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Supersymmetry Breaking Scalar Masses and Trilinear Soft Terms in Generalized Minimal Supergravity

Csaba Balazs[1], **Tianjun Li**[2,3], **Dimitri V. Nanopoulos**[2,4,5], **Fei Wang**[1]

[1] *School of Physics, Monash University, Melbourne Victoria 3800, Australia*
[2] *George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA*
[3] *Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China*
[4] *Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA*
[5] *Academy of Athens, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece*

Abstract: In the generalized minimal supergravity (GmSUGRA) scenario, we systematically study the supersymmetry breaking scalar masses, Standard Model fermion Yukawa coupling terms, and trilinear soft terms in $SU(5)$ models with the Higgs fields in the **24** and **75** representations, and in $SO(10)$ models where the gauge symmetry is broken down to the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry, $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry, George-Glashow $SU(5) \times U(1)'$ gauge symmetry, flipped $SU(5) \times U(1)_X$ gauge symmetry, and $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ gauge symmetry. Most importantly, we for the first time consider the scalar and gaugino mass relations, which can be preserved from the unification scale to the electroweak scale under one-loop renormalization group equation running, in the $SU(5)$ models, the Pati-Salam models and flipped $SU(5) \times U(1)_X$ models arising from $SO(10)$ models. With such interesting relations, we may distinguish the minimal supergravity (mSUGRA) and GmSUGRA scenarios if the supersymmetric particle spectrum can be measured at the LHC and ILC. Thus, it provides us with another important window of opportunity at the Planck scale.

Contents

1. Introduction

Supersymmetry naturally solves the gauge hierarchy problem of the Standard Model (SM). The unification of the three gauge couplings $SU(3)_C, SU(2)_L$ and $U(1)_Y$ in the supersymmetric Standard Model at about 2×10^{16} GeV [1] strongly suggests the existence of Grand Unified Theories (GUTs). In addition, supersymmetric GUTs such as $SU(5)$ [2] or $SO(10)$ [3] models give us deep insights into the other SM problems such as the emergence of the fundamental forces, the assignments and quantization of their charges, the fermion masses and mixings, and beyond. Although supersymmetric GUTs are attractive it is challenging to test them at the Large Hadron Collider (LHC), the future International Linear Collider (ILC), and other experiments.

In traditional supersymmetric SMs supersymmetry is broken in the hidden sector and the supersymmetry breaking effects can be mediated to the observable sector via gravity [4], gauge interactions [5, 6], or super-Weyl anomaly [7, 8, 9], or other mechanisms. However, the relations between the supersymmetric particle (sparticle) spectra and the fundamental theories can be very complicated and model dependent. An important observation is that compared to the supersymmetry breaking soft masses of squarks and sleptons (scalar masses), gaugino masses have a simpler form and are less model dependent [10, 11]. In the minimal supergravity (mSUGRA) scenario [4] supersymmetry breaking is mediated by gravity and gauge couplings and gaugino masses are unified at the GUT scale. Thus, a relation holds between the the gauge couplings and the gaugino masses at the GUT scale M_{GUT}:

$$\frac{1}{\alpha_3} = \frac{1}{\alpha_2} = \frac{1}{\alpha_1} , \tag{1.1}$$

$$\frac{M_3}{\alpha_3} = \frac{M_2}{\alpha_2} = \frac{M_1}{\alpha_1} , \tag{1.2}$$

where α_3, α_2, and $\alpha_1 \equiv 5\alpha_Y/3$ (M_3, M_2, and M_1) are gauge couplings (gaugino masses) for the $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ gauge symmetries, respectively. Because M_i/α_i are constant under renormalization group evolution, the gaugino mass relation in Eq. (1.2) is valid from the GUT scale to the electroweak scale at one loop. Two-loop renormalization group effects on gaugino masses are very small, thus, we can test this gaugino mass relation at the LHC and ILC where the gaugino masses can be measured [12, 13]. Recently, considering GUTs with high-dimensional operators [5, 10, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23] and F-theory GUTs with $U(1)$ fluxes [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35], two of us (TL and DN) proposed the generalized mSUGRA (GmSUGRA) scenario, and studied the generic gaugino mass relations and defined their indices [36]. The gaugino mass relations and their indices have also been studied for general gauge and anomaly mediated supersymmetry breaking in GUTs with vector-like particles [37].

In this paper, we consider the supersymmetry breaking scalar masses and trilinear soft terms in the GmSUGRA. We briefly review GUTs and consider the general gravity mediated supersymmetry breaking. With the high-dimensional operators including the GUT Higgs fields, we systematically calculate the supersymmetry breaking scalar masses, SM fermion Yukawa coupling terms, and trilinear soft terms in $SU(5)$ models with GUT Higgs fields in the **24** and **75** representations, and in $SO(10)$ models where the gauge symmetry is broken down to the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry, $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry, George-Glashow $SU(5) \times U(1)'$ gauge symmetry, flipped $SU(5) \times U(1)_X$ gauge symmetry [38, 39, 40], and $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ gauge symmetry. We examine the scalar and gaugino mass relations, which are valid from the GUT scale to the electroweak scale under one-loop renormalization group running, in the $SU(5)$ models, the Pati-Salam models and flipped $SU(5) \times U(1)_X$ models arising from the $SO(10)$ model. With these relations, we may distinguish the mSUGRA and GmSUGRA scenarios if the supersymmetric particle spectrum can be measured at the LHC and ILC.

This paper is organized as follows. In Section 2, we briefly review four-dimensional GUTs. In Section 3, we explain the general gravity mediated supersymmetry breaking. In Section 4, we discuss the scalar masses, the SM fermion Yukawa coupling terms, and trilinear soft terms in the $SU(5)$ model. For models arising from $SO(10)$, we derive the scalar masses in Section 5, and the SM fermion Yukawa coupling terms and trilinear soft terms in Section 6. In Section 7 we consider the scalar and gaugino mass relations. Section 8 contains our conclusions.

2. Brief Review of Grand Unified Theories

In this Section we explain our conventions. In supersymmetric SMs, we denote the left-handed quark doublets, right-handed up-type quarks, right-handed down-type quarks, left-handed lepton doublets, right-handed neutrinos and right-handed charged leptons as Q_i, U_i^c, D_i^c, L_i, N_i^c, and E_i^c, respectively. Also, we denote one pair of Higgs doublets as H_u and H_d, which give masses to the up-type quarks/neutrinos and the down-type quarks/charged leptons, respectively.

First, we briefly review the $SU(5)$ model. We define the $U(1)_Y$ hypercharge generator in $SU(5)$ as follows

$$T_{\mathrm{U(1)_Y}} = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right) \ . \tag{2.1}$$

Under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, the $SU(5)$ representations are decomposed as follows

$$\mathbf{5} = (\mathbf{3}, \mathbf{1}, \mathbf{-1/3}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{1/2}) \ , \tag{2.2}$$

$$\overline{\mathbf{5}} = (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1/3}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{-1/2}) \ , \tag{2.3}$$

$$\mathbf{10} = (\mathbf{3}, \mathbf{2}, \mathbf{1/6}) \oplus (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{-2/3}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{1}) \ , \tag{2.4}$$

$$\overline{\mathbf{10}} = (\overline{\mathbf{3}}, \mathbf{2}, \mathbf{-1/6}) \oplus (\mathbf{3}, \mathbf{1}, \mathbf{2/3}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{-1}) \ , \tag{2.5}$$

$$\mathbf{24} = (\mathbf{8}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{0}) \oplus (\mathbf{3}, \mathbf{2}, \mathbf{-5/6}) \oplus (\overline{\mathbf{3}}, \mathbf{2}, \mathbf{5/6}) \ . \tag{2.6}$$

There are three families of the SM fermions whose quantum numbers under $SU(5)$ are

$$F_i' = \mathbf{10}, \ \overline{f}_i' = \overline{\mathbf{5}}, \ N_i^c = \mathbf{1} \ , \tag{2.7}$$

where $i = 1, 2, 3$ for three families. The SM particle assignments in F_i' and $\bar{f}_i'$ are

$$F_i' = (Q_i, U_i^c, E_i^c) \ , \ \overline{f}_i' = (D_i^c, L_i) \ . \tag{2.8}$$

To break the $SU(5)$ gauge symmetry and electroweak gauge symmetry, we introduce the adjoint Higgs field and one pair of Higgs fields whose quantum numbers under $SU(5)$ are

$$\Phi' \ = \ \mathbf{24} \ , \quad h' \ = \ \mathbf{5} \ , \quad \overline{h}' \ = \ \bar{\mathbf{5}} \ , \tag{2.9}$$

where h' and $\overline{h}'$ contain the Higgs doublets H_u and H_d, respectively.

Second, we briefly review the flipped $SU(5) \times U(1)_X$ model [38, 39, 40]. The gauge group $SU(5) \times U(1)_X$ can be embedded into $SO(10)$. We define the generator $U(1)_{Y'}$ in $SU(5)$ as

$$T_{\mathrm{U(1)_{Y'}}} = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right). \tag{2.10}$$

The hypercharge is given by

$$Q_Y = \frac{1}{5}\left(Q_X - Q_{Y'}\right). \tag{2.11}$$

There are three families of the SM fermions whose quantum numbers under $SU(5) \times U(1)_X$ are

$$F_i = (\mathbf{10}, \mathbf{1}), \ \bar{f}_i = (\bar{\mathbf{5}}, \mathbf{-3}), \ \bar{l}_i = (\mathbf{1}, \mathbf{5}), \tag{2.12}$$

where $i = 1, 2, 3$. The particle assignments for the SM fermions are

$$F_i = (Q_i, D_i^c, N_i^c) \ , \quad \overline{f}_i = (U_i^c, L_i) \ , \quad \bar{l}_i = E_i^c \ . \tag{2.13}$$

To break the GUT and electroweak gauge symmetries, we introduce two pairs of Higgs fields whose quantum numbers under $SU(5) \times U(1)_X$ are

$$H = (\mathbf{10}, \mathbf{1}) \ , \quad \overline{H} = (\overline{\mathbf{10}}, \mathbf{-1}) \ , \quad h = (\mathbf{5}, \mathbf{-2}) \ , \quad \overline{h} = (\bar{\mathbf{5}}, \mathbf{2}) \ , \tag{2.14}$$

where h and $\overline{h}$ contain the Higgs doublets H_d and H_u, respectively.

Moreover, the flipped $SU(5) \times U(1)_X$ models can be embedded into $SO(10)$. Under the $SU(5) \times U(1)_X$ gauge symmetry, the $SO(10)$ representations are decomposed as follows

$$\mathbf{10} \ = \ (\mathbf{5}, \mathbf{-2}) \oplus (\overline{\mathbf{5}}, \mathbf{2}) \ , \tag{2.15}$$

$$\mathbf{16} \ = \ (\mathbf{10}, \mathbf{1}) \oplus (\overline{\mathbf{5}}, \mathbf{-3}) \oplus (\mathbf{1}, \mathbf{5}) \ , \tag{2.16}$$

$$\mathbf{45} \ = \ (\mathbf{24}, \mathbf{0}) \oplus (\mathbf{1}, \mathbf{0}) \oplus (\mathbf{10}, \mathbf{-4}) \oplus (\overline{\mathbf{10}}, \mathbf{4}) \ . \tag{2.17}$$

Third, we briefly review the Pati-Salam model. The gauge group is $SU(4)_C \times SU(2)_L \times SU(2)_R$ which can also be embedded into $SO(10)$. There are three families of the SM fermions whose quantum numbers under $SU(4)_C \times SU(2)_L \times SU(2)_R$ are

$$F_i^L = (\mathbf{4}, \mathbf{2}, \mathbf{1}) \ , \quad F_i^{Rc} = (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \ , \tag{2.18}$$

where $i = 1, 2, 3$. Also, the particle assignments for the SM fermions are

$$F_i^L = (Q_i, L_i) \ , \quad F_i^{Rc} = (U_i^c, D_i^c, E_i^c, N_i^c) \ . \tag{2.19}$$

To break the Pati-Salam and electroweak gauge symmetries, we introduce one pair of Higgs fields and one bidoublet Higgs field whose quantum numbers under $SU(4)_C \times SU(2)_L \times SU(2)_R$ are

$$\Phi = (\mathbf{4}, \mathbf{1}, \mathbf{2}) \ , \quad \overline{\Phi} = (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \ , \quad H' = (\mathbf{1}, \mathbf{2}, \mathbf{2}) \ , \tag{2.20}$$

where H' contains one pair of the Higgs doublets H_d and H_u.

