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The Origin of Space-Time as W Symmetry Breaking in String Theory

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Abstract

Physics in the neighbourhood of a space-time metric singularity is described by a world-sheet topological gauge field theory which can be represented as a twisted $N = 2$ superconformal Wess-Zumino model with a $W_{1+\infty} \otimes W_{1+\infty}$ bosonic symmetry. The measurable W -hair associated with the singularity is associated with Wilson loop integrals around gauge defects. The breaking of $W_{1+\infty} \otimes W_{1+\infty} \rightarrow W_{1+\infty}$ is associated with expectation values for open Wilson lines that make the metric non-singular away from the singularity. This symmetry breaking is accompanied by massless discrete ‘tachyon’ states that appear as leg poles in S -matrix elements. The triviality of the S -matrix in the high-energy limit of the $c = 1$ string model, after renormalisation by the leg pole factors, is due to the restoration of double W -symmetry at the singularity.

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1 Introduction and Summary

One of the deepest and most fascinating problems in quantum gravity is the nature of singularities in space - as inside a black hole - or time - as at the beginning of conventional Big bang cosmology. These singularities raise severe challenges for physics as we understand it so far: the known laws of physics are no longer applicable there, and they may cause observable deviations from the known laws even in non-singular regions of space-time. For example, quantum coherence cannot presumably be maintained in any local field theory in the presence of a space-time singularity [1].

String theory provides a powerful framework for addressing and potentially solving these and other problems in quantum gravity. In particular, related consistent string models have been formulated which are able to describe the early stage of Big Bang cosmology [2], and a spherically-symmetric black hole [3, 4]. It has been shown that quantum coherence is maintained in the presence of such a stringy black hole, thanks to an exact W-symmetry which provides an infinite number of conserved gauge charges, W-hair, sufficient to label and distinguish all the black hole states [5],[6], [7]. The W-charges of a string black hole are in principle measurable by scattering experiments similar to meson-Skyrmion or quark-monopole scattering, or by analogues of Aharonov-Bohm interference measurements [8]. However, this W-symmetry is just part of an infinite set of string gauge symmetries [9, 10] whose full extent and character are not yet understood. The string black hole has a singularity at the origin, and one could expect a fuller set of gauge symmetries to be restored there, as occurs at the core of a gauge monopole [11].

In this paper we demonstrate the existence of a double W-symmetry at the singularity, show how it is broken by Wilson lines down to the observable W-symmetry of the black hole solution away from the singularity, demonstrate the relation of tachyon leg-poles to this symmetry breakdown, and interpret the known triviality of the high-momentum $c=1$ string S-matrix as a manifestation of this higher string symmetry. The singularity at the core of the string black hole serves as a window on the nature of string before its higher symmetries are broken by the appearance of an expectation value for the target-space metric.

Our reasoning is as follows. We start from the observation [3, 12], that, in string theory, physics in the neighbourhood of a space-time singularity is described by a topological gauge theory (TFT) on the world-sheet. It is known that such a TFT can be represented as a twisted $N=2$ superconformal Wess-Zumino model [13]. As such [14], it possesses a super-W-symmetry, which includes $W_{1+\infty} \times W_{1+\infty}$ as bosonic symmetry, to be compared with the simple observable $W_{1+\infty}$ symmetry of the string black hole. The target space-time singularity reflects the presence of a *gauge defect* on the world-sheet [15], and the observable W-charges are Wilson line integrals around loops enclosing this world-sheet defect, which can be viewed as a world-sheet

vortex . The pure gauge TFT describes physics at the target-space singularity, where the physical metric vanishes. A non-vanishing metric corresponds to an expectation value for an open Wilson line terminating on ‘matter’ fields of the TFT, analogous to quark-antiquark condensation in QCD. Similarly to the way the quark condensate breaks chiral $SU(N) \otimes SU(N) \rightarrow SU(N)$ [16], the ‘matter’ condensate breaks the TFT $W_{1+\infty} \otimes W_{1+\infty} \rightarrow W_{1+\infty}$. The analogues of the Goldstone bosons of chiral symmetry breaking are mass-zero states with discrete momenta, explaining the leg-poles of the string $c=1$ model and black hole S-matrices, which were previously mysterious [17, 18]¹. In the high-momentum limit, the $c=1$ model S-matrix elements sample the singular region alone, and the higher string symmetry there explains the triviality found after absorbing the leg pole factors into the vertex operators for the scattered particles [19].

