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August 1990**PHASE TRANSITIONS IN QCD AND STRING THEORY**Bruce A. Campbell¹⁾ and John Ellis
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LAPP-Annecy**Abstract**

We develop a unified effective field theory approach to the high-temperature phase transitions in QCD and string theory, incorporating winding modes (time-like Polyakov loops, vortices) as well as low-mass states (pseudoscalar mesons and glueballs, matter and dilaton supermultiplets). Anomalous scale invariance and the Z_3 structure of the centre of $SU(3)$ decree a first-order phase transition with simultaneous deconfinement and Polyakov loop condensation in QCD, whereas string vortex condensation is a second-order phase transition breaking a Z_2 symmetry. We argue that vortex condensation is accompanied by a dilaton phase transition to a strong coupling régime, and comment on the possible role of soliton degrees of freedom in the high-temperature string phase.

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Nowadays there is much discussion about the nature of string at high temperatures, short distance and high energies, and its relation to the low-temperature, low-energy phase of string that we experience daily. In particular, the possible transition(s) between low-and high-temperature phases somewhere around the string Hagedorn temperature is (are) the focus of much debate [1]. These discussions echo eerily those of 20 years ago on the true nature of the strong interactions and on what might lie beyond the hadronic Hagedorn temperature [2]. We now know that this is QCD, and understand that it may be represented at low energies and temperatures by an effective field theory with approximate chiral and scale invariance [3]. Moreover, even now this effective field theory can give us extra insight into the phase transition(s) between low-and high-temperature QCD [4]. In the case of string, we understand the general form of the effective low-energy field theory, namely some generalized $N = 1$ no-scale supergravity [5], [6] containing a model-independent dilaton supermultiplet [7]. In this paper we study its finite-temperature properties in parallel with those of the low-energy QCD meson theory, seeking insights into the nature(s) of the string phase transition(s), and possibly into the nature of the high-temperature string phase.

There are many analogies, but some differences, between the QCD and string cases. In both cases there are light elementary excitations-pseudoscalar mesons and a scalar pseudo-dilaton glueball in a chiral Lagrangian derived from QCD [3], analogous to the matter and dilaton supermultiplets in a low-energy supergravity theory derived from string theory [7]. Also, in both cases there are massive winding modes, time-like Polyakov loops [8] and vortices [9], that are expected to condense in the high-temperature phase. In this paper we construct and analyze in parallel models coupling the elementary glueball/dilaton and winding Polyakov/vortex modes, looking for similarities and differences between the two cases. In some respects our prior knowledge is deeper in the QCD case, for example there is a lot of lattice information on the nature of the QCD phase transition(s) [10]. But, amusingly, in some respects we know more about string, for example the nature of the high-temperature tachyonic mode and its coupling to the dilaton are calculable in string theory [9], whereas we only have qualitative symmetry arguments for the corresponding features of the QCD case.

We find in this case that simultaneous first-order transitions in the order variable (the gluon condensate) and the disorder variable (the time-like Polyakov loop) are enforced by the symmetry properties of QCD (anomalous scale invariance and the Z_3 centre of $SU(3)$, respectively). In the case of string, we argue that the dilaton v.e.v. is first driven towards infinity, i.e. the string coupling is driven towards zero, below the string Hagedorn temperature. Then vortices condense in a second-order phase transition at the string Hagedorn temperature, breaking a Z_2 duality symmetry spontaneously. This is followed by a transition to strong coupling above the Hagedorn temperature. We conclude with some speculations on the nature of this phase, in particular on the possible role that soliton quantum numbers [11] might play in the high-temperature phase.

We first review and extend previous work on the low-energy effective field theory analysis of the QCD phase transition [4], emphasizing points of comparison with the corresponding string case. Since we will be interested in closed string theory, we will not include quark degrees of freedom. In this case, the only low-energy QCD order parameter is the scalar gluon condensate $\langle 0 | G_{\mu\nu} G^{\mu\nu} | 0 \rangle$: $G_{\mu\nu} G^{\mu\nu} = \frac{25}{\beta} \theta_\mu^\mu$ where θ_μ^μ is the trace of the energy-momentum tensor and divergence of the scale current.

