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Flat Directions in Left–Right Symmetric String Derived Models

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Abstract

The only string models known to reproduce the Minimal Supersymmetric Standard Model in the low energy effective field theory are those constructed in the free fermionic formulation. We demonstrate the existence of quasi–realistic free fermionic heterotic–string models in which supersymmetric singlet flat directions do not exist. This raises the possibility that supersymmetry is broken perturbatively in such models by the one–loop Fayet–Iliopoulos term. We show, however, that supersymmetric flat directions that utilize VEVs of some non–Abelian fields in the massless string spectrum do exist in the model. We argue that hidden sector condensates lift the flat directions and break supersymmetry hierarchically.

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1 Introduction

The only string models known to produce the minimal supersymmetric standard model in the low energy effective field theory are those constructed in the free fermionic formulation [1]. The first free fermionic string models that were constructed included the flipped $SU(5)$ string models [2] (FSU5), the standard–like string models [3, 4] (SLM) and the Pati–Salam string models [5] (PS). Recently, we constructed such three generation free fermionic models with the Left–Right Symmetric (LRS) gauge group, $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [6]. The massless spectrum of the FSU5, SLM and PS free fermionic string models that have been constructed always contained an "anomalous" $U(1)$ symmetry. The anomalous $U(1)_A$ is broken by the Green–Schwarz–Dine–Seiberg–Witten mechanism [7] in which a potentially large Fayet–Iliopoulos D–term ξ is generated by the VEV of the dilaton field. Such a D–term would, in general, break supersymmetry, unless there is a direction $\hat{\phi} = \sum \alpha_i \phi_i$ in the scalar potential for which $\sum Q^i_A |\alpha_i|^2 < 0$ and that is D–flat with respect to all the non–anomalous gauge symmetries along with F–flat. If such a direction exists, it will acquire a VEV, canceling the Fayet–Iliopoulos ξ–term, restoring supersymmetry and stabilizing the vacuum. The set of D and F flat constraints is given by,

$$\langle D_A \rangle = \langle D_\alpha \rangle = 0; \quad \langle F_i \equiv \frac{\partial W}{\partial \eta_i} \rangle = 0 \tag{1.1}$$

$$D_A = \left[K_A + \sum Q^k_A |\chi_k|^2 + \xi \right] \tag{1.2}$$

$$D_\alpha = \left[K_\alpha + \sum Q^k_\alpha |\chi_k|^2 \right] \ , \ \alpha \neq A \tag{1.3}$$

$$\xi = \frac{g^2 (\mathrm{Tr} Q_A)}{192 \pi^2} M^2_{\mathrm{Pl}} \tag{1.4}$$

where χ_k are the fields which acquire VEVs of order $\sqrt{\xi}$, while the K–terms contain fields η_i like squarks, sleptons and Higgs bosons whose VEVs vanish at this scale. Q^k_A and Q^k_α denote the anomalous and non–anomalous charges, and $M_{\mathrm{Pl}} \approx 2 \times 10^{18}$ GeV denotes the reduced Planck mass. The solution (*i.e.* the choice of fields with non–vanishing VEVs) to the set of equations (1.1)–(1.3), though nontrivial, is not unique. Therefore in a typical model there exist a moduli space of solutions to the F and D flatness constraints, which are supersymmetric and degenerate in energy [8]. Much of the study of the superstring models phenomenology (as well as non–string supersymmetric models [9]) involves the analysis and classification of these flat directions. The methods for this analysis in string models have been systematized in the past few years [10, 1].

It has in general been assumed in the past that in a given string model there should exist a supersymmetric solution to the F and D flatness constraints. The simpler type of solutions utilize only fields that are singlets of all the non–Abelian groups in a given model (type I solutions). More involved solutions (type II solutions), that utilize also non–Abelian fields, have also been considered [1], as well as recent inclusion of non–Abelian fields in systematic methods of analysis [1].

In contrast to the case of the FSU5, SLM and PS string models, the LRS string models [6] gave rise to models in which all the Abelian $U(1)$ symmetries are anomaly free. These models, therefore, are at a stable point in the moduli space, and the vacuum remains unshifted. In ref. [6] we discussed the characteristic features of the LRS string models that resulted in models completely free of Abelian anomalies. On the other hand, some of the LRS string models that were constructed in ref. [6] did contain an anomalous $U(1)$. In this paper we examine the supersymmetric flat directions in the LRS models that do contain an anomalous $U(1)$ symmetry, and find some surprising results. The immediate observation is that in the LRS string model that we study, there in fact does not exist a type I solution. Namely, there is no supersymmetric vacuum that is obtained solely by utilizing singlet VEVs! Specifically, there is no solution to the D–term constraints that utilizes solely singlet VEVs! If a D–flat vacuum exists it necessitates the induction of a non–vanishing VEV for some non–Abelian fields in the spectrum. We then show that in fact there exists a D–flat solution that utilizes non–Abelian VEVs. While in the past non–Abelian VEVs have been advocated for phenomenological considerations [12], this is the first instance were non–Abelian VEVs are necessitated by requiring the existence of a supersymmetric vacuum at the Planck scale. This is an interesting outcome for the following reason. It has been argued in the past that NA VEVs necessarily induce a mass term in the superpotential that results in hierarchical supersymmetry breaking [11]. While in the past the motivation for the non–Abelian VEVs was purely phenomenological [12], here we have an example were the non–Abelian VEVs are enforced if we require that there is no supersymmetry breaking VEVs of the Planck scale order. The new features of the LRS string model may therefore have important bearing on the issue of supersymmetry breaking.

Several further comments are in order. First we comment that the LRS symmetric model under study cannot give rise to a realistic model for the following reasons. First, all the Higgs doublets from the Neveu–Schwarz sector are projected out by the GSO projections, which renders the prospect of generating realistic fermion mass spectrum rather problematic. Second, and more importantly, the non–Abelian fields $\mathcal{L}$ that are used to cancel the anomalous $U(1)$ D–term carry fractional electric charge and consequently the supersymmetric vacuum state cannot be realistic.

The new features of the LRS string models have interesting potential implications from additional perspectives. First, as discussed above, the model does not admit singlet flat directions. The question then is whether a given string model always admits a supersymmetric vacuum. This has always been assumed in the past, but there is no theorem to this effect. In the present model we do show that there exists a supersymmetric vacuum at the Planck scale, but the new observations prompt us to examine the issue in closely related models which will be reported in a future publication. However, lets assume that a supersymmetric flat direction does not exist.

The implication would be that the vacuum is necessarily non–supersymmetric and the supersymmetry breaking is of the order of the string scale. However, this breaking is unlike the breaking that can be induced by projecting the remaining gravitino with a GSO projection. While in the later the supersymmetry breaking is at the level of the spectrum, in the former the tree level spectrum remains supersymmetric. In fact, as a result, also the one–loop partition function, and hence the one–loop cosmological constant, is vanishing. One still expects, however, a two–loop contribution to the vacuum energy due to the possible non–vanishing D–term [13]. These new features of the LRS string models may therefore have important bearing on the issue of supersymmetry breaking as well as on that of the vacuum energy. Although we find that supersymmetric flat directions do exist in the model, we in general expect that hidden sector condensates break supersymmetry and lift the flat directions [11, 1]. This is unlike the case in type I solutions in which one often finds solutions that are not lifted by hidden sector condensates. Thus, the LRS models may reveal a situation in which NA VEVs are enforced by requiring that the vacuum is supersymmetric at the Planck scale. Which, in turn, induces hierarchical SUSY breaking by the hidden sector condensates. In such a situation, the hierarchical breaking of supersymmetry is no longer a choice, but is enforced in the vacuum.

2 The string model

The realistic free fermionic string models are constructed by specifying a set of boundary condition basis vectors and one–loop GSO projection coefficients [14]. The rules for extracting the superpotential terms were derived in ref. [15] The general structure of the models have been discussed in detail in the past. The basis vectors which generate the models are divided into two groups. The first five consist of the NAHE set [16] and are common to all the semi–realistic free fermionic models. The second consists of three additional boundary condition basis vectors. Specific details on the construction of the LRS free fermionic string models are given in ref. [6].