2 Symmetries of the Topological Field Theory at the Singularity

Although the discussion outlined in the Introduction should apply to any point-like isolated singularity in 4-dimensional target space-time, so far it can be developed explicitly only for the singularity at the centre of a spherically-symmetric 4-dimensional black hole. This is described by the Schwarzschild metric, which can be written as [20]

$$ds^2 = -\frac{32M^3 e^{-\frac{r}{2M}}}{r} dudv + r(u, v)^2 (\sin^2 \tilde{\theta} d\phi^2 + d\tilde{\theta}^2) \quad (1)$$

where u and v are Kruskal-Szekeres coordinates. Discarding the radial part of (1) and making a trivial change of coordinates, one finds that the metric can be written as a conformally-rescaled form of the 2-dimensional black hole metric [3]

$$ds_{2bh}^2 = \frac{1}{1-uv} dudv \quad (2)$$

which is singular when $uv = 1$. The 2-dimensional (4-dimensional spherically-symmetric [4]) string black hole is described by an exact conformal theory which can be written as an $SL(2, R)/U(1)$ coset Wess-Zumino (WZ) model on the world-sheet [3]:

$$\begin{aligned} S_{WZ} = & \frac{k}{8\pi} \int d^2z Tr(\partial_i g \partial_i g^{-1}) + i \frac{k}{12\pi} \int d^3x \epsilon^{ijk} Tr(g^{-1} \partial_i g g^{-1} \partial_j g g^{-1} \partial_k g) \\ & + \frac{k}{2\pi} \int d^2z Tr(B_z \sigma_3 g^{-1} \partial_z g + B_z \sigma_3 \bar{\partial}_z g g^{-1} + B_z \sigma_3 B_{\bar{z}} + B_z \sigma_3 g B_{\bar{z}} \sigma_3 g^{-1}) \end{aligned} \quad (3)$$

¹Continuum leg poles were first found in a Wick-rotated version in the last paper of ref. [2].

where g is a 2×2 matrix parametrized by

$$g = \begin{pmatrix} a & u \\ -v & b \end{pmatrix}, \quad ab + uv = 1 \quad (4)$$

from which we see that the space-time singularity $uv = 1$ corresponds to $ab = 0$. The action (3) is invariant under the U(1) gauge transformation

$$\begin{aligned} \delta g &= \epsilon(\sigma_3 g + g \sigma_3), & \delta B_i &= -\partial_i \epsilon \\ (\delta a &= 2\epsilon a, & \delta b &= -2\epsilon b, & \delta u &= \delta v = 0) \end{aligned} \quad (5)$$

Eliminating the gauge field B_z from the action (3), the latter can be written to leading order in k as [3]

$$S = \frac{-k}{4\pi} \int d^2x \frac{\partial_i u \partial_i v}{1 - uv} \quad (6)$$

where we see explicitly that the WZ model (3) corresponds to the black hole metric (2) with its target-space singularity at $uv = 1$. However, the WZ model is well-defined even in the neighbourhood of the singularity [3, 12]. Parametrizing u and v in the forms $u = e^w$, $v = e^{-w}$, the action (3) there becomes

$$S = -\frac{k}{4\pi} \int d^2x D_i a D_i b + i \frac{k}{2\pi} \int d^2x w \epsilon^{ij} G_{ij} \quad (7)$$

which has the form of a U(1) topological gauge field theory (TFT) on the world-sheet, with matter fields a, b , and G_{ij} is the gauge field strength.

