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HIGGS EFFECTS ON THE RELIC SUPERSYMMETRIC PARTICLE DENSITY

John Ellis

CERN - Geneva

Leszek Roszkowski

Center for Theoretical Physics, Department of Physics,
Texas A&M University, College Station, TX 77843-4242, USA
and
Astroparticle Physics Group, Houston Advanced Research Center (HARC)
The Woodlands, TX 77381, USA

and

Zygmunt Lalak

Physics Department, Warsaw University, Warsaw, Poland

ABSTRACT

We explore in the minimal supersymmetric extension of the Standard Model the effects of Higgs exchange on the cosmological relic density $\tilde{\rho}$ of the lightest supersymmetric particle (LSP), taking into account the constraints on the supersymmetric Higgs masses imposed by LEP. We find in general a significant enlargement of the region of parameter space in which $\tilde{\rho} < \tilde{\rho}_{70} = 10^{-29}\,\mathrm{g\,cm^{-3}}$, the closure density if the Hubble constant is $70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. This effect is potentially important if the ratio of Higgs v.e.v.'s $\tan\beta \equiv v_t/v_b \sim 2$ to 4 as in many realistic models. The restricted region of parameter space in which closure density is obtained for a Hubble constant between 50 and $70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is particularly sensitive to the Higgs contribution.

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1. INTRODUCTION

There is increasing astrophysical evidence that most of the matter in the Universe is invisible[1] and there are arguments that most of this dark matter is not baryonic.[2] Many candidates for this dark matter have been proposed, one of the most plausible of which is the lightest supersymmetric particle (LSP) denoted by χ. There are good theoretical reasons[3] based on the hierarchy problem and naturalness to think that supersymmetry plays a role in particle physics. Most supersymmetric theories conserve R-parity,[4] which ensures that the LSP is absolutely stable. The relic cosmological density of LSPs can be calculated given a spectrum of supersymmetric particles, and is often found to be of the same order of magnitude as that required to close the Universe.[5,6,7] The LSP χ differs in this latter respect from other well-motivated dark matter candidates such as axions, for which there is no good theoretical reason to expect a density within an order of magnitude of the closure density.

Calculating the relic LSP abundance requires knowledge of the low-energy LSP pair-annihilation cross-section.[6,8] This has normally been calculated using sfermion and Z exchange diagrams giving $\bar{f}f$ final states, and these are usually the most important. However, under some circumstances Higgs (H) exchange diagrams giving $\bar{f}f$ final states could be important, or annihilation to pairs of Higgs bosons. These mechanisms were originally considered in Ref. 9, where they were said to be of marginal significance, but detailed results were not presented.

It is easy to give an approximate estimate of the relative magnitudes of the annihilation rates due to $\tilde{f}$ and H exchange, and therefore guess when the later could be important. We recall that the relic particle density may be approximated by[6]

$$\tilde{\rho} \simeq 4.5 \times 10^{-40} \left(\frac{T_\chi}{T_\gamma}\right)^3 \left(\frac{T_\gamma}{2.7^\circ\,\mathrm{K}}\right)^3 N_F^{1/2} \left(\frac{\mathrm{GeV}^{-2}}{\tilde{a}x_f + \frac{1}{2}\tilde{b}x_f^2}\right) \mathrm{g/cm^3} \qquad (1)$$

where $T_\gamma \simeq 2.7^\circ$ K is the relic photon temperature, $(T_\chi/T_\gamma)^3$ is a reheating factor tabulated together with the effective number of particle degrees of freedom N_f

at the freeze-out temperature T_f in Ref. 10, $x_f = O(1/20)$ is a scaled freeze-out temperature $x_f \equiv T_f/m_\chi$, and $\tilde{a}, \tilde{b}$ parametrize the averaged annihilation rate at low temperatures:

$$\langle \sigma v_{\rm rel} \rangle \simeq \tilde{a} + \tilde{b}x\,. \qquad (2)$$

Comparing the values of $\tilde{a}$ for Higgs and sfermion exchanges calculated later, we find

$$\left.\frac{\langle \sigma_H\, v_{\rm rel}\rangle}{\langle \sigma_{\tilde{f}}\, v_{\rm rel}\rangle}\right|_{x=0} \simeq \frac{\left(m_{\tilde{f}}^2 + m_\chi^2\right)^2}{\left(4m_\chi^2 - m_H^2\right)^2} \left(\frac{m_f}{m_W}\right)^2 \frac{\theta_H^2}{\theta_{\tilde{f}}^2} \qquad (3)$$

where θ_H and $\theta_{\tilde{f}}$ are some generic mixing angle factors. We therefore see that the potential relevance of the Higgs exchange diagram is greatly increased for large $m_{\tilde{f}}$, and for m_χ close to $m_H/2$. Experiments during the last few years have pushed the lower limits on $m_{\tilde{f}}$ up significantly,[11] and suggest indirectly an increased lower limit on m_χ. (For example, one expects[6] that $m_{\tilde{\gamma}}/m_{\tilde{g}} \simeq 8\alpha_{\rm em}/3\alpha_s$, so an increased lower bound on $m_{\tilde{g}}$ suggests an increased lower bound on m_χ, if it is approximately a photino.) The ratio (3) permits the Higgs exchange contribution to be as important as the $\tilde{f}$ exchange contribution, even taking into account the recent LEP limits on supersymmetric Higgs boson masses,[12] which could therefore still allow substantial Higgs effects on the LSP density.