The Pati-Salam model can be embedded into $SO(10)$ as well. Under $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry, the $SO(10)$ representations are decomposed as follows

$$\mathbf{10} = (\mathbf{6}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{2}) \ , \tag{2.21}$$

$$\mathbf{16} = (\mathbf{4}, \mathbf{2}, \mathbf{1}) \oplus (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \ , \tag{2.22}$$

$$\mathbf{45} = (\mathbf{15}, \mathbf{1}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{6}, \mathbf{2}, \mathbf{2}) \ . \tag{2.23}$$

3. General Gravity Mediated Supersymmetry Breaking

The supegravity scalar potential can be written as [4]

$$V = M_*^4 e^G \left[G^i (G^{-1})_i^j G_j - 3 \right] + \frac{1}{2} \mathrm{Re} \left[(f^{-1})_{ab} \hat{D}^a \hat{D}^b \right] \ , \tag{3.1}$$

where M_* is the fundamental scale, D-terms are

$$\hat{D}^a \equiv -\, G^i (T^a)_i^j \phi_j = -\phi^{j*} (T^a)_j^i G_i \ , \tag{3.2}$$

and the Kähler function G as well as its derivatives and metric G_i^j are

$$G \equiv \frac{K}{M_*^2} + \ln\left(\frac{W}{M_*^3}\right) + \ln\left(\frac{W^*}{M_*^3}\right) \ , \tag{3.3}$$

$$G^i = \frac{\delta G}{\delta \phi_i} \ , \quad G_i = \frac{\delta G}{\delta \phi_i^*} \ , \quad G_i^j = \frac{\delta^2 G}{\delta \phi_i^* \delta \phi_j} \ , \tag{3.4}$$

where K is Kähler potential and W is superpotential.

Because the gaugino masses have been studied previously [36], we only consider the supersymmetry breaking scalar masses and trilinear soft terms in this paper. To break supersymmetry, we introduce a chiral superfield S in the hidden sector whose F term acquires a vacuum expectation value (VEV), $i.e$, $\langle S \rangle = \theta^2 F_S$. To calculate the scalar masses and trilinear soft terms, we consider the following superpotential and Kähler potential

$$W = \frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k + \alpha \frac{S}{M_*} \left(\frac{1}{6} y^{ijk} \phi_i \phi_j \phi_k \right) \ , \tag{3.5}$$

$$K = \phi_i^\dagger \phi_i + \beta \frac{S^\dagger S}{M_*^2} \phi_i^\dagger \phi_i \ , \tag{3.6}$$

where y^{ijk}, α, and β are Yukawa couplings. Thus, we obtain the universal supersymmetry breaking scalar mass m_0 and trilinear soft term A of mSUGRA

$$m_0^2 \;=\; \beta\frac{|F_S|^2}{M_*^2}~, \quad A \;=\; \alpha\frac{F_S}{M_*}~. \tag{3.7}$$

When we break the GUT gauge symmetry by giving VEV to the Higgs field Φ, we can have the general superpotential and Kähler potential

$$\begin{aligned} W =\; & \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k + \frac{1}{6}\left(h^{ijk}\frac{\Phi}{M_*}\phi_i\phi_j\phi_k\right) + \alpha\frac{S}{M_*}\left(\frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k\right) \\ & + \alpha'\frac{T}{M_*}\left(\frac{1}{6}y^{ijk}\frac{\Phi}{M_*}\phi_i\phi_j\phi_k\right)~, \end{aligned} \tag{3.8}$$

$$K = \phi_i^\dagger\phi_i + \frac{1}{2}h'\phi_i^\dagger\left(\frac{\Phi}{M_*}+\frac{\Phi^\dagger}{M_*}\right)\phi_i + \beta\frac{S^\dagger S}{M_*^2}\phi_i^\dagger\phi_i + \frac{1}{2}\beta'\frac{T^\dagger T}{M_*^2}\phi_i^\dagger\left(\frac{\Phi}{M_*}+\frac{\Phi^\dagger}{M_*}\right)\phi_i~, \tag{3.9}$$

where h^{ijk}, α', β' and h' are Yukawa couplings, and T can be S or another chiral superfield with non-zero F term, *i.e*, $\langle T\rangle = \theta^2 F_T$. Therefore, after the GUT gauge symmetry is broken by the VEV of Φ, we obtain the non-universal supersymmetry breaking scalar masses and trilinear soft terms, which will be studied in the following. For simplicity, we assume $h' = 0$ in the following discussions since we can redefine the fields and the SM fermion Yukawa couplings.

4. Scalar Masses and Trilinear Soft Terms in the $SU(5)$ Model

First, we study the supersymmetry breaking scalar masses. In order to construct gauge invariant high-dimensional operators, we need the decompositions of the following tensor products

$$\bar{\mathbf{5}}\otimes\mathbf{5} \;=\; \mathbf{1}\oplus\mathbf{24}~, \tag{4.1}$$

$$\overline{\mathbf{10}}\otimes\mathbf{10} = \mathbf{1}\oplus\mathbf{24}\oplus\mathbf{75}~. \tag{4.2}$$

Thus, the adjoint Higgs field can give scalar masses to both F_i' and $\overline{f}_i'$, while the Higgs field in the **75** representation can only give soft masses to F_i'. The VEVs of the Higgs field $\Phi_{\mathbf{24}}$ in the adjoint representation are expressed as 5×5 and 10×10 matrices

$$\langle\Phi_{\mathbf{24}}\rangle \;=\; v\sqrt{\frac{3}{5}}\mathrm{diag}\left(-\frac{1}{3},-\frac{1}{3},-\frac{1}{3},\frac{1}{2},\frac{1}{2}\right)~, \tag{4.3}$$

$$\langle\Phi_{\mathbf{24}}\rangle \;=\; v\sqrt{\frac{3}{5}}\mathrm{diag}(\underbrace{-\frac{2}{3},\cdots,-\frac{2}{3}}_{3},\underbrace{\frac{1}{6},\cdots,\frac{1}{6}}_{6},\;1)~, \tag{4.4}$$

which are normalized to $c=1/2$ and $c=3/2$, respectively. Thus, we obtain the following scalar masses

$$m^2_{\widetilde{Q}_i} = (m_0^U)^2 + \sqrt{\frac{3}{5}}\beta'_{\mathbf{10}}\frac{1}{6}(m_0^N)^2 \ , \tag{4.5}$$

$$m^2_{\widetilde{U}^c_i} = (m_0^U)^2 - \sqrt{\frac{3}{5}}\beta'_{\mathbf{10}}\frac{2}{3}(m_0^N)^2 \ , \tag{4.6}$$

$$m^2_{\widetilde{E}^c_i} = (m_0^U)^2 + \sqrt{\frac{3}{5}}\beta'_{\mathbf{10}}(m_0^N)^2 \ , \tag{4.7}$$

$$m^2_{\widetilde{D}^c_i} = (m_0^U)^2 + \sqrt{\frac{3}{5}}\beta'_{\bar{\mathbf{5}}}\frac{1}{3}(m_0^N)^2 \ , \tag{4.8}$$

$$m^2_{\widetilde{L}_i} = (m_0^U)^2 - \sqrt{\frac{3}{5}}\beta'_{\bar{\mathbf{5}}}\frac{1}{2}(m_0^N)^2 \ , \tag{4.9}$$

where we introduced

$$(m_0^U)^2 \ \equiv \ \frac{\beta}{M_*^2}F_S^*F_S \ , \quad (m_0^N)^2 = \frac{v}{M_*^3}F_T^*F_T \ . \tag{4.10}$$

Because the second non-universal terms are proportional to the hypercharge for each fields, we obtain general relations among the supersymmetry breaking scalar masses

$$\frac{Y_{L_i}m^2_{\widetilde{D}^c_i} - Y_{D^c_i}m^2_{\widetilde{L}_i}}{Y_{L_i} - Y_{D^c_i}} = \frac{Y_{U^c_i}m^2_{\widetilde{Q}_i} - Y_{Q_i}m^2_{\widetilde{U}^c_i}}{Y_{U^c_i} - Y_{Q_i}} = \frac{Y_{E^c_i}m^2_{\widetilde{Q}_i} - Y_{Q_i}m^2_{\widetilde{E}^c_i}}{Y_{E^c_i} - Y_{Q_i}} = \frac{Y_{U^c_i}m^2_{\widetilde{E}^c_i} - Y_{E^c_i}m^2_{\widetilde{U}^c_i}}{Y_{U^c_i} - Y_{E^c_i}} \ ,$$

which give the scalar mass relations at the GUT scale M_U

$$3m^2_{\widetilde{D}^c_i} + 2m^2_{\widetilde{L}_i} = 4m^2_{\widetilde{Q}_i} + m^2_{\widetilde{U}^c_i} = 6m^2_{\widetilde{Q}_i} - m^2_{\widetilde{E}^c_i} = 2m^2_{\widetilde{E}^c_i} + 3m^2_{\widetilde{U}^c_i} \ . \tag{4.11}$$

Next, we consider the Higgs field $\Phi^{[ij]}_{kl}$ in the **75** representation. Because the Higgs fields $\Phi_{\mathbf{24}}$ and $\Phi^{[ij]}_{kl}$ belong to the decomposition of the tensor product representation of $\overline{\mathbf{10}} \times \mathbf{10}$, their VEVs must be orthogonal to each other. Thus, we obtain the VEV of $\Phi^{[ij]}_{kl}$ in terms of the 10×10 matrix

$$\langle \Phi^{[ij]}_{kl} \rangle \ = \ \frac{v}{2\sqrt{3}}\mathrm{diag}\left(\underbrace{1, \cdots, \ 1}_{3}, \underbrace{-1, \cdots, -1}_{6}, 3 \right) \ . \tag{4.12}$$

So we obtain scalar masses

$$\begin{aligned}
m^2_{\widetilde{Q}_i} &= (m_0^U)^2 - \frac{\beta'_{\mathbf{75}}}{2\sqrt{3}}(m_0^N)^2 \ , \\
m^2_{\widetilde{U}^c_i} &= (m_0^U)^2 + \frac{\beta'_{\mathbf{75}}}{2\sqrt{3}}(m_0^N)^2 \ , \\
m^2_{\widetilde{E}^c_i} &= (m_0^U)^2 + 3\frac{\beta'_{\mathbf{75}}}{2\sqrt{3}}(m_0^N)^2 \ , \\
m^2_{\widetilde{D}^c_i} &= m^2_{\widetilde{L}_i} \ = \ (m_0^U)^2 \ ,
\end{aligned} \tag{4.13}$$

which respect the scalar mass relation at M_U

$$m^2_{\widetilde{E}^c_i} + m^2_{\widetilde{Q}_i} = 2m^2_{\widetilde{U}^c_i} ~. \tag{4.14}$$

Second, we study the supersymmetry breaking trilinear soft terms. For simplicity, we assume that the Yukawa couplings are diagonal. To get the possible high-dimensional operators for the trilinear soft terms, we need to consider the decompositions of the tensor products for the SM fermion Yukawa coupling terms [41]

$$\begin{aligned}
\mathbf{10} \otimes \mathbf{10} \otimes \mathbf{5} &= (\bar{\mathbf{5}} \oplus \overline{\mathbf{45}} \oplus \overline{\mathbf{50}}) \otimes \mathbf{5} \\
&= (\mathbf{1} \oplus \mathbf{24}) \oplus (\mathbf{24} \oplus \mathbf{75} \oplus \mathbf{126}) \oplus (\mathbf{75} \oplus \mathbf{175'}) ~, \qquad (4.15) \\
\mathbf{10} \otimes \bar{\mathbf{5}} \otimes \bar{\mathbf{5}} &= \mathbf{10} \otimes (\overline{\mathbf{10}} \oplus \overline{\mathbf{15}}) = (\mathbf{1} \oplus \mathbf{24} \oplus \mathbf{75}) \oplus (\mathbf{24} \oplus \overline{\mathbf{126}}) ~. \qquad (4.16)
\end{aligned}$$

Because the Higgs fields in the $\mathbf{126}$, $\overline{\mathbf{126}}$ and $\mathbf{175'}$ do not have the $SU(3)_C \times SU(2)_L$ singlets [41], we do not consider them in the following discussions. Thus, we only consider the Higgs fields in the $\mathbf{24}$ and $\mathbf{75}$ representations.