The left–right symmetric free fermionic heterotic string model that we consider is specified in the tables below. The boundary conditions of the three basis vectors which extend the NAHE set are shown in Table (2.1). Also given in Table (2.1) are the pairings of left– and right–moving real fermions from the set $\{y, \omega | \bar{y}, \bar{\omega}\}$. These fermions are paired to form either complex, left– or right–moving fermions, or Ising model operators, which combine a real left–moving fermion with a real right–moving fermion. The generalized GSO coefficients determining the physical massless states of Model 1 appear in matrix (2.2).

LRS Model 1 Boundary Conditions:

	ψ^μ	χ^{12}	χ^{34}	χ^{56}	$\bar\psi^{1,...,5}$	$\bar\eta^1$	$\bar\eta^2$	$\bar\eta^3$	$\bar\phi^{1,...,8}$
α	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
β	0	0	0	0	1 1 1 0 0	0	0	0	1 1 1 1 0 0 0 0
γ	0	0	0	0	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 0 0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0 1 1 1 1 1 1 0

	y^3y^6	$y^4\bar y^4$	$y^5\bar y^5$	$\bar y^3\bar y^6$	$y^1\omega^5$	$y^2\bar y^2$	$\omega^6\bar\omega^6$	$\bar y^1\bar\omega^5$	$\omega^2\omega^4$	$\omega^1\bar\omega^1$	$\omega^3\bar\omega^3$	$\bar\omega^2\bar\omega^4$
α	1	1	1	0	1	1	1	0	1	1	1	0
β	0	1	0	1	0	1	0	1	1	0	0	0
γ	0	0	1	1	1	0	0	0	0	1	0	1

(2.1)

LRS Model 1 Generalized GSO Coefficients:

$$\begin{array}{c} \\ \mathbf{1} \\ \mathbf{S} \\ \\ \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \\ \\ \alpha \\ \beta \\ \gamma \end{array} \begin{array}{c} \begin{array}{cccccccc} \mathbf{1} & \mathbf{S} & \mathbf{b}_1 & \mathbf{b}_2 & \mathbf{b}_3 & \alpha & \beta & \gamma \end{array} \\ \left(\begin{array}{cc|ccc|ccc} 1 & 1 & -1 & -1 & -1 & 1 & 1 & i \\ 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 \\ & & & & & & & \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\ -1 & -1 & -1 & -1 & -1 & -1 & -1 & i \\ & & & & & & & \\ 1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 & 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & -1 & 1 & -1 & -1 & 1 \end{array}\right) \end{array} \qquad (2.2)$$

In matrix (2.2) only the entries above the diagonal are independent and those below and on the diagonal are fixed by the modular invariance constraints. Blank lines are inserted to emphasize the division of the free phases between the different sectors of the realistic free fermionic models. Thus, the first two lines involve only the GSO phases of $c\binom{\{\mathbf{1},\mathbf{S}\}}{\mathbf{a}_i}$. The set $\{\mathbf{1},\mathbf{S}\}$ generates the $N=4$ model with $\mathbf{S}$ being the space–time supersymmetry generator and therefore the phases $c\binom{\mathbf{S}}{\mathbf{a}_i}$ are those that control the space–time supersymmetry in the superstring models. Similarly, in the free fermionic models, sectors with periodic and anti–periodic boundary conditions, of the form of $\mathbf{b}_i$, produce the chiral generations. The phases $c\binom{\mathbf{b}_i}{\mathbf{b}_j}$ determine the chirality of the states from these sectors.

We note that the boundary condition basis vectors that generate the string model are those of Model 3 of ref. [6]. The two models differ in the GSO phase $c\binom{\mathbf{b}_3}{\beta}$, with $c\binom{\mathbf{b}_3}{\beta} = -1$ in the string model above and $c\binom{\mathbf{b}_3}{\beta} = +1$ in Model 3 of [6]. As we elaborate below, the consequence of this GSO phase change is that the gauge symmetry is enhanced, with one of the Abelian generators being absorbed into the enhanced non–Abelian gauge symmetry. Consequently, the number of Abelian group factors is reduced, which simplifies somewhat the analysis of the D–flat directions. However, the results that we discuss here are independent of this simplification and therefore also hold in Model 3 of ref. [6].

The final gauge group of the string model arises as follows: In the observable sector the NS boundary conditions produce gauge group generators for

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_C \times U(1)_{1,2,3} \times U(1)_{4,5,6} \tag{2.3}$$

Thus, the $SO(10)$ symmetry is broken to $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_C$, where,

$$U(1)_C = \mathrm{Tr}\, U(3)_C \;\Rightarrow\; Q_C = \sum_{i=1}^{3} Q(\bar{\psi}^i). \tag{2.4}$$

The flavor $SO(6)^3$ symmetries are broken to $U(1)^{3+n}$ with $(n = 0, \cdots, 6)$. The first three, denoted by $U(1)_j$ $(j = 1, 2, 3)$, arise from the world–sheet currents $\bar{\eta}^j \bar{\eta}^{j^*}$. These three $U(1)$ symmetries are present in all the three generation free fermionic models which use the NAHE set. Additional horizontal $U(1)$ symmetries, denoted by $U(1)_j$ $(j = 4, 5, ...)$, arise by pairing two real fermions from the sets $\{\bar{y}^{3,\cdots,6}\}$, $\{\bar{y}^{1,2}, \bar{\omega}^{5,6}\}$, and $\{\bar{\omega}^{1,\cdots,4}\}$. The final observable gauge group depends on the number of such pairings. In this model there are the pairings, $\bar{y}^3\bar{y}^6$, $\bar{y}^1\bar{\omega}^5$ and $\bar{\omega}^2\bar{\omega}^4$, which generate three additional $U(1)$ symmetries, denoted by $U(1)_{4,5,6}$*.

In the hidden sector, which arises from the complex world–sheet fermions $\bar{\phi}^{1\cdots 8}$, the NS boundary conditions produce the generators of

$$SU(3)_{H_1} \times U(1)_{H_1} \times U(1)_{7'} \times SU(3)_{H_2} \times U(1)_{H_2} \times U(1)_{8'}\,. \tag{2.5}$$

$U(1)_{H_1}$ and $U(1)_{H_2}$ correspond to the combinations of the world–sheet charges

$$Q_{H_1} = Q(\bar{\phi}^1) - Q(\bar{\phi}^2) - Q(\bar{\phi}^3) + Q(\bar{\phi}^4) - \sum_{i=5}^{7} Q(\bar{\phi})^i + Q(\bar{\phi})^8, \tag{2.6}$$

$$Q_{H_2} = \sum_{i=1}^{4} Q(\bar{\phi})^i - Q(\bar{\phi}^5) + \sum_{i=6}^{8} Q(\bar{\phi})^i. \tag{2.7}$$

The sector $\zeta \equiv 1 + \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$ produces the representations $(3,1)_{-5,0} \oplus (\bar{3},1)_{5,0}$ and $(1,3)_{0,-5} \oplus (1,\bar{3})_{0,5}$ of $SU(3)_{H_1} \times U(1)_{H_1}$ and $SU(3)_{H_2} \times U(1)_{H_2}$. Thus, the E_8 symmetry reduces to $SU(4)_{H_1} \times SU(4)_{H_2} \times U(1)^2$. The additional $U(1)$'s in $SU(4)_{H_{1,2}}$ are given by the combinations in eqs. (2.6) and (2.7), respectively. The remaining $U(1)$ symmetries in the hidden sector, $U(1)_{7'}$ and $U(1)_{8'}$, correspond to

*It is important to note that the existence of these three additional $U(1)$ currents is correlated with a superstringy doublet–triplet splitting mechanism [17]. Due to these extra $U(1)$ symmetries the color triplets from the NS sector are projected out of the spectrum by the GSO projections while the electroweak doublets remain in the light spectrum.

