Vortex Plasma in a Superconducting Film with Magnetic Dots

D.E. Feldman[1,2], I.F. Lyuksyutov[1] and V.L. Pokrovsky[1,2]
[1] *Department of Physics, Texas A&M University, College Station, Texas 77843-4242*
[2] *Landau Institute for Theoretical Physics, Chernogolovka, Moscow region 142432, Russia*

We consider a superconducting film, placed upon a magnetic dot array. Magnetic moments of the dots are normal to the film and randomly oriented. We determine how the concentration of the vortices in the film depends on the magnetic moment of a dot at low temperatures. The concentration of the vortices, bound to the dots, is proportional to the density of the dots and depends on the magnetization of a dot in a step-like way. The concentration of the unbound vortices oscillates about a value, proportional to the magnetic moment of the dots. The period of the oscillations is equal to the width of a step in the concentration of the bound vortices.

PACS numbers: 74.60.Ge, 74.25.Dw, 74.76.-w

Superconductivity of thin films was studied for a long time [1]. An important difference of the two-dimensional superconductors from the three-dimensional ones is related with the topological defects. Vortices appear in thin films of the superconductors which are of the first kind in the bulk [2]. They can appear spontaneously even in the absence of the magnetic field. In specially prepared films with the size about the effective screening length, unbound vortices appear above the Berezinskii-Kosterlitz-Thouless transition [3]- [5].

A recent surge of interest to this problem is associated with advances in preparation of magnetic nanostructures interacting with the superconducting films [6]- [9]. Magnetic field from the magnetic nanostructures (dots) gives rise to vortices and pins them. As a result, in a superconducting film placed upon an array of magnetic dots, a periodic field dependence of the magnetoresistance and superconducting transition temperature was observed [7], [8].

Theoretically the problem of a superconducting film supplied by a periodic array of magnetic dots was studied in Ref. [10]. It was shown that the properties of such a system depend on the orientation of the magnetization of the dots, their mutual distances and the coercive field. In the case of strong coercive force, the reorientation of the magnetic moments is a slow process. Hence, a random array of magnetic moments occurs at zero-field cooling below the Curie temperature. If the easy magnetic axes are perpendicular to the film, each dot favors creation of a vortex [10]. Thus, a random vortex structure appears in the superconductor. The vortices pinned by the dots induce a random potential in the film. If the period of the dot array is small enough, the random potential may be sufficient for creation of additional unpinned vortices. The resulting vortex plasma phase is expected to have a much larger resistance, than the pinned vortex state [10].

In the present paper we consider a toy model of the vortex plasma state which reproduces the phenomena predicted in [10], but leads also to new predictions of a rich phase diagram and elementary excitations. We study the dependence of the concentration of the vortices on the magnetic moment of a dot. The concentration of the pinned vortices is found to be proportional to the density of the dots. The concentration of the unbound vortices is less to the factor of order a/λ_{eff}, where λ_{eff} is the effective screening length [1], a the lattice constant of the dots. Both concentrations grow as the magnetic moment of a dot increases. The concentration of the pinned vortices depends on the magnetic moment in a step-like way. The concentration of the unpinned vortices has linear and oscillating components.

We consider first a thin film of the size $\sim \lambda_{eff} = \lambda^2/d$, where λ is the London penetration length, d the thickness of the film. Then we discuss what happens in a film, which is larger than the effective screening length λ_{eff}. We argue that the behavior of the vortex plasma does not depend qualitatively on the size of the system.