For the Higgs field $\Phi_{\mathbf{24}}$ in the $\mathbf{24}$ representation, we consider the following superpotential for the additional contributions to the Yukawa coupling terms and trilinear soft terms

$$\begin{aligned}
W \supset & \Big(h^{Ui} \epsilon^{mnpql} (F'_i)_{mn} (F'_i)_{pq} (h')_k (\Phi_{\mathbf{24}})^k_l + h'^{Ui} \epsilon^{mnpkl} (F'_i)_{mn} (F'_i)_{pq} (h')_k (\Phi_{\mathbf{24}})^q_l \\
& + h^{DEi} (F'_i)_{mn} (\overline{f}'_i \otimes \overline{h}')^{ml}_{Sym} (\Phi_{\mathbf{24}})^n_l + h'^{DEi} (F'_i)_{mn} (\overline{f}'_i \otimes \overline{h}')^{ml}_{Asym} (\Phi_{\mathbf{24}})^n_l \Big) \\
& + \alpha' \frac{T}{M_*} \Big(y^{Ui} \epsilon^{mnpql} (F'_i)_{mn} (F'_i)_{pq} (h')_k (\Phi_{\mathbf{24}})^k_l \\
& + y'^{Ui} \epsilon^{mnpkl} (F'_i)_{mn} (F'_i)_{pq} (h')_k (\Phi_{\mathbf{24}})^q_l + y^{DEi} (F'_i)_{mn} (\overline{f}'_i \otimes \overline{h}')^{ml}_{Sym} (\Phi_{\mathbf{24}})^n_l \\
& + y'^{DEi} (F'_i)_{mn} (\overline{f}'_i \otimes \overline{h}')^{ml}_{Asym} (\Phi_{\mathbf{24}})^n_l \Big) ~,
\end{aligned} \tag{4.17}$$

where the subscripts Sym and $Asym$ denote the symmetric and anti-symmetric products of two $\bar{\mathbf{5}}$ representations. After $\Phi_{\mathbf{24}}$ acquires a VEV, we obtain the Yukawa coupling terms in the superpotential

$$\begin{aligned}
W \supset \frac{v}{M_*} \sqrt{\frac{3}{5}} & \left(-2h^{Ui} Q_i U^c_i H_u - h'^{Ui} Q_i U^c_i H_u \right. \\
& \left. - \frac{1}{6} h'^{DEi} Q_i D^c_i H_d - h'^{DEi} L_i E^c_i H_d \right) ~.
\end{aligned} \tag{4.18}$$

We also obtain the supersymmetry breaking trilinear soft terms

$$\begin{aligned}
-\mathcal{L} \supset \alpha' \frac{F_T v}{M^2_*} \sqrt{\frac{3}{5}} & \left(-2y^{Ui} \widetilde{Q}_i \widetilde{U}^c_i H_u - y'^{Ui} \widetilde{Q}_i \widetilde{U}^c_i H_u \right. \\
& \left. - \frac{1}{6} y'^{DEi} \widetilde{Q}_i \widetilde{D}^c_i H_d - y'^{DEi} \widetilde{L}_i \widetilde{E}^c_i H_d \right) ~.
\end{aligned} \tag{4.19}$$

As a double check, we can obtain these results by choosing the VEVs of $\mathbf{24}$ dimensional Higgs field $\Phi_{\mathbf{24}}$ as appropriate 5×5 and 10×10 matrices.

We can write the VEV of the **75** dimensional Higgs field $\Phi_{jl}^{[ik]}$ as [10]

$$\langle\Phi_{jl}^{[ik]}\rangle \;=\; \frac{v}{2\sqrt{3}}\left[\Delta_{cj}^{[i}\Delta_{cl}^{k]}+2\Delta_{wj}^{[i}\Delta_{wl}^{k]}-\frac{1}{2}\delta_j^{[i}\delta_l^{k]}\right] \;, \tag{4.20}$$

where

$$\Delta_c \;=\; \mathrm{diag}(\,1,\;1,\;1,\;0,\;0)\;,\quad \Delta_w \;=\; \mathrm{diag}(\,0,\;0,\;0,\;1,\;1)\;. \tag{4.21}$$

We consider the following superpotential for the additional contributions to the Yukawa coupling terms and trilinear soft terms

$$\begin{aligned} W \supset \Big(& h^{Ui}\epsilon^{mnpjl}(F_i')_{mn}(F_i')_{pq}(h')_k\Phi_{jl}^{[qk]} + h'^{Ui}\epsilon^{jlpqk}(F_i')_{mn}(F_i')_{pq}(h')_k\Phi_{jl}^{[mn]} \\ & + h^{DEi}(F_i')_{mn}(\overline{f}_i')^p(\overline{h}')^q\Phi_{pq}^{[mn]}\Big) + \alpha'\frac{T}{M_*}\left(y^{Ui}\epsilon^{mnpjl}(F_i')_{mn}(F_i')_{pq}(h')_k\Phi_{jl}^{[qk]}\right. \\ & \left. + y'^{Ui}\epsilon^{jlpqk}(F_i')_{mn}(F_i')_{pq}(h')_k\Phi_{jl}^{[mn]} + y^{DEi}(F_i')_{mn}(\overline{f}_i')^p(\overline{h}')^q\Phi_{pq}^{[mn]}\right)\;. \end{aligned} \tag{4.22}$$

After $\Phi_{jl}^{[ik]}$ acquires a VEV, we obtain the Yukawa coupling terms in the superpotential

$$W \supset \frac{v}{M_*}\frac{1}{2\sqrt{3}}\left(-h'^{DEi}Q_iD_i^cH_d + 3h'^{DEi}L_iE_i^cH_d\right)\;, \tag{4.23}$$

and the supersymmetry breaking trilinear soft terms

$$-\mathcal{L} \supset \alpha'\frac{F_Tv}{M_*^2}\frac{1}{2\sqrt{3}}\left(-y'^{DEi}\widetilde{Q}_i\widetilde{D}_i^cH_d + 3y'^{DEi}\widetilde{L}_i\widetilde{E}_i^cH_d\right)\;. \tag{4.24}$$

These results can also be obtained by considering the VEV of **75** dimensional Higgs field as an appropriate 10×10 matrix. Due to the arbitrariness of the coefficients in the Yukawa coupling terms and the trilinear soft terms, we will not discuss the relations among the trilinear soft terms.

5. Scalar Masses in the $SO(10)$ Model

In order to calculate the scalar masses, we need to decompose the tensor product of $\overline{\mathbf{16}}\otimes\mathbf{16}$ which gives

$$\overline{\mathbf{16}}\otimes\mathbf{16} = \mathbf{1}\oplus\mathbf{45}\oplus\mathbf{210}\;. \tag{5.1}$$

Thus we need to consider the Higgs fields in the **45** and **210** representations to determine the scalar masses. The $SO(10)$ gauge symmetry can be broken down to the Pati-Salam $SU(4)_C\times SU(2)_L\times SU(2)_R$ gauge symmetry by the Higgs fields in the **45**, **210**, and **770** representations, and can be (further) broken down to the $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ gauge symmetry by the Higgs field in the $(\mathbf{15},\mathbf{1},\mathbf{1})$ component of the $SU(4)_C\times SU(2)_L\times SU(2)_R$ under the **45** and **210** representations. In addition, the $SO(10)$ gauge symmetry can be broken down to the Georgi-Glashow $SU(5)\times U(1)'$ and flipped $SU(5)\times U(1)_X$ gauge symmetries by the Higgs fields in the **45** and **210** representations, and can be (further) broken down to the $SU(3)_C\times SU(2)_L\times U(1)_1\times U(1)_2$ gauge symmetries by the Higgs field in the $(\mathbf{24},\mathbf{0})$ component of the $SU(5)\times U(1)$ under the **45** representation, or by the $(\mathbf{24},\mathbf{0})$ or $(\mathbf{75},\mathbf{0})$ component under the **210** representation. Thus, in the following, we consider these breaking chains.

5.1 The Pati-Salam Model

From the decomposition of the **16** dimensional spinor representation under the $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry, we obtain the VEV (of the $(\mathbf{1},\mathbf{1},\mathbf{1})$ component) of the 210 dimensional Higgs field $\Phi_{\mathbf{210}}$ in terms of the 16×16 matrix

$$< \Phi_{\mathbf{210}} >= \frac{v}{2\sqrt{2}}\mathrm{diag}(\underbrace{1,\cdots,\ 1}_{8},\underbrace{-1,\cdots,-1}_{8})\ , \tag{5.2}$$

with the normalization $c = 2$. From this we get the scalar masses

$$M^2(\tilde{F}_i^L) = (m_0^U)^2 + \frac{v}{2\sqrt{2}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3}\ , \tag{5.3}$$

$$M^2(\tilde{F}_i^{Rc}) = (m_0^U)^2 - \frac{v}{2\sqrt{2}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3}\ . \tag{5.4}$$

In components, we have

$$\begin{aligned}
m^2_{\widetilde{Q}_i} &= (m_0^U)^2 + \frac{v}{2\sqrt{2}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3}\ ,\\
m^2_{\widetilde{U}_i^c} &= (m_0^U)^2 - \frac{v}{2\sqrt{2}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3}\ ,\\
m^2_{\widetilde{E}_i^c} &= (m_0^U)^2 - \frac{v}{2\sqrt{2}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3}\ ,\\
m^2_{\widetilde{D}_i^c} &= (m_0^U)^2 - \frac{v}{2\sqrt{2}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3}\ ,\\
m^2_{\widetilde{L}_i} &= (m_0^U)^2 + \frac{v}{2\sqrt{2}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3}\ .
\end{aligned} \tag{5.5}$$

5.2 The $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ Model

The $SO(10)$ gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ symmetry by giving VEVs to the $(\mathbf{15},\mathbf{1},\mathbf{1})$ components of the Higgs field in the **45** and **210** representations of $SU(4)_C \times SU(2)_L \times SU(2)_R$. The decomposition of **16** under the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ group is

$$\mathbf{16} = (\mathbf{3},\mathbf{2},\mathbf{1},\mathbf{1/6}) \oplus (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{-1/2}) \oplus (\bar{\mathbf{3}},\mathbf{1},\bar{\mathbf{2}},\mathbf{-1/6}) \oplus (\mathbf{1},\mathbf{1},\bar{\mathbf{2}},\mathbf{1/2})\ . \tag{5.6}$$