Scale invariance is exact in classical quark-less QCD, but is broken by renormalization. The minimal dynamical model incorporating gluon condensation and anomalous scale invariance contains a single scalar glueball field χ [3]:

$$L_{\text{eff}}(\chi) = \frac{1}{2}(\partial_\mu \chi)^2 - V(\chi) : V(\chi) = B\chi^4 \ln(\chi/e^{1/4}\chi_0) \quad (1)$$

where $\chi_0 \equiv \langle 0|\chi|0 \rangle$ and $\theta_\mu^\mu = \frac{\beta}{2g} G_{\mu\nu} G^{\mu\nu} = 4B\chi^4$. When the temperature T increases, the potential $V(\chi)$ acquires finite-temperature corrections $\delta V(\chi, T)$. Because of the Coleman-Weinberg-like flatness of $V(\chi)$ in equation (1), these could be expected to induce a first-order phase transition at some critical temperature T_c , and this has been verified in model calculations [4].

In the high-temperature phase of QCD, a key role is played [8] by the centre Z_3 of the $SU(3)$ gauge group. Time-like Polyakov loops P take values in Z_3 , and can be described by an effective Hamiltonian density

$$H_{\text{eff}}(P) = |\nabla P|^2 + V(P) : V(P) = \mu^2 |P|^2 + \frac{A}{2} (P^3 + P^{*3}) + \lambda |P|^4 \quad (2)$$

where the form of the effective potential $V(P)$ is dictated by the Z_3 symmetry [8]. It is easy to see that $\langle 0|P|0 \rangle = 0$ when $\mu^2 > \frac{1}{4}A^2$, corresponding to the low-temperature phase, but $\langle 0|P|0 \rangle \sim \exp(2\pi i n/3)$ when $\mu^2 < \frac{1}{4}A^2$, corresponding to the high-temperature phase. The phase transition is first order, with

$$\Delta |\langle 0|P|0 \rangle| = \mu = \frac{A}{2} \quad (3)$$

at the phase boundary : $\mu = A/2$.

For a coupled description of the order and disorder variables, we consider the full effective potential $V(\chi, P) = V(\chi) + V(P)$, and include χ -and T -dependence in the parameter μ^2 ¹⁾. By analogy with the well-understood string case [9], [1], we expect that

$$\mu^2 \simeq \hat{\mu}^2(\chi) \left(\frac{1}{T^2} - \frac{1}{T_H^2} \right) \quad (4)$$

where T_H is the Hagedorn temperature. The expression (4) has the feature that the P field has infinite mass-squared when $T \rightarrow 0$, but becomes a tachyon at the Hagedorn temperature, like the Z_2 vortex to be discussed later in the string case. The precise form of $\hat{\mu}^2$ in (4) is not essential for our argument: for simplicity we take it to be $\hat{\mu}^2 = \hat{\mu}_0^2 \chi^2 : \mu_0^2 > 0$. It is easy to see that in this case the first-order phase transition in $\langle 0|P|0 \rangle$ at fixed χ occurs when

$$T = T_P : T_P = T_H \sqrt{\frac{\mu_0^2 \chi^2}{\mu_0^2 \chi^2 + \frac{1}{4}A^2 T_H^2}} \quad (5)$$

which is always less than the Hagedorn temperature, reflecting the fact that $\Delta |\langle 0|P|0 \rangle| \neq 0$ when $\mu^2 > 0$, because of the trilinear term in $V(P)$ (2).

