$$z^i = x^i + \tau^{ij} y^j, \tag{2.25}$$

are global holomorphic coordinates. The explicit orientifold is $T^6/\Omega R(-1)^{F_L}$, where R is the involution which changes the sign of all torus coordinates, $R : (x^i, y^i) \to -(x^i, y^i)$.

The holomorphic three-form is

$$\Omega = dz^1 \wedge dz^2 \wedge dz^3, \tag{2.26}$$

and the metric is

$$ds^2 = dz^i d\bar{z}^{\bar{i}}. \tag{2.27}$$

We choose the following orientation

$$\int_{T^6} dx^1 \wedge dx^2 \wedge dx^3 \wedge dy^1 \wedge dy^2 \wedge dy^3 = 1, \tag{2.28}$$

and the basis of $H^3(T^6, \mathbb{Z})$:

$$\begin{aligned} \alpha_0 &= dx^1 \wedge dx^2 \wedge dx^3 \\ \alpha_{ij} &= \frac{1}{2}\varepsilon_{ilm} dx^l \wedge dx^m \wedge dy^j, \quad 1 \le i, j \le 3 \\ \beta^{ij} &= -\frac{1}{2}\varepsilon_{jlm} dy^l \wedge dy^m \wedge dx^i, \quad 1 \le i, j \le 3 \\ \beta^0 &= dy^1 \wedge dy^2 \wedge dy^3 \end{aligned} \tag{2.29}$$

which satisfies $\int_{T^6} \alpha_I \wedge \beta^J = \delta_I^J$. The fluxes can be expanded in this basis

$$\begin{aligned} \frac{1}{(2\pi)^2\alpha'} H_{RR} &= a^0 \alpha_0 + a^{ij}\alpha_{ij} + b_{ij}\beta^{ij} + b_0 \beta^0 \qquad (2.30) \\ \frac{1}{(2\pi)^2\alpha'} H_{NS} &= c^0 \alpha_0 + c^{ij}\alpha_{ij} + d_{ij}\beta^{ij} + d_0 \beta^0. \end{aligned}$$

Here we take $a^0, a^{ij}, b_0, b_{ij}, c^0, c^{ij}, d_0, d_{ij}$ to be even integers, so that all the O3-planes are of the standard type and the issues regarding flux quantization discussed in ref. [8] can be avoided. In this case, the total number of O3-planes is 64 and each plane has D3-brane charge $-1/4$. For simplicity we only turn on the diagonal components of the flux, so that we can set the off-diagonal components of τ^{ij} equal to zero at the critical points. This condition can be imposed by restricting to an enhanced symmetry locus on the moduli space of the T^6 [4]. For example, we will consider configurations which are symmetric under

$$\begin{aligned} R_1 : (x^1, x^2, x^3, y^1, y^2, y^3) &\rightarrow (-x^1, -x^2, x^3, -y^1, -y^2, y^3) \\ R_2 : (x^1, x^2, x^3, y^1, y^2, y^3) &\rightarrow (x^1, -x^2, -x^3, y^1, -y^2, -y^3) \end{aligned} \tag{2.31}$$

Only the diagonal components of the complex structure τ^{ij}, and the three forms $\alpha_0, \alpha_{ii}, \beta^0, \beta^{ii}$ are preserved under these symmetries, so that the only non-vanishing flux components are a^0, a^{ii}, b_0, b_{ii} and c^0, c^{ii}, d_0, d_{ii}. We are left with 3 non-vanishing complex moduli and the axio-dilaton.

To use the conditions (2.16) and (2.17) which we derived in subsection 2.2, we need to transform the scalar potential (2.6) into the standard $\mathcal{N}=1$ supergravity formula (2.9). For tori having a general complex structure the result is complicated (see for example [15] and [4]). However for tori with diagonal complex structure, we can express the scalar potential in the form

$$V = e^{\mathcal{K}} \left(\sum_{i,j=1}^{3} g^{i\bar{j}} \mathcal{D}_{\tau_i} W \overline{\mathcal{D}_{\tau_j} W} + g^{\tau\bar{\tau}} \mathcal{D}_\tau W \overline{\mathcal{D}_\tau W} \right), \tag{2.32}$$

with superpotential (2.7) and "Kähler potential",

$$\mathcal{K} = -3\log[-i(\rho-\bar{\rho})] - \log[-i(\tau-\bar{\tau})] - \log[i(\tau_1-\bar{\tau}_1)(\tau_2-\bar{\tau}_2)(\tau_3-\bar{\tau}_3)], \tag{2.33}$$

where we used τ_i to replace τ^{ii}. Before we proceed we have one more comment. Generally we can only set $\tau^{ij}=0$ (for $i \neq j$), after computing the first derivative of the scalar potential (2.6), but on the symmetric locus, the criticality conditions $\partial_{\tau^{ij}} V = 0$ (for $i \neq j$) are automatically satisfied. As a result we can set $\tau^{ij}=0$ (for $i \neq j$) at the beginning of the computation and only deal with the conditions $\partial_{\tau^{ii}} V = 0$. However, when computing the second derivatives we can not set $\tau^{ij}=0$ before we differentiate, as there are non-vanishing terms of the form $\partial^2_{\tau^{ij}} V$, which will disappear if we set $\tau^{ij}=0$ (for $i \neq j$) at the beginning.