To see the symmetries of this TFT, we recall that it can be rewritten as a twisted $N = 2$ supersymmetric Wess-Zumino (SWZ) model [13]. This is obtained from the superconformal version of the $SL(2, R)/U(1)$ model (3):

$$S_{susy} = S_{WZ} + \frac{i}{2\pi} \int d^2z (Tr \psi D_{\bar{z}} \psi + Tr \bar{\psi} D_z \bar{\psi}) \quad (8)$$

where ψ and $\bar{\psi}$ are coset fermions. The model (8) is twisted by adding to the stress tensor a piece proportional to the U(1) current, so that it has zero central charge, one of the supersymmetry operators is turned into a BRST operator, the fermions are converted into ghost fields α, β of spins 0,1, and the action becomes

$$S_{WZ}^{twisted} = S_{WZ} + \frac{i}{2\pi} \int d^2z Tr(\beta_z D_{\bar{z}} \alpha + \bar{\beta}_{\bar{z}} D_z \bar{\alpha}) \quad (9)$$

Physical states are annihilated by the BRST transformation:

$$Q|physical\rangle = 0 \quad (10)$$

and are given by the chiral ring of the $N = 2$ theory.

It will be convenient in the following to use the twisted $N = 2$ SWZ action in the form given by Nojiri [21]:

$$\begin{aligned}
S^{(1)} &= \frac{k}{\pi} \int d^2z \tanh^2 r (\partial\theta\bar{\partial}\theta - \partial\bar{\psi}\bar{\psi} + \psi\bar{\partial}\psi) \\
&- 2 \frac{\sinh r}{\cosh^3 r} (\eta\psi\bar{\partial}\theta + \bar{\eta}\bar{\psi}\partial\theta) + 4 \tanh^2 r \eta\bar{\eta}\psi\bar{\psi} \\
&+ \partial r \bar{\partial} r - \partial\bar{\eta}\bar{\eta} + \eta\bar{\partial}\eta)
\end{aligned} \tag{11}$$

where r and θ are the physical coordinates related to the Kruskal-Szekeres coordinates introduced in equation (1), and $\{\eta, \psi, \bar{\eta}, \bar{\psi}\}$ are ghosts.

Now we come to our first key new point: since this is an SWZ model, it has a super- $W_{1+\infty}$ symmetry. As such, its bosonic symmetry is of the form $W_{1+\infty} \otimes W_{1+\infty}$ [22, 14], whose structure can be seen conveniently via the super-KP hierarchy [22]:

$$\Lambda = \rho^2 + \sum_{r=0}^{\infty} U_r \rho^{-r-1} \tag{12}$$

where $\rho \equiv \partial_\zeta + \zeta \partial_x$, $\rho^2 = \partial_x$, (ζ, x) are the odd- and even-parity components of (1,1) superspace, and $U_i \equiv v_i(x) + \zeta u_i(x)$. Redefining

$$\tilde{u}_{2i} = u_{2i} + v_{2i+1} \tag{13}$$

we see that $\{\tilde{u}_{2m}, v_{2r}\} = 0$, and hence the bosonic currents $\{\tilde{u}_{2m}\}$ and $\{v_{2r}\}$ constitute independent bosonic KP hierarchies. Hence they together realize a direct product $W_{1+\infty} \otimes W_{1+\infty}$ bosonic symmetry, whilst the currents u_{2k+1} and v_{2r} are fermionic operators which we discuss in the next section.

3 W-symmetry Breaking

The double W -symmetry we have found above is larger than the single W -symmetry of the black hole solution [5, 6, 23, 24]. Therefore, our next step is to identify the pattern of symmetry breaking for this double W -symmetry (cf the breaking of $SU(N)_L \otimes SU(N)_R \rightarrow SU(N)_V$ in QCD), and the mechanism responsible (cf $q\bar{q}$ condensation in QCD). Additional twists are needed, beyond those leading to (9), to construct a topological version of the *super*- $W_{1+\infty}$ algebra. This is because, although the currents v_{2r} are nilpotent, as follows directly from their anticommutation relations, the currents u_{2r+1} become nilpotent only after twisting as follows [22]:

$$\tilde{u}_{2r+1} = u_{2r+1} - v_{2r+2} \tag{14}$$

This construction leads to two infinite sets of cohomology operators, and the fermionic currents \tilde{u}_{2r+1} and v_{2r} become ghosts. Thus there are two BRST operators:

$$\begin{aligned}
Q_1 &= \int_y \tilde{u}_1(y) \\
Q_2 &= \int_y v_0(y)
\end{aligned} \tag{15}$$

which do not anticommute:

$$\{Q_1, Q_2\} = \int_x u_0(x) \quad (16)$$

The physical states are those annihilated by just one of the two BRST charges (15). It is precisely the non-anticommutativity property (16) that guarantees that the physical states are not doubled, an indication of spontaneous symmetry breaking in one of the sectors.