Indeed, we find that including Higgs exchange has a significant effect on the range of LSP parameter space that is allowed by cosmology or favored by astrophysics, even taking into account the LEP constraints[12] on Higgs masses. We take the view that present data on the expansion of the Universe allow at most the closure density for relic LSPs,

$$\tilde{\rho} < \tilde{\rho}_{h_0} \equiv 2 \times 10^{-29}\mathrm{g\,cm^{-3}} \left(\frac{h_0}{100\,\mathrm{km\,s^{-1}Mpc^{-1}}}\right)^2 \qquad (4)$$

where h_0 is the present Hubble expansion rate normally thought to be in the range $50\,\mathrm{km\,s^{-1}Mpc^{-1}} \lesssim h_0 \lesssim 70\,\mathrm{km\,s^{-1}Mpc^{-1}}$.[1] There is increasing evidence that the mass density in the universe approaches the closure density when averaged over a sufficiently large scale, and certainly astrophysicists appear to favor a dark matter density of a least 1/4 of the closure density.[1] Accordingly, we favor the range

$$5 \times 10^{-30} \mathrm{g\,cm}^{-3} < \tilde{\rho} < 10^{-29} \mathrm{g\,cm}^{-3} \tag{5}$$

$$\tilde{\rho}_{50} < \tilde{\rho} < \tilde{\rho}_{70} \qquad \text{or} \qquad \frac{1}{2}\tilde{\rho}_{70} < \tilde{\rho} < \tilde{\rho}_{70} \tag{6}$$

The inclusion of Higgs exchange clearly expands the allowed range. We find that the effect is significant, though not in general substantial. Most significant, perhaps, is the fact that the favored values of m_χ in which the conditions (6) are both satisfied may be shifted so that some previously-favored domains are now disfavored, and .[13] This is potentially important for attempts[14] to constrain supersymmetric model parameters by using the astrophysical constraints (6), though their exploration is beyond the scope of this paper.

2. BASIC FORMULAE

The cosmological relic LSP density was already given in Eq. (1), where (T_χ/T_γ) and N_f are determined by the freeze-out temperature $T_f = x_f m_\chi$, and are tabulated in Ref. 10. For our calculations, we will determine x_f by an iterative procedure described in Ref. 6 and identify the quark-hadron phase transition temperature T_H and the effective strange quark mass: $T_H = m_s = 250$ MeV. We first review the particle physics previously included in the calculation of the $\chi\chi \to f\bar{f}$ annihilation cross-section coefficients $\tilde{a}$, $\tilde{b}$ of Eq. (2).

We assume that the LSP is a mixture of spin-$\frac{1}{2}$ supersymmetric fermions determined by the mass mixing matrix[6]

$$\begin{pmatrix} M_2 & 0 & -m_Z\cos\theta_W\sin\beta & m_Z\cos\theta_W\cos\beta \\ 0 & \frac{5}{3}M_2\tan^2\theta_W & m_Z\sin\theta_W\sin\beta & -m_Z\sin\theta_W\cos\beta \\ -m_Z\cos\theta_W\sin\beta & m_Z\sin\theta_W\sin\beta & 0 & -\mu \\ m_Z\cos\theta_W\cos\beta & -m_Z\sin\theta_W\cos\beta & -\mu & 0 \end{pmatrix} \tag{7}$$

where we have assumed the normal GUT relation between the $SU(2)$ and $U(1)$ gaugino masses. The quantities M_2 and μ are unknown gauge mass and Higgs mixing parameters, and we will explore the allowed and favored ranges of values for them. The ratio $\tan\beta = v_t/v_b$, where the Higgs v.e.v.'s v_t and v_b give masses to the charge $+\frac{2}{3}$ and $-\frac{1}{3}$ quarks, respectively, is also unknown, and we will consider a discrete set of representative values for $\tan\beta$. The mixing matrix (7) determines a lightest mass eigenstate, the LSP, which we parametrize by

$$\chi = \alpha\tilde{W}^3 + \beta\tilde{B}^0 + \gamma\tilde{H}_t^0 + \delta\tilde{H}_b^0 \tag{8}$$

in the following. We note that by the lightest mass eigenstate we mean the state whose eigenvalue has the smallest absolute value. There are well-known procedures[15,16] for dealing with negative LSP masses.