Next we consider a flux in the (2,1)+(1,2) direction, so the conditions (2.17) and (2.16) take the form

$$\kappa_{ijk} A^j B^k = 0 \qquad \text{and} \qquad g_{i\bar{j}} A^i \bar{B}^{\bar{j}} = 0. \tag{2.34}$$

Since we are working with a torus we set $\kappa_{123}=1$ and one solution to the above condition is

$$A^3 = B^3 = 0, \qquad A^1 B^2 = -B^1 A^2, \qquad \frac{A^1 \bar{B}^1}{(\mathrm{Im}\tau_1)^2} + \frac{A^2 \bar{B}^2}{(\mathrm{Im}\tau_2)^2} = 0. \tag{2.35}$$

For the concrete torus orientifold we are considering the tadpole cancelation condition takes the form

$$\frac{i}{2\mathrm{Im}\tau (2\pi)^4 \alpha'^2} \int_{T^6} G \wedge \overline{G} = 32. \tag{2.36}$$

In the following we will present a concrete solution of Eq. (2.35). For simplicity we redefine the parameters according to

$$A^i = -2i\mathrm{Im}\tau_i\mathrm{Im}\tau\tilde{A}^i, \qquad \text{and} \qquad \bar{B}^{\bar{i}} = 2i\mathrm{Im}\tau_i\mathrm{Im}\tau\tilde{\bar{B}}^{\bar{i}} \tag{2.37}$$

and drop the tilde in the following. The conditions (2.35) and (2.36) can be written as

$$A^1B^2 = -B^1A^2, \qquad B^1\bar{B}^1 = B^2\bar{B}^2, \qquad (A^2\bar{A}^2 - B^2\bar{B}^2)\mathrm{Im}\tau\prod_{i=1}^{3}\mathrm{Im}\tau_i = 4 \tag{2.38}$$

and the non-vanishing components of H_{RR} and H_{NS} are

$$\begin{aligned}
a^0 &= -\mathrm{Im}[\bar{\tau}(A^1 + A^2 + \bar{B}^1 + \bar{B}^2)] \\
a^{11} &= -\mathrm{Im}[\bar{\tau}(A^1\bar{\tau}_1 + A^2\tau_1 + \bar{B}^1\tau_1 + \bar{B}^2\bar{\tau}_1)] \\
a^{22} &= -\mathrm{Im}[\bar{\tau}(A^1\bar{\tau}_2 + A^2\tau_2 + \bar{B}^1\tau_2 + \bar{B}^2\bar{\tau}_2)] \\
a^{33} &= -\mathrm{Im}[\bar{\tau}(A^1\bar{\tau}_3 + A^2\tau_3 + \bar{B}^1\tau_3 + \bar{B}^2\bar{\tau}_3)] \\
b_0 &= -\mathrm{Im}[\bar{\tau}(A^1\bar{\tau}_1\tau_2\tau_3 + A^2\tau_1\bar{\tau}_2\tau_3 + \bar{B}^1\tau_1\bar{\tau}_2\bar{\tau}_3 + \bar{B}^2\bar{\tau}_1\tau_2\bar{\tau}_3)] \\
b_{11} &= \mathrm{Im}[\bar{\tau}(A^1\tau_2\tau_3 + A^2\bar{\tau}_2\tau_3 + \bar{B}^1\bar{\tau}_2\bar{\tau}_3 + \bar{B}^2\tau_2\bar{\tau}_3)] \\
b_{22} &= \mathrm{Im}[\bar{\tau}(A^1\bar{\tau}_1\tau_3 + A^2\tau_1\tau_3 + \bar{B}^1\tau_1\bar{\tau}_3 + \bar{B}^2\bar{\tau}_1\bar{\tau}_3)] \\
b_{33} &= \mathrm{Im}[\bar{\tau}(A^1\bar{\tau}_1\tau_2 + A^2\tau_1\bar{\tau}_2 + \bar{B}^1\tau_1\bar{\tau}_2 + \bar{B}^2\bar{\tau}_1\tau_2)] \\
c^0 &= -\mathrm{Im}[A^1 + A^2 + \bar{B}^1 + \bar{B}^2] \\
c^{11} &= -\mathrm{Im}[A^1\bar{\tau}_1 + A^2\tau_1 + \bar{B}^1\tau_1 + \bar{B}^2\bar{\tau}_1] \\
c^{22} &= -\mathrm{Im}[A^1\bar{\tau}_2 + A^2\tau_2 + \bar{B}^1\tau_2 + \bar{B}^2\bar{\tau}_2] \\
c^{33} &= -\mathrm{Im}[A^1\bar{\tau}_3 + A^2\tau_3 + \bar{B}^1\tau_3 + \bar{B}^2\bar{\tau}_3] \\
d_0 &= -\mathrm{Im}[A^1\bar{\tau}_1\tau_2\tau_3 + A^2\tau_1\bar{\tau}_2\tau_3 + \bar{B}^1\tau_1\bar{\tau}_2\bar{\tau}_3 + \bar{B}^2\bar{\tau}_1\tau_2\bar{\tau}_3] \\
d_{11} &= \mathrm{Im}[A^1\tau_2\tau_3 + A^2\bar{\tau}_2\tau_3 + \bar{B}^1\bar{\tau}_2\bar{\tau}_3 + \bar{B}^2\tau_2\bar{\tau}_3] \\
d_{22} &= \mathrm{Im}[A^1\bar{\tau}_1\tau_3 + A^2\tau_1\tau_3 + \bar{B}^1\tau_1\bar{\tau}_3 + \bar{B}^2\bar{\tau}_1\bar{\tau}_3] \\
d_{33} &= \mathrm{Im}[A^1\bar{\tau}_1\tau_2 + A^2\tau_1\bar{\tau}_2 + \bar{B}^1\tau_1\bar{\tau}_2 + \bar{B}^2\bar{\tau}_1\tau_2].
\end{aligned} \tag{2.39}$$