But is χ fixed at the P transition point? Precisely because $\mu^2 \propto \chi^2 > 0$ at this point, this term alone in the effective potential would favour $\chi \rightarrow 0$ when $P \neq 0$. However, to determine whether this in fact occurs, we must consider the energy difference

1) We could also include such dependences in A and λ , but this would not introduce any new issues of principle.

between $\chi = \chi_0$ and $\chi = 0$ given by $V(\chi)$. Treating the latter as a perturbation on $V(P)$, we find as a first approximation that there is a simultaneous first-order transition in P and χ at

$$T = T_c : T_c \simeq T_H \left(\frac{1}{1 + \left(\frac{1}{4}A^2 - \frac{B\chi_0^4}{A^2} \right) \left(\frac{T_H^2}{\mu_0^2 \chi_0^2} \right)} \right)^{\frac{1}{2}} \quad (6)$$

which is again below the Hagedorn temperature in the limit of small B .

We summarize this QCD analysis as follows. The form of the effective potential decreed by the symmetries of the order and disorder variables (anomalous scale invariance and Z_3 respectively) enforce first-order transitions in both the dilaton field χ and the ‘‘vortex’’ field P . A plausible model for the coupling of these fields indicates that these two first-order phase transitions occur simultaneously at some temperature below the Hagedorn temperature.

Before extending the above QCD analysis to the string case, we first recall relevant aspects of supergravity at zero and finite temperature. A general $N = 1$ supergravity theory is characterized by the Kähler potential $\mathcal{G}(\phi, \phi^*)$ and the gauge kinetic function $f_{\alpha\beta}(\phi)$, where the ϕ are chiral superfields [12]. It is customary to split the Kähler potential into parts which can (cannot) be written as holomorphic and antiholomorphic functions of the chiral fields:

$$\mathcal{G}(\phi, \phi^*) = G(\phi, \phi^*) + F(\phi) + F^*(\phi^*) \quad (7)$$

where F is called the superpotential. The effective scalar potential can then be written as

$$V = e^{\mathcal{G}} \left[\mathcal{G}^i (\mathcal{G}^{-1})^j_i \mathcal{G}_j - 3 \right] + \frac{1}{2} \text{Re} f_{\alpha\beta}^{-1} D^\alpha D^\beta \quad (8)$$

where $\mathcal{G}_i \equiv \partial \mathcal{G} / \partial \phi^i$, etc., and $D^\alpha \equiv g \mathcal{G}_i (T^\alpha)^i_j \phi^j$. In string theory [7], G has a model-dependent generalization of the original no-scale form [5], [6]:

$$G = -\ln(S + S^*) + \hat{G} : \hat{G} = -3 \ln \left(T + T^* - \frac{1}{3} \phi^i \phi_i^* \right) \quad (9)$$

and

$$f_{\alpha\beta} = S \delta_{\alpha\beta} : S_R \equiv \text{Re} S = \frac{1}{g^2} \quad (10)$$

is model-independent. (We use natural units: $\kappa^2 = 8\pi G_N \equiv 1$). The generalized no-scale piece \hat{G} of the Kähler potential (9) fills in the (-3) hole in the potential V (8), so that it is positive semi-definite.

The v.e.v. of the dilaton S , and hence (10) the value of the gauge and string coupling g , is believed to be fixed at zero temperature by gaugino condensation $\langle 0 | \lambda \lambda | 0 \rangle \neq 0$ due to non-perturbative gauge dynamics in one or more hidden sectors [13]. We expect finite-temperature effects in the corresponding strongly-coupled gauge theories to dissolve such condensates when the temperature rises above their strong coupling scales $\Lambda_H \sim \langle 0 | \lambda \lambda | 0 \rangle^{1/3}$, just as happened to the gluon condensate field χ in QCD. What happens to the S field and the string coupling once they are freed from their zero-temperature fixed values?