Let us first consider $\chi\chi$ annihilation to $\bar{f}f$ via Z-exchange. It is convenient to introduce effective Lagrange couplings[6]

$$A_Z = \frac{g_2^2(\gamma^2-\delta^2)}{4\cos^2\theta_W}\frac{(T^3_{f_L} - e_f\sin^2\theta_W)}{m_Z^2}, \tag{9}$$

and

$$B_Z = \frac{g_2^2(\gamma^2-\delta^2)}{4\cos^2\theta_W}\frac{(-e_f\sin^2\theta_W)}{m_Z^2}, \tag{10}$$

where g_2 is the $SU(2)$ gauge coupling constant and $e_f = T_f^3 + Y_f/2$ is the electric charge of a given fermion final state. In terms of (9), (10) one can write a Lagrangian for $\chi\chi$ annihilation into $\bar{f}f$ via Z-exchange as

$$\mathcal{L}_Z = \sum_f [\bar{\chi}\gamma^\mu\gamma_5\chi]\frac{(g_{\mu\nu} - q_\mu q_\nu/m_Z^2)\,m_Z^2}{s - m_Z^2 + i\Gamma_Z m_Z}\,[\bar{f}\gamma^\nu(A_Z P_L + B_Z P_R)f]\,. \tag{11}$$

In equation (11) we denote by q the sum of the initial or final momenta: $s = q^2$. Notice that we have not neglected momentum-dependent terms in the propagator.

In fact, the magnitude of the term $q_\mu q_\nu/m_Z^2$ is of order s/m_Z^2 which is not negligible for m_χ comparable with m_Z. Using the prescription for thermal averaging introduced in Ref. 8, one then obtains

$$\tilde{a}_Z = \sum_f \frac{c_f}{2\pi}\sqrt{1-\frac{m_f^2}{m_\chi^2}}\frac{m_Z^4}{(m_Z^2-4m_\chi^2)^2+\Gamma_Z^2 m_Z^2}\, m_f^2(A_Z-B_Z)^2\left[1-4\left(\frac{m_\chi}{m_Z}\right)^2\right]^2, \tag{12}$$

and

$$\begin{aligned}\tilde{b}_Z = \sum_f \frac{c_f}{2\pi}\sqrt{1-\frac{m_f^2}{m_\chi^2}}\frac{m_Z^4}{(m_Z^2-4m_\chi^2)^2+\Gamma_Z^2 m_Z^2}\Bigg\{&4m_\chi^2(A_Z^2+B_Z^2)\\ &-4m_f^2\left[A_Z^2-3A_ZB_Z+B_Z^2-3\left(\frac{m_\chi}{m_Z}\right)^2(A_Z-B_Z)^2\right]\\ &+\left(\frac{3}{4}\beta_f+12\gamma_f\right)\, m_f^2\,(A_Z-B_Z)^2\left[1-4\left(\frac{m_\chi}{m_Z}\right)^2\right]^2\Bigg\},\end{aligned} \tag{13}$$

where

$$\beta_f \equiv \frac{m_f^2}{m_\chi^2-m_f^2} \tag{14}$$

and

$$\gamma_f \equiv \frac{m_\chi^2(m_Z^2-4m_\chi^2)}{[(m_Z^2-4m_\chi^2)^2+\Gamma_Z^2 m_Z^2]}. \tag{15}$$

The color factor c_f is equal to 3 when $f = q$, and 1 for $f = l$. In equations (12), (13), terms proportional to m_χ/m_Z arise from keeping $q_\mu q_\nu/m_Z^2$ in the Z-propagator and, as mentioned above, in general their contribution is significant. The term proportional to γ_f is due to keeping the s-dependence in the denominator and is not significant for m_χ much smaller than m_Z.

For the sfermion annihilation channel, it is sufficient to consider only the effective Lagrangian given in Refs. 6 and 17, namely

$$\mathcal{L}_{\tilde{f}}^{eff} = \sum_f (\bar{\chi}\gamma_\mu\gamma_5\chi)\left[\bar{f}\gamma^\mu(A_{\tilde{f}}P_L+B_{\tilde{f}}P_R)f\right] + C_{\tilde{f}}\left[(\bar{\chi}\chi)(\bar{f}f)+(\bar{\chi}\gamma_5\chi)(\bar{f}\gamma_5 f)\right], \tag{16}$$

where the effective Lagrange couplings are[6]

$$A_{\tilde{f}} = \frac{(T_{f_L}^3\alpha g_2+\frac{1}{2}Y_{f_L}\beta g_1)^2}{2(m_{\tilde{f}_L}^2+m_\chi^2)} + \frac{(g_2 m_f d_f/m_W)^2}{8(m_{\tilde{f}_R}^2+m_\chi^2)}, \tag{17}$$

and

$$B_{\tilde{f}} = -\frac{(\frac{1}{2}Y_{f_R}\beta g_1)^2}{2(m_{\tilde{f}_R}^2+m_\chi^2)} - \frac{(g_2 m_f d_f/m_W)^2}{8(m_{\tilde{f}_L}^2+m_\chi^2)}, \tag{18}$$

where g_1 is the $U(1)$ gauge coupling constant and

$$d_f = \begin{cases}\gamma/\sin\beta & \text{(u,s,t)}\\ \delta/\cos\beta & (\text{e},\mu,\tau,\text{d,s,b})\end{cases} \tag{19}$$