Usually one starts with certain flux numbers and then determines the values of moduli fields. Here we solve the inverse problem, namely, we start with the value of the moduli and determine the flux numbers which stabilize the moduli at the given values. To solve Eq. (2.38) using even flux numbers (2.39) we use the ansatz

$$\mathrm{Im}\tau = 4, \qquad \mathrm{Im}\tau_1 = \mathrm{Im}\tau_2 = \mathrm{Im}\tau_3 = 1 \tag{2.40}$$

So one solution of Eq. (2.38) is

$$A^1 = -3i, \qquad A^2 = 3i, \qquad \bar{B}^1 = 2 + 2i, \qquad \bar{B}^2 = 2 + 2i \tag{2.41}$$

From Eq. (2.39), we can explicitly compute the flux numbers and obtain

$$\begin{aligned}
(a^0, a^{11}, a^{22}, a^{33}) &= (16, -24, 24, 16), \\
(b_0, b_{11}, b_{22}, b_{33}) &= (16, 0, 0, -16) \\
(c^0, c^{11}, c^{22}, c^{33}) &= (-4, 0, 0, 4), \\
(d_0, d_{11}, d_{22}, d_{33}) &= (4, 6, -6, 4)
\end{aligned} \tag{2.42}$$

which are all even integrals.

3 Type IIB mirrors of type IIA strings compactified on rigid Calabi-Yau three-folds

In this section we would like to generalize the previous analysis to type IIB theories which arise as mirrors of type IIA models compactified on rigid Calabi-Yau three-folds, *i.e.* with $h_{2,1} = 0$. On the type IIB side these correspond to models with $h_{1,1} = 0$ and consequently are not ordinary Calabi-Yau manifolds since a Kähler form is missing but can nevertheless be described using conformal field theory techniques. Here we will be interested in the properties of the resulting four-dimensional theories which contain $h_{2,1}+1$ four-dimensional $\mathcal{N} = 1$ chiral superfields originating from the complex structure moduli and the axio-dilaton. The number of these fields will in general be reduced if we consider an orientifold projection.

It has been shown in ref. [1] that for compactifications of type IIB strings on backgrounds with no Kähler structure the Kähler potential for the axio-dilaton and the complex structure is

$$\mathcal{K} = -4 \log\left[-i(\tau - \bar{\tau})\right] - \log\left[-i \int \Omega \wedge \bar{\Omega}\right], \tag{3.1}$$

which differs by a subtle factor 4 from the conventional Kähler potential for the axio-dilaton. This unconventional factor 4 has the consequence that supersymmetric flux configurations are no longer required to be ISD [1]. The Kähler potential (3.1) also causes the scalar potential to display new and interesting properties. In order to illustrate this imagine one considers a real three-form flux, *i.e.* a flux configuration with $H_{NS} = 0$. Then

$$W = W_{RR} = \int H_{RR} \wedge \Omega, \tag{3.2}$$

and the scalar potential can be written in the form

$$V = e^{\mathcal{K}} \left(g^{i\bar{j}} D_i W_{RR} \overline{D_j W_{RR}} + \mid W_{RR} \mid^2 \right), \tag{3.3}$$

which is positive definite and depends non-trivially on the complex structure. If

$$\partial_i V = 0 \qquad \text{for} \qquad i = 1, \ldots, h_{2,1}, \tag{3.4}$$

the potential is critical in all the complex structure directions. So for example, one solution of Eq. (3.4) is given by

$$H_{RR} = a \left(\Omega + \bar{\Omega} \right), \tag{3.5}$$

where a is some real constant. This equation determines the complex structure moduli. Indeed, it turns out that this is nothing else than the equation defining a rank 1 attractor which is well known from black hole physics. Eq. (3.5) can, for example, be explicitly solved in the large complex structure limit as has been shown by Shmakova in ref. [9] (see also ref. [10]). These critical points are stable since the only non-vanishing entries of the Hessian matrix are

$$\partial_{\bar{i}} \partial_j V = 2 e^{\mathcal{K}} g_{\bar{i}j} |W_{RR}|^2. \tag{3.6}$$

