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Geodesic Scattering of Solitonic Strings †

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We compute the metric on moduli space for the Dabholkar-Harvey string soliton in $D = 4$ to lowest nontrivial order in the string tension. The metric is found to be flat, which implies trivial scattering of the solitons. This result is consistent with an earlier test-string calculation of the leading order dynamical force and a computation of the Veneziano amplitude for the scattering of macroscopic strings.

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1. Introduction

In [1], Dabholkar *et al.* constructed static multi-soliton "cosmic string" solutions of the low-energy supergravity equations of the heterotic string. The existence of this solution owes to the cancellation of the long-range forces due to exchange of axions, gravitons and dilatons and is reminiscent of the cancellation of gauge and Higgs forces between BPS monopoles [2] . Since these static properties are also possessed by fundamental strings winding around an infinitely large compactified dimension, Dabholkar *et al.* conjectured [3] that the soliton is actually the exterior solution for an infinitely long fundamental string.

More recently [4], the scattering of these solitons was examined using the "test string" approximation: From the sigma model action describing the motion of a point-like test string in a general background of axion, graviton and dilaton fields, an effective action was derived for the motion of the center of mass coordinate of the test string in the special background provided by a string soliton. In addition to the zero static force, it was found that the $O(v^2)$ velocity-dependent forces vanish as well. This result, which implies trivial scattering of the solitons, was found to be consistent with expectations from the Veneziano formula.

In the low-velocity limit, multi-soliton solutions trace out geodesics in the static solution manifold, with distance defined by the Manton metric on moduli space manifold [5]. In the absence of a full time-dependent solution to the equations of motion, these geodesics represent a good approximation to the low-energy dynamics of the solitons. For BPS monopoles, the Manton procedure was implemented by Atiyah and Hitchin [6,7].

In this letter we compute the Manton metric on moduli space for the scattering of the soliton string solutions in $D=4$. We find that the metric is flat to lowest nontrivial order in the string tension. This result implies trivial scattering of the string solitons and is consistent with the results of [4], and thus provides even more compelling evidence for the identification of the string soliton with the underlying fundamental string.

2. Metric on Moduli Space

We begin with a brief review of the solution of [1]. The action for the massless spacetime fields (graviton, axion and dilaton) in the presence of a source string can be written as [1]

$$S=\frac{1}{2\kappa^2}\int d^Dx\sqrt{g}\left(R-\frac{1}{2}(\partial\phi)^2-\frac{1}{12}e^{-2\alpha\phi}H^2\right)+S_\sigma, \tag{2.1}$$

with the source terms contained in the sigma model action S_σ,

$$S_\sigma = -\frac{\mu}{2}\int d^2\sigma(\sqrt{\gamma}\gamma^{mn}\partial_m X^\mu \partial_n X^\nu g_{\mu\nu} e^{\alpha\phi} + \epsilon^{mn}\partial_m X^\mu \partial_n X^\nu B_{\mu\nu}), \tag{2.2}$$

with $\alpha = \sqrt{2/(D-2)}$, $H = dB$ and γ_{mn} a worldsheet metric to be determined. The static solution to the equations of motion is given by [1]

$$\begin{aligned} ds^2 &= e^A[-dt^2 + (dx^1)^2] + e^B d\vec{x}\cdot d\vec{x} \\ A &= \frac{D-4}{D-2}E(r) \qquad B = -\frac{2}{D-2}E(r) \\ \phi &= \alpha E(r) \qquad\qquad B_{01} = -e^{E(r)}, \end{aligned} \tag{2.3}$$

where x^1 is the direction along the string, $r = \sqrt{\vec{x}\cdot\vec{x}}$ and

$$e^{-E(r)} = \begin{cases} 1 + \frac{M}{r^{D-4}} & D > 4 \\ 1 - 8G\mu \ln(r) & D = 4 \end{cases} \tag{2.4}$$

for a single static string source. The solution can be generalized to an arbitrary number of static string sources by linear superposition of solutions of the $(D-2)$-dimensional Laplace's equation. In this letter we compute the Manton metric on moduli space for the case $D = 4$, although we expect that the same result will hold for arbitrary $D \geq 4$. Note that for $D = 4$, $\phi = E$ and the metric simplifies to

$$ds^2 = -dt^2 + (dx^1)^2 + e^{-E} d\vec{x}\cdot d\vec{x}. \tag{2.5}$$

Manton's procedure may be summarized as follows. We first invert the constraint equations of the system (Gauss' law for the case of BPS monopoles). The corresponding time dependent field configuration does not in general satisfy the time dependent field equations, but provides an initial data point for the fields and their time derivatives. Another way of saying this is that the initial motion is tangent to the set of exact static solutions. The resultant kinetic action obtained by replacing the solution to the constraints in the action defines a metric on the parameter space of static solutions. This metric defines geodesic motion on the moduli space[5].

We now assume that each string source possesses velocity $\vec{\beta}_n, n = 1, ..., N$ in the two-dimensional transverse space (23). This will appear in the contribution of the sigma-model source action to the equations of motion in the form of "moving" δ-functions $\delta^{(2)}(\vec{x}-\vec{a}_n(t))$, where $\vec{a}_n(t) \equiv \vec{A}_n + \vec{\beta}_n t$ (here $\vec{A}_n$ is the initial position of the nth string source).