The answer to this question involves finite-temperature corrections to the $N=1$ supergravity effective potential (8), which arise from loops of matter fields, gauge bosons and gauginos, and gravitinos. These are given at the one-loop level by

$$\Delta V_T = \frac{-\pi^2}{48} N_B T^4 + \frac{1}{24} \text{Tr} \left(m_B^2 + \frac{1}{2} m_F^2 \right) T^2 + \dots \quad (11)$$

where N_B is the number of bosonic degrees of freedom, which is equal to the number of fermionic degrees of freedom, and m_B^2 (m_F^2) is the boson (fermion) mass-squared matrix. The complete expression for ΔV_T has been calculated for general \mathcal{G} and for minimal $f_{\alpha\beta}$ [14]. The resulting expression is simplified by taking the limit of large N , the total number of chiral superfields. The large- N limit selects the contributions to ΔV_T from \mathcal{G}_k^k , leading to [15]

$$\Delta V_T \simeq \frac{1}{12} N T^2 e^{\mathcal{G}} (\mathcal{G}_i \mathcal{G}^i - 2) \quad (12)$$

in minimal supergravity, and [15]

$$\Delta V_T \simeq \frac{1}{18} N T^2 V \quad (13)$$

where V is given by equation (8), in no-scale supergravity.

Here we are interested in the dilaton dependence of the finite-temperature corrections, which for the Kähler potential (9) takes the form

$$\Delta V_T \propto N T^2 / S_R \quad (14)$$

This suggests that at high temperatures $S_R \rightarrow \infty$ and $g^2 \rightarrow 0$, the weak coupling limit. This effect is also seen in a toy example where we set $N=1$, i.e. we only include the S fields (while assuming that the other matter fields, moduli, etc., still play their role of cancelling the (-3) in V (8)). In this case with $G \ni -\ln(S + S^+)$ the scalar potential (8) becomes

$$V(S_R) = \frac{1}{2S_R} \quad (15)$$

and

$$\text{Tr} m_B^2 = 2/S_R, \text{Tr} m_{1/2}^2 = 0, \text{Tr} m_{3/2}^2 = -2/S_R$$

so that

$$\Delta V_T = \frac{1}{24} T^2 / S_R \quad (16)$$

displaying again the $1/S_R$ dependence that pushes string towards weak coupling as the temperature increases towards the string Hagedorn temperature.

To discuss the transition to the high-temperature phase of string, we must also include the new effects due to its extended nature. As in field theory, the free energy of thermal strings can be computed by doing the path integral for propagation on $R^{D-1} \otimes S^1$, where S^1 has circumference $\beta = 1/T$. In closed string theory, this raises the novel possibility [9] that the string “wraps” some number of times around the S^1 time coordinate, analogously to time-like Polyakov loops in finite-temperature QCD [8]. These “vortex” configurations represent sectors topologically distinct from the “unwrapped” string states that dominate the path integral at low temperatures, and lead to intrinsically stringy effects at high temperatures $T = 0(m_{\text{Planck}})$.

The contribution of these sectors to the genus-one partition function for finite-temperature string theory is restricted by modular invariance [16]. World-sheet modular transformations can interchange contributions from “wrapped” strings and strings with quantized time-like momenta. Modular invariance then relates the contributions to the free energy from these sectors. In string mode computations, modular invariance imposes the level matching constraint $L_0 - \bar{L}_0 = 0$. In the case of the bosonic string, the mode expansion for the compactified time coordinate is (where we now use units in which the string tension $\alpha'^{-1} = 2$)

$$X^0(\sigma, \tau) = X^0 + \frac{2\pi m\tau}{\beta} + \frac{n\beta\sigma}{\pi} + \text{oscillators} \quad (17)$$

and the mass-shell condition is

$$\frac{1}{4}M^2 = N_L + \frac{1}{2}\left(\frac{m\pi}{\beta} + \frac{n\beta}{2\pi}\right)^2 - 1 = N_R + \frac{1}{2}\left(\frac{m\pi}{\beta} - \frac{n\beta}{2\pi}\right)^2 - 1 \quad (18)$$

where N_L and N_R are the left and right oscillator mode numbers. Level-matching enforces