The scalar potential (3.3) has been studied before in the literature in the context of non-supersymmetric attractors (for a partial list of references on non-supersymmetric attractors see [11]). In particular, the critical points of the potential are the solutions of

$$H_{RR} = 2\mathrm{Im} \left[e^{\mathcal{K}_{cs}} \left(\Omega \bar{W} - \bar{F}^i \chi_i \right) \right], \tag{3.7}$$

subjected to the constraint

$$Z_{ij} \bar{F}^j + 2 F_i \bar{W} = 0 \tag{3.8}$$

which can be written as

$$2 F_i \bar{W} \int \Omega \wedge \overline{\Omega} + \kappa_{ijk} \bar{F}^j \bar{F}^k = 0. \tag{3.9}$$

Moreover, these critical points are stable since the Hessian matrix written in terms of[1]

$$d\Sigma^2 = 2e^{\mathcal{K}} \left(g^{\gamma\sigma} Z_{\alpha\gamma} \overline{Z}_{\beta\sigma} dw^\alpha dw^\beta + \bar{F}_\alpha F_\beta dw^\alpha dw^\beta \right), \tag{3.10}$$

is positive definite. In this form the critical points correspond non-supersymmetric attractor points as described in ref. [12]. This indicates that within a non-geometric model with $h_{1,1} = 0$ the proposal of ref. [4] leads to an interesting new class of backgrounds in which all the complex structure moduli can be stablized in terms of RR fluxes only with no need of negative energy sources like orientifold planes.

Using the solution (3.5) shows that the potential at the minimum satisfies

$$V_\star > 0, \tag{3.11}$$

if $a \neq 0$ so the external space is dS. However, before we can conclude that supersymmetry is spontaneously broken by the solution (3.5) we should take into account the dependence on the axio-dilaton arising from the overall factor $e^{\mathcal{K}} \sim (\mathrm{Im}\tau)^{-4}$. This factor causes the potential to slope to zero at infinity so a supersymmetric state is gained back at infinity and as it stands the theory has no ground state at all. Here (as in [1]) we will simply assume that perturbative corrections to the Kähler potential and non-perturbative corrections to the superpotential could achieve this stabilization and lead to a metastable ground state.

In order to stabilize the axio-dilaton using perturbative fluxes the only possibility is to use a non-vanishing H_{NS} flux. By including RR and NS three-form fluxes one obtains a four-dimensional superpotential which does depend non-trivially on all moduli fields. Any geometric compactification would lead to a superpotential which is independent of the Kähler moduli and consequently the radial modulus would slide off to infinity. As a result even in the absence of any type of corrections moduli stabilization may be possible within the non-geometric model by including all possible fluxes. Moreover, in order to obtain moduli fields which are heavy enough we may have to break supersymmetry [1]. But note that once the NS flux is non-vanishing the scalar potential is no longer positive definite and it is not obvious that supersymmetry breaking vacua, and in particular the phenomenologically interesting vacua leading to a positive cosmological constant, exist. As an illustrative toy example lets consider a non-geometric model with $h_{2,1} = 0$, *i.e.* a model with only one massless scalar field,

[1] Here the indices α, β label the complex structure moduli and their complex conjugates.

the axio-dilaton, with a Kähler potential

$$\mathcal{K} = -4\log\left[-i(\tau - \bar{\tau})\right], \tag{3.12}$$

and a superpotential

$$W = W_{RR} - \tau W_{NS}, \tag{3.13}$$

where W_{RR} and W_{NS} are constants. The condition for unbroken supersymmetry has one solution only

$$\tau = \frac{1}{W_{NS}\bar{W}_{NS}}\left[\mathrm{Re}(\bar{W}_{NS}W_{RR}) + 2i\mathrm{Im}(\bar{W}_{NS}W_{RR})\right]. \tag{3.14}$$

However, it is not difficult to see that the scalar potential is also critical if

$$\tau = \frac{1}{W_{NS}\bar{W}_{NS}}\left[\mathrm{Re}(\bar{W}_{NS}W_{RR}) - \frac{i}{2}\mathrm{Im}(\bar{W}_{NS}W_{RR})\right], \tag{3.15}$$

which leads to $D_\tau W \neq 0$ so that supersymmetry is broken. Moreover, the scalar potential at the minimum is negative so that the external space is AdS. As a result supersymmetry breaking critical points of the potential do exist even though in this case they lead to an AdS space. However, it is interesting that a single four-dimensional chiral field with a Kähler potential of the form (3.12) avoids the no-go theorem of ref. [16] (see also [17]) according to which dS or Minkowski space vacua with a broken supersymmetry are never possible in a theory with a single chiral field for any superpotential if the Kähler potential is $\mathcal{K} = -n\log\left[-i(\tau - \bar{\tau})\right]$ with $1 \leq n \leq 3$. As a result stable dS vacua are no longer excluded. It will be very interesting to see if by considering a 'realistic' model with a non-vanishing number of complex structure moduli fields stable critical points of the potential at which supersymmetry is broken can be found. We leave this topic for future research.