The energy of the system of the vortices is composed of three components. These are the single vortex energies, the energy of the vortex-vortex interaction and the coupling energy between the vortices and the dots. A single vortex energy is [1]

$$E_i = \epsilon_0 n_i^2 \ln \frac{\lambda_{eff}}{\xi}, \tag{1}$$

n_i being the vorticity, ξ the core size and $\epsilon_0 = \Phi_0^2/16\pi^2\lambda_{eff}$, where Φ_0 is the magnetic flux quantum. The interaction of a pair of vortices separated by a distance r decreases fast at $r > \lambda_{eff}$, while at $r < \lambda_{eff}$ it depends on the distance logarithmically [4]

$$E_{ij} = 2\epsilon_0 n_i n_j \ln \frac{\lambda_{eff}}{r}, \tag{2}$$

where n_i, n_j are the vorticities. The interaction of a vortex and a dot is estimated in Ref. [10] as

$$E_i^d = \epsilon_0 \frac{\Phi_d}{\Phi_0} n_i, \tag{3}$$

where Φ_d is the magnetic flux generated by the dot. The total Hamiltonian of the system reads

$$H = \sum_i E_i + \sum_{i>j} E_{ij} + \sum_i E_i^d. \tag{4}$$

The vortex plasma appears when the distance between the dots $a \ll \lambda_{eff}$ [10]. Then the number of the dots in the film of the size λ_{eff} is $N_D = (\lambda_{eff}/a)^2 \gg 1$. In this case the system, governed by the Hamiltonian (4), displays strong collective effects. For simplification, we approximate the slow logarithmic dependence (2) at the scales $a < r < \lambda_{eff}$ with a constant. Then the model Hamiltonian mimicking the real Hamiltonian (4) has a form

$$H = -U \sum_i \sigma_i n_i + \sum_i E_i + 2\epsilon_0 \sum_{i>j} n_i n_j, \tag{5}$$

where $\sigma_i = \pm 1$ describe two possible orientations of the magnetizations of the dots, $U = \epsilon_0 \Phi_d/\Phi_0$. This Hamiltonian can be rewritten as

$$H = -U \sum_i \sigma_i n_i + \epsilon \sum n_i^2 + \epsilon_0 \left(\sum_i n_i \right)^2, \tag{6}$$

where $\epsilon = \epsilon_0 \ln(a/\xi)$.

The minimum of the energy (6) corresponds to zero total vorticity $Q = \sum_i n_i$. Indeed, if ΔQ vortices are removed, so that the vorticity become $Q' = Q - \Delta Q$, the last term in (6) would decrease by the value proportional to $Q\Delta Q$. At the same time, the maximal possible energy loss due to the first two terms of the Hamiltonian is proportional to ΔQ. Hence, decreasing the total vorticity is favorable unless $Q \sim 1$. In the system with the number of vortices $\sim (\lambda_{eff}/a)^2 \gg 1$ this is practically zero. Thus, the "neutrality" condition must be satisfied:

$$\sum_i n_i = 0. \tag{7}$$

Our aim is to find the ground state of the Hamiltonian (6) with the constraint (7). The ground state depends on the parameter $\kappa = U/\epsilon$. The discussion below is limited by the case $\kappa \ll \lambda_{eff}/a$.

Let us assume that there are N "positive" dots favoring creating positive vortices, and $N + K$ "negative" dots which favor negative vortices. Obviously, $N \approx \lambda_{eff}^2/(2a^2)$ and $K \sim \lambda_{eff}/a$ are random. Note that in the ground state all the unbound vortices have the same sign and the unit vorticity. Below we assume that the unbound vortices are positive. As seen below, this assumption is equivalent to the condition $K > 0$. The case of the negative vorticity is completely analogous.

Let us consider an arbitrary dot with occupancy n. The neutrality condition (7) allows to change the occupancy by ± 1 and simultaneously create an unbound vortex or antivortex. In the ground state these excitations can not decrease the energy. This gives a restriction on the possible values of the occupancy. The energies of the excitations are

$$\Delta E = \epsilon[1 + (n \pm 1)^2 - (n \pm 1)\kappa] - \epsilon[n^2 - n\kappa] = \epsilon[2 \pm (2n - \kappa)] \geq 0. \tag{8}$$

Hence, $2n - 2 \leq \kappa \leq 2n + 2$. Thus, at $\kappa = 2(q + \delta)$, where q is integer, $0 < \delta < 1$, the only possible values of n are q and $q + 1$. At $\kappa = 2q$ an additional possible occupancy is $q - 1$.