Following Griest,[17] the relative sign between the first and the second part in $A_{\tilde{f}}, B_{\tilde{f}}$ has been corrected, and relative to that in Ref. 6 the part of the Lagrangian proportional to $C_{\tilde{f}}$ has been taken into account, where the Lagrange coupling $C_{\tilde{f}}$ reads

$$C_{\tilde{f}} = \epsilon\frac{g_2 m_f d_f}{4m_W}\left[\frac{T_{f_L}^3\alpha g_2+\frac{1}{2}Y_{f_L}\beta g_1}{m_{\tilde{f}_L}^2+m_\chi^2} - \frac{\frac{1}{2}Y_{f_R}\beta g_1}{m_{\tilde{f}_R}^2+m_\chi^2}\right] \tag{20}$$

and ϵ is the sign of the LSP mass m_χ resulting from diagonalizing the mass mixing matrix (7). Notice that we have approximated t and u in the sfermion propagators by $-m_\chi^2$, since the values of $m_{\tilde{f}}$ that we consider are always much larger than the LSP mass which in turn is typically significantly larger than the masses of the final state fermions. In terms of $A_{\tilde{f}}$, $B_{\tilde{f}}$ and $C_{\tilde{f}}$, and using the method of thermal averaging derived in Ref. 8, we obtain

$$\tilde{a}_{\tilde{f}} = \sum_f \frac{c_f}{2\pi}\sqrt{1-\frac{m_f^2}{m_\chi^2}}\left[m_f(A_{\tilde{f}}-B_{\tilde{f}})+2m_\chi C_{\tilde{f}}\right]^2 , \tag{21}$$

and

$$\tilde{b}_{\tilde{f}} = \sum_f \frac{c_f}{2\pi}\sqrt{1-\frac{m_f^2}{m_\chi^2}}\Bigg[4m_\chi^2\left(A_{\tilde{f}}^2+B_{\tilde{f}}^2+\frac{3}{2}C_{\tilde{f}}^2\right) -4m_f^2\left(A_{\tilde{f}}^2-3A_{\tilde{f}}B_{\tilde{f}}+B_{\tilde{f}}^2+\frac{3}{2}C_{\tilde{f}}^2\right) -6(A_{\tilde{f}}-B_{\tilde{f}})C_{\tilde{f}}m_\chi m_f+\frac{3}{4}\beta_f\left[m_f(A_{\tilde{f}}-B_{\tilde{f}})+2m_\chi^2 C_{\tilde{f}}\right]^2\Bigg]. \tag{22}$$

The values of $\tilde{\rho}$ resulting from formulae (12), (13) as well as (21) and (22) will later be discussed and compared with the Higgs exchange contribution.

3. HIGGS EXCHANGE

The general Lagrangian for the three neutral Higgs exchange contributions to $\chi\chi \to h, H, A \to \bar{f}f$ annihilation can be written in the form

$$\mathcal{L}_{Higgs} = \mathcal{L}_h + \mathcal{L}_H + \mathcal{L}_A = \sum_f \left[(\bar{\chi}\chi)\frac{D_h}{s-m_h^2}(\bar{f}f)+(\bar{\chi}\chi)\frac{D_H}{s-m_H^2}(\bar{f}f)+(\bar{\chi}\gamma_5\chi)\frac{D_A}{s-m_A^2}(\bar{f}\gamma_5 f)\right]. \tag{23}$$

We note that because one of the Higgs bosons with scalar couplings to matter fermions must weigh less than m_Z in the minimal supersymmetric extension of the Standard Model,[15,16] and since the pseudoscalar boson may also be quite light, we have not neglected the LSP mass in writing (23). We will consider the scalar, h and H, and pseudoscalar, A, contributions to $\chi\chi \to \bar{f}f$ annihilation, but will not consider $\chi\chi \to$ Higgs pair annihilation processes, nor $\chi\chi \to Zh$, even though these could also contribute to $(\tilde{a}, \tilde{b})$ even in the region of LSP masses below m_W

to which we confine ourselves here. Using the same prescription as before, we find the annihilation cross-section coefficients

$$\tilde{a}_{Higgs} = \sum_f \frac{2c_f}{\pi}\sqrt{1-\frac{m_f^2}{m_\chi^2}}\frac{m_\chi^2 D_A^2}{(4m_\chi^2-m_A^2)^2} \tag{24}$$

and

$$\tilde{b}_{Higgs} = \sum_f \frac{2c_f}{\pi}\sqrt{1-\frac{m_f^2}{m_\chi^2}}\Bigg[\frac{3}{2}\left(\frac{D_h}{4m_\chi^2-m_h^2}+\frac{D_H}{4m_\chi^2-m_H^2}\right)^2(m_\chi^2-m_f^2) + \left(\frac{3}{4}\beta_f - 12\frac{m_\chi^2}{4m_\chi^2-m_A^2}\right)\frac{m_\chi^2 D_A^2}{(4m_\chi^2-m_A^2)^2}\Bigg]. \tag{25}$$

In principle, one should consider the effects of interference[17] between the amplitudes given by the $\tilde{f}$, the Z, and the Higgs exchanges. However, because of the different helicity structures such effects are important only in a limited range of m_χ just above m_f (e.g., $m_b \simeq 5$ GeV) which is not of great significance, and where final-state strong interaction effects modify the free-quark formulae in any case.