First, let us consider the Higgs field $\Phi_{\mathbf{45}}$ in the **45** representation. The VEV of $\Phi_{\mathbf{45}}$ can be written in terms of a 16×16 matrix as follows

$$\langle \Phi_{\mathbf{45}} \rangle = \frac{v}{2\sqrt{6}}\mathrm{diag}(\underbrace{1,\ 1,\ 1,-3}_{2},\underbrace{-1,-1,-1,\ 3}_{2})\ , \tag{5.7}$$

which is normalized as $c = 2$. Thus, the scalar masses are

$$m^2_{\widetilde{Q}_i} = (m_0^U)^2 + \frac{v}{2\sqrt{6}}\beta'_{\mathbf{45}}\frac{v|F_T|^2}{M_*^3}\ ,$$

$$
\begin{aligned}
m^2_{\widetilde{U}^c_i} &= (m_0^U)^2 - \frac{v}{2\sqrt{6}}\beta'_{\mathbf{45}}\frac{v|F_T|^2}{M_*^3} ~, \\
m^2_{\widetilde{E}^c_i} &= (m_0^U)^2 + \frac{3v}{2\sqrt{6}}\beta'_{\mathbf{45}}\frac{v|F_T|^2}{M_*^3} ~, \\
m^2_{\widetilde{D}^c_i} &= (m_0^U)^2 - \frac{v}{2\sqrt{6}}\beta'_{\mathbf{45}}\frac{v|F_T|^2}{M_*^3} ~, \\
m^2_{\widetilde{L}_i} &= (m_0^U)^2 - \frac{3v}{2\sqrt{6}}\beta'_{\mathbf{45}}\frac{v|F_T|^2}{M_*^3} ~.
\end{aligned}
\tag{5.8}
$$

Second, we consider the Higgs field $\Phi_{\mathbf{210}}$ in the **210** representation. The VEV of $\Phi_{\mathbf{210}}$ in terms of a 16×16 matrix is

$$
\langle \Phi_{\mathbf{210}} \rangle ~=~ \frac{v}{2\sqrt{6}} \mathrm{diag}(\underbrace{1,~1,~1,-3}_{4}) ~,
\tag{5.9}
$$

which is normalized as $c = 2$. Thus, the scalar masses are

$$
\begin{aligned}
m^2_{\widetilde{Q}_i} &= (m_0^U)^2 + \frac{v}{2\sqrt{6}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3} ~, \\
m^2_{\widetilde{U}^c_i} &= (m_0^U)^2 + \frac{v}{2\sqrt{6}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3} ~, \\
m^2_{\widetilde{E}^c_i} &= (m_0^U)^2 - \frac{3v}{2\sqrt{6}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3} ~, \\
m^2_{\widetilde{D}^c_i} &= (m_0^U)^2 + \frac{v}{2\sqrt{6}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3} ~, \\
m^2_{\widetilde{L}_i} &= (m_0^U)^2 - \frac{3v}{2\sqrt{6}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3} ~.
\end{aligned}
\tag{5.10}
$$

5.3 The Georgi-Glashow $SU(5) \times U(1)'$ and Flipped $SU(5) \times U(1)_X$ Models

The $SO(10)$ gauge symmetry can also be broken down to the $SU(5) \times U(1)$ gauge symmetry by the **45** and **210** dimensional Higgs fields $\Phi_{\mathbf{45}}$ and $\Phi_{\mathbf{210}}$. The decomposition of the **16** spinor representation under $SU(5) \times U(1)$ is

$$
\mathbf{16} = (\mathbf{10},~\mathbf{1}) \oplus (\overline{\mathbf{5}},~-\mathbf{3}) \oplus (\mathbf{1},~\mathbf{5}) ~.
\tag{5.11}
$$

First, we consider the Higgs field $\Phi_{\mathbf{45}}$. From Eq. (5.11), we obtain the VEV of $\Phi_{\mathbf{45}}$ in terms of a 16×16 matrix

$$
\langle \Phi_{\mathbf{45}} \rangle ~=~ \frac{v}{2\sqrt{10}} \mathrm{diag}(\underbrace{-3,\cdots,-3}_{5},\underbrace{1,\cdots,1}_{10},5) ~,
\tag{5.12}
$$

which is normalized as $c = 2$. Consequently, we obtain the scalar masses in the Georgi-Glashow $SU(5) \times U(1)'$ and flipped $SU(5) \times U(1)_X$ models:

- The Georgi-Glashow $SU(5) \times U(1)'$ Model

$$
M^2(\widetilde{F}'_i) = (m_0^U)^2 + \beta'_{\mathbf{45}} \frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~,
$$

$$M^2(\widetilde{\overline{f}}'_i) = (m_0^U)^2 - 3\beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~,$$

$$M^2(\widetilde{N}_i^c) = (m_0^U)^2 + 5\beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~. \tag{5.13}$$

In components, we have

$$\begin{aligned} m^2_{\widetilde{Q}_i} &= (m_0^U)^2 + \beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~, \\ m^2_{\widetilde{U}_i^c} &= (m_0^U)^2 + \beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~, \\ m^2_{\widetilde{E}_i^c} &= (m_0^U)^2 + \beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3}, \\ m^2_{\widetilde{D}_i^c} &= (m_0^U)^2 - 3\beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~, \\ m^2_{\widetilde{L}_i} &= (m_0^U)^2 - 3\beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~. \end{aligned} \tag{5.14}$$

In this paper, we will not consider the scalar masses for right-handed sneutrinos because the heavy Majorana neutrino masses will give the dominant contributions.

- The Flipped $SU(5) \times U(1)_X$ Model

$$M^2(\widetilde{F}_i) = (m_0^U)^2 + \beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~,$$

$$M^2(\widetilde{\overline{f}}_i) = (m_0^U)^2 - 3\beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~,$$

$$M^2(\widetilde{\overline{l}}_i) = (m_0^U)^2 + 5\beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~. \tag{5.15}$$

In components, this gives

$$\begin{aligned} m^2_{\widetilde{Q}_i} &= (m_0^U)^2 + \beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~, \\ m^2_{\widetilde{U}_i^c} &= (m_0^U)^2 - 3\beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~, \\ m^2_{\widetilde{E}_i^c} &= (m_0^U)^2 + 5\beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~, \\ m^2_{\widetilde{D}_i^c} &= (m_0^U)^2 + \beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~, \\ m^2_{\widetilde{L}_i} &= (m_0^U)^2 - 3\beta'_{\mathbf{45}}\frac{v|F_T|^2}{2\sqrt{10}M_*^3} ~. \end{aligned} \tag{5.16}$$

Second, let us consider the Higgs field $\Phi_{\mathbf{210}}$. Because the VEVs of $\Phi_{\mathbf{45}}$ and $\Phi_{\mathbf{210}}$ are orthogonal to each other, we obtain the VEV of $\Phi_{\mathbf{210}}$ in terms of the 16×16 matrix

$$< \Phi >= \frac{v}{2\sqrt{5}}\mathrm{diag}(\underbrace{1, \cdots, ~1}_{5}, \underbrace{-1, \cdots, -1}_{10}, ~5) ~, \tag{5.17}$$

which is normalized as $c = 2$. From this, we obtain the scalar masses in the Georgi-Glashow $SU(5) \times U(1)'$ and flipped $SU(5) \times U(1)_X$ models:

- The Georgi-Glashow $SU(5) \times U(1)'$ Model

$$\begin{aligned} M^2(\widetilde{F}_i') &= (m_0^U)^2 - \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~, \\ M^2(\widetilde{\overline{f}}_i') &= (m_0^U)^2 + \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~, \\ M^2(\widetilde{N}_i^c) &= (m_0^U)^2 + 5\beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~. \end{aligned} \tag{5.18}$$

In components

$$\begin{aligned} m^2_{\widetilde{Q}_i} &= (m_0^U)^2 - \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~, \\ m^2_{\widetilde{U}_i^c} &= (m_0^U)^2 - \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~, \\ m^2_{\widetilde{E}_i^c} &= (m_0^U)^2 - \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3}, \\ m^2_{\widetilde{D}_i^c} &= (m_0^U)^2 + \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~, \\ m^2_{\widetilde{L}_i} &= (m_0^U)^2 + \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~. \end{aligned} \tag{5.19}$$

- The Flipped $SU(5) \times U(1)_X$ Model

$$\begin{aligned} M^2(\widetilde{F}_i) &= (m_0^U)^2 - \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~, \\ M^2(\widetilde{\overline{f}}_i) &= (m_0^U)^2 + \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~, \\ M^2(\widetilde{\overline{l}}_i) &= (m_0^U)^2 + 5\beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~. \end{aligned} \tag{5.20}$$

In components

$$\begin{aligned} m^2_{\widetilde{Q}_i} &= (m_0^U)^2 - \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~, \\ m^2_{\widetilde{U}_i^c} &= (m_0^U)^2 + \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~, \\ m^2_{\widetilde{E}_i^c} &= (m_0^U)^2 + 5\beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~, \\ m^2_{\widetilde{D}_i^c} &= (m_0^U)^2 - \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~, \\ m^2_{\widetilde{L}_i} &= (m_0^U)^2 + \beta'_{\mathbf{210}} \frac{v|F_T|^2}{2\sqrt{5}M_*^3} ~. \end{aligned} \tag{5.21}$$

5.4 The $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ Model

The $SO(10)$ gauge symmetry can also be broken down to $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ by VEVs of the $(\mathbf{24}, \mathbf{0})$ component Higgs field which is in the $\mathbf{45}$ representation under $SU(5) \times U(1)$, or by the $(\mathbf{24}, \mathbf{0})$ or $(\mathbf{75}, \mathbf{0})$ component Higgs fields in the $\mathbf{210}$ representation.

First, we consider the Higgs field $\Phi_{\mathbf{45}}$ in the $\mathbf{45}$ representation. The VEV of $\Phi_{\mathbf{45}}$ is

$$\langle \Phi_{\mathbf{45}} \rangle = v\sqrt{\frac{3}{5}} \mathrm{diag}(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}, \underbrace{\frac{1}{6}, \cdots, \frac{1}{6}}_{6}, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, 1, 0) , \tag{5.22}$$

which is normalized to $c = 2$.

Thus, we obtain the scalar masses in the Georgi-Glashow $SU(5) \times U(1)'$ and flipped $SU(5) \times U(1)_X$ models:

- The Georgi-Glashow $SU(5) \times U(1)'$ Model

$$\begin{aligned} m^2_{\widetilde{Q}_i} &= (m_0^U)^2 + \sqrt{\frac{3}{5}} \beta'_{\mathbf{45}} \frac{1}{6} (m_0^N)^2 , \\ m^2_{\widetilde{U}^c_i} &= (m_0^U)^2 - \sqrt{\frac{3}{5}} \beta'_{\mathbf{45}} \frac{2}{3} (m_0^N)^2 , \\ m^2_{\widetilde{E}^c_i} &= (m_0^U)^2 + \sqrt{\frac{3}{5}} \beta'_{\mathbf{45}} (m_0^N)^2 , \\ m^2_{\widetilde{D}^c_i} &= (m_0^U)^2 + \sqrt{\frac{3}{5}} \beta'_{\mathbf{45}} \frac{1}{3} (m_0^N)^2 , \\ m^2_{\widetilde{L}_i} &= (m_0^U)^2 - \sqrt{\frac{3}{5}} \beta'_{\mathbf{45}} \frac{1}{2} (m_0^N)^2 , \end{aligned} \tag{5.23}$$

  where $(m_0^U)^2$ and $(m_0^N)^2$ are given in Eq. (4.10).