We believe that the physical states are those generated by the Q_1 operator, so the physical fields are functionals of the v sector of the model. This follows from the fact that the unbroken symmetry contains the entire \tilde{u} sector. This in turn follows from the observation that the super-KP operator Λ , when acting on bosonic functions of x alone, yields a KP hierarchy with respect to the coefficients \tilde{u} in (13). Since we are interested only in the physical non-ghost sector, we discard the ghost contributions in (12), and are left with

$$\Lambda^{phys} = \partial_x + \sum_{k'=0}^{+\infty} (u_{2k'} + v_{2k'+1}) \partial_x^{-(k'+1)} \quad (17)$$

It is the coefficients of this operator that appear in higher orders of the OPE between parafermions of the bosonic WZ coset model of level parameter $k \geq 2$ [23, 24], and generate the $W_{1+\infty}$ of the theory that is measurable in target space and responsible, in our interpretation, for the maintenance of quantum coherence [5, 6].

Since the world-sheet, where the $W_{1+\infty} \otimes W_{1+\infty}$ symmetry appears in the TFT and is subsequently broken, is 2-dimensional, the mechanism for W-symmetry breaking cannot be simple condensation accompanied by the appearance of Nambu-Goldstone bosons [25], but must be some generalization of Kosterlitz-Thouless vortex condensation with an infinite set of non-zero order parameters appearing. The starting-point for a discussion of this possibility is the configuration of the U(1) gauge field on the world-sheet in the model (7). Consider the following potential for a vortex-antivortex configuration placed at the origin and the north pole of the world-sheet spherical surface :

$$\theta = \frac{1}{2}(\text{Im} \ln z - \text{Im} \ln \bar{z}) \quad (18)$$

We can define an embedding of the S^2 world-sheet in a 2-dimensional target space $X^\mu(z, \bar{z}) : \mu = 1, 2$ by the relation [15]

$$2z = e^\omega - e^{-\omega} \quad : \quad \omega \equiv r + i\theta \quad (19)$$

It is then easy to see that the target-space metric $ds^2 = g^{ij} dX_i dX_j = \frac{1}{1+z\bar{z}} dz d\bar{z}$ is of (Euclidean) black hole type:

$$ds^2 = dr^2 + \tanh^2 r d\theta^2 \quad (20)$$

This demonstrates that a singularity in space-time corresponds to a topological defect in the $U(1)$ gauge theory on the world-sheet. The dynamics of symmetry breaking and the values of the measurable W quantum numbers are associated with Wilson line integrals in this gauge theory, as we now see.

To acquire a physical picture of the breaking of $W_{1+\infty} \otimes W_{1+\infty} \rightarrow W_{1+\infty}$, we recall from the parametrization (4) of the $SL(2, R)$ matrix g that $ab + uv = 1$, so that a non-zero expectation value for the product ab corresponds to $uv \neq 1$, and hence a non-singular region of target-space. Experience with 2-dimensional gauge theories tells us that in a TFT such as (7) we should expect $\langle ab \rangle \neq 0$ when $r \neq 0$, which is the case away from the world-sheet defect. Thus gauge dynamics, reminiscent of that responsible for chiral symmetry breaking in QCD [16], *generates* space-time away from the singularity.

To see this formally, it is convenient to use Nojiri's representation (11) of the *TFT* action in twisted $N = 2$ *SWZ* form. At the singularity, $r \rightarrow 0$, we see that it becomes a free field theory,

$$S^{(1)} = \frac{k}{\pi} \int d^2z [\partial r \bar{\partial} r - \partial \bar{\eta} \bar{\eta} + \eta \bar{\partial} \eta] \quad (21)$$

whilst $r \neq 0$ corresponds to a flat direction of the the effective potential along which the effective Thirring four-fermion interaction acquires a non-zero coefficient (consistently with conformal invariance). The free action (21) clearly possesses super- $W_{1+\infty}$ symmetry, which is absent when the other terms in $S^{(1)}$ (11) are switched on at $r \neq 0$.