$$N_L - N_R = mn \quad (19)$$

The tachyonic ground state of the bosonic string has $N_L = 0 = N_R$ and $n = 0 = m$. There are in addition time-like excitations of the tachyon with $m = \pm 1$, $n = 0$ that are also tachyonic at low temperatures. Dual to these modes, in the sense of modular transformations, are the vortex modes that wind once around S^1 time: $n = \pm 1$, $m = 0$ with $N_L = 0 = N_R$. Their masses-squared have an inverse dependence on the temperature:

$$M^2 = -4 + \frac{\beta^2}{2\pi^2} \quad (20)$$

and so become tachyonic when

$$T = T_H \equiv \frac{1}{2\sqrt{2}\pi} \quad (21)$$

which is the Hagedorn temperature of the bosonic string.

In the case of heterotic string, modular invariance imposes a generalized GSO projection [1], [16] that removes the tachyon from the theory. The mass-shell condition for the physical states analogous to (18) is:

$$\frac{M^2}{8} = N_R - \frac{1}{2} + \frac{1}{2}\left(\frac{\pi m}{\beta} - \frac{n\beta}{2\pi}\right)^2 = N_L - 1 + \frac{1}{2}\left(\frac{\pi m}{\beta} + \frac{n\beta}{2\pi}\right)^2 \quad (22)$$

where we consider right-moving modes in the Neveu-Schwarz sector, for which modular invariance requires us to take half-integer values [1], [16]. The vortex modes whose dynamics determines the Hagedorn temperature for the heterotic string are $N_L = 0 = N_R$ with $n = \pm 1$, $m = \pm \frac{1}{2}$, which we denote hereafter by ϕ^\pm . They are self-dual, with

$$M^2 = -6 + \frac{\pi^2}{\beta^2} + \frac{\beta^2}{\pi^2} \quad (23)$$

and become tachyonic when

$$T = T_H \equiv \frac{1}{\pi} (\sqrt{2} - 1) \quad (24)$$

the Hagedorn temperature of the heterotic string [17].

The Hagedorn phase transition is characterized by condensation of these vortices, which we may represent in the effective field theory language as the development of a non-zero v.e.v. for ϕ^\pm . To discuss whether this phase transition is first- or second-order, and whether the v.e.v.'s of other order parameters change at the same temperature, we must construct an effective Lagrangian which takes other light degrees of freedom into account as we did in the QCD case (1, 2, 4). This remark applies in particular to the scalar component $S_R \equiv e^\sigma$ of the dilaton supermultiplet, which is an inescapable feature of the effective low energy field theory (9), and is not heavy in the limit $T \rightarrow T_H - \epsilon$, where equations (14, 15, 16) tell us that $S_R = e^\sigma \rightarrow \infty^1$

Noting that the couplings of the σ field are completely dictated by the underlying conformal invariance of the theory, it can be shown from general arguments that all the potential terms in the effective Lagrangian description of string at the tree level appear scaled by $1/S_R = e^{-\sigma}$, including mass terms for massive modes. Therefore the finite-temperature effective potential for the vortex modes takes the form

$$V_{eff}^{vortex} = e^{-\sigma} \left[\left(-6 + \frac{\beta^2}{\pi^2} + \frac{\pi^2}{\beta^2} \right) \phi^+ \phi^- + \frac{3}{2} (\phi^+ \phi^-)^2 \right] \quad (25)$$

where the coefficient of the quartic interaction is the result of a simple conformal field theory calculation. This potential exhibits a second-order Kosterlitz-Thouless [18] phase transition due to vortex condensation at the heterotic string Hagedorn temperature T_H (24) where M^2 (23) vanishes. The second-order nature of the phase transition is enforced by the Z_2 symmetry of the order parameter (contrast the Z_3 QCD case (2)), and by the positivity of the quartic term²). Minimization of the effective potential (25) for any fixed value of the dilaton field σ gives a negative contribution to the vacuum energy

$$V_{eff}^{MIN} = e^{-\sigma} \left[-\frac{\left(-6 + \frac{\beta^2}{\pi^2} + \frac{\pi^2}{\beta^2} \right)^2}{6} \right] \quad (26)$$

which depends non-trivially on the dilaton σ .