- The Flipped $SU(5) \times U(1)_X$ Model

$$\begin{aligned} m^2_{\widetilde{Q}_i} &= (m_0^U)^2 + \sqrt{\frac{3}{5}} \beta'_{\mathbf{45}} \frac{1}{6} (m_0^N)^2 , \\ m^2_{\widetilde{U}^c_i} &= (m_0^U)^2 + \sqrt{\frac{3}{5}} \beta'_{\mathbf{45}} \frac{1}{3} (m_0^N)^2 , \end{aligned} \tag{5.24}$$

$$\begin{aligned} m^2_{\widetilde{E}^c_i} &= (m_0^U)^2 , \\ m^2_{\widetilde{D}^c_i} &= (m_0^U)^2 - \sqrt{\frac{3}{5}} \beta'_{\mathbf{45}} \frac{2}{3} (m_0^N)^2 , \\ m^2_{\widetilde{L}_i} &= (m_0^U)^2 - \sqrt{\frac{3}{5}} \beta'_{\mathbf{45}} \frac{1}{2} (m_0^N)^2 . \end{aligned} \tag{5.25}$$

Second, we consider the Higgs field $\Phi^{\mathbf{24}}_{\mathbf{210}}$ in the $(\mathbf{24}, \mathbf{0})$ component of the $\mathbf{210}$ representation that acquires a VEV as follows

$$\langle \Phi^{\mathbf{24}}_{\mathbf{210}} \rangle = \frac{v}{\sqrt{5}} \mathrm{diag}(-1, -1, -1, \frac{3}{2}, \frac{3}{2}, \underbrace{\frac{1}{6}, \cdots, \frac{1}{6}}_{6}, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, 1, 0) , \tag{5.26}$$

which is normalized to $c=2$. From this, we obtain the scalar masses in the Georgi-Glashow $SU(5)\times U(1)'$ and the flipped $SU(5)\times U(1)_X$ models:

- The Georgi-Glashow $SU(5)\times U(1)'$ Model

$$
\begin{aligned}
m^2_{\widetilde{Q}_i} &= (m_0^U)^2 + \frac{1}{\sqrt{5}}\beta'^{\mathbf{24}}_{\mathbf{210}}\frac{1}{6}(m_0^N)^2 ~, \\
m^2_{\widetilde{U}^c_i} &= (m_0^U)^2 - \frac{1}{\sqrt{5}}\beta'^{\mathbf{24}}_{\mathbf{210}}\frac{2}{3}(m_0^N)^2 ~, \\
m^2_{\widetilde{E}^c_i} &= (m_0^U)^2 + \frac{1}{\sqrt{5}}\beta'^{\mathbf{24}}_{\mathbf{210}}(m_0^N)^2 ~, \\
m^2_{\widetilde{D}^c_i} &= (m_0^U)^2 - \frac{1}{\sqrt{5}}\beta'^{\mathbf{24}}_{\mathbf{210}}(m_0^N)^2 ~, \\
m^2_{\widetilde{L}_i} &= (m_0^U)^2 + \frac{1}{\sqrt{5}}\beta'^{\mathbf{24}}_{\mathbf{210}}\frac{3}{2}(m_0^N)^2 ~.
\end{aligned}
\tag{5.27}
$$

- The Flipped $SU(5)\times U(1)_X$ Model

$$
\begin{aligned}
m^2_{\widetilde{Q}_i} &= (m_0^U)^2 + \frac{1}{\sqrt{5}}\beta'^{\mathbf{24}}_{\mathbf{210}}\frac{1}{6}(m_0^N)^2 ~, \\
m^2_{\widetilde{U}^c_i} &= (m_0^U)^2 - \frac{1}{\sqrt{5}}\beta'^{\mathbf{24}}_{\mathbf{210}}(m_0^N)^2 ~, \\
m^2_{\widetilde{E}^c_i} &= (m_0^U)^2 ~, \\
m^2_{\widetilde{D}^c_i} &= (m_0^U)^2 - \frac{1}{\sqrt{5}}\beta'^{\mathbf{24}}_{\mathbf{210}}\frac{2}{3}(m_0^N)^2 ~, \\
m^2_{\widetilde{L}_i} &= (m_0^U)^2 + \frac{1}{\sqrt{5}}\beta'^{\mathbf{24}}_{\mathbf{210}}\frac{3}{2}(m_0^N)^2 ~.
\end{aligned}
\tag{5.28}
$$

Third, we consider that the $(\mathbf{75},\mathbf{0})$ component Higgs field $\Phi^{\mathbf{75}}_{\mathbf{210}}$ in the $\mathbf{210}$ representation acquires a VEV as follows

$$
\langle\Phi^{\mathbf{75}}_{\mathbf{210}}\rangle ~=~ \frac{v}{3}{\rm diag}(~0,~0,~0,~0,~0,\underbrace{-1,\cdots,-1}_{6},~1,~1,~1,~3,~0)~,
\tag{5.29}
$$

which is normalized to $c=2$. From this, we obtain the following scalar masses in the Georgi-Glashow $SU(5)\times U(1)'$ and the flipped $SU(5)\times U(1)_X$ models:

- The Georgi-Glashow $SU(5)\times U(1)'$ Model

$$
\begin{aligned}
m^2_{\widetilde{Q}_i} &= (m_0^U)^2 - \frac{1}{3}\beta'^{\mathbf{75}}_{\mathbf{210}}(m_0^N)^2 ~, \\
m^2_{\widetilde{U}^c_i} &= (m_0^U)^2 + \frac{1}{3}\beta'^{\mathbf{75}}_{\mathbf{210}}(m_0^N)^2 ~, \\
m^2_{\widetilde{E}^c_i} &= (m_0^U)^2 + \beta'^{\mathbf{75}}_{\mathbf{210}}(m_0^N)^2 ~, \\
m^2_{\widetilde{D}^c_i} &= (m_0^U)^2 ~, \\
m^2_{\widetilde{L}_i} &= (m_0^U)^2 ~.
\end{aligned}
\tag{5.30}
$$

- The Flipped $SU(5) \times U(1)_X$ Model

$$
\begin{aligned}
m^2_{\widetilde{Q}_i} &= (m_0^U)^2 - \frac{1}{3}\beta'^{\mathbf{75}}_{\mathbf{210}}(m_0^N)^2 , \\
m^2_{\widetilde{U}^c_i} &= (m_0^U)^2 , \\
m^2_{\widetilde{E}^c_i} &= (m_0^U)^2 , \\
m^2_{\widetilde{D}^c_i} &= (m_0^U)^2 + \frac{1}{3}\beta'^{\mathbf{75}}_{\mathbf{210}}(m_0^N)^2 , \\
m^2_{\widetilde{L}_i} &= (m_0^U)^2 .
\end{aligned}
\tag{5.31}
$$

6. The Yukawa Coupling Terms and Trilinear Soft Terms in the $SO(10)$ Model

There are several kinds of the renormalizable Yukawa coupling terms for the SM fermions in the $SO(10)$ model. For example, we can use **120** or **126** Higgs fields to obtain reasonable SM fermion masses and mixings. In this paper we choose the simplest Higgs field $H_{\mathbf{10}}$ in the $SO(10)$ fundamental representation. To obtain the non-renormalizable contributions to the Yukawa coupling terms and trilinear soft terms, we need to know the decompositions of the tensor product $\mathbf{16} \otimes \mathbf{16} \otimes \mathbf{10}$ [41]

$$
\mathbf{16} \otimes \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126} , \tag{6.1}
$$

$$
\mathbf{16} \otimes \mathbf{16} \otimes \mathbf{10} = (\mathbf{1} \oplus \mathbf{45} \oplus \mathbf{54}) \oplus (\mathbf{45} \oplus \mathbf{210} \oplus \mathbf{945}) \oplus (\mathbf{210} \oplus \mathbf{1050}) . \tag{6.2}
$$

Because the **945** and **1050** representations do not have $SU(5)\times U(1)$ or $SU(4)_C \times SU(2)_L \times SU(2)_R$ singlets [41], we only consider the Higgs fields in the **45**, **54** and **210** representations.

6.1 The Pati-Salam Model

The $SO(10)$ gauge symmetry can be broken down to the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry by giving VEVs to the Higgs fields in the **54** and **210** representations.

For the Higgs field $\Phi_{\mathbf{54}}$ in the **54** representation, we can write the VEV in terms of a $\mathbf{10} \times \mathbf{10}$ matrix

$$
\langle \Phi_{\mathbf{54}} \rangle = \frac{v}{2\sqrt{15}} \mathrm{diag}(\underbrace{2, \cdots, 2}_{6}, \underbrace{-3, \cdots, -3}_{4}) , \tag{6.3}
$$

which is normalized to $c = 1$.

To calculate the additional contributions to the Yukawa coupling terms and trilinear soft terms, we consider the following superpotential

$$
W \supset \frac{1}{M_*} h^i (\mathbf{16_i} \otimes \mathbf{16_i})^m_{\mathbf{10}} (\Phi_{\mathbf{54}})_{mn} \mathbf{10}^n + \alpha' \frac{T}{M_*^2} y^i (\mathbf{16_i} \otimes \mathbf{16_i})^m_{\mathbf{10}} (\Phi_{\mathbf{54}})_{mn} \mathbf{10}^n . \tag{6.4}
$$

After $\Phi_{\mathbf{54}}$ acquires a VEV, we obtain the additional contributions to the Yukawa coupling terms

$$W \supset -h^i \frac{3v}{\sqrt{15}M_*} \left[Q_i U_i^c H_u + L_i N_i^c H_u + Q_i D_i^c H_d + L_i E_i^c H_d\right] . \tag{6.5}$$

The extra supersymmetry breaking trilinear soft terms are

$$-\mathcal{L} \supset -y^i \frac{3F_T v}{\sqrt{15}M_*^2} \left[\widetilde{Q}_i \widetilde{U}_i^c H_u + \widetilde{L}_i \widetilde{N}_i^c H_u + \widetilde{Q}_i \widetilde{D}_i^c H_d + \widetilde{L}_i \widetilde{E}_i^c H_d\right] . \tag{6.6}$$

For the Higgs field $\Phi_{\mathbf{210}}$ in the **210** representation, we can write the VEV in terms of a $\mathbf{16} \times \mathbf{16}$ matrix

$$\langle \Phi_{\mathbf{210}} \rangle = \frac{v}{2\sqrt{2}} \mathrm{diag}(\underbrace{1, \cdots, 1}_{8}, \underbrace{-1, \cdots, -1}_{8}) , \tag{6.7}$$

which is normalized to $c = 2$. We consider the following superpotential

$$\begin{aligned} W \supset\ & \frac{1}{M_*} \left[h^i (\mathbf{16_i} \otimes \mathbf{16_i})^{mnl}_{\mathbf{120}} (\Phi_{\mathbf{210}})_{mnlk} \mathbf{10}^k + h'^i (\mathbf{16_i} \otimes \mathbf{16_i})^{mnlpq}_{\mathbf{126}} (\Phi_{\mathbf{210}})_{mnlp} \mathbf{10}_q \right] \\ & + \alpha' \frac{T}{M_*^2} \left[y^i (\mathbf{16_i} \otimes \mathbf{16_i})^{mnl}_{\mathbf{120}} (\Phi_{\mathbf{210}})_{mnlk} \mathbf{10}^k \right. \\ & \left. + y'^i (\mathbf{16_i} \otimes \mathbf{16_i})^{mnlpq}_{\mathbf{126}} (\Phi_{\mathbf{210}})_{mnlp} \mathbf{10}_q \right] . \end{aligned} \tag{6.8}$$

We can show that the above superpotential will not contribute to the SM fermion Yukawa coupling terms and trilinear soft terms.

6.2 The $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ Model

The $SO(10)$ gauge symmetry can also be broken down to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ by giving VEVs to the $(\mathbf{15},\ \mathbf{1},\ \mathbf{1})$ components of the Higgs fields in the **45** and **210** representations under $SU(4)_C \times SU(2)_L \times SU(2)_R$.

For the Higgs field $\Phi_{\mathbf{45}}$ in the **45** representation, we can write the VEV in terms of a 10×10 matrix as follows

$$\langle \Phi_{\mathbf{45}} \rangle = \frac{v}{2\sqrt{6}} \mathrm{diag}(\underbrace{2, \cdots, 2}_{3}, \underbrace{-2, \cdots, -2}_{3}, \underbrace{0, \cdots, 0}_{4}) , \tag{6.9}$$

which is normalized as $c = 1$.

To calculate the additional contributions to the Yukawa coupling terms and trilinear soft terms, we consider the following superpotential

$$\begin{aligned} W \supset\ & \frac{1}{M_*} \left[h^i (\mathbf{16_i} \otimes \mathbf{16_i})^{m}_{\mathbf{10}} (\Phi_{\mathbf{45}})_{mn} \mathbf{10}^n + h'^i (\mathbf{16_i} \otimes \mathbf{16_i})^{mnl}_{\mathbf{120}} (\Phi_{\mathbf{45}})_{mn} \mathbf{10}_l \right] \\ & + \alpha' \frac{T}{M_*^2} \left[y^i (\mathbf{16_i} \otimes \mathbf{16_i})^{m}_{\mathbf{10}} (\Phi_{\mathbf{45}})_{mn} \mathbf{10}^n + y'^i (\mathbf{16_i} \otimes \mathbf{16_i})^{mnl}_{\mathbf{120}} (\Phi_{\mathbf{45}})_{mn} \mathbf{10}_l \right] . \end{aligned} \tag{6.10}$$

We can show that the above superpotential will not contribute to the SM fermion Yukawa coupling terms and trilinear soft terms.