the combination of world–sheet charges

$$Q_{7'} = Q(\bar{\phi}^1) - Q(\bar{\phi}^7), \tag{2.8}$$

$$Q_{8'} = Q(\bar{\phi}^1) - \sum_{i=2}^{4} Q(\bar{\phi})^i + \sum_{i=5}^{7} Q(\bar{\phi})^i + Q(\bar{\phi}^8). \tag{2.9}$$

In addition to the NS and ζ sector the string model contains a combination of non–NAHE basis vectors with $\mathbf{X}_L \cdot \mathbf{X}_L = 0$, which therefore may give rise to additional space–time vector bosons. The vector combination is given by $\mathbf{X} \equiv \zeta + 2\gamma$, where $\zeta \equiv 1 + \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3$. This combination arises only from the NAHE set basis vectors plus 2γ, with γ inducing the left–right symmetry breaking pattern $SO(6) \times SO(4) \to SU(3) \times U(1) \times SU(2)_L \times SU(2)_R$, and is independent of the assignment of periodic boundary conditions in the basis vectors α, β and γ. This vector combination is therefor generic for the pattern of symmetry breaking $SO(10) \to SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R$, in NAHE based models.

The sector $\mathbf{X}$ gives rise to six additional space–time vector bosons which are charged with respect to the world–sheet $U(1)$ currents, and transform as $3 \oplus \bar{3}$ under $SU(3)_C$. These additional gauge bosons enhance the $SU(3)_C \times U(1)_{C'}$ symmetry to $SU(4)_C$, where $U(1)_{C'}$ is given by the combination of world–sheet charges,

$$Q_{C'} = Q(\bar{\psi}^1) - Q(\bar{\psi}^2) - Q(\bar{\psi}^3) - \sum_{i=1}^{3} Q(\bar{\eta})^i + Q(\bar{\phi}^7) - Q(\bar{\phi}^8). \tag{2.10}$$

The remaining orthogonal $U(1)$ combinations are

$$\begin{aligned} Q_{1'} &= Q_1 - Q_2, \\ Q_{2'} &= Q_1 + Q_2 - 2Q_3, \\ Q_{3'} &= 3Q_C - (Q_1 + Q_2 + Q_3), \\ Q_{7''} &= Q_C + 3(Q_1 + Q_2 + Q_3) + 5Q_{7'}. \end{aligned} \tag{2.11}$$

and $Q_{4,5,6,8'}$ are unchanged. Thus, the full massless spectrum transforms under the final gauge group, $SU(4)_C \times SU(2)_L \times SU(2)_R \times U(1)_{1',2',3'} \times U(1)_{4,5,6} \times SU(4)_{H_1} \times SU(4)_{H_2} \times U(1)_{7'',8'}$.

In addition to the graviton, dilaton, antisymmetric sector and spin–1 gauge bosons, the NS sector gives three pairs of $SO(10)$ singlets with $U(1)_{1,2,3}$ charges; and three singlets of the entire four dimensional gauge group.

The states from the sectors $\mathbf{b}_j \oplus \mathbf{b}_j + \mathbf{X}$ ($j = 1, 2, 3$) produce the three light generations. The states from these sectors and their decomposition under the entire gauge group are shown in Table 1 of the Appendix. The leptons (and quarks) are singlets of the color $SU(4)_{H_1,H_2}$ gauge groups and the $U(1)_{8'}$ symmetry of eq. (2.9) becomes a gauged leptophobic symmetry. The leptophobic $U(1)$ symmetry arises from a combination of the $U(1)_{B-L}$ symmetry with a family universal combination of the flavor and hidden $U(1)$ symmetries [18]. The remaining massless states in the model and their quantum numbers are also given in Table 1.

We next turn to the definition of the weak–hypercharge in this LRS model. Due to the enhanced symmetry there are several possibilities to define a weak–hypercharge combination which is still family universal and reproduces the correct charge assignment for the Standard Model fermions. One option is to define the weak–hypercharge with the standard $SO(10)$ embedding, as in eq. (2.12),

$$U(1)_Y \;=\; \frac{1}{3}U(1)_C \;+\; \frac{1}{2}U(1)_L\;. \tag{2.12}$$

This is identical to the weak–hypercharge definition in $SU(3)\times SU(2)\times U(1)_Y$ free fermionic models, which do not have enhanced symmetries, as for example in Model 3 of ref. [6]. The weak hypercharge definition of eq. (2.12) reproduces the canonical MSSM normalization of the weak hypercharge, $k_Y = 5/3$. Alternatively, we can define the weak–hypercharge to be the combination

$$U(1)_Y \;=\; \frac{1}{2}U(1)_L \;-\; \frac{1}{10}(U(1)_{3'} \;+\; \frac{1}{3}U_{7''}) \tag{2.13}$$

where $U(1)_{3'}$ and $U(1)_{7''}$ are given in (2.11). This combination still reproduces the correct charge assignment for the Standard Model states. The reason being that the states from the sectors $\mathbf{b}_i$ $i = 1,2,3$ which are identified with the Standard Model states, are not charged with respect to the additional Cartan subgenerators that form the modified weak hypercharge definition. In some models it is found that such alternative definitions allow all massless exotic states to be integrally charged. The price, however, is that the Kač–Moody level of the weak hypercharge current as defined in eq. (2.13) is no longer the canonical $SO(10)$ normalization and the simple unification picture is lost. We conclude that the model admits a sensible weak–hypercharge definition, which for our purpose here is sufficient. We stress that our objective here is not to present the LRS string model as a semi–realistic model, but rather to study the new features pertaining to the existence of a supersymmetric vacuum.

3 Anomalous $U(1)$

The string model contains an anomalous $U(1)$ symmetry. The anomalous $U(1)$ is a combination of $U(1)_4$, $U(1)_5$ and $U(1)_6$, which are generated by the world–sheet complex fermions $\bar{y}^3\bar{y}^6$, $\bar{y}^1\bar{\omega}^5$ and $\bar{\omega}^2\bar{\omega}^4$, respectively. The anomalous $U(1)$ that arises in this model is therefore of a different origin than the one that typically arises in the FSU5, SLM and PS free fermionic string models. The difference between the two cases is discussed in detail in ref. [6]. In short, the main distinction is that in the case of the FSU5, SLM and PS string models, the $U(1)$ symmetries, $U(1)_1$, $U(1)_2$ and $U(1)_3$ which are embedded in the observable E_8 are necessarily anomalous because of the symmetry breaking pattern $E_6 \to SO(10) \times U(1)_A$ [19], whereas in the LRS models they are necessarily anomaly free because the models do not admit the pattern $E_6 \to SO(10) \times U(1)_A$. Consequently, in the LRS string models the anomalous $U(1)$ can only arise from $U(1)$ currents that arise from the six dimensional internal manifold, rather than from the $U(1)$ currents of the observable E_8. This distinction, as we demonstrate further below, is potentially important because typically the non–Abelian singlets that arise from the untwisted sector, and are used to cancel the $U(1)_A$ D–term equation, are charged with respect to $U(1)_{1,2,3}$, but not with respect to $U(1)_{4,5,6}$, which arise from the internal "compactified" degrees of freedom.

The anomalous $U(1)$ generates a Fayet–Iliopoulos D–term, which breaks supersymmetry and destabilizes the vacuum. Stabilization of the vacuum implies that the vacuum is shifted by a VEV which cancels the anomalous $U(1)$ D–term and restores supersymmetry. If such a direction in the scalar potential does not exist, it would imply that supersymmetry is necessarily broken and a supersymmetric vacuum does not exist. Additionally a supersymmetric vacuum also requires that F–flatness is also respected in the vacuum. The anomalous $U(1)_A$ combination is given by

$$U_A \equiv U_4 + U_5 + U_6, \tag{3.1}$$

with $\text{Tr}Q_A = -72$. The two orthogonal linear combinations,

$$\begin{aligned} U_{4'} &= U_4 - U_5 \\ U_{5'} &= U_4 + U_5 - 2U_6 \end{aligned} \tag{3.2}$$

are both traceless.