4 Conclusion and speculations

In this paper we have analyzed stability conditions of the no-scale scalar potential determining the dynamics of the complex structure moduli and the axio-dilaton in geometric flux compactification of type IIB strings to four dimensions. In order to obtain critical points which do not preserve supersymmetry an essential ingredient is the appearance of IASD flux components. But not any IASD flux is allowed. Fluxes have to satisfy the conditions (2.16) and (2.17) to stabilize all fields except the Kähler structure moduli.

Searching for the critical points of the scalar potential obtained from compactifications of type IIB strings on mirrors of rigid Calabi-Yau three-folds we discovered a fascinating and unexpected analogy to black hole physics and, in particular, to non-supersymmetric attractors. This mapping was possible because of the peculiarities of the axio-dilaton Kähler potential in the non-geometric setting derived recently in ref. [1]. The similarities between the attractor mechanism and flux compactifications have been known for some time (see for example ref. [13]) but an explicit mapping of the scalar potentials is new. Complex structure moduli stabilization can be achieved in terms of H_{RR} only, *i.e.* in terms of a real three-form. It will be interesting to further explore if lessons learned from black hole physics can be used to discover properties of flux vacua with a small and positive cosmological constant.

Moreover, compactifications of type IIB strings on mirrors of rigid Calabi-Yau manifolds lead to a flux induced superpotential which depends non-trivially on all scalar fields even in the absence of any non-perturbative effects once the RR and NS three-form fluxes are included. This leads to the interesting possibility that perturbative fluxes alone may stabilize all moduli fields once supersymmetry is broken.

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5 Appendix A

In this appendix, we present the details of some of the computations presented in this paper. To set up our notation we start by reviewing a few basic formulas regarding Calabi-Yau manifolds [14]. On a Calabi-Yau three-fold, there exists a unique harmonic (3,0) form Ω, whose first derivatives satisfy

$$\frac{\partial\Omega}{\partial z^i} = K_i\Omega + \chi_i \qquad \text{and} \qquad \frac{\partial\Omega}{\partial \bar{z}^i} = 0 \tag{5.1}$$

where χ_i is an harmonic (2,1) form. The Kähler potential on the complex structure moduli space is

$$K_{cs} = -\log[-i \int \Omega \wedge \overline{\Omega}]. \tag{5.2}$$

As is easy to check

$$\partial_i K_{cs} = -K_i \qquad \text{and} \qquad g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K_{cs} = -\frac{\int \chi_i \wedge \bar{\chi}_{\bar{j}}}{\int \Omega \wedge \overline{\Omega}}. \tag{5.3}$$

One important property of the (3,0) form Ω is that it is undefined up to multiplication by a holomorphic function $f(z)$

$$\Omega \to f(z)\Omega. \tag{5.4}$$

Under (5.4) the Kähler potential transforms as

$$K_{cs} \to K_{cs} - \log f(z) - \log \bar{f}(\bar{z}), \tag{5.5}$$

which leaves the metric on moduli space invariant. For convenience, we can define a gauge covariant derivative

$$\chi_i = \mathcal{D}_i \Omega = \partial_i \Omega + \partial_i K_{cs} \Omega, \tag{5.6}$$

and thus under the Kähler transformation, it transforms according to $\mathcal{D}_i \Omega \to f \mathcal{D}_i \Omega$, *i.e.* $\chi_i \to f \chi_i$. One can also generalize the definition of the covariant derivative to other quantities which transform like

$$\Psi^{(a,b)} \to f^a \bar{f}^b \Psi^{(a,b)} \tag{5.7}$$

under the Kähler transformation. In this case the covariant derivatives take the form

$$\begin{aligned} \mathcal{D}_i \Psi^{(a,b)} &= (\partial_i + a \partial_i K_{cs}) \Psi^{(a,b)} \\ \mathcal{D}_{\bar{j}} \Psi^{(a,b)} &= (\partial_{\bar{j}} + b \partial_{\bar{j}} K_{cs}) \Psi^{(a,b)}. \end{aligned} \tag{5.8}$$

The partial derivatives ∂_i and $\partial_{\bar{i}}$ are to be replaced by ordinary covariant derivatives ∇_i, $\nabla_{\bar{j}}$ when acting on tensors. It is easy to see that under Kähler transformations

$$\mathcal{D}_i \Psi^{(a,b)} \to f^a \bar{f}^b \mathcal{D}_i \Psi^{(a,b)} \qquad \text{and} \qquad \mathcal{D}_{\bar{j}} \Psi^{(a,b)} \to f^a \bar{f}^b \mathcal{D}_{\bar{j}} \Psi^{(a,b)}. \tag{5.9}$$

We also require

$$[\mathcal{D}_i, \mathcal{D}_{\bar{j}}]\Omega = -g_{i\bar{j}}\Omega, \qquad \text{and} \qquad \mathcal{D}_k g_{i\bar{j}} = 0. \tag{5.10}$$