The equations of motion following from S are complicated and nonlinear (see [1]), and it is remarkable that such a simple ansatz as above could provide a solution in the static limit. In the time dependent case, we are even less likely to be so fortunate. In order to make headway in solving even the $O(\beta)$ time dependent constraints, we make the simplifying assumption that $8G\mu << 1$ (this is equivalent to assuming that each cosmic string produces a small deficit angle). It turns out that to linear order in μ an $O(\beta)$ solution to the constraint equations is given by

$$\begin{aligned}
e^{-E(\vec{x},t)} &= 1 - 8G\mu \sum_{n=1}^{N} \ln(\vec{x} - \vec{a}_n(t)) \\
g_{00} &= -g_{11} = -1, \qquad g^{00} = -g^{11} = -1 \\
g_{ij} &= e^{-E}\delta_{ij}, \qquad g^{ij} = e^{E}\delta_{ij} \\
g_{0i} &= 8G\mu \sum_{n=1}^{N} \vec{\beta}_n \cdot \hat{x}_i \ln(\vec{x} - \vec{a}_n(t)), \qquad g^{0i} = e^{E} g_{0i} \\
H_{10j} &= \partial_j e^{E} \\
H_{1ij} &= \partial_i g_{0j} - \partial_j g_{0i},
\end{aligned} \tag{2.6}$$

where $i, j = 2, 3$.

The kinetic Lagrangian is obtained by replacing the expressions for the fields in (2.6) in S. Since (2.6) is a solution to order β, the leading order terms in the action (after the quasi-static part) is of order β^2. The source part of the action (S_2) now represents moving string sources, and its only contribution to the kinetic Lagrangian density is of the form $(\mu/2)\beta_n^2$ for each source. The nontrivial elements of the metric on moduli space must therefore be read off from the $O(\beta^2)$ part of the massless fields effective action. In principle one must add a Gibbons-Hawking surface term (GHST) in order to cancel the double derivative terms in S (see [8–12]). In this case, however, the GHST vanishes to $O(\beta^2)$. To lowest nontrivial order in μ, the kinetic Lagrangian density is computed to be

$$\mathcal{L}_{kin} = \frac{1}{2\kappa^2}\left(2\dot{E}^2 - (\partial_m g_{0k})^2\right). \tag{2.7}$$

Henceforth we simplify to the case of two strings with velocities $\vec{\beta}_1$ and $\vec{\beta}_2$ and positions $\vec{a}_1$ and $\vec{a}_2$. Let $\vec{X}_n \equiv \vec{x} - \vec{a}_n, n = 1, 2$. Our moduli space consists of the configuration space of the relative separation vector $\vec{a} \equiv \vec{a}_2 - \vec{a}_1$. We now compute the metric on moduli space by integrating (2.7) over the (23) space. It turns out that the self-terms vanish on integration over the two-space. We are then left with the interaction terms, which may be written as

$$\mathcal{L}_{int} = \frac{64G^2\mu^2}{\kappa^2}\left[\frac{2(\vec{\beta}_1\cdot\vec{X}_1)(\vec{\beta}_2\cdot\vec{X}_2)}{X_1^2X_2^2} - \frac{(\vec{\beta}_1\cdot\vec{\beta}_2)(\vec{X}_1\cdot\vec{X}_2)}{X_1^2X_2^2}\right]. \tag{2.8}$$

The most general answer obtained by integrating (2.8) over the transverse two-space is of the form

$$L_{int}(\vec{a}) = 2f(a)\vec{\beta}_1\cdot\vec{\beta}_2 + 2g(a)(\vec{\beta}_1\cdot\hat{a})(\vec{\beta}_2\cdot\hat{a}). \tag{2.9}$$

We compute f and g by integrating (2.8) for only two configurations. In both cases, $\vec{\beta}_1$ is parallel to $\vec{\beta}_2$. The first case has the velocities parallel to $\vec{a}$ and yields

$$L_{int}(a) = (2f+2g)\beta_1\beta_2 \tag{2.10}$$

while the second case has the velocities perpendicular to $\vec{a}$ and yields

$$L_{int}(a) = 2f\beta_1\beta_2. \tag{2.11}$$

In this way we can compute both f and g. A slightly tedious but straightforward computation yields

$$g = -2f = -\frac{64g^2\mu^2\pi}{\kappa^2}\left(1-\frac{\ln 2}{2}\right), \tag{2.12}$$

and thus all the metric elements are constant. In two-dimensions, this implies that the metric on moduli space is flat (being of the form $dr^2 + Ar^2d\theta^2$, where A is a constant), and therefore has straight-line geodesics in the static solution manifold. To this approximation, then, the low-energy scattering is trivial, i.e. the strings do not deviate from their initial trajectories.

3. Discussion

The above result is in perfect agreement with those of [4]. A flat metric was also found [12] for the recently constructed heterotic multimonopole solution of [13]. It should be noted that in [4] the "test-fivebrane" approach also yielded a zero dynamical force, and consequently trivial scattering, in the low-energy limit for the class of string solitons with fivebrane structure[14–18]. A different result, however, was found in [11] for the metric on moduli space for the scattering of heterotic fivebranes. In that case, however, the self-terms were found to be divergent and we were compelled to extract information from the convergent interaction terms in what amounted to a regularization procedure. This difficulty with the regularization is probably the cause of the apparent contradiction between the two results for fivebrane scattering. Fortunately, for both the string and monopole solutions, we do not encounter any divergence in the self-terms and do not have to resort to regularization. In both cases, we can explicitly compute the metric on moduli space, which turns out to be flat, in direct agreement with test-string and test-monopole calculations.

More recently in [19], a flat metric was also found for the fivebranes using a different gauge choice than in [11]. For the fivebrane analogue of the the Veneziano amplitude calculation of [4] (which yielded trivial scattering for macroscopic winding state strings), we must await the construction of the conjectured dual theory of fundamental fivebranes[20,16,14,15].

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