Since $\text{Tr}Q_A < 0$, the sign for the Fayet–Iliopoulos term is negative. Requiring D–flatness then implies that that there must exist a direction in the scalar potential in which a field (or a combination of fields) with positive total $U(1)_A$ charge, gets a VEV and cancels the $U(1)_A$ D–term. Examining the massless spectrum of the model, given in Table 1, we immediately note that the model does not contain any non–Abelian singlet fields with such a charge. Therefore, if a supersymmetric vacuum exists, some non–Abelian fields must get a VEV. From Table 1 it is seen that the only states that carry positive $U(1)_A$ charge are the $SU(2)_L$ and $SU(2)_R$ doublets from the three sectors $\mathbf{b}_k + \zeta + 2\gamma \equiv \mathbf{1} + \mathbf{b}_i + \mathbf{b}_j + 2\gamma$, $(i, j, k = 1, 2, 3)$ with i, j, k all distinct. This then implies that $SU(2)_L$ or $SU(2)_R$ must be broken in the vacuum. The same result holds also in Model 3 of ref. [6], in which there is no gauge enhancement from the sector $\zeta + 2\gamma$. We note, however, that the doublets from these sectors carry fractional $\pm 1/2$ charge with respect to electric charge as defined in Eq. (2.12). While there exist alternative definitions of the weak–hypercharge that allow these states to be integrally charged, the primary question of interest here is whether a supersymmetric vacuum exist at all! In the model under consideration this is contingent on finding D–flat direction, which are also F–flat. We next turn to examine whether a D-flat direction exist in this model.

4 D–Flat Directions

In Tables 1 and 2 we have listed all of the massless states that appear in our LRS string model. There are a total of 68 fields, 38 of which may be used to form 19 sets of vector–like pairs of fields. Of these 19 vector–like pairs, 13 pairs are singlets under all non–Abelian gauge groups, while three pairs are $\mathbf{-4/4}$ sets under $SU(4)_C$ and two pairs are $\mathbf{6/6}$'s sets under $SU(4)_{H_1}$. The 30 non–vector–like fields are all non–Abelian reps. That is, all singlets occur in vector–like pairs.

The anomalous charge trace of $U(1)_A$ is negative for the LRS Model 1. Thus, the anomaly can only be cancelled by fields with positive anomalous charge. In this model *none* of the NA singlets carry anomalous $Q^{(A)}$. The only fields with positive anomalous charge are three $SU(2)_L$ doublets, $\mathcal{L}_{L1,L2,L3}$ and three $SU(2)_R$ doublets, $\mathcal{L}_{R1,R2,R3}$. Hence any possible flat solutions automatically break the initial $SU(2)_L \times SU(2)_R$ symmetry to a subgroup. Good phenomenology would clearly prefer the subgroup $SU(2)_L \times U(1)_R$.

Since we have clearly shown that no non–Abelian singlet D–flat directions are possible for this model, it is possible that no D–flat directions exist. That is, the severity of non–Abelian D–flat constraints may allow for no solutions. Thus, this model either (i) automatically breaks $SU(2)_L \times SU(2)_R$ or (ii) has no flat directions and, therefore, breaks supersymmetry at the string scale. In either case, this is a very interesting model.

To systematically study D–flatness for this model, we first generate a complete basis of directions D–flat for all non–anomalous Abelian symmetries. We provide this basis in Table 2 of the Appendix. For a given row in Table 2, the first column entry denotes the name of the D–flat basis direction. The next row specifies the anomalous charge of the basis direction. The following seven entries specify the ratios of the norms of the VEVs of the fields common to these directions. The first five of these fields have vector–like partners. For these, a negative norm indicates the vector–partner acquires the VEV, rather than the field specified at the top of the respective column. The last two of these seven fields are not vector–like. Thus, the norm must be non–negative for each of these for a flat direction formed from a linear combination of basis directions to be physical. The next to last entry specifies the norm of the VEV of the field unique to a given basis direction, while the identity of the unique field is given by the last entry.

We have labeled these D–flat directions as $\mathbf{D}_1$ through $\mathbf{D}_{41}$. The first eight D–flat directions ($\mathbf{D}_1$ to $\mathbf{D}_8$) carry a positive net anomalous charge. The next fourteen ($\mathbf{D}_9$ to $\mathbf{D}_{22}$) carry a negative net anomalous charge, while the remaining nineteen ($\mathbf{D}_{23}$ to $\mathbf{D}_{41}$) lack a net anomalous charge. There are two classes of basis vectors lacking anomalous charge. The first class contains six directions for which the unique field is non–vector–like. These directions also contain VEVs for $\mathcal{H}_1$ ($\bar{\mathcal{H}}_1$) and/or $\mathcal{H}_2$ ($\bar{\mathcal{H}}_2$). The second class contains thirteen basis directions wherein the unique fields are vector–like and which do not contain $\bar{H}_{2'}$ and/or $\bar{H}_{4'}$. Thus, these thirteen directions are themselves vector–like and are denoted as such by a superscript "v". For each vector–like basis direction, $\mathbf{D}^v$ there is a corresponding $-\mathbf{D}^v$ direction, wherein the fields in $\mathbf{D}^v$ are replaced by their respective vector–like partners.

None of the positive $Q^{(\mathrm{A})}$ directions are good in themselves because one or both of $|\left\langle \bar{H}_{2'} \right\rangle|^2$ and $|\left\langle \bar{H}_{4'} \right\rangle|^2$ are non–zero and negative while $|\left\langle \bar{H}_{2'} \right\rangle|^2$ and $|\left\langle \bar{H}_{4'} \right\rangle|^2$ are not vector–like reps. In particular, the $Q^{(\mathrm{A})} = 12$ directions have either $|\left\langle \bar{H}_{2'} \right\rangle|^2 = -2$ and $|\left\langle \bar{H}_{4'} \right\rangle|^2 = 0$ or $|\left\langle \bar{H}_{2'} \right\rangle|^2 = 0$ and $|\left\langle \bar{H}_{4'} \right\rangle|^2 = -2$, while the $Q^{(\mathrm{A})} = 24$ directions all have $|\left\langle \bar{H}_{2'} \right\rangle|^2 = |\left\langle \bar{H}_{4'} \right\rangle|^2 = -4$. In this basis we also find that the $|\left\langle \bar{H}_{2'} \right\rangle|^2$ and $|\left\langle \bar{H}_{4'} \right\rangle|^2$ charges of all of the $Q^{(\mathrm{A})} = 0$ directions are zero or negative. So the $|\left\langle \bar{H}_{2'} \right\rangle|^2$ and $|\left\langle \bar{H}_{4'} \right\rangle|^2$ negative charges on the positive $Q^{(\mathrm{A})}$ directions cannot be made zero or positive by adding $Q^{(\mathrm{A})} = 0$ directions to $Q^{(\mathrm{A})} > 0$ directions. In contrast, all of the $Q^{(\mathrm{A})} = -12$ directions have either $|\left\langle \bar{H}_{2'} \right\rangle|^2 = 2$ and $|\left\langle \bar{H}_{4'} \right\rangle|^2 = 0$ or $|\left\langle \bar{H}_{2'} \right\rangle|^2 = 0$ and $|\left\langle \bar{H}_{4'} \right\rangle|^2 = 2$; the $Q^{(\mathrm{A})} = 24$ directions all have either $|\left\langle \bar{H}_{2'} \right\rangle|^2 = |\left\langle \bar{H}_{4'} \right\rangle|^2 = 4$ or $|\left\langle \bar{H}_{2'} \right\rangle|^2 = |\left\langle \bar{H}_{4'} \right\rangle|^2 = 2$; while the $Q^{(\mathrm{A})} = -48$ directions all have $|\left\langle \bar{H}_{2'} \right\rangle|^2 = |\left\langle \bar{H}_{4'} \right\rangle|^2 = 4$. Therefore, physical D–flat directions must necessarily be formed from linear combinations of $Q^{(\mathrm{A})} > 0$ and $Q^{(\mathrm{A})} < 0$ directions such that the net $Q^{(\mathrm{A})}$, $|\left\langle \bar{H}_{2'} \right\rangle|^2$, and $|\left\langle \bar{H}_{4'} \right\rangle|^2$ are all positive. Physical D–flat directions may also contain $Q^{(\mathrm{A})} = 0$ components that keep $|\left\langle \bar{H}_{2'} \right\rangle|^2$, $|\left\langle \bar{H}_{4'} \right\rangle|^2 \geq 0$.