For the Higgs field $\Phi_{\mathbf{210}}$ in the **210** representation, we can write the VEV in terms of a 16×16 matrix as follows

$$\langle \Phi_{\mathbf{210}} \rangle \;=\; \frac{v}{2\sqrt{6}} \mathrm{diag}(\underbrace{\;1,\;1,\;1,-3}_{4}) \;, \tag{6.11}$$

which is normalized as $c = 2$. We consider the following superpotential

$$\begin{aligned} W \supset\; & \frac{1}{M_*}\left[h^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi_{\mathbf{210}})_{mnlk}\mathbf{10}^k + h'^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnlpq}_{\mathbf{126}}(\Phi_{\mathbf{210}})_{mnlp}\mathbf{10}_q\right] \\ & +\alpha'\frac{T}{M_*^2}\left[y^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi_{\mathbf{210}})_{mnlk}\mathbf{10}^k \right. \\ & \left. +y'^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnlpq}_{\mathbf{126}}(\Phi_{\mathbf{210}})_{mnlp}\mathbf{10}_q\right] \;. \end{aligned} \tag{6.12}$$

After $\Phi_{\mathbf{210}}$ acquires a VEV, we obtain the additional contributions to the Yukawa coupling terms

$$W \supset h'^i \frac{v}{\sqrt{6}M_*}\left[Q_i U_i^c H_u - 3L_i N_i^c H_u + Q_i D_i^c H_d - 3L_i E_i^c H_d\right] \;. \tag{6.13}$$

The extra supersymmetry breaking trilinear soft terms are

$$-\mathcal{L} \supset y'^i \frac{F_T v}{\sqrt{6}M_*^2}\left[\widetilde{Q}_i \widetilde{U}_i^c H_u - 3\widetilde{L}_i \widetilde{N}_i^c H_u + \widetilde{Q}_i \widetilde{D}_i^c H_d - 3\widetilde{L}_i \widetilde{E}_i^c H_d\right] \;. \tag{6.14}$$

6.3 The Georgi-Glashow $SU(5)\times U(1)'$ Model

The $SO(10)$ gauge symmetry can be broken down to the Georgi-Glashow $SU(5)\times U(1)'$ gauge symmetry by giving VEVs to the Higgs fields in the **45** and **210** representations.

For the Higgs field $\Phi_{\mathbf{45}}$ in the **45** representation, we can write the VEV as a $\mathbf{10}\times\mathbf{10}$ matrix:

$$\langle \Phi_{\mathbf{45}} \rangle \;=\; \frac{v}{\sqrt{10}} \mathrm{diag}(\underbrace{\;1,\cdots,\;1}_{5},\underbrace{-1,\cdots,-1}_{5}) \;, \tag{6.15}$$

where the normalization is $c = 1$. Using the conventions in [42] we obtain the non-zero components

$$(\Phi_{\mathbf{45}})_{12} = (\Phi_{\mathbf{45}})_{34} = (\Phi_{\mathbf{45}})_{56} = (\Phi_{\mathbf{45}})_{78} = (\Phi_{\mathbf{45}})_{90} = \frac{v}{\sqrt{10}} \;. \tag{6.16}$$

To calculate the additional contributions to the Yukawa coupling terms and trilinear soft terms, we consider the following superpotential

$$\begin{aligned} W \supset\; & \frac{1}{M_*}\left[h^i(\mathbf{16_i}\otimes\mathbf{16_i})^{m}_{\mathbf{10}}(\Phi_{\mathbf{45}})_{mn}\mathbf{10}^n + h'^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi_{\mathbf{45}})_{mn}\mathbf{10}_l\right] \\ & +\alpha'\frac{T}{M_*^2}\left[y^i(\mathbf{16_i}\otimes\mathbf{16_i})^{m}_{\mathbf{10}}(\Phi_{\mathbf{45}})_{mn}\mathbf{10}^n + y'^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi_{\mathbf{45}})_{mn}\mathbf{10}_l\right] \;. \end{aligned} \tag{6.17}$$

Note that **120** is anti-symmetric representation, the h'^i and y'^i terms will not contribute to the SM fermion Yukawa coupling terms and trilinear soft terms. After $\Phi_{\mathbf{210}}$ acquires a VEV, we obtain the additional contributions to the Yukawa coupling terms

$$W \supset h^i \frac{2v}{\sqrt{10}M_*}\left[Q_i U_i^c H_u + L_i N_i^c H_u - Q_i D_i^c H_d - L_i E_i^c H_d\right] \;. \tag{6.18}$$

The extra supersymmetry breaking trilinear soft terms are

$$-\mathcal{L} \supset y^i \frac{2F_T v}{\sqrt{10}M_*^2} \left[\widetilde{Q}_i \widetilde{U}_i^c H_u + \widetilde{L}_i \widetilde{N}_i^c H_u - \widetilde{Q}_i \widetilde{D}_i^c H_d - \widetilde{L}_i \widetilde{E}_i^c H_d\right] . \tag{6.19}$$

For the Higgs field $\Phi_{\mathbf{210}}$ in the **210** representation, we can write the VEV in the form of a $\mathbf{16} \times \mathbf{16}$ matrix as follows

$$\langle \Phi_{\mathbf{210}} \rangle = \frac{v}{2\sqrt{5}} \mathrm{diag}(\underbrace{1, \cdots, 1}_{5}, \underbrace{-1, \cdots, -1}_{10}, 5) , \tag{6.20}$$

where the normalization is $c = 2$. This VEV can be written in components as follows

$$\begin{aligned}(\Phi_{\mathbf{210}})_{1234} &= (\Phi_{\mathbf{210}})_{1256} = (\Phi_{\mathbf{210}})_{1278} = (\Phi_{\mathbf{210}})_{1290} = (\Phi_{\mathbf{210}})_{3456} = (\Phi_{\mathbf{210}})_{3478} \\ &= (\Phi_{\mathbf{210}})_{3490} = (\Phi_{\mathbf{210}})_{5678} = (\Phi_{\mathbf{210}})_{5690} = (\Phi_{\mathbf{210}})_{7890} = -\frac{v}{2\sqrt{5}} . \end{aligned} \tag{6.21}$$

We consider the following superpotential

$$\begin{aligned} W \supset \; & \frac{1}{M_*} \left[h^i (\mathbf{16_i} \otimes \mathbf{16_i})_{\mathbf{120}}^{mnl} (\Phi_{\mathbf{210}})_{mnlk} \mathbf{10}^k + h'^i (\mathbf{16_i} \otimes \mathbf{16_i})_{\mathbf{126}}^{mnlkp} (\Phi_{\mathbf{210}})_{mnlk} \mathbf{10}_p\right] \\ & + \alpha' \frac{T}{M_*^2} \left[y^i (\mathbf{16_i} \otimes \mathbf{16_i})_{\mathbf{120}}^{mnl} (\Phi_{\mathbf{210}})_{mnlk} \mathbf{10}^k \right. \\ & \left. + y'^i (\mathbf{16_i} \otimes \mathbf{16_i})_{\mathbf{126}}^{mnlkp} (\Phi_{\mathbf{210}})_{mnlk} \mathbf{10}_p \right] . \end{aligned} \tag{6.22}$$

After $\Phi_{\mathbf{45}}$ acquires a VEV, we obtain the additional contributions to the Yukawa coupling terms

$$W \supset h'^i \frac{v}{\sqrt{5}M_*} \left[3L_i N_i^c H_u - Q_i U_i^c H_u\right] . \tag{6.23}$$

The extra supersymmetry breaking trilinear soft terms are

$$-\mathcal{L} \supset y'^i \frac{F_T v}{\sqrt{5}M_*^2} \left[3\widetilde{L}_i \widetilde{N}_i^c H_u - \widetilde{Q}_i \widetilde{U}_i^c H_u\right] . \tag{6.24}$$

6.4 The Flipped $SU(5) \times U(1)_X$ Model

The discussion for the flipped $SU(5) \times U(1)_X$ model is similar to those for the Georgi-Glashow $SU(5) \times U(1)'$ model except that we make the following transformations

$$Q_i \leftrightarrow Q_i \; , \;\; U_i^c \leftrightarrow D_i^c \; , \;\; L_i \leftrightarrow L_i \; , \;\; N_i^c \leftrightarrow E_i^c \; , \;\; H_d \leftrightarrow H_u \; . \tag{6.25}$$

Therefore, for the Higgs field in the **45** representation, we obtain the additional contributions to the SM fermion Yukawa coupling terms and trilinear soft terms

$$W \supset h^i \frac{2v}{\sqrt{10}M_*} \left[Q_i D_i^c H_d + L_i E_i^c H_d - Q_i U_i^c H_u - L_i N_i^c H_u\right] , \tag{6.26}$$

$$-\mathcal{L} \supset y^i \frac{2F_T v}{\sqrt{10}M_*^2} \left[\widetilde{Q}_i \widetilde{D}_i^c H_d + \widetilde{L}_i \widetilde{E}_i^c H_d - \widetilde{Q}_i \widetilde{U}_i^c H_u - \widetilde{L}_i \widetilde{N}_i^c H_u\right] . \tag{6.27}$$

For the Higgs field in the **210** representation, we have

$$W \supset h'^i \frac{v}{\sqrt{5}M_*} \left[3L_i E_i^c H_d - Q_i D_i^c H_d\right] , \tag{6.28}$$

$$-\mathcal{L} \supset y'^i \frac{F_T v}{\sqrt{5}M_*^2} \left[3\widetilde{L}_i \widetilde{E}_i^c H_d - \widetilde{Q}_i \widetilde{D}_i^c H_d\right] . \tag{6.29}$$

6.5 The $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ Model

The $SO(10)$ gauge symmetry can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ gauge symmetry by giving VEVs to the $(\mathbf{24}, \mathbf{0})$ component of the Higgs fields in the $\mathbf{45}$, $\mathbf{54}$ and $\mathbf{210}$ representations under $SU(5) \times U(1)$, or to the $(\mathbf{75}, \mathbf{0})$ component of the Higgs field in the $\mathbf{210}$ representation. In this subsection, we only study the Yukawa coupling terms and trilinear soft terms in the Georgi-Glashow $SU(5) \times U(1)'$ model, *i.e.* the gauge symmetry is $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$. The Yukawa coupling terms and trilinear soft terms in the flipped $SU(5) \times U(1)_X$ model can be obtained from those in the Georgi-Glashow $SU(5) \times U(1)'$ model by making the transformation in Eq. (6.25).