To use the formulae (24), (25), we must specify the (h, H, A) couplings to $\chi\chi$ and $\bar{f}f$. From Ref. 16 we derive effective Lagrange couplings

$$D_h = \frac{g_2^2 m_f}{4m_W}(\alpha-\beta\tan\theta_W)(\gamma\cos\bar{\alpha}+\delta\sin\bar{\alpha})\begin{cases}-\frac{\cos\bar{\alpha}}{\sin\beta} & \text{charge } \frac{2}{3}\\ \frac{\sin\bar{\alpha}}{\cos\beta} & \text{charge } -\frac{1}{3}\end{cases} , \tag{26}$$

$$D_H = \frac{g_2^2 m_f}{4m_W}(\alpha-\beta\tan\theta_W)(\gamma\sin\bar{\alpha}-\delta\cos\bar{\alpha})\begin{cases}\frac{\sin\bar{\alpha}}{\sin\beta} & \text{charge } \frac{2}{3}\\ -\frac{\cos\bar{\alpha}}{\cos\beta} & \text{charge } -\frac{1}{3}\end{cases} , \tag{27}$$

and

$$D_A = \frac{g_2^2 m_f}{4m_W}(\alpha-\beta\tan\theta_W)(\gamma\cos\beta-\delta\sin\beta)\begin{cases}\cot\beta & \text{charge } \frac{2}{3}\\ \tan\beta & \text{charge } -\frac{1}{3}\end{cases} \tag{28}$$

where $\bar{\alpha}$ is the mixing angle between h and H to be specified shortly, and the (h, H, A) couplings to charged leptons have forms identical to those for charge $-1/3$ quarks.

To fix the normalization of the effective Higgs exchange contribution (24), (25) we must also fix $m_{h,H,A}$. Once one of them is specified, the other ones are also fixed for any given value of $\tan\beta$:[15]

$$m^2_{h,H} = \frac{1}{2}\left[m^2_Z + m^2_A \mp \sqrt{(m^2_Z + m^2_A)^2 - 4m^2_Z m^2_A \cos^2 2\beta}\right]. \tag{29}$$

Moreover, the choice of m_A fixes the value of the mixing angle $\bar{\alpha}$ introduced earlier

$$\begin{aligned} \cos 2\bar{\alpha} &= -\cos 2\beta \left(\frac{m^2_A - m^2_Z}{m^2_H - m^2_h}\right), \\ \sin 2\bar{\alpha} &= -\sin 2\beta \left(\frac{m^2_H + m^2_h}{m^2_H - m^2_h}\right). \end{aligned} \tag{30}$$

Our procedure will actually be to fix m_A and $\tan\beta$, determine $m_{h,H}$ using (29) and $\bar{\alpha}$ using (30) and then use these dependent values in the above formulae for the Higgs contributions to $\chi\chi \to \bar{f}f$.

4. DISCUSSION OF RESULTS

Similarly as in Ref. 6, we will present our results as density contours in the (μ, M_2) plane for certain discrete choices of $\tan\beta$. We remind the reader that there are two essentially different possible sets of relative signs for these parameters, so we keep $M_2 > 0$ and present two sets of results, for $\mu > 0$ and for $\mu < 0$. As previously,[6] we choose to present results for $\tan\beta = 2$ and 4. In interpreting the density results, it will be convenient to have at hand the LSP mass contours in the (μ, M_2) plane which are given for completeness in Fig. 1 for the same choices of $\tan\beta$. We remind the reader that the almost vertical portions of the contours correspond to almost pure higgsino states whereas the almost horizontal portions along the bottom correspond to almost pure gaugino (bino) states. In the subsequent figures presenting relic density contours we have discarded regions where $m_{\tilde{W}} < 45$ GeV,[18] which can also be supplemented by other LEP constraints[19] on the gaugino/higgsino sector parameters.

It is clear that the relic LSP density depends sensitively on the matter sfermion masses $m_{\tilde{f}}$. The experimental lower limits on the $m_{\tilde{f}}$ have improved significantly[11] since the publication of Ref. 6, and here we consider $m_{\tilde{f}} = 200$ GeV. The relative significance of Higgs exchange would be larger (smaller) for larger (smaller) values of $m_{\tilde{f}}$, particularly in the regions: $|\mu| > M_2$ where the LSP contains larger gaugino components. We consider two choices for the Higgs boson masses, specified by $m_A = 40$ GeV in Fig. 2, and $m_A = 90$ GeV in Fig. 3. For each choice of m_A and value of $\tan\beta$, m_h and m_H are determined using Eq. (29) and noted on the figures.