The specific values of $Q^{(\mathrm{A})}$, $|\left\langle \bar{H}_{2'} \right\rangle|^2$, and $|\left\langle \bar{H}_{4'} \right\rangle|^2$ in the basis directions indicate that the roots of all physical flat directions must contain either $\mathbf{D}_{19}$ or $\mathbf{D}_{20}$ and combinations of basis vectors $\mathbf{D}_1$, $\mathbf{D}_2$, $\mathbf{D}_3$, and $\mathbf{D}_4$ of the form

$$n_1\mathbf{D}_1 + n_2\mathbf{D}_2 + n_3\mathbf{D}_3 + n_4\mathbf{D}_4 + n_{19}\mathbf{D}_{19} + n_{20}\mathbf{D}_{20}, \tag{4.1}$$

where the non–negative integers n_1, n_2, n_3, n_4, n_{19}, n_{20} satisfy the constraints

$$n_1 + n_2 + n_3 + n_4 - 2n_{19} - 2n_{20} > 0; \tag{4.2}$$

$$-n_1 - n_2 + 2n_{19} + 2n_{20} \geq 0; \tag{4.3}$$

$$-n_3 - n_4 + 2n_{19} + 2n_{20} \geq 0. \tag{4.4}$$

For example, one of the simplest D–flat solutions for all Abelian gauge groups is $n_1 = n_2 = n_3 = n_4 = 2$, $n_{19} = n_{20} = 1$. This direction is simply $|\langle \mathcal{L}_{L1} \rangle|^2 = |\langle \mathcal{L}_{L2} \rangle|^2 = |\langle \mathcal{L}_{L3} \rangle|^2 = |\langle \mathcal{L}_{R1} \rangle|^2 = |\langle \mathcal{L}_{R2} \rangle|^2 = |\langle \mathcal{L}_{R3} \rangle|^2$. The corresponding fields are three exotic $SU(2)_L$ doublets, $\mathcal{L}_{L1}$, $\mathcal{L}_{L2}$, and $\mathcal{L}_{L3}$, and three exotic $SU(2)_R$ doublets, $\mathcal{L}_{R1}$, $\mathcal{L}_{R2}$, and $\mathcal{L}_{R3}$. These six fields are singlets under all other non–Abelian groups.

For this model any D–flat direction must contain $SU(2)_L$ or $SU(2)_R$ fields. Thus, let us examine more closely $SU(2)$ D–flat constraints. The only $SU(2)$ fields in this model are doublet representations, which we generically denote L_i. Thus, the related three $SU(2)$ D–terms,

$$D^{SU(2)}_{a=1,2,3} \equiv \sum_m L_i^\dagger T^{SU(2)}_{a=1,2,3} L_i \ , \tag{4.5}$$

contain matrix generators $T_a^{SU(2)}$ that take on the values of the three Pauli matrices,

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{4.6}$$

respectively.

As discussed in [20] and [21], each component of the vector $\vec{D}^{SU(2)}$ is the total "spin expectation value" in the given direction of the internal space, summed over all $SU(2)$ doublet fields of the gauge group. Thus, for all of the $\left\langle D_a^{SU(2)} \right\rangle$ to vanish, the $SU(2)$ VEVs must be chosen such that the total $\hat{x}, \hat{y}$, and $\hat{z}$ expectation values are zero. Abelian D–flatness constraints from any extra $U(1)$ charges carried by the doublet generally restrict the normalization length, $L_i^{\dagger} L_i$, of a "spinor" L_i to integer units. Thus, since each spinor has a length and direction associated with it, D–flatness requires the sum, placed tip–to–tail, to be zero. Let us choose for an explicit representation of a generic $SU(2)$ doublet $L(\theta, \phi)$ that used in [20]:

$$L(\theta, \phi) \equiv A \begin{pmatrix} \cos\frac{\theta}{2}\, e^{-i\frac{\phi}{2}} \\ \sin\frac{\theta}{2}\, e^{+i\frac{\phi}{2}} \end{pmatrix} , \tag{4.7}$$

where A is the overall amplitude of the VEV. The range of physical angles, $\theta = 0 \to \pi$ and $\phi = 0 \to 2\pi$ provide for the most general possible doublet. (Note the ϕ phase freedom for $\theta = 0, \pi$.) Each such θ, ϕ combination carries a one–to–one geometrical correspondence.

The contribution of $L(\theta, \phi)$ to each $SU(2)$ D–term is,

$$D_1^{SU(2)}(L) \;\; \equiv \;\; L^{\dagger} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} L = |A|^2 \sin\theta \; \cos\phi \tag{4.8}$$

$$D_2^{SU(2)}(L) \;\; \equiv \;\; L^{\dagger} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} L = |A|^2 \sin\theta \; \sin\phi \tag{4.9}$$

$$D_3^{SU(2)}(L) \;\; \equiv \;\; L^{\dagger} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} L = |A|^2 \cos\theta \; . \tag{4.10}$$

A doublet's D-term contribution for any extra $U(1)$ charges carried by it is,

$$D_{U(1)}(L) \;\; \equiv \;\; Q^{U(1)}|L|^2 = Q^{U(1)}|A|^2 \; . \tag{4.11}$$

From this we can see that the VEVS of three $SU(2)$ doublets $\mathcal{L}_{i=1,2,3}$ with equal norms $|A_1|^2 = |A_2|^2 = |A_3|^2 \equiv |A|^2$ can, indeed, produce an $SU(2)$ D–flat direction with the choice of angles (in radians), $\theta_1 = 0$, $\theta_2 = \theta_3 = 2\pi/3$, $\phi_2 = 0$, $\phi_3 = \pi$. For these angles, the $SU(2)$ D–vectors added tip–to–tail for the three doublets form an equilateral triangle with starting and ending points at the origin. In other words, the total $\hat{x}$, $\hat{y}$, and $\hat{z}$ expectation values are all zero!

This flat direction gives a specific example of what will occur for every flat direction of this model: non–Abelian VEVs (for at least $SU(2)_L$ or $SU(2)_R$ doublets) are a necessary effect of the retention of spacetime supersymmetry. This has profound implications for this model! First, $SU(2)_L \times SU(2)_R$ gauge symmetry is automatically broken at the FI scale. Second, non–Abelian condensates necessarily form.

Let us consider (ignoring effects of other possible field condensates that might develop significantly below the FI–scale) the status of F–flatness for the particular flat direction $|\langle\mathcal{L}_{L1}\rangle|^2 = |\langle\mathcal{L}_{L2}\rangle|^2 = |\langle\mathcal{L}_{L3}\rangle|^2 = |\langle\mathcal{L}_{R1}\rangle|^2 = |\langle\mathcal{L}_{R2}\rangle|^2 = |\langle\mathcal{L}_{R3}\rangle|^2$. From gauge invariance arguments, we find that F–flatness remains to all finite order. No related dangerous terms appear in the superpotential. We do however expect dangerous F–breaking terms to appear at finite orders for all but a few of the more complicated D–flat directions.

For a generic $SU(N_c)$ gauge group containing N_f flavors of matter states in vector–like pairings $H_i\bar{H}_i$, $i = 1, \ldots N_f$, the gauge coupling g_s, though weak at the string scale $M_{\rm string}$, becomes strong for $N_f < N_c$ at a condensation scale defined by

$$\Lambda = M_{Pl} {\rm e}^{8\pi^2/\beta g_s^2}\,, \tag{4.12}$$

where the β–function is given by,

$$\beta = -3N_c + N_f\,. \tag{4.13}$$

The N_f flavors counted are only those that ultimately receive masses $m \ll \Lambda$.