Let us show that a non-zero number of unbound vortices appears when κ is equal to an even integer only. Indeed, for the system in the ground state, the energy must not decrease when an unbound vortex is placed onto a dot, or when an unbound vortex and a bound antivortex are removed. The consideration, similar to the derivation of Eq. (8), leads to a condition

$$2m_{min} \geq \kappa \geq 2n_{max}, \tag{9}$$

where m_{min} and n_{max} are the minimal occupancy of the positive dots and the maximal occupancy of the negative ones respectively. Eq. (9) is compatible with (7) only if the inequalities in (9) are actually equalities. Indeed, the number of the positive vortices $V^+ \geq N m_{min}$ and the negative vortex number $V^- \leq (N + K)n_{max}$. Since $K \ll N$, it follows from the neutrality condition $V^+ = V^-$ that $m_{min} = n_{max}$. Hence, $\kappa = 2k$ where k is an integer. Note that Eq. (9) does not contradict Eq. (7) if $K \geq 0$ only. We assume below that this is the case.

Let us consider the case of $\kappa = 2(q + \delta)$, $0 < \delta < 1$. At these values of κ unbound vortices are absent. Let the number of the positive dots with occupancy q be S. The numbers of the negative dots with occupancies q and $q + 1$

are then determined by the neutrality condition. The energy as a function of S is $E(S) = \text{constant} + 2S\epsilon[\kappa - 2q - 1]$. Depending on κ, the minimum of the energy $E(S)$ corresponds to $S = 0$ or $S = N$. At $\kappa = 2q + 1$ the ground state is degenerate.

Now we study the case of $\kappa = 2q$. We denote the numbers of the positive dots with occupancies $q - 1$ and $q + 1$ as A_+ and B_+ respectively, the numbers of the negative dots with occupancies $q-1$ and $q+1$ as A_- and B_- respectively. The number of the unbound vortices is determined by the neutrality. The energy dependence on $A_\pm, B_\pm$ is given by the expression $E = \text{constant} + 2\epsilon[A_+ + B_-]$. Thus, in the ground state $A_+ = B_- = 0$. At the same time the energy does not depend on A_- and B_+.

Now we are in position to describe all the ground states. Below we consider the case when the number of the negative dots $(N + K)$ is larger than the number of the positive dots N. The opposite case is analogous.

1) At $\kappa < 1$ the vortices are absent.

2) At $\kappa = 2n - 1$ the ground state is degenerate. All the dots can be divided into 4 groups with occupancies n positive, $(n - 1)$ positive, $(n - 1)$ negative and n negative vortices on each dot, respectively. The numbers of dots in these groups are $N - S$, S, $nK + S$ and $N - (n - 1)K - S$, respectively, where S is any integer satisfying inequality $S \leq N - (n - 1)K$.

3) At $2n - 1 < \kappa < 2n$, n vortices are bound with each positive dot and with $N - (n - 1)K$ negative dots. Each of the other nK negative dots is occupied by $(n - 1)$ vortices.

4) At $\kappa = 2n$ the ground state is degenerate. There are 4 groups of dots with occupancies $(n - 1)$ negative, n negative, $(n + 1)$ positive and n positive vortices on each dot. The groups contain S, $N + K - S$, $nK - S - P$ and $N + S + P - nK$ dots, respectively, where integer P and S satisfy inequality $S + P \leq nK$. Besides, there are P unbound vortices.

5) At $2n < \kappa < (2n + 1)$ each negative dot is occupied by n vortices. nK positive dots are occupied by $(n + 1)$ vortices and the other $N - nK$ positive dots are occupied by n vortices.