We certainly expect the relative importance of the Higgs exchange diagrams to be greater for large $m_{\tilde{f}}$ and small m_A, and this is borne out in the figures. As a general rule, the inclusion of Higgs exchange expands the allowed domain in which $\tilde{\rho} < \tilde{\rho}_{70}$, and can shift significantly the preferred domain in which $\tilde{\rho}_{50} < \tilde{\rho} < \tilde{\rho}_{70}$.[13]

We use the following notation for the regions of parameter space in Figs. 2 and 3, to characterize our results according to the values of the relic density given by $\tilde{f}$ and Z exchange alone ($\tilde{\rho}_{Z,\tilde{f}}$) and including (h,H,A) exchanges ($\tilde{\rho}_{sum}$):

$$\begin{aligned} A &: \tilde{\rho}_{Z,\tilde{f}},\ \tilde{\rho}_{sum} < \tilde{\rho}_{50} \\ B &: \tilde{\rho}_{sum} < \tilde{\rho}_{50} < \tilde{\rho}_{Z,\tilde{f}} < \tilde{\rho}_{70} \\ C &: \tilde{\rho}_{Z,\tilde{f}} > \tilde{\rho}_{70},\ \tilde{\rho}_{sum} < \tilde{\rho}_{50} \\ D &: \tilde{\rho}_{50} < \tilde{\rho}_{Z,\tilde{f}},\ \tilde{\rho}_{sum} < \tilde{\rho}_{70} \\ E &: \tilde{\rho}_{50} < \tilde{\rho}_{sum} < \tilde{\rho}_{70} < \tilde{\rho}_{Z,\tilde{f}} \\ F &: \tilde{\rho}_{Z,\tilde{f}},\ \tilde{\rho}_{sum} > \tilde{\rho}_{70} \end{aligned} \tag{31}$$

We also denote by L regions excluded by LEP,[18,19] and by H regions where $m_\chi > m_W = 80\ GeV$, which we do not study in this paper (for heavier relic masses, see Ref. 20). Recent results[11] indicate that $m_t > 89\ GeV$, and so we do not consider relic annihilations into $\bar{t}t$ final states.

Regions which were disallowed before the inclusion of the (h, H, A) exchanges because $\tilde{\rho}_{Z,\tilde{f}} > \tilde{\rho}_{70}$ and allowed after inclusion because $\tilde{\rho}_{sum} < \tilde{\rho}_{70}$ are those denoted by C and E. Regions which were formerly favored ($\tilde{\rho}_{50} < \tilde{\rho}_Z < \tilde{\rho}_{70}$) are denoted by B and D. Regions favored when (h, H, A) exchanges are included are denoted by D and E. Hence the letters denoting shifts in the favored region are B and E.

$\underline{m_A = 40\ GeV}$

We see for $\tan\beta = 2$ (Fig. 2a) when $\mu > 0$ there is a large central region A where the relic density is below $\tilde{\rho}_{50}$, whether the Higgs exchanges are included or not, and a region F at a large negative μ where the relic density is always above $\tilde{\rho}_{70}$. At the boundary between these regions, and also at moderate values of M_2, there is a smaller region D where the relic density is always in the preferred range between $\tilde{\rho}_{50}$ and $\tilde{\rho}_{70}$. There is also a smaller over-dense region F at large M_2, bounded by another preferred region D. The main effect of introducing Higgs exchanges is to provide a new strip of allowed parameter values C, E where $\chi\chi \to$ Higgs $\to \bar{f}f$ is almost on the Higgs mass-shell, but there are also marginal changes in the boundaries of the A and F regions. When $\tan\beta = 2$ and $\mu > 0$ there are larger regions E where the relic density is reduced into the preferred range, and regions B, C where it is reduced below $\tilde{\rho}_{50}$. In this case only a relatively small region D is stable in the preferred range between $\tilde{\rho}_{50}$ and $\tilde{\rho}_{70}$. Turning now to $\tan\beta = 4$ (Fig. 2b), we see that when $\mu < 0$ there are extensive regions C and E where Higgs exchange reduce the relic density into the allowed and preferred ranges. Again the stable part D of the preferred region is relatively small. Finally, the case of $\tan\beta = 4$, $\mu > 0$ is qualitatively similar to that of $\tan\beta = 2$, $\mu > 0$.

$\underline{m_A = 90\ GeV}$

In the case $\tan\beta = 2$ (Fig. 3a), when $\mu < 0$ the main difference from the $m_A = 40\ GeV$ choice (Fig. 2a) is the disappearance of the previous separate strip C, E of low density due to annihilation through an almost on-shell Higgs. However, there is now a substantial increase in the region B where the exchanges of Higges with $m \sim m_Z$ reduce the relic density below the relic density below the preferred range between $\tilde{\rho}_{50}$ and $\tilde{\rho}_{70}$. The case $\tan\beta = 2$ and $\mu > 0$ is qualitatively

similar to the $m_A = 40\ GeV$ choice (Fig. 2a), with the exception that the stable preferred range D is considerably smaller, and there is a larger region B where Higgs exchanges reduce the relic density below $\tilde{\rho}_{50}$. Turning to the case $\tan\beta = 4$ (Fig. 3b), we see that, when $\mu < 0$, by comparison with the case of $m_A = 40\ GeV$ (Fig. 2b), a large part of the region C at small M_2 previously allowed has now reverted to being disallowed (F) because of the absence of annihilation through light Higgs bosons. There is, however, a large region B at larger M_2 where the relic density is pushed below $\tilde{\rho}_{50}$ by annihilation through Higgs channels. The region D in which the relic density remains stable in the preferred range is very small. Even more strikingly, in the case $\tan\beta = 4$ and $\mu > 0$ there is no region D. There are, however, large regions C where Higgs exchange reduces the relic density into the preferred range, and B where the relic density is reduced below $\tilde{\rho}_{50}$.