To get the full dilaton-dependent potential at high temperature, we will add to the above the finite-temperature effective potential terms due to the light degrees of freedom (equations (14, 15, 16)), for which two remarks are in order. First, we note that the form of ΔV_T from equation (16) does not exhibit the $T \leftrightarrow 1/T$ duality that we expect in the full string theory. So, in addition to the term of equation (16) there must be terms whose inclusion results in corrections to ΔV_T which provide the required $T \leftrightarrow 1/T$ invariance. These extra terms must behave as $(N_L/24) \left(\frac{\beta^2}{\pi^4} - \frac{1}{\beta^2} \right)$ at $T \gg T_H$ and may not be negligible even for $T \sim T_H$. However, the argument that we make below holds even if these terms are comparable to the $N_L/24\beta^2$ term that we retain. Secondly we remark that before adding ΔV_T (16) to V_{eff}^{MIN} (26) we must correctly restore their units, as they have been derived from calculations with different conventions. The string

1) On this point we differ from Atick and Witten [1].

2) We also differ from Atick and Witten [1] on this point. See also [19] for an alternative discussion.

winding mode potential has been computed with a choice of units such that the string tension was fixed: $\alpha' = 1/2$, whereas the contributions from the light modes have been calculated in natural physical units where Newton's constant is fixed and $\kappa = 1$. Since $\alpha' = 2\kappa^2 S_R$, we can only hold one of these variables fixed while studying functional dependence on S . If we write the two contributions in string units, using $\kappa^2 = \alpha'/2S_R$, we get for the total effective potential governing the dilaton critical dynamics:

$$V_{eff}(\sigma) \simeq e^{-\sigma} \left[\frac{-\left(-6 + \frac{\beta^2}{2\alpha'\pi^2} + \frac{2\alpha'\pi^2}{\beta^2}\right)^2}{6(2\alpha')^2} \right] + \frac{N_L}{12\beta^2\alpha'} \quad (27)$$

whereas if we convert it to natural units with $\kappa^2 = 1$ fixed (and write $S_R = e^\sigma$ as before) we get:

$$V_{eff}(S_R) \simeq \frac{-1}{24\kappa^4 S_R^3} \left(-6 + \frac{\beta^2}{4\pi^2\kappa^2 S_R} + \frac{4\pi^2\kappa^2 S_R}{\beta^2} \right)^2 + \frac{1}{S_R} \frac{N_L}{24\kappa^2\beta^2} \quad (28)$$

Because of the dilaton-dependent relation $\alpha' = 2\kappa^2 S_R$, and because the Hagedorn temperature (where the vortices become tachyonic) is fixed in terms of the string tension: $T_H = \frac{(\sqrt{2}-1)}{\sqrt{2}\pi} \frac{1}{\sqrt{\alpha'}}$, then in natural units where Newton's constant is fixed ($\kappa = 1$) the Hagedorn temperature is a function of the dilaton v.e.v. S_R .

So, at any finite temperature there will be a region of (large) S_R space where the Hagedorn temperature is less than our physical temperature (measured in gravitational units with $\kappa = 1$); in this region at large S_R , the full expression (28) for the effective potential of S_R must be used, as in this region ($T > T_H$) the vortex mode carries its v.e.v.. For smaller values of S_R , where $T < T_H$, the vortex mode no longer has a v.e.v., and we only retain from (28) the contribution due to the light modes (14, 15, 16). In the figure we plot $V_{eff}(S_R)$ for different temperatures (related to the Hagedorn temperature at $S_R = .25$, where $\alpha' = 2\kappa^2 S_R$ implies $\alpha' = \frac{1}{2}$ and $\kappa = 1$ so both systems of units agree). We see that while the minimum occurs at large S_R at low temperature ($S_R \rightarrow \infty$ as $T \rightarrow 0$ unless it is fixed by some nonperturbative potential), as we raise the temperature through the Hagedorn temperature (at the comparison value $S_R = .25$) the minimum shifts rapidly to small values of S_R where the string is strongly coupled. This rapid, but continuous, transition to a strongly-coupled regime at $T \gg T_H$ eventually leads us to a domain in which our methods of analysis are no longer reliable.