Using the above formulas, we can get the results quoted in the table below

$$\begin{array}{|c|c|}
\hline
\textit{Derivatives of the basis} & \textit{Spans} \\
\hline
\Omega & (3,0) \\
\mathcal{D}_i\Omega = \chi_i & (2,1) \\
\mathcal{D}_i\chi_j = \frac{1}{\int \Omega\wedge\overline{\Omega}}\kappa_{ij}{}^{\bar{k}}\bar{\chi}_{\bar{k}} & (1,2) \\
\mathcal{D}_i\bar{\chi}_{\bar{j}} = g_{i\bar{j}}\overline{\Omega} & (0,3) \\
\mathcal{D}_i\overline{\Omega} = 0 & \\
\hline
\end{array} \tag{5.11}$$

where the Yukawa couplings are defined as

$$\kappa_{ijk} = \int \Omega \wedge \mathcal{D}_i\mathcal{D}_j\mathcal{D}_k\Omega. \tag{5.12}$$

The above results are the tools needed to compute the first derivative of scalar potential (2.10). Because the scalar potential is invariant under Kähler transformation, *i.e.* $a = b = 0$, we can transform the ordinary derivatives into covariant derivatives

$$\partial_I V = \mathcal{D}_I V = e^{\mathcal{K}}\left(Z_{IJ}\bar{F}^J + F_I\overline{W}\right) \tag{5.13}$$

with the notation (2.11). To obtain an explicit expression for $\partial_I V = 0$, we need to compute a few quantities,

$$\begin{aligned}
F_i &= \mathcal{D}_i W = \int_{\mathcal{M}_6} G\wedge\chi_i \\
F_\tau &= \mathcal{D}_\tau W = \partial_\tau W + \partial_\tau\mathcal{K}W = -\frac{1}{\tau-\bar{\tau}}\int_{\mathcal{M}_6}\overline{G}\wedge\Omega \\
Z_{ij} &= \mathcal{D}_i\mathcal{D}_j W = \frac{\kappa_{ijk}}{\int\Omega\wedge\overline{\Omega}}\int_{\mathcal{M}_6} G\wedge\bar{\chi}^k \\
Z_{\tau i} &= \mathcal{D}_\tau\mathcal{D}_i W = -\frac{1}{\tau-\bar{\tau}}\int_{\mathcal{M}_6}\overline{G}\wedge\chi_i \\
Z_{\tau\tau} &= \mathcal{D}_\tau\mathcal{D}_\tau W = \partial_\tau F_\tau - \Gamma^\tau_{\tau\tau}F_\tau + \partial_\tau\mathcal{K}F_\tau = 0.
\end{aligned} \tag{5.14}$$

As a result the critical condition $\partial_I V = 0$ can be explicitly written as

$$\begin{cases}
\int\overline{G}\wedge\chi_i\int\overline{G}\wedge\bar{\chi}^i + \int\overline{G}\wedge\Omega\int\overline{G}\wedge\overline{\Omega} = 0 \\
\\
\int G\wedge\bar{\chi}^k\int\overline{G}\wedge\bar{\chi}^i\left(\frac{\kappa_{ijk}}{\int\Omega\wedge\overline{\Omega}}\right) + \int G\wedge\chi_j\int\overline{G}\wedge\overline{\Omega} + \int\overline{G}\wedge\chi_j\int G\wedge\overline{\Omega} = 0
\end{cases} \tag{5.15}$$

After using the Hodge decomposition for G

$$G = A\Omega + A^i\chi_i + \bar{B}^{\bar{i}}\bar{\chi}_{\bar{i}} + \bar{B}\overline{\Omega} \tag{5.16}$$

the condition (5.15) can be further written in the form

$$\begin{cases} \int G \wedge \star G = 0 \\ \\ (B\bar{B}_k + A\bar{A}_k) \int \Omega \wedge \overline{\Omega} + \kappa_{ijk} A^i B^j = 0 \end{cases} \tag{5.17}$$

which are Eq.(2.16) and (2.17). To derive these equations, we have used the property that the harmonic (2,1) and (0,3) forms are imaginary self-dual, and the harmonic (1,2) and (3,0) forms are imaginary anti-self-dual on Calabi-Yau three-fold.

Now we are going to compute the second derivative of scalar potential by noting that

$$\partial_I \partial_J V = \mathcal{D}_I \mathcal{D}_J V, \qquad \partial_I \partial_{\bar{J}} V = \mathcal{D}_I \mathcal{D}_{\bar{J}} V \tag{5.18}$$

at the critical point $\partial_I V = 0$. After a little algebra, the second derivatives of the scalar potential (2.10) are

$$\begin{aligned} \partial_I \partial_J V &= e^{\mathcal{K}} \left(U_{IJK} \bar{F}^K + 2 Z_{IJ} \overline{W} \right) \\ \partial_I \partial_{\bar{J}} V &= e^{\mathcal{K}} (U_{\bar{J}IK} \bar{F}^K + F_I \bar{F}_{\bar{J}} + Z_{IL} \bar{Z}_{\bar{J}\bar{K}} g^{L\bar{K}} + g_{I\bar{J}} |W|^2), \end{aligned} \tag{5.19}$$

where $U_{\bar{J}IK} = \mathcal{D}_{\bar{J}} \mathcal{D}_I \mathcal{D}_K W$. The above formula can be easily transformed to (2.22) by using the identity:

$$[\mathcal{D}_I, \mathcal{D}_{\bar{J}}] F_K = -g_{I\bar{J}} F_K + R_{I\bar{J}K}{}^L F_L \tag{5.20}$$