First, for the Higgs field $\Phi_{\mathbf{45}}$ in the $\mathbf{45}$ representation, we can write the VEV in the form of a $\mathbf{10} \times \mathbf{10}$ matrix as follows

$$\langle \Phi_{\mathbf{45}} \rangle \; = \; v\sqrt{\frac{3}{5}}\mathrm{diag}(\, \frac{1}{3}, \, \frac{1}{3}, \, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \, \frac{1}{2}, \, \frac{1}{2}) \; , \tag{6.30}$$

which is normalized to $c = 1$. It can also be written in components as follows

$$3(\Phi_{\mathbf{45}})_{12} = 3(\Phi_{\mathbf{45}})_{34} = 3(\Phi_{\mathbf{45}})_{56} = -2(\Phi_{\mathbf{45}})_{78} = -2(\Phi_{\mathbf{45}})_{90} = v\sqrt{\frac{3}{5}} \; . \tag{6.31}$$

To calculate the additional contributions to the Yukawa coupling terms and trilinear soft terms, we consider the following superpotential

$$\begin{aligned} W \supset & \frac{1}{M_*}\left[h^i(\mathbf{16_i} \otimes \mathbf{16_i})^m_{\mathbf{10}}(\Phi_{\mathbf{45}})_{mn}\mathbf{10}^n + h'^i(\mathbf{16_i} \otimes \mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi_{\mathbf{45}})_{mn}\mathbf{10}_l\right] \\ & +\alpha' \frac{T}{M_*^2}\left[y^i(\mathbf{16_i} \otimes \mathbf{16_i})^m_{\mathbf{10}}(\Phi_{\mathbf{45}})_{mn}\mathbf{10}^n + y'^i(\mathbf{16_i} \otimes \mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi_{\mathbf{45}})_{mn}\mathbf{10}_l\right] \; . \end{aligned} \tag{6.32}$$

After $\Phi_{\mathbf{45}}$ acquires a VEV, we obtain the additional contributions to the Yukawa coupling terms

$$W \supset h^i \frac{v}{2M_*}\sqrt{\frac{3}{5}}\left[Q_i U^c_i H_u + L_i N^c_i H_u - Q_i D^c_i H_d - L_i E^c_i H_d\right] \; . \tag{6.33}$$

The extra supersymmetry breaking trilinear soft terms are

$$-\mathcal{L} \supset y^i \frac{F_T v}{2M_*^2}\sqrt{\frac{3}{5}}\left[\widetilde{Q}_i \widetilde{U}^c_i H_u + \widetilde{L}_i \widetilde{N}^c_i H_u - \widetilde{Q}_i \widetilde{D}^c_i H_d - \widetilde{L}_i \widetilde{E}^c_i H_d\right] \; . \tag{6.34}$$

The new contributions to the low-energy Yukawa coupling terms and trilinear soft terms are the same as the $SU(5) \times U(1)'$ models.

Second, for the Higgs field $\Phi_{\mathbf{54}}$ in the $\mathbf{54}$ representation, we can write the VEV in the form of a $\mathbf{10} \times \mathbf{10}$ matrix as follows

$$\langle \Phi_{\mathbf{54}} \rangle \; = \; v\sqrt{\frac{3}{5}}\mathrm{diag}(\, \frac{1}{3}, \, \frac{1}{3}, \, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}, \, \frac{1}{3}, \, \frac{1}{3}, \, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2}) \; , \tag{6.35}$$

which is normalized to $c = 1$. We consider the following superpotential

$$W \supset \frac{1}{M_*}h^i(\mathbf{16_i} \otimes \mathbf{16_i})^m_{\mathbf{10}}(\Phi_{\mathbf{54}})_{mn}\mathbf{10}^n + \alpha' \frac{T}{M_*^2}y^i(\mathbf{16_i} \otimes \mathbf{16_i})^m_{\mathbf{10}}(\Phi_{\mathbf{54}})_{mn}\mathbf{10}^n \; . \tag{6.36}$$

After $\Phi_{\mathbf{54}}$ acquires a VEV, we obtain the additional contributions to the Yukawa coupling terms

$$W \supset -h^i \frac{v}{2M_*}\sqrt{\frac{3}{5}}\left[Q_i U_i^c H_u + L_i N_i^c H_u + Q_i D_i^c H_d + L_i E_i^c H_d\right] \ . \tag{6.37}$$

The extra supersymmetry breaking trilinear soft terms are

$$-\mathcal{L} \supset -y^i \frac{F_T v}{2M_*^2}\sqrt{\frac{3}{5}}\left[\widetilde{Q}_i \widetilde{U}_i^c H_u + \widetilde{L}_i \widetilde{N}_i^c H_u + \widetilde{Q}_i \widetilde{D}_i^c H_d + \widetilde{L}_i \widetilde{E}_i^c H_d\right] \ . \tag{6.38}$$

Third, we consider that the $(\mathbf{24}, \mathbf{0})$ component of the Higgs field $\Phi_{\mathbf{210}}^{\mathbf{24}}$ in the $\mathbf{210}$ representation obtains a VEV. We can write its VEV in the $\mathbf{16} \times \mathbf{16}$ matrix as follows

$$\langle \Phi_{\mathbf{210}}^{\mathbf{24}} \rangle \ = \ \frac{v}{\sqrt{5}}\mathrm{diag}(-1, -1, -1, \ \frac{3}{2}, \ \frac{3}{2}, \underbrace{\frac{1}{6}, \cdots, \ \frac{1}{6}}_{6}, -\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}, \ 1, \ 0) \ , \tag{6.39}$$

which is normalized to $c = 2$. In components we have

$$\begin{aligned} 6(\Phi_{\mathbf{210}}^{\mathbf{24}})_{1278} &= 6(\Phi_{\mathbf{210}}^{\mathbf{24}})_{3478} = 6(\Phi_{\mathbf{210}}^{\mathbf{24}})_{5678} = 6(\Phi_{\mathbf{210}}^{\mathbf{24}})_{1290} \\ &= 6(\Phi_{\mathbf{210}}^{\mathbf{24}})_{3490} = 6(\Phi_{\mathbf{210}}^{\mathbf{24}})_{5690} = -\frac{3}{2}(\Phi_{\mathbf{210}}^{\mathbf{24}})_{1234} \\ &= = -\frac{3}{2}(\Phi_{\mathbf{210}}^{\mathbf{24}})_{1256} = -\frac{3}{2}(\Phi_{\mathbf{210}}^{\mathbf{24}})_{3456} = (\Phi_{\mathbf{210}}^{\mathbf{24}})_{7890} = \frac{v}{\sqrt{5}} \ . \end{aligned} \tag{6.40}$$

We consider the following superpotential

$$\begin{aligned} W \supset \ & \frac{1}{M_*}\left[h^i(\mathbf{16_i} \otimes \mathbf{16_i})_{\mathbf{120}}^{mnl}(\Phi_{\mathbf{210}}^{\mathbf{24}})_{mnlk}\mathbf{10}^k + h'^i(\mathbf{16_i} \otimes \mathbf{16_i})_{\mathbf{126}}^{mnlpq}(\Phi_{\mathbf{210}}^{\mathbf{24}})_{mnlp}\mathbf{10}_q\right] \\ & +\alpha' \frac{T}{M_*^2}\left[y^i(\mathbf{16_i} \otimes \mathbf{16_i})_{\mathbf{120}}^{mnl}(\Phi_{\mathbf{210}}^{\mathbf{24}})_{mnlk}\mathbf{10}^k \right. \\ & \left. +y'^i(\mathbf{16_i} \otimes \mathbf{16_i})_{\mathbf{126}}^{mnlpq}(\Phi_{\mathbf{210}}^{\mathbf{24}})_{mnlp}\mathbf{10}_q\right] \ . \end{aligned} \tag{6.41}$$

After $\Phi_{\mathbf{210}}^{\mathbf{24}}$ acquires a VEV, we obtain the additional contributions to the Yukawa coupling terms

$$W \supset h'^i \frac{v}{M_*}\frac{1}{6\sqrt{5}}\left[-3Q_i U_i^c H_u + 9L_i N_i^c H_u - 5Q_i D_i^c H_d + 15L_i E_i^c H_d\right] \ . \tag{6.42}$$

The extra supersymmetry breaking trilinear soft terms are

$$-\mathcal{L} \supset y'^i \frac{F_T v}{M_*^2}\frac{1}{6\sqrt{5}}\left[-3\widetilde{Q}_i \widetilde{U}_i^c H_u + 9\widetilde{L}_i \widetilde{N}_i^c H_u - 5\widetilde{Q}_i \widetilde{D}_i^c H_d + 15\widetilde{L}_i \widetilde{E}_i^c H_d\right] \ . \tag{6.43}$$

Finally, we consider that the $(\mathbf{75}, \mathbf{0})$ component of the Higgs field $\Phi_{\mathbf{210}}^{\mathbf{75}}$ in the $\mathbf{210}$ representation obtains a VEV. We can write its VEV in the $\mathbf{16} \times \mathbf{16}$ matrix as follows

$$\langle \Phi_{\mathbf{210}}^{\mathbf{75}} \rangle \ = \ \frac{v}{3}\mathrm{diag}(\ 0, \ 0, \ 0, \ 0, \ 0, \underbrace{-1, \cdots, -1}_{6}, \ 1, \ 1, \ 1, \ 3, \ 0) \ , \tag{6.44}$$

which is normalized as $c = 2$. In components we have

$$
\begin{aligned}
(\Phi^{\mathbf{75}}_{\mathbf{210}})_{1278} &= (\Phi^{\mathbf{75}}_{\mathbf{210}})_{3478} = (\Phi^{\mathbf{75}}_{\mathbf{210}})_{5678} = (\Phi^{\mathbf{75}}_{\mathbf{210}})_{1290} \\
&= (\Phi^{\mathbf{75}}_{\mathbf{210}})_{3490} = (\Phi^{\mathbf{75}}_{\mathbf{210}})_{5690} = -(\Phi^{\mathbf{75}}_{\mathbf{210}})_{1234} \\
&= = -(\Phi^{\mathbf{75}}_{\mathbf{210}})_{1256} = -(\Phi^{\mathbf{75}}_{\mathbf{210}})_{3456} = -\frac{1}{3}(\Phi^{\mathbf{75}}_{\mathbf{210}})_{7890} = -\frac{v}{3} \ .
\end{aligned} \tag{6.45}
$$

We consider the following superpotential

$$
\begin{aligned}
W \supset\ & \frac{1}{M_*}\left[h^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi^{\mathbf{75}}_{\mathbf{210}})_{mnlk}\mathbf{10}^k + h'^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnlpq}_{\mathbf{126}}(\Phi^{\mathbf{75}}_{\mathbf{210}})_{mnlp}\mathbf{10}_q\right] \\
& +\alpha'\frac{T}{M_*^2}\left[y^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnl}_{\mathbf{120}}(\Phi^{\mathbf{75}}_{\mathbf{210}})_{mnlk}\mathbf{10}^k\right. \\
& \left.+y'^i(\mathbf{16_i}\otimes\mathbf{16_i})^{mnlpq}_{\mathbf{126}}(\Phi^{\mathbf{75}}_{\mathbf{210}})_{mnlp}\mathbf{10}_q\right] \ .
\end{aligned} \tag{6.46}
$$

After $\Phi^{\mathbf{75}}_{\mathbf{210}}$ acquires a VEV, we obtain the additional contributions to the Yukawa coupling terms

$$
W \supset h'^i\frac{v}{3M_*}\left[-Q_iD^c_iH_d + 3L_iE^c_iH_d\right] \ . \tag{6.47}
$$

The extra supersymmetry breaking trilinear soft terms are

$$
-\mathcal{L} \supset y'^i\frac{F_Tv}{3M_*^2}\left[-\widetilde{Q}_i\widetilde{D}^c_iH_d + 3\widetilde{L}_i\widetilde{E}^c_iH_d\right] \ . \tag{6.48}
$$