Our model contains three vector–like $\mathbf{4} - \bar{\mathbf{4}}$ pairs for each hidden sector $SU(4)_{H_{1,2}}$ gauge group, along with an additional $\mathbf{6} - \mathbf{6}$ pair for $SU(4)_{H_2}$. We have computed all possible mass terms for the non–Abelian fields resulting from our simplest flat direction VEV. From gauge invariance we find that the only mass terms appearing are for the hidden sector $\mathbf{4}$ and $\bar{\mathbf{4}}$ fields.[†] For these fields all gauge invariant mass terms are also allowed by picture–changed worldsheet charge invariance, being of worldsheet $[2_R, 2_R, 2_R]$ class. These mass terms are all sixth order and have coupling constants of equal magnitude. The latter results from the $\mathbb{Z}_2 \times \mathbb{Z}_2$ worldsheet symmetry still present among these terms. The specific terms are:

$$\begin{aligned}
&(H_1\bar{H}_{1'} + H_2\bar{H}_{2'})\,\langle\mathcal{L}_{L2}\mathcal{L}_{L3}\mathcal{L}_{R2}\mathcal{L}_{R3}\rangle\\
&(H_1\bar{H}_{3'} + H_2\bar{H}_{4'})\,\langle\mathcal{L}_{L2}\mathcal{L}_{L3}\mathcal{L}_{R1}\mathcal{L}_{R3}\rangle\,,\\
&(H_1\bar{H}_{5'} + H_2\bar{H}_{6'})\,\langle\mathcal{L}_{L2}\mathcal{L}_{L3}\mathcal{L}_{R1}\mathcal{L}_{R2}\rangle\,,\\
&(H_3\bar{H}_{1'} + H_4\bar{H}_{2'})\,\langle\mathcal{L}_{L1}\mathcal{L}_{L3}\mathcal{L}_{R2}\mathcal{L}_{R3}\rangle\,,\\
&(H_3\bar{H}_{3'} + H_4\bar{H}_{4'})\,\langle\mathcal{L}_{L1}\mathcal{L}_{L3}\mathcal{L}_{R1}\mathcal{L}_{R3}\rangle\,,\\
&(H_3\bar{H}_{5'} + H_4\bar{H}_{6'})\,\langle\mathcal{L}_{L1}\mathcal{L}_{L3}\mathcal{L}_{R1}\mathcal{L}_{R2}\rangle\,,\\
&(H_5\bar{H}_{1'} + H_6\bar{H}_{2'})\,\langle\mathcal{L}_{L1}\mathcal{L}_{L2}\mathcal{L}_{R2}\mathcal{L}_{R3}\rangle\,,\\
&(H_5\bar{H}_{3'} + H_6\bar{H}_{4'})\,\langle\mathcal{L}_{L1}\mathcal{L}_{L2}\mathcal{L}_{R1}\mathcal{L}_{R3}\rangle\,,\\
&(H_5\bar{H}_{5'} + H_6\bar{H}_{6'})\,\langle\mathcal{L}_{L1}\mathcal{L}_{L2}\mathcal{L}_{R1}\mathcal{L}_{R2}\rangle\,.
\end{aligned} \tag{4.14}$$

[†]No $\mathbf{4}$ and $\bar{\mathbf{4}}$ fields of $SU(4)_c$ mass terms of this type appear either.

Assuming identical phase factors for each of these terms, the eigenstates and mass eigenvalues for the $SU(4)_{H_1}$ mass matrix are

$$H_a = \tfrac{1}{6}(2H_2 - H_4 - H_6), \quad \bar{H}_a = \tfrac{1}{6}(2\bar{H}_{2'} - \bar{H}_{4'} - \bar{H}_{6'}); \quad M_a^2 = 0 \tag{4.15}$$
$$H_b = \tfrac{1}{2}(H_4 - H_6), \quad \bar{H}_b = \tfrac{1}{2}(\bar{H}_{4'} - \bar{H}_{6'}); \quad M_b^2 = 0 \tag{4.16}$$
$$H_c = \tfrac{1}{3}(H_2 + H_4 + H_6), \quad \bar{H}_c = \tfrac{1}{3}(\bar{H}_{2'} + \bar{H}_{4'} + \bar{H}_{6'}); \quad M_c^2 \approx \tfrac{1}{100}M_{Pl}^2 . \tag{4.17}$$

Thus, for $SU(4)_{H_1}$ we have $N_c = 4$ and $N_f = 2$. This yields $\beta = -10$ and results in an $SU(4)_{H_1}$ condensation scale

$$\Lambda_{H_1} = \mathrm{e}^{-15.8} M_{Pl} \sim 3 \times 10^{11} \text{ GeV}. \tag{4.18}$$

The $SU(4)_{H_2}$ eigenstates and eigenvalues may be converted from those of $SU(4)_{H_1}$ by exchanging field subscripts $(1 \leftrightarrow 2)$, $(3 \leftrightarrow 4)$, and $(5 \leftrightarrow 6)$ and adding one massless $\mathbf{6}-\mathbf{6}$ vector pair. The additional vector pair slightly lowers the $SU(4)_{H_2}$ condensation scale to around

$$\Lambda_{H_2} = \mathrm{e}^{-17.5} M_{Pl} \sim 6 \times 10^{10} \text{ GeV}. \tag{4.19}$$

Hidden sector condensation scale of this order, together with the hidden sector matter condensates and the superpotential terms eq. (4.14), can indeed induce supersymmetry breaking at a phenomenologically viable scale [11, 1]. As has been argued above, the LRS string model discussed here does not give rise to a phenomenologically viable vacuum. The new interesting feature of our LRS string model (2.1,2.2) is the fact that supersymmetry is hierarchically broken in the vacuum because of the necessity to utilize non–Abelian VEVs.

5 Discussion

The free fermionic heterotic string models are the only known string models that reproduced the Minimal Supersymmetric Standard Model in the effective low energy field theory. The important characteristics of such models is the generation of solely the MSSM spectrum in the effective low energy field theory, as well as the canonical normalization of the weak–hypercharge, and the general GUT embedding of the Standard Model spectrum, like the $SO(10)$ embedding. These characteristics are well motivated by the structure of the Standard Model itself, as well as the MSSM gauge coupling unification prediction. This should be contrasted with the case of type I string models in which one does not obtain the compelling GUT picture, but in which the Standard Model gauge group arises from a product of $U(n)$ groups [22]. The phenomenological success of the free fermionic models gives rise to the possibility that the true string vacuum lies in the vicinity of these models and justifies the continued efforts to understand the general properties of these models.

In this paper we examined the existence of a supersymmetric vacuum in the LRS free fermionic heterotic string models. This class of models exhibits new features with respect to the anomalous $U(1)$ symmetry. In ref. [6], in contrast to the case of the FSU5, SLM and PS free fermionic string models, the existence of some LRS string models with vanishing $U(1)_A$ was demonstrated, whereas some LRS models did contain an anomalous $U(1)$ symmetry. In this paper we observed the absence of singlet flat directions in the LRS string model which contained an anomalous $U(1)$. This observation prompted the exciting possibility that a supersymmetric vacuum does not exist in this model, and that a non–vanishing Fayet–Iliopoulos D–term is generated at one–loop in string perturbation theory, whereas the one–loop partition function still vanishes because of the one–loop fermion–boson degeneracy. However, we demonstrated that a D–flat vacuum, as well as F–flat to all finite orders, does exist if some non–Abelian fields in the massless string spectrum obtain a non–vanishing VEV. This is the first instance in the study of the realistic free fermionic models in which non–Abelian VEVs are enforced by the requirement of a stable vacuum rather than by other phenomenological considerations. This situation still raises interesting prospects for the issue of supersymmetry breaking for the following reasons. It has been shown that if one utilizes solely singlet VEVs in the cancellation of the Fayet–Iliopoulos D–term then the vacuum can remain supersymmetric to all orders. In contrast it has been argued in the past that utilizing non–Abelian VEVs necessarily results in generation of superpotential mass terms via hidden sector matter condensates that results in hierarchical supersymmetry breaking [11, 1]. The study of these issues in closely related models is therefore of further interest and will be reported in future publications.