To see how the measurable W -charges of the residual $W_{1+\infty}$ are determined we recall that in higher-dimensional gauge theories formulated on space-times with non-trivial topologies, gauge symmetry breaking can be achieved via non-trivial boundary conditions for quantum fields transported around a Wilson loop enclosing a defect in the space-time [26]. Specifically, in theories with a non-Abelian gauge symmetry G and

$$\langle F_{\mu\nu} \rangle = 0 \quad (22)$$

in vacuo in a non-simply-connected space-time, the choice of vacuum

$$\langle A_\mu \rangle = V^\dagger \partial_\mu V \quad (23)$$

can be physically distinct from the symmetric vacuum

$$\langle A_\mu \rangle = 0 \quad (24)$$

This is achieved by imposing non-trivial boundary conditions for transport around a closed loop enclosing a defect:

$$A_\mu(x + L) = U A_\mu(x) U^\dagger \quad (25)$$

where U is a constant matrix, via a non-zero Wilson loop integral around the defect:

$$W(L) = P \exp(i \int_L A_\mu dx^\mu) U = V^\dagger U^{symm} V \quad (26)$$

The gauge transformations $\Omega(x)$ that preserve the vacuum (24) are those that satisfy

$$[V(x+L)UV(x)^\dagger, \Omega(x)] = 0 \quad (27)$$

so the gauge symmetry is reduced [26].

What about our case? In general in two dimensions there is tunnelling between the vacua, induced by instanton effects, which prevents A_μ from taking a definite local vacuum expectation value. The ground-state wave-function is not a delta-function, but has smooth support, and no definite vacuum choice is made. The quantities U and V are not well-defined, and the Wilson integrals (26) do not act as order parameters for the breaking of the symmetry. However, in the case of the TFT of interest to us, we believe on the basis of the absence of instanton effects in the $c = 1$ string model [27], and because of supersymmetry [28], that tunnelling effects are suppressed, so the above mechanism for symmetry breaking can be applied to one of the $W_{1+\infty}$ symmetries.

Remember that all the discrete quasi-topological states of the two-dimensional string in a black hole background can be regarded as generalized gauge states related to singular gauge transformations [29]. This means that one can define quantities of the form (23) for discrete values of the energy and momentum, which are the only remnants in two dimensions of string states in higher-dimensional target spaces. The above symmetry-breaking mechanism applies away from, but arbitrarily close to, the defect on the world-sheet, and breaks down at the defect, where the representation (23) makes no sense. Hence, the symmetry-breaking pattern that we describe below does not apply at the target-space singularity, where the full symmetry of the TFT is recovered. The complete uncertainty in space-time location of these higher string states is to be expected on the basis of the Mermin-Wagner-Coleman theorem [25] in two-dimensional space-times, which forbids symmetry breaking via a local condensate.

Our scenario is the following. At the singularity there is no concept of space-time, only the world-sheet on which the WZ model is defined. The Wilson loops that break the symmetry are defined on the world-sheet as line integrals for the string gauge states. These loops are well-defined small-radius ($R \rightarrow 0$) limits of the boundary operators (around closed loops γ) introduced in ref. [30]

$$W(L) = \lim_{R \rightarrow 0} W(\gamma) \quad : \quad W(\gamma) = \int_\gamma e^{\beta\phi} O(X) \quad (28)$$

where $O(X)$ are matter operators, β has the appropriate value to ensure that the boundary dimension is one, as required by conformal invariance, and the Liouville

field ϕ contains a singular logarithmic piece, lnz , as well as regular parts [30]. The logarithmic part is cancelled, in the small radius limit, by the vanishing measure of the line integration, so the limit is a well-defined local operator with discrete energy and momentum. The world-sheet defect enables one to impose non-trivial boundary conditions which are elements of the global world-sheet $W_{1+\infty}$ generated by the v sector of the super-KP hierarchy discussed in section 2. This is consistent with the previously-mentioned fact that the physical states are constructed out of the v sector. The remaining exact W symmetry is realized by the states of the TFT, which are singular gauge transformations of the form (23).