To get expression (2.24), we need to generalize the definition of U_{IJK} and Z_{IJ} to

$$U_{\alpha\beta\gamma} = \mathcal{D}_\alpha \mathcal{D}_\beta \mathcal{D}_\gamma W \qquad \text{and} \qquad \overline{U}_{\bar{\alpha}\bar{\beta}\bar{\gamma}} = \overline{U_{\alpha\beta\gamma}} \tag{5.21}$$

and

$$Z_{\alpha\beta} = \mathcal{D}_\alpha \mathcal{D}_\beta W \qquad \text{and} \qquad \overline{Z}_{\bar{\alpha}\bar{\beta}} = \overline{Z_{\alpha\beta}}, \tag{5.22}$$

where α,β,and γ label all coordinates, *i.e.* the axio-dilaton, complex structure moduli and their complex conjugates. Using the results quoted in the table (5.11), we have

$$\begin{aligned} U_{ijk} &= \mathcal{D}_i \mathcal{D}_j \mathcal{D}_k W = \frac{\int G \wedge \overline{\Omega}}{\int \Omega \wedge \overline{\Omega}} \kappa_{ijk} \\ U_{ij\tau} &= \mathcal{D}_i \mathcal{D}_j \mathcal{D}_\tau W = -\frac{\int \overline{G} \wedge \bar{\chi}^k}{\int \Omega \wedge \overline{\Omega}} \frac{\kappa_{ijk}}{\tau - \bar{\tau}} \\ U_{\bar{k}ij} &= -\frac{1}{(\int \Omega \wedge \overline{\Omega})^2} \kappa_{ij}{}^{\bar{m}} \bar{\kappa}_{\bar{k}\bar{m}\bar{n}} F^{\bar{n}} \end{aligned} \tag{5.23}$$

One consequence of Eq. (5.23) and Eq. (5.14) is

$$\begin{aligned}
&\bar{F}^{\tau} U_{ij\tau} = \bar{F}^{k} U_{ijk}, \\
&Z_{\bar{J}I} = g_{I\bar{J}} W, \\
&Z_{J\bar{I}} = 0, \\
&U_{K\bar{J}I} = g_{I\bar{J}} F_K, \\
&U_{\tau\tau i} = U_{\tau\tau\tau} = U_{\bar{K}\bar{J}I} = U_{\alpha\bar{j}\tau} = 0.
\end{aligned} \tag{5.24}$$

The above expressions are useful to show the equivalence of (2.24) and (5.19).

6 Appendix B

In this appendix we explicitly show the appearance of the two superpotentials

$$W = \int G \wedge \Omega, \qquad \text{and} \qquad \widetilde{W} = \int \overline{G} \wedge \Omega, \tag{6.1}$$

by dimensional reduction of ten-dimensional supergravity theories. Our convention is $\varepsilon_{01....9} = 1$, and

$$\star\, dx^{m_0} \wedge ... \wedge dx^{m_n} = \frac{1}{(9-n)!} \epsilon_{m_{n+1}...m_9}{}^{m_0...m_n} dx^{m_{n+1}} \wedge ... \wedge dx^{m_9} \tag{6.2}$$

We take the type IIB effective action (2.1) together with the local terms are

$$S_{loc} = -\int_{R^4\times\Sigma} d^{p+1}\xi T_p \sqrt{-\hat{G}} + \mu_p \int_{R^4\times\Sigma} C_{p+1} \tag{6.3}$$

To perform the dimensional reduction, we assume that the metric is independent of external coordinates

$$ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + e^{-2A(y)} \tilde{g}_{mn}(y) dy^m dy^n \tag{6.4}$$

The Einstein equation is

$$R_{MN} = k_{10}^2 \left(T_{MN} - \frac{1}{8} g_{MN} T \right) \qquad \text{with} \qquad T_{MN} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{MN}}, \tag{6.5}$$

The non-compact components of the Einstein equation can be written as

$$R_{\mu\nu} = \left[-\frac{1}{8\mathrm{Im}\tau} |G|^2 - \frac{1}{4} e^{-8A} (\partial_m \alpha)^2 \right] g_{\mu\nu} + k_{10}^2 \left(T^{loc}_{\mu\nu} - \frac{1}{8} T^{loc} g_{\mu\nu} \right) \tag{6.6}$$

On the other hand, using the metric (6.4), we obtain

$$R_{\mu\nu} = -e^{2A}\tilde{\nabla}^2 A g_{\mu\nu} \tag{6.7}$$

which yields

$$\tilde{\nabla}^2 A = \frac{1}{8\mathrm{Im}\tau}e^{-2A}|G|^2 + \frac{1}{4}e^{-10A}|\partial_m\alpha|^2 + \frac{1}{8}k_{10}^2 e^{-2A}[T^m_m - T^\mu_\mu]^{loc}. \tag{6.8}$$