Thus, the concentration c_b of the bound vortices obeys a step-like low

$$c_b = \left[\frac{\kappa + 1}{2}\right] \frac{1}{a^2} + O\left(\frac{1}{\lambda_{eff}a}\right), \tag{10}$$

where the square brackets denote the integer part. The unbound vortices exist only at $\kappa = 2n$, and their concentration c_u satisfies a relation

$$c_u \sim \frac{n|K|}{\lambda_{eff}^2} \tag{11}$$

where $|K|$ is the absolute value of the difference between the numbers of the positive and negative dots. The disorder average of $|K|$ is $2\lambda_{eff}/(\sqrt{\pi}a)$.

The fact, that the concentration of the unbound vortices is proportional to $1/(\lambda_{eff}a)$, can be understood in the framework of the approach [10]. It was argued in Ref. [10] that the bound vortices induce a random potential with the characteristic variation $\epsilon_0\lambda_{eff}/a$. This leads to creation of unbound vortices screening the random potential. The concentration at which appearance of new vortices becomes unfavorable is $c_u \sim 1/(\lambda_{eff}a)$.

The model (5) is oversimplified in two respects. First, within the model the potential created by the vortices is completely screened. It is natural that in the ground state the potential is screened at the scales $r > a$, larger than the intervortex distance. However, the potential can not be screened at the scales $r \leq a$. This unscreened potential provides an additional contribution to the Hamiltonian (5) leading to dependence of the vortex energy on the position. This contribution lifts the degeneracy of the ground state at even κ and fixes the number of the unbound vortices. It also makes the creation of the unbound vortices favorable at non-integer values of κ. This is a consequence of the fact that the low-lying states are almost degenerate at $\kappa \approx 2n$, and the distances between the low-lying levels may turn out to be less than the value of the unscreened potential variation. Still the maxima of the unbound vortex density correspond to the integer even values of κ. Another effect of the incomplete screening is the lifting of the ground state degeneracy at $\kappa = 2n + 1$. In this case the total number of the bound vortices is determined by the unscreened potential. This potential smears the concentration steps at the odd integer values of κ.

The second simplification consists in the choice of the energy of a vortex upon a dot in the form $E = E_i + E_i^d$, where E_i and E_i^d are given by Eqs. (1,3). This value of E provides only an upper boundary for the energy of a vortex pinned by a dot. Since the energies of the bound vortices are lower than it is assumed in Eq. (5), the creation of the unbound vortices is less favorable. As a consequence their concentration in a more realistic model is lower than (11).

We expect that the behavior of a large film is qualitatively the same as the behavior of the film of the size λ_{eff}. The interaction of the vortices at the distance $r > \lambda_{eff}$ can be calculated with the method of Ref. [2]. The main contribution originates from the interaction of the magnetic fields, induced by the vortices. It depends on the distance as $V \sim 1/r$. Due to the screening, the blocks of the size λ_{eff} are to be considered not as free charges, but as dipoles. Their potential obeys $1/r^2$ law. Since the orientation of the dipoles is random, the interaction of the distant blocks is irrelevant. Thus, to cut-off the intervortex interaction at some scale $R \sim \lambda_{eff}$ is a reasonable approximation. Then a qualitatively correct picture is given by the following model. The system is divided into blocks of the size R. The interaction of the vortices from the different blocks is neglected. Inside a block the Hamiltonian (4) is valid. The main features of this model are the same as in the film of the size λ_{eff}.

Besides the ground states, we have determined the spectrum of elementary excitations. In particular, the energy cost of an unbound vortex is $\epsilon_0|\kappa - 2[(\kappa+1)/2]|$, where [...] denotes the integer part. Our approach is similar to the idea of Efros and Shklovskii [11,12]. They found a soft Coulomb gap in the doped semiconductor. In our case a slower dependence of the interaction on the distance leads to a gap of the finite width. Within the toy-model the gap disappears at the even integer κ. Although this result is most probably an artifact of the model, we expect that the gap is minimal at the even values of κ. An interesting question concerns the role of the collective excitations. For the problem of Coulomb blockade it was recently discussed in Ref. [13].