Although we have not studied in this paper the regions H where $m_\chi > m_W$, we expect that annihilation into W^+W^- and Z^0Z^0 will usually suppress the relic density below $\tilde{\rho}_{50}$. This expectation is consistent with the results of Ref. 20.

5. CONCLUSIONS

The general estimate (5) indicates that the Higgs exchange contribution is not necessarily negligible when calculating the relic LSP density. Indeed, we have found that it can reduce the relic density substantially, particularly in the case of the relatively large sfermion masses $m_{\tilde{f}} \gtrsim 120$ GeV now required by experiment,[11] even taking into account the LEP constraints on supersymmetric Higgs boson masses.[12] The preferred values of LSP masses that give relic densities in the range $\tilde{\rho}_{50} < \tilde{\rho} < \tilde{\rho}_{70}$ of interest to cosmology and astrophysics[1] are often altered significantly. We note that the range of parameters which give $\tilde{\rho}$ in the preferred range is also substantially shifted and reduced.[13] These results confirm that Higgs exchange cannot be neglected when calculating LSP relic densities. It was found in a recent study[14] that only a restricted range of supersymmetric model parameter space was compatible with all possible theoretical, experimental, astrophysical and cosmological constraints. The preferred domain of parameter space may now be shifted significantly, and it is planned to study this point in a future publication.[21]

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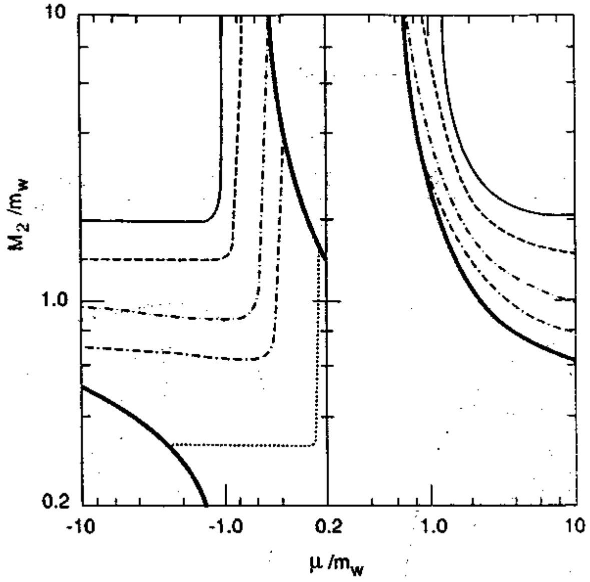
FIGURE CAPTIONS

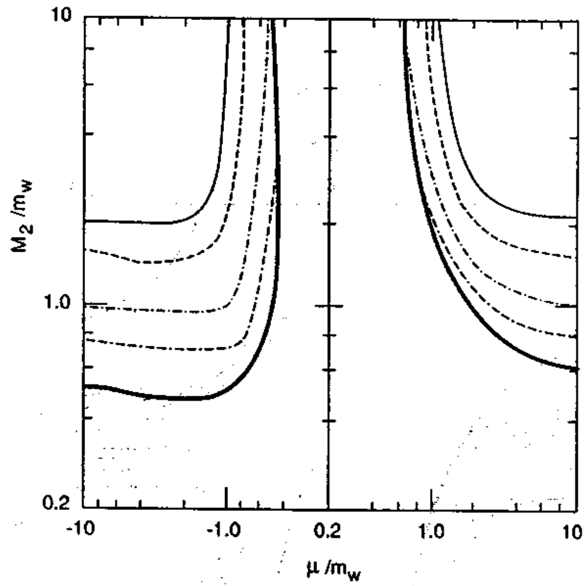
1) Contours[6] of the LSP mass (solid: 80 GeV, dashed: 60 GeV, dash-dotted: 40 GeV, double-dash-dotted: 30 GeV, dotted: 15 GeV) in the (μ, M_2) plane for different choices of $\tan\beta \equiv v_t/v_b$ and the sign of μ: (a) $\tan\beta = 2$, (b) $\tan\beta = 4$. Also shown as thick lines are the bounds $m_{\tilde{W}^\pm} = 45\ GeV$ established by LEP.[18]

2) Cosmologically excluded $(\tilde{\rho} > \tilde{\rho}_{70})$, favored $(\tilde{\rho}_{50} < \tilde{\rho} < \tilde{\rho}_{70})$ and other allowed $(\tilde{\rho} < \tilde{\rho}_{50})$ domains of the (μ, M_2) plane for the same choices of $\tan\beta$ and the sign of μ as in Fig. 1. The domains excluded (favored) (allowed) if just Z and $\tilde{f}$ exchange are taken into account or (h, H, A) exchanges are included are indicated by the letters between A and F as explained in the text. The region marked by L is excluded by the LEP limit[18] on $m_{\tilde{W}}$ and the region H corresponds to $m_\chi > m_W$, which we do not consider in this paper. In this figure, $m_{\tilde{f}} = 200$ GeV and $m_A = 40$ GeV are chosen.