The world-sheet W -symmetry can be elevated to space-time, as discussed in [4], where it becomes a local gauge symmetry [5, 31]. Accordingly, because of the embedding (19) of the world-sheet in the space-time, the small loops (28) can be regarded as small space-time integrals, and hence the above global world-sheet symmetry breaking mechanism is elevated to the familiar [26] local gauge symmetry breaking mechanism. The non-trivial boundary conditions are constant group elements of the full $W_{1+\infty} \otimes W_{1+\infty}$, and the physical gauge symmetry of the space-time away from the singularity is due to those generators of this symmetry group that commute with the Wilson line integrals around the defect on the world-sheet. This mechanism leaves unbroken the $W_{1+\infty}$ symmetry generated by the \tilde{u} sector of the TFT, which commutes with the v sector.

It should be noted, on the basis of the the commutation relations of the generators W_s of the $W_{1+\infty}$ algebra constructed out of the world-sheet currents v_r , that there are always elements with non-vanishing commutators, i.e., the broken algebra is centreless. For instance, exploiting the bi-Hamiltonian structure of the KP hierarchy in the no-ghost sector [32], we observe that the anticommutators of the first Hamiltonian structure that generate $W_{1+\infty}$ contain

$$\int dz' \{W_r(z), W_2(z')\} = \partial_z W_r(z) \quad (29)$$

Since the $W_r(z)$ constitute the full set of generators of $W_{1+\infty}$, it is clearly always possible to construct boundary condition matrices (23) such that (27) is not satisfied, and hence the entire $W_{1+\infty}$ symmetry group is broken.

It is natural to ask what determines the choice of boundary conditions, which seems arbitrary at first sight. In our picture, they are determined dynamically by the appropriate choice of vacuum, though we cannot yet give a rigorous proof of this statement, since a complete description of the TFT is not yet available. However, it should be possible, within the context of string theory, to understand these boundary conditions as specifying some sort of sigma-model vacuum configuration. Its determination is of course as difficult as any other issue related to lifting the string vacuum degeneracy, and will not be pursued further in this paper. We note, however, a close analogy with the problem of fixing the boundary conditions for

the wave-function of the Universe at the initial singularity [33]. Indeed, we would say that this is the same problem, since the initial cosmological singularity presumably shares the black hole singularity features discussed here. The difference in our approach, compared to that of Hawking, is that our boundary conditions should be specified on the world-sheet, with the space-time properties emerging as derived quantities.

4 Leg Poles as Mermin-Wagner-Coleman Bosons

One of the outstanding puzzles of the string $c=1$ and black hole models has been the appearance of poles in the external leg momenta of tachyon scattering amplitudes [17, 18]. These are distinct from the massive discrete states found by factorization of the tachyon S-matrix, which are known to be associated with topological states that are in 1-to-1 correspondence with the generators of the measurable W-algebra [34]. The leg poles appear at light-like momenta, $\sqrt{\alpha'}p = n^+$, where α' is the Regge slope, and n^+ is a positive integer, and hence correspond to discrete massless tachyon states. The fact that their momenta are discrete is consistent with the failure of the Goldstone-Higgs mechanism in two dimensions. In local field theories, the breaking of a gauge symmetry does not yield a Goldstone boson, since the latter become the longitudinal components of the vector bosons, and as such are gauged out of the physical spectrum. However, at the special values of momentum where the leg poles appear one cannot gauge away longitudinal components of string gauge states [18]. Hence discrete tachyon states appear, which are non-local in target space, and associated with the breaking of the gauge symmetry. It is for this reason that we call them Mermin-Wagner-Coleman bosons.

The appearance of these Mermin-Wagner-Coleman bosons can also be seen via the Nojiri's action $S^{(1)}$ (11). The Thirring interaction term in $S^{(1)}$ may be expanded with coefficients in the following series at large r :

$$\sum_{m=0}^{+\infty} A_m e^{-2mr} \quad (30)$$

After appropriate normalisation, we observe that the terms in this series correspond (after Fourier transformation) to background values of plane-wave tachyons with definite energies, at precisely the values that correspond to the different leg poles ².