We conclude with a few comments on the high-temperature string phase, inspired in part by our knowledge of the high-temperature phase of QCD. In that case, there are solitons of the low-energy field theory that can be identified with baryons - the Skyrmions [20]. They carry a topological quantum number that can be identified with baryon number, and get larger and lighter as the QCD phase transition temperature is approached from below. Baryon number is a precursor of the appearance of free quarks in the high-temperature phase, and the increase in the Skyrmion size can be interpreted as a precursor of deconfinement in the high-temperature phase. String theory also has solitons, the five-branes of [11], which are in one sense dual to strings. The five-brane's mass per unit volume \hat{M}_5 decreases as the string coupling increases: $\hat{M}_5 \propto 1/\sqrt{g}$, so that the five-brane description becomes more relevant as the temperature approaches T_H from below, just like Skyrmions in QCD. As in the case of the baryon number carried by Skyrmions, the (from the point of view of the low-energy string theory) "topological"

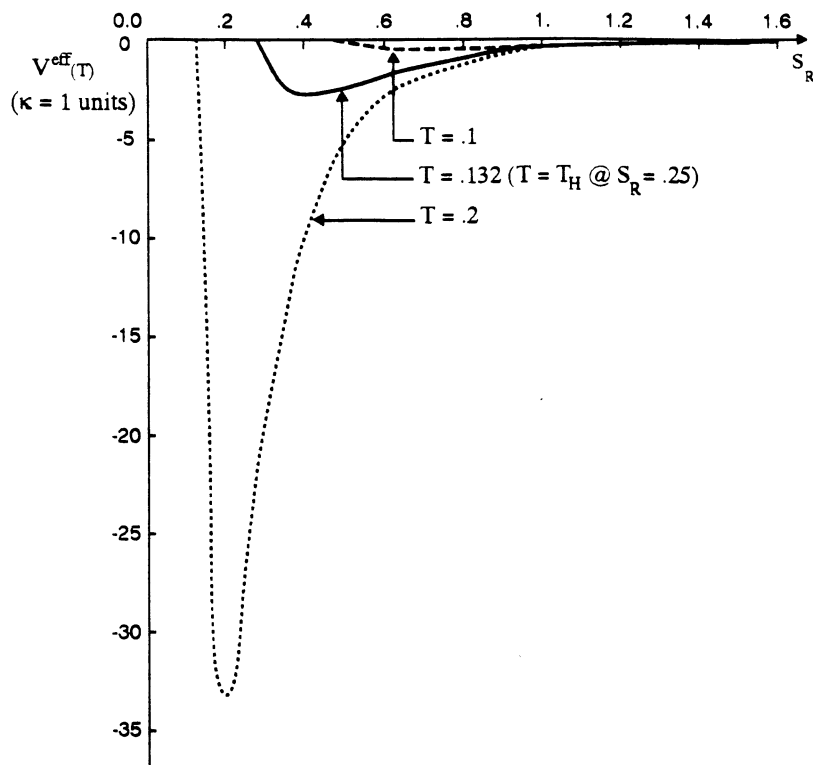


Figure 1: The S_R effective potential at different temperatures, in $\kappa = 1$ Planck units, when the number of light degrees of freedom is taken as $N = 100$. Curves shown are for the range of S for which the temperatures plotted are beyond the Hagedorn temperature.

charge carried by the five-branes may become deconfined above T_H and play a key rôle in characterizing the high-temperature phase, analogous to the presence and importance of quarks in QCD. This uncanny resemblance between the phase structure of QCD and string, coupled with an understanding of string /five-brane dynamics, and in particular the role played by duality and its breaking, will surely be useful in future explorations of the high-temperature phase of string theory.

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