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Left–Right Symmetric Model 1 Fields

F	SEC	$(C;L;R)$	Q_A	$Q_{1'}$	$Q_{2'}$	$Q_{3'}$	$Q_{4'}$	$Q_{5'}$	$Q_{7''}$	$Q_{8'}$	$SU(4)_{H_{1;2}}$
Q_{L_1}		$(4,2,1)$	-2	2	2	4	-2	-2	0	8	$(1,1)$
Q_{R_1}	$\mathbf{b}_1\oplus$	$(\bar{4},1,2)$	-2	-2	-2	-4	-2	-2	0	-8	$(1,1)$
L_{L_1}	$\mathbf{b}_1+\zeta+2\gamma$	$(1,2,1)$	-2	2	2	-20	-2	-2	0	0	$(1,1)$
L_{R_1}		$(1,1,2)$	-2	-2	-2	20	-2	-2	0	0	$(1,1)$
$\mathcal{L}_{L_1}$		$(1,2,1)$	2	2	2	4	-2	2	0	-32	$(1,1)$
$\mathcal{L}_{R_1}$		$(1,1,2)$	2	-2	-2	-4	2	2	0	32	$(1,1)$
Q_{L_2}		$(4,2,1)$	-2	-2	2	4	2	-2	0	8	$(1,1)$
Q_{R_2}	$\mathbf{b}_2\oplus$	$(\bar{4},1,2)$	-2	2	-2	-4	2	-2	0	-8	$(1,1)$
L_{L_2}	$\mathbf{b}_2+\zeta+2\gamma$	$(1,2,1)$	-2	-2	2	-20	2	-2	0	0	$(1,1)$
L_{R_2}		$(1,1,2)$	-2	2	-2	20	2	-2	0	0	$(1,1)$
$\mathcal{L}_{L_2}$		$(1,2,1)$	2	-2	2	4	-2	2	0	-32	$(1,1)$
$\mathcal{L}_{R_2}$		$(1,1,2)$	2	2	-2	-4	2	2	0	32	$(1,1)$
Q_{L_3}		$(4,2,1)$	-2	0	-4	4	0	4	0	8	$(1,1)$
Q_{R_3}	$\mathbf{b}_3\oplus$	$(\bar{4},1,2)$	-2	0	4	-4	0	4	0	-8	$(1,1)$
L_{L_3}	$\mathbf{b}_3+\zeta+2\gamma$	$(1,2,1)$	-2	0	-4	-20	0	4	0	0	$(1,1)$
L_{R_3}		$(1,1,2)$	-2	0	4	20	0	4	0	0	$(1,1)$
$\mathcal{L}_{L_3}$		$(1,2,1)$	2	0	-4	4	0	-4	0	-32	$(1,1)$
$\mathcal{L}_{R_3}$		$(1,1,2)$	2	0	4	-4	0	-4	0	32	$(1,1)$
Φ_1		$(1,1,1)$	0	0	0	0	0	0	0	0	$(1,1)$
Φ_2		$(1,1,1)$	0	0	0	0	0	0	0	0	$(1,1)$
Φ_3		$(1,1,1)$	0	0	0	0	0	0	0	0	$(1,1)$
Φ_{12}	Neveu-	$(1,1,1)$	0	-8	0	0	0	0	0	0	$(1,1)$
$\bar{\Phi}_{12}$	Schwarz	$(1,1,1)$	0	8	0	0	0	0	0	0	$(1,1)$
Φ_{23}		$(1,1,1)$	0	4	-12	0	0	0	0	0	$(1,1)$
$\bar{\Phi}_{23}$		$(1,1,1)$	0	-4	12	0	0	0	0	0	$(1,1)$
Φ_{31}		$(1,1,1)$	0	-4	-12	0	0	0	0	0	$(1,1)$
$\bar{\Phi}_{31}$		$(1,1,1)$	0	4	12	0	0	0	0	0	$(1,1)$
D_3		$(4,1,1)$	0	0	4	8	0	0	0	-24	$(1,1)$
$\bar{D}_3$		$(\bar{4},1,1)$	0	0	-4	-8	0	0	0	24	$(1,1)$
$\phi_{\alpha\beta}$		$(1,1,1)$	0	0	-12	0	0	0	0	0	$(1,1)$
$\bar{\phi}_{\alpha\beta}$	$\xi\equiv\mathbf{S}+\mathbf{b}_1+\mathbf{b}_2+$	$(1,1,1)$	0	0	12	0	0	0	0	0	$(1,1)$
ϕ_1	$\alpha+\beta$	$(1,1,1)$	0	4	0	0	0	0	0	0	$(1,1)$
$\bar{\phi}_1$	$\oplus$	$(1,1,1)$	0	-4	0	0	0	0	0	0	$(1,1)$
ϕ_2	$\xi+\zeta$	$(1,1,1)$	0	4	0	0	0	0	0	0	$(1,1)$
$\bar{\phi}_2$		$(1,1,1)$	0	-4	0	0	0	0	0	0	$(1,1)$
S_8		$(1,1,1)$	0	0	4	-16	0	0	0	-32	$(1,1)$
$\bar{S}_8$		$(1,1,1)$	0	0	-4	16	0	0	0	32	$(1,1)$