This can also be written in the form

$$\tilde{\nabla}^2 e^{4A} = \frac{1}{2\mathrm{Im}\tau}e^{2A}|G|^2 + e^{-6A}\Big[(\partial_m\alpha)^2 + (\partial_m e^{4A})^2\Big] + \frac{1}{2}k_{10}^2 e^{2A}\Big(T^m_m - T^\mu_\mu\Big)^{loc} \tag{6.9}$$

To compute the equation of motion for C_4 we only need to consider a few terms in the action namely

$$\frac{1}{8\kappa_{10}^2}\int \tilde{F}_{(5)} \wedge \star\tilde{F}_{(5)} - \frac{1}{8i\kappa_{10}^2}\int \frac{C_{(4)} \wedge G \wedge \overline{G}}{\mathrm{Im}\tau} + \frac{\mu_p}{2}\int_{R^4\times\Sigma} C_{p+1} \tag{6.10}$$

The appearance of extra factor $\frac{1}{2}$ is a consequence of the self-duality of the five form. The Bianchi identity is

$$d\star\tilde{F}_{(5)} = -\frac{G\wedge\overline{G}}{2i\mathrm{Im}\tau} + 2k_{10}^2 T_3\rho_3^{loc} \tag{6.11}$$

As $\tilde{F}_{(5)}$ is self-dual, we have

$$\tilde{F}_5 = (1+\star)d\alpha\wedge dx^0\wedge dx^1\wedge dx^2\wedge dx^3 \tag{6.12}$$

and the Bianchi identity becomes

$$\tilde{\nabla}^2\alpha = \frac{i}{12\mathrm{Im}\tau}e^{2A}G_{mnp}\star\overline{G}^{mnp} + 2e^{-6A}\partial_m e^{4A}\partial^m\alpha + 2k_{10}^2 T_3\rho_3^{loc} \tag{6.13}$$

By summing or subtracting equations (6.9)and (6.13), we get

$$\begin{aligned}\tilde{\nabla}^2(e^{4A}\pm\alpha) =& \frac{1}{2\mathrm{Im}\tau}e^{2A}|G\mp i\star G|^2 + e^{-6A}|\partial_m\alpha\pm\partial_m e^{4A}|^2 \\ &+ 2k_{10}^2 e^{2A}\left(\frac{1}{4}(T^m_m - T^\mu_\mu)^{loc} \pm T_3\rho_3^{loc}\right).\end{aligned} \tag{6.14}$$

The left hand side of the above equation vanishes when integrated over a compact manifold $\mathcal{M}_6$. As a result there are two solutions

$$\begin{aligned}&\star_6\, G = -iG, \qquad \alpha = -e^{4A}, \qquad \text{with} \qquad \overline{O}3, \overline{D}3\\ &\star_6\, G = +iG, \qquad \alpha = +e^{4A}, \qquad \text{with} \qquad O3, D3.\end{aligned} \tag{6.15}$$

Notice that we can not have $O3$ and $\overline{D}3$ at the same time.

Using the results above we can perform the dimensional reduction

$$\int d^{10}x\sqrt{-g}\mathcal{R} = \int d^4x\sqrt{-g_4}\int d^6y\sqrt{g_6}\left[-8(\nabla A)^2 e^{4A}\right]. \tag{6.16}$$

Taking into account the fact the self-duality of the five-form we get

$$\int d^{10}x\sqrt{-g}\frac{\tilde{F}^2_{(5)}}{4} = \int d^4x\sqrt{-g_4}\int d^6y\sqrt{g_6}\frac{e^{-4A}}{2}(\partial_m\alpha)^2 \tag{6.17}$$

Since $\alpha = \mp e^{4A}$, this term gives the same contribution as the Einstein term

$$\begin{aligned}
&\int d^{10}x\sqrt{-g}\left(\mathcal{R} - \frac{\tilde{F}^2_{(5)}}{4}\right) = \int d^4x\sqrt{-g_4}\int d^6y\sqrt{g_6}\left(-(\partial_m\alpha)^2 e^{4A}\right) \\
&= \int d^4x\sqrt{-g_4}\int d^6y\sqrt{g_6}\Big(\mp(\partial_m\alpha)^2 \pm 4\partial_m\alpha\partial^m A\Big) \\
&= \int d^4x\sqrt{-g_4}\int d^6y\sqrt{g_6}\Big(\pm\frac{1}{12i\mathrm{Im}\tau}e^{4A}G_{mnp}\star\overline{G}^{mnp} \mp 2e^{4A}\kappa^2_{10}T_3\rho^{loc}_3\Big)
\end{aligned} \tag{6.18}$$

Where we have used the Bianchi identity (6.13). The second term in the last equation of (6.18) will cancel the first term of S_{loc}, and the CS term cancels the second term of S_{loc}. At the end, the scalar potential is

$$S_v = \frac{1}{2\kappa^2_{10}}\int d^4x\sqrt{-g_4}\int\frac{e^{4A}}{2\mathrm{Im}\tau}G\wedge\star_6(\overline{G}\pm i\star\overline{G}) \tag{6.19}$$

From this expression, we can write the scalar potential in the standard form with

$$\widetilde{W} = \int\overline{G}\wedge\Omega, \qquad \text{or} \qquad W = \int G\wedge\Omega \tag{6.20}$$

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