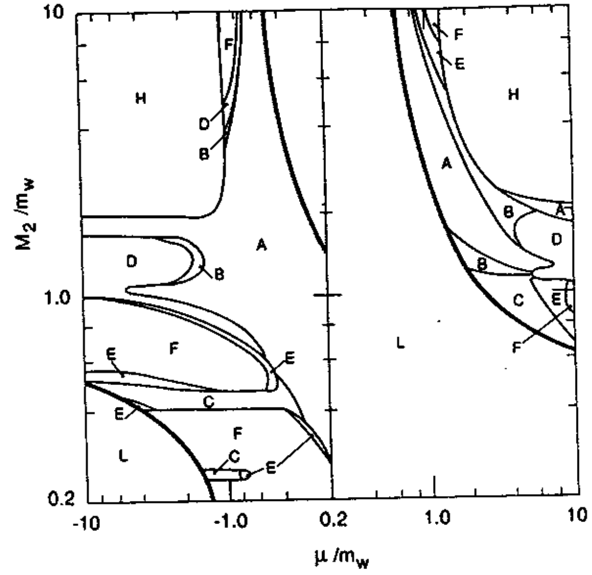
3) As for Fig. 2, but with $m_{\tilde{f}} = 200$ GeV and $m_A = 90$ GeV.



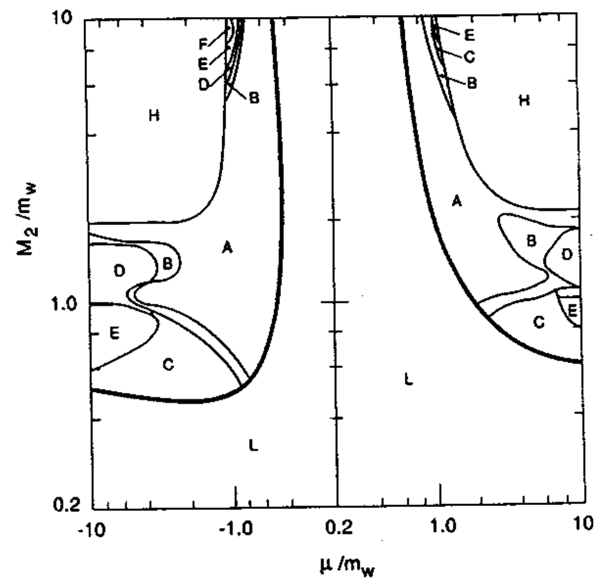
- Figure 1a -



- Figure 1b -



- Figure 2a -



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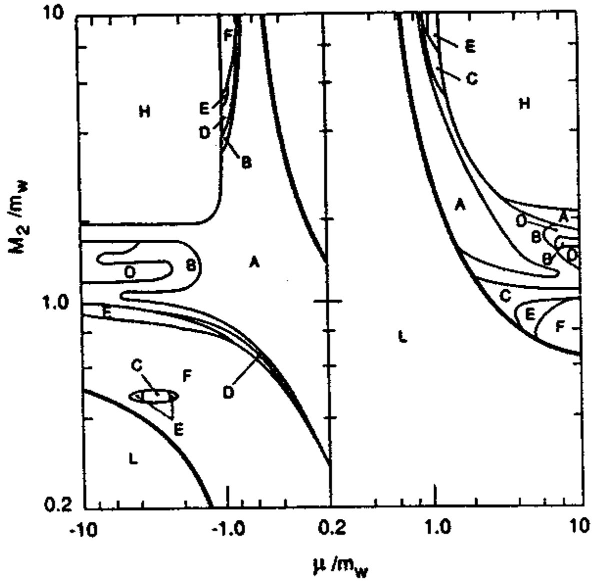
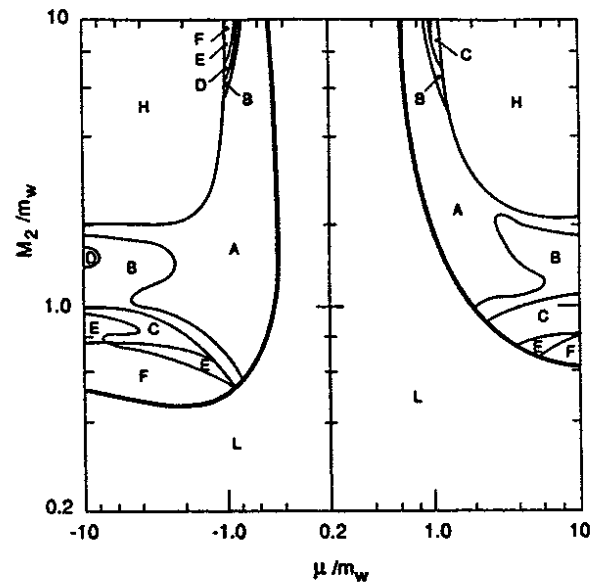


- Figure 3a -



- Figure 3b -