Thirring interactions are known to preserve conformal invariance of the pertinent σ model for arbitrary expectation values of the (target space) scalar fields coupled to them [35]. Hence, from the equivalence of the vanishing β -function conditions to

²Because we use the concept of plane waves, one should be careful in subtracting the background charge terms from the definition of Liouville energies, when one defines mass-shell conditions and/or discusses background values [2].

stationary points of the target space effective action [36], we observe that, on account of the non-propagating, quasi-topological nature of the leg-pole states, each term in the series (30) corresponds to an independent *flat direction* of the target-space effective potential of the $N = 2$ superconformal theory. These are the analogues of Goldstone-Higgs bosons in higher-dimensions [37], but here they are delocalized massless states. They transform non-trivially under the broken $W_{1+\infty}$, analogously to pions and chiral symmetry in QCD .

5 Triviality of the High-Momentum Limit of the $c=1$ String Model S-Matrix

We are now in a position to interpret the observation that, after absorbing the external leg poles, the S-matrix of the $c=1$ string model is trivial in the high-momentum limit [19]. This is because the rapidly-varying phase factors of high-momentum tachyons do not sample the non-singular regions of target space-time, but only the singular region, in which the $W_{1+\infty} \otimes W_{1+\infty}$ symmetry is so high that no non-trivial S-matrix elements are allowed. This is what happens in the topological world-sheet models that possess this symmetry, and have no bulk contributions to the scattering matrix.

This can be seen explicitly using the Nojiri's action $S^{(1)}$ (11). In the limit $r \rightarrow 0$ in which the $SL(2, R)/SO(1, 1)$ model becomes the $c = 1$ string model, we retain just the leading term in the expansion (30), and

$$S^{(1)} = \frac{k}{\pi} \int d^2z (\partial\theta\bar{\partial}\theta + \partial r\bar{\partial}r + \dots) \quad (31)$$

which is just the action of a theory of free tachyons. Hence there is no scattering of the tachyons in the $c = 1$ model, and hence the S -matrix $S \rightarrow 1$ in this limit. However, we understand from the previous discussion that this is just an asymptotic statement about the high-momentum limit in which $\frac{p}{M} \rightarrow 0$ for the mass M of any quasitopological state. At subasymptotic energies, the S -matrix is non-trivial [38], and reflects the double W -symmetry, just as pion couplings reflect the $SU(N) \otimes SU(N)$ symmetry of QCD .

6 Comments on the Symmetric Phase of String Theory

We would like to conclude with a few general comments about the symmetric phase of string theory on the basis of the above discussion. (1) The symmetric phase of

string theory has often been conjectured to correspond to a topological field theory [39]. Our analysis of singularities in string theory confirms that the relevant *TFT* is one formulated on the *world-sheet*, not in space-time. (2) The relevant *TFT* is a *topological gauge field theory*, not a topological gravity theory as one might have thought. So far at least, the metric on the world-sheet does not dissolve (but see ref. [40]). However, defects in the effective gauge theory on the world-sheet can form, and appear at space-time singularities. (3) So far we have only discussed a single, isolated black hole singularity, corresponding to a single gauge defect on the world-sheet. This could be adapted to discuss the initial cosmological singularity. (4) One should be able to generalize the analysis to the case with more defects on the world-sheet, and to higher genera, corresponding to multi-black-hole solutions. Such a development would open the way to a more precise and quantitative description of space-time foam, in which many microscopic space-time singularities presumably co-exist. It has not escaped our notice that the Reissner-Nordstrom extremity of string black holes [3] suggests compliance with the ‘no-net-force’ condition, sufficient to ensure *superposition* of the solutions in target space [41]. In view of our world-sheet defect interpretation of target-space singularities, it would be interesting to exploit the world-sheet version of the ‘no-net-force’ condition. (5) In this way one should also be able to discuss a high-temperature transition to the symmetric phase of string theory, which is presumably analogous to the unconfined phase of a two-dimensional gauge field theory.

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