The superconducting film includes regions of the size $\sim \lambda_{eff}$ with correlated positive or negative values of the random potential. The behavior of the vortices near the borders of the regions is relevant for the transport properties. In particular, an important process for the resistivity is the transport of the free vortices between the regions with the same sign of the random potential through the points of intersection of the borders. Another important process is the transport along the borders, since the borders constitute a percolating equipotential cluster. The resistivity depends on the temperature, potential barriers and concentration of the unbound vortices. It increases as the number of the unbound vortices grows. At low temperatures a complicated energy landscape may lead to the glassy dynamical behavior.

In conclusion, we obtain that both, the concentrations of the bound and unbound vortices, increase as the magnetization of a dot increases. The concentration of the bound vortices depends on the magnetization in a step-like way and is proportional to the density of the dots. The concentration of the unbound vortices is proportional to $1/(\lambda_{eff}a)$. Its dependence on the magnetic moment of a dot can be represented as oscillations about a value, proportional to the dot magnetization. The period of the oscillations is the same as the width of a step of the bound vortices concentration.

This work was partly supported by grants DEFG03-96ER45598, NSF DMR-97-05182, THECB ARP 010366-003. The research of DEF was partly supported by the Russian Program of Leading Scientific Schools, grant 96-15-96756.

---

[1] A.A. Abrikosov, *Introduction to the Theory of Metals* (North Holland, Amsterdam, 1986).

[2] J. Pearl, J. Appl. Phys. Lett. **5**, 65 (1964).

[3] D. Nelson, in *Phase transitions ant Critical Phenomena* (C. Domb and J. Lebowitz, eds.) v. **7** (1983). P. Minnhangen, Rev. Mod. Phys. **59**, 1001 (1987).

[4] S. Doniach and B.A. Hubermann, Phys. Rev. Lett. **42**, 1169 (1979).

[5] A.F. Hebard and A.T. Fiori, Phys. Rev. Lett. **44**, 291 (1980); *ibid.* **50**, 1603 (1983).

[6] V.V. Metlushko, M. Baert, R. Janckheere *et al.*, Solid State Comm. **91**, 331 (1994). M. Baert, V.V. Metlushko, R. Janckheere *et al.*, Phys. Rev. Lett. **74**, 3269 (1995). V.V. Moshchalkov, M. Baert, V.V. Metlushko *et al.*, Phys. Rev. B **54**, 7385 (1996). V.V. Metlushko, L.E. DeLong, M. Baert *et al.*, Europhys. Lett. **41**, 333 (1998).

[7] O. Geoffroy, D. Givord, Y. Otani *et al.*, J. Magn. Magn. Matter **121**, 223 (1993). Y. Otani, B. Pannetier, J.P. Nozieres, and D. Givord, J. Magn. Magn. Matter **126**, 622 (1993). Y. Nozaki, Y. Otani, K. Runge, H. Miyajima *et al.*, J. Apll. Phys. **79**, 8571 (1996).

[8] J.I. Martin, M. Velez, J. Nogues and I.K. Schuller, Phys. Rev. Lett. **79**, 1929 (1997).

[9] D.J. Morgan and J.B. Ketterson, Phys. Rev. Lett. **80**, 3614 (1998).

[10] I.F. Lyuksyutov and V.L. Pokrovsky, Phys. Rev. Lett. **81**, 2344 (1998).

[11] B.I. Shklovskii and A.L. Efros, *Electronic Properties of Doped Semiconductors* (Springer, New York, 1984).

[12] U.C. Tauber, H.J. Dai, D.R. Nelson *et al.*, Phys. Rev. Lett. **74**, 5132 (1995). U.C. Tauber and D.R. Nelson, Phys. Rev. B **52**, 16106 (1995).

[13] L.S. Levitov and A.V. Shytov, *JETP Lett.* **66**, 214 (1997).