Table 1: *Model 1 fields.*

Left–Right Symmetric Model 1 Fields Continued

F	SEC	$(C;L;R)$	Q_A	$Q_{1'}$	$Q_{2'}$	$Q_{3'}$	$Q_{4'}$	$Q_{5'}$	$Q_{7''}$	$Q_{8'}$	$SU(4)_{H_{1;2}}$
D_1		$(4,1,1)$	0	2	2	4	0	0	-8	8	$(1,1)$
$\bar{D}_1$		$(\bar{4},1,1)$	0	-2	-2	-4	0	0	8	-8	$(1,1)$
S_1	$\xi \equiv \mathbf{S}+\mathbf{b}_2+\mathbf{b}_3+$	$(1,1,1)$	0	-2	6	-12	0	0	8	16	$(1,1)$
$\bar{S}_1$	$\beta+\gamma$	$(1,1,1)$	0	2	-6	12	0	0	-8	-16	$(1,1)$
S_2	$\oplus$	$(1,1,1)$	0	2	-6	-12	0	0	8	16	$(1,1)$
$\bar{S}_2$	$\xi+\zeta+2\gamma$	$(1,1,1)$	0	-2	6	12	0	0	-8	-16	$(1,1)$
$\mathcal{H}_1$		$(1,1,1)$	0	2	2	-8	0	0	0	-16	$(1,6)$
$\bar{\mathcal{H}}_1$		$(1,1,1)$	0	-2	-2	8	0	0	0	16	$(1,6)$
D_2		$(4,1,1)$	0	-2	2	4	0	0	-8	8	$(1,1)$
$\bar{D}_2$		$(\bar{4},1,1)$	0	2	-2	-4	0	0	8	-8	$(1,1)$
S_3	$\xi \equiv \mathbf{S}+\mathbf{b}_1+\mathbf{b}_3+$	$(1,1,1)$	0	2	6	-12	0	0	8	16	$(1,1)$
$\bar{S}_3$	$\alpha+\gamma$	$(1,1,1)$	0	-2	-6	12	0	0	-8	-16	$(1,1)$
S_4	$\oplus$	$(1,1,1)$	0	-2	-6	-12	0	0	8	16	$(1,1)$
$\bar{S}_4$	$\xi+\zeta+2\gamma$	$(1,1,1)$	0	2	6	12	0	0	-8	-16	$(1,1)$
$\mathcal{H}_2$		$(1,1,1)$	0	-2	2	-8	0	0	0	-16	$(1,6)$
$\bar{\mathcal{H}}_2$		$(1,1,1)$	0	2	-2	8	0	0	0	16	$(1,6)$
H_1	$\xi \equiv \mathbf{S}+\mathbf{b}_2+\mathbf{b}_3+$	$(1,1,1)$	-4	-2	-2	2	2	2	-4	-16	$(1,4)$
$\bar{H}_{1'}$	$\alpha+2\gamma$	$(1,1,1)$	-4	2	2	-2	2	2	4	16	$(1,\bar{4})$
H_2	$\oplus$	$(1,1,1)$	-4	-2	-2	2	2	2	4	-16	$(4,1)$
$\bar{H}_{2'}$	$\xi+\zeta$	$(1,1,1)$	-4	2	2	-2	2	2	-4	16	$(\bar{4},1)$
H_3	$\xi \equiv \mathbf{S}+\mathbf{b}_1+\mathbf{b}_3+$	$(1,1,1)$	-4	2	-2	2	-2	2	-4	-16	$(1,4)$
$\bar{H}_{3'}$	$\alpha+2\gamma$	$(1,1,1)$	-4	-2	2	-2	-2	2	4	16	$(1,\bar{4})$
H_4	$\oplus$	$(1,1,1)$	-4	2	-2	2	-2	2	4	-16	$(4,1)$
$\bar{H}_{4'}$	$\xi+\zeta$	$(1,1,1)$	-4	-2	2	-2	-2	2	-4	16	$(\bar{4},1)$
H_5	$\xi \equiv \mathbf{S}+\mathbf{b}_1+\mathbf{b}_2+$	$(1,1,1)$	-4	0	4	2	0	-4	-4	-16	$(1,4)$
$\bar{H}_{5'}$	$\alpha+2\gamma$	$(1,1,1)$	-4	0	-4	-2	0	-4	4	16	$(1,\bar{4})$
H_6	$\oplus$	$(1,1,1)$	-4	0	4	2	0	-4	4	-16	$(4,1)$
$\bar{H}_{6'}$	$\xi+\zeta+2\gamma$	$(1,1,1)$	-4	0	-4	-2	0	-4	-4	16	$(\bar{4},1)$
S_5		$(1,1,1)$	0	0	-8	-16	0	0	0	-32	$(1,1)$
$\bar{S}_5$		$(1,1,1)$	0	0	8	16	0	0	0	32	$(1,1)$
S_6	$\mathbf{S}+\zeta+2\gamma$	$(1,1,1)$	0	-4	4	-16	0	0	0	-32	$(1,1)$
$\bar{S}_6$		$(1,1,1)$	0	4	-4	16	0	0	0	32	$(1,1)$
S_7		$(1,1,1)$	0	4	4	-16	0	0	0	-32	$(1,1)$
$\bar{S}_7$		$(1,1,1)$	0	-4	-4	16	0	0	0	32	$(1,1)$

Table 1: *Model 1 fields continued.*

Basis	$Q^{(\mathrm{A})}$	$S_5/\bar{S}_5$	$\bar{D}_3/D_3$	$\bar{D}_2/D_2$	$\mathcal{H}_2/\bar{\mathcal{H}}_2$	$\mathcal{H}_1/\bar{\mathcal{H}}_1$	$\bar{H}_{4'}$	$\bar{H}_{2'}$		
$\mathbf{D}_1$	12	0	-1	1	-2	3	0	-2	2	$\mathcal{L}_{R1}$
$\mathbf{D}_2$	12	-1	3	-1	0	1	0	-2	2	$\mathcal{L}_{L2}$
$\mathbf{D}_3$	12	-1	3	-1	0	1	-2	0	2	$\mathcal{L}_{L1}$
$\mathbf{D}_4$	12	0	-1	-1	2	-1	-2	0	2	$\mathcal{L}_{R2}$
$\mathbf{D}_7$	24	-1	4	-4	-2	2	-4	-4	4	Q_{R3}
$\mathbf{D}_5$	24	-3	4	-4	2	6	-4	-4	4	Q_{L3}
$\mathbf{D}_6$	24	-1	8	-4	2	6	-4	-4	4	L_{R3}
$\mathbf{D}_8$	24	-3	0	-4	-2	2	-4	-4	2	L_{L3}
$\mathbf{D}_9$	-12	0	-1	1	0	-1	2	0	2	Q_{R2}
$\mathbf{D}_{10}$	-12	1	-3	1	-4	-1	2	0	2	L_{L2}
$\mathbf{D}_{11}$	-12	1	-1	1	-2	1	2	0	2	Q_{L2}
$\mathbf{D}_{13}$	-12	0	1	1	2	1	2	0	2	L_{R2}
$\mathbf{D}_{12}$	-12	0	1	1	2	1	0	2	2	L_{R1}
$\mathbf{D}_{14}$	-12	1	-3	1	0	-5	0	2	2	L_{L1}
$\mathbf{D}_{15}$	-12	0	-1	1	0	-1	0	2	2	Q_{R1}
$\mathbf{D}_{16}$	-12	1	-1	1	2	-3	0	2	2	Q_{L1}
$\mathbf{D}_{17}$	-24	1	-3	1	0	-1	2	2	2	$\bar{H}_{5'}$
$\mathbf{D}_{18}$	-24	1	-1	3	0	-3	2	2	2	H_5
$\mathbf{D}_{19}$	-24	-1	0	4	2	-2	4	4	4	L_{L3}
$\mathbf{D}_{20}$	-24	5	-8	4	-2	-6	4	4	4	L_{R3}
$\mathbf{D}_{21}$	-48	1	-6	6	2	-4	4	4	4	$\bar{H}_{6'}$
$\mathbf{D}_{22}$	-48	3	-2	2	-2	-4	4	4	4	H_6
$\mathbf{D}_{40}$	0	-3	4	0	-2	6	0	-4	4	H_1
$\mathbf{D}_{37}$	0	1	0	-4	-2	2	0	-4	4	$\bar{H}_{1'}$
$\mathbf{D}_{41}$	0	-3	4	0	6	-2	-4	0	4	H_3
$\mathbf{D}_{29}$	0	1	0	-4	-2	2	-4	0	4	$\bar{H}_{3'}$
$\mathbf{D}_{34}$	0	-1	2	-2	-2	4	0	-2	2	H_2
$\mathbf{D}_{39}$	0	-1	2	-2	2	0	0	-2	2	H_4
$\mathbf{D}^v_{32}$	0	0	0	0	-2	2	0	0	1	Φ_{12}
$\mathbf{D}^v_{35}$	0	-1	0	0	2	0	0	0	1	Φ_{23}
$\mathbf{D}^v_{28}$	0	-1	0	0	0	2	0	0	1	Φ_{31}
$\mathbf{D}^v_{31}$	0	-1	0	0	1	1	0	0	1	S_8
$\mathbf{D}^v_{25}$	0	0	0	0	-1	-1	0	0	1	$\phi_{\alpha\beta}$
$\mathbf{D}^v_{33}$	0	0	0	0	-1	1	0	0	1	$\bar{\phi}_{1,2}$
$\mathbf{D}^v_{30}$	0	0	-1	-1	-1	1	0	0	1	S_4
$\mathbf{D}^v_{27}$	0	0	-1	-1	0	0	0	0	1	S_2
$\mathbf{D}^v_{36}$	0	1	-1	-1	-1	-1	0	0	1	S_3
$\mathbf{D}^v_{24}$	0	1	-1	-1	-2	0	0	0	1	S_1
$\mathbf{D}^v_{23}$	0	0	0	0	0	-2	0	0	1	S_7
$\mathbf{D}^v_{38}$	0	0	0	-2	0	0	0	0	1	S_6
$\mathbf{D}^v_{26}$	0	0	0	-1	-1	1	0	0	1	$\bar{D}_1$

Table 2: *D–Flat Direction Basis Set for Model 3.*