7. Scalar and Gaugino Mass Relations

In order to study the scalar and gaugino mass relations that are invariant under one-loop renormalization group running, we need to know the renormalization group equations (RGEs) of the supersymmetry breaking scalar masses and gaugino masses. For simplicity, we only consider the one-loop RGE running since the two-loop RGE running effects are small [35]. In particular, for the first two generations, we can neglect the contributions from the Yukawa coupling terms and trilinear soft terms, and then the RGEs for the scalar masses are [43]

$$
16\pi^2\frac{dm^2_{\widetilde{Q}_j}}{dt} = -\frac{32}{3}g_3^2M_3^2 - 6g_2^2M_2^2 - \frac{2}{15}g_1^2M_1^2 + \frac{1}{5}g_1^2S \ , \tag{7.1}
$$

$$
16\pi^2\frac{dm^2_{\widetilde{U}^c_j}}{dt} = -\frac{32}{3}g_3^2M_3^2 - \frac{32}{15}g_1^2M_1^2 - \frac{4}{5}g_1^2S \ , \tag{7.2}
$$

$$
16\pi^2\frac{dm^2_{\widetilde{D}^c_j}}{dt} = -\frac{32}{3}g_3^2M_3^2 - \frac{8}{15}g_1^2M_1^2 + \frac{2}{5}g_1^2S \ , \tag{7.3}
$$

$$
16\pi^2\frac{dm^2_{\widetilde{L}_j}}{dt} = -6g_2^2M_2^2 - \frac{6}{5}g_1^2M_1^2 - \frac{3}{5}g_1^2S \ , \tag{7.4}
$$

$$
16\pi^2\frac{dm^2_{\widetilde{E}^c_j}}{dt} = -\frac{24}{5}g_1^2M_1^2 + \frac{6}{5}g_1^2S \ , \tag{7.5}
$$

where $j = 1, 2$, and $t = \ln\mu$ and μ is the renormalization scale. Also, S is given by

$$S = Tr[Y_{\phi_i} m^2(\phi_i)] = m^2_{H_u} - m^2_{H_d} + Tr[M^2_{\widetilde{Q}_i} - M^2_{\widetilde{L}_i} - 2M^2_{\widetilde{U}^c_i} + M^2_{\widetilde{D}^c_i} + M^2_{\widetilde{E}^c_i}] \ . \tag{7.6}$$

The one-loop RGEs for gauge couplings g_i and gaugino masses M_i are

$$\frac{d}{dt} g_i = \frac{1}{16\pi^2} b_i g_i^3 \ , \quad \frac{d}{dt} M_i = \frac{1}{8\pi^2} b_i g_i^2 M_i \ , \tag{7.7}$$

where $g_1 \equiv \sqrt{5} g_Y/\sqrt{3}$, and b_1, b_2 and b_3 are one-loop beta functions for $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively. For the supersymmetric SM, we have

$$b_3 = -3 \ , \ b_2 = 1 \ , \ b_1 = \frac{33}{5} \ . \tag{7.8}$$

Therefore, we obtain

$$\begin{aligned} \frac{d}{dt}\left[\frac{MSQj}{Y_{Q_j}}\right] = \frac{d}{dt}\left[\frac{MSUj}{Y_{U^c_j}}\right] &= \frac{d}{dt}\left[\frac{MSDj}{Y_{D^c_j}}\right] \\ = \frac{d}{dt}\left[\frac{MSLj}{Y_{L_j}}\right] &= \frac{d}{dt}\left[\frac{MSEj}{Y_{E^c_j}}\right] \ , \end{aligned} \tag{7.9}$$

where

$$MSQj = 4m^2_{\widetilde{Q}_j} + \frac{32}{3b_3} M_3^2 + \frac{6}{b_2} M_2^2 + \frac{2}{15b_1} M_1^2 \ , \tag{7.10}$$

$$MSUj = 4m^2_{\widetilde{U}^c_j} + \frac{32}{3b_3} M_3^2 + \frac{32}{15b_1} M_1^2 \ , \tag{7.11}$$

$$MSDj = 4m^2_{\widetilde{D}^c_j} + \frac{32}{3b_3} M_3^2 + \frac{8}{15b_1} M_1^2 \ , \tag{7.12}$$

$$MSLj = 4m^2_{\widetilde{L}_j} + \frac{6}{b_2} M_2^2 + \frac{6}{5b_1} M_1^2 \ , \tag{7.13}$$

$$MSEj = 4m^2_{\widetilde{E}^c_j} + \frac{24}{5b_1} M_1^2 \ . \tag{7.14}$$

In addition, we obtain the most general scalar and gaugino mass relations that are valid from the GUT scale to the electroweak scale under one-loop RGE running for the first two families

$$\gamma_{Q_j}\frac{MSQj}{Y_{Q_j}} + \gamma_{U^c_j}\frac{MSUj}{Y_{U^c_j}} + \gamma_{D^c_j}\frac{MSDj}{Y_{D^c_j}} + \gamma_{L_j}\frac{MSLj}{Y_{L_j}} + \gamma_{E^c_j}\frac{MSEj}{Y_{E^c_j}} = C_o \ , \tag{7.15}$$

where C_o denotes the invariant constant under one-loop RGE running, and γ_{Q_j}, $\gamma_{U^c_j}$, $\gamma_{D^c_j}$, γ_{L_j}, and $\gamma_{E^c_j}$ are real or complex numbers that satisfy

$$\gamma_{Q_j} + \gamma_{U^c_j} + \gamma_{D^c_j} + \gamma_{L_j} + \gamma_{E^c_j} = 0 \ . \tag{7.16}$$

In this paper, we shall study the following scalar and gaugino mass relations

$$C_o^{AB} = 3m^2_{\widetilde{D}^c_j} + 2m^2_{\widetilde{L}_j} - 4m^2_{\widetilde{Q}_j} - m^2_{\widetilde{U}^c_j} - \left[\frac{16}{3b_3} M_3^2 + \frac{3}{b_2} M_2^2 - \frac{1}{3b_1} M_1^2\right] \ , \tag{7.17}$$

$$C_o^{AC} = 3m^2_{\widetilde{D}^c_j} + 2m^2_{\widetilde{L}_j} + m^2_{\widetilde{E}^c_j} - 6m^2_{\widetilde{Q}_j} - \left[\frac{8}{b_3}M_3^2 + \frac{6}{b_2}M_2^2 - \frac{2}{b_1}M_1^2\right] , \tag{7.18}$$

$$C_o^{AD} = 3m^2_{\widetilde{D}^c_j} + 2m^2_{\widetilde{L}_j} - 3m^2_{\widetilde{U}^c_j} - 2m^2_{\widetilde{E}^c_j} + \left[\frac{3}{b_2}M_2^2 - \frac{3}{b_1}M_1^2\right] , \tag{7.19}$$

$$C_o^{BC} = m^2_{\widetilde{U}^c_j} + m^2_{\widetilde{E}^c_j} - 2m^2_{\widetilde{Q}_j} - \left[\frac{8}{3b_3}M_3^2 + \frac{3}{b_2}M_2^2 - \frac{5}{3b_1}M_1^2\right] , \tag{7.20}$$

$$C_o^{X} = m^2_{\widetilde{Q}_j} + 3m^2_{\widetilde{L}_j} + m^2_{\widetilde{E}^c_j} - 2m^2_{\widetilde{U}^c_j} - 3m^2_{\widetilde{D}^c_j} - \left[\frac{32}{3b_3}M_3^2 - \frac{6}{b_2}M_2^2 - \frac{2}{3b_1}M_1^2\right] . \tag{7.21}$$

In short, we can obtain the scalar and gaugino mass relations that are valid from the GUT scale to the electroweak scale at one loop. Such relations will be useful to distinguish between the mSUGRA and GmSUGRA scenarios.

The scalar and gaugino mass relations can be simplified by the scalar and gaugino mass relations at the GUT scale. Because the high-dimensional operators can contribute to gauge kinetic functions after GUT symmetry breaking, the SM gauge couplings may not be unified at the GUT scale. Thus, we will have two contributions to the gaugino masses at the GUT scale: the universal gaugino masses as in the mSUGRA, and the non-universal gaugino masses due to the high-dimensional operators. In particular, for the scenarios studied in Refs. [17, 18, 19, 20, 21, 22, 23] where the universal gaugino masses are assumed to be zero, *i.e.*, $M_i/\alpha_i = a_i M'_{1/2}$, we obtain the gaugino mass relation at one loop [36]

$$\frac{M_3}{a_3\alpha_3} = \frac{M_2}{a_2\alpha_2} = \frac{M_1}{a_1\alpha_1} . \tag{7.22}$$

We can calculate the scalar and gaugino mass relations in the mSUGRA and GmSUGRA scenarios, and compare them in different cases.

7.1 The $SU(5)$ Model

In the following, we consider the RGE running for the scalar masses of the first two families in the $SU(5)$ model with the Higgs fields in the **24** and **75** representations.

- The $SU(5)$ Model with a **24** Dimensional Higgs Field

  Let us consider the scalar and gaugino mass relations C_o^{AB}, C_o^{AC}, C_o^{AD}, and C_o^{BC} in the mSUGRA and GmSUGRA scenarios for the first two generations. In the mSUGRA scenario with universal gaugino masses and scalar masses, we obtain the C_o^{AB}, C_o^{AC}, C_o^{AD}, and C_o^{BC} as follows

$$(C_o^{AB})^U = -\frac{116}{99}\left(\frac{M_1^2(\mu)}{g_1^4(\mu)}\right) g_1^4(M_U) , \tag{7.23}$$

$$(C_o^{AC})^U = -\frac{100}{33}\left(\frac{M_1^2(\mu)}{g_1^4(\mu)}\right) g_1^4(M_U) , \tag{7.24}$$

$$(C_o^{AD})^U = \frac{28}{11}\left(\frac{M_1^2(\mu)}{g_1^4(\mu)}\right) g_1^4(M_U) , \tag{7.25}$$

$$(C_o^{BC})^U = -\frac{184}{99}\left(\frac{M_1^2(\mu)}{g_1^4(\mu)}\right) g_1^4(M_U) . \tag{7.26}$$

In the GmSUGRA scenario, we consider the scenario in Refs. [17, 18, 19, 20, 21, 22, 23]. At the GUT scale we have

$$\frac{M_3}{2} = \frac{M_2}{-3} = \frac{M_1}{-1} ~. \tag{7.27}$$

Thus, with Eq. (4.11) for the non-universal scalar mass relations at the GUT scale in the GmSUGRA scenario, we obtain the scalar and gaugino mass relations

$$(C_o^{AB})^{NU} = -\frac{1964}{99}\left(\frac{M_1^2(\mu)}{g_1^4(\mu)}\right) g_1^4(M_U) ~, \tag{7.28}$$

$$(C_o^{AC})^{NU} = -\frac{1420}{33}\left(\frac{M_1^2(\mu)}{g_1^4(\mu)}\right) g_1^4(M_U) ~, \tag{7.29}$$

$$(C_o^{AD})^{NU} = \frac{292}{11}\left(\frac{M_1^2(\mu)}{g_1^4(\mu)}\right) g_1^4(M_U) ~, \tag{7.30}$$

$$(C_o^{BC})^{NU} = -\frac{2296}{99}\left(\frac{M_1^2(\mu)}{g_1^4(\mu)}\right) g_1^4(M_U) ~. \tag{7.31}$$

Thus, with precise enough measurements, we may distinguish the mSUGRA and GmSUGRA scenarios. In particular, we can consider the ratios of these one-loop RGE invariant constants and then distinguish the mSUGRA and GmSUGRA scenarios, for example, $(C_o^{AC})^U/(C_o^{AB})^U = 2.586$, while $(C_o^{AC})^{NU}/(C_o^{AB})^{NU} = 2.169$. Similarly, we can discuss the other scalar and gaugino mass relations for the first two generations in mSUGRA and GmSUGRA.

- The $SU(5)$ Model with a **75** Dimensional Higgs Field

In the mSUGRA scenario with universal gaugino and scalar masses, we obtain the one-loop RGE invariant constant C_o^X at the GUT scale

$$(C_o^X)^U = \frac{956}{99}\left(\frac{M_3^2(\mu)}{g_3^4(\mu)}\right) g_3^4(M_U) ~. \tag{7.32}$$

In the GmSUGRA scenario with non-universal gaugino and scalar masses, we consider the non-universal gaugino mass ratios in Refs. [17, 18, 19, 20, 21, 22, 23]

$$\frac{M_3}{1} = \frac{M_2}{3} = \frac{M_1}{-5} ~. \tag{7.33}$$

With the non-universal scalar masses in Eq. (4.13), we obtain

$$(C_o^X)^{NU} = \frac{5948}{99}\left(\frac{M_3^2(\mu)}{g_3^4(\mu)}\right) g_3^4(M_U) ~. \tag{7.34}$$

Assuming that there are no threshold corrections from the electroweak scale to the GUT scale, we can calculate the gauge couplings at the GUT scale and check these scalar and gaugino mass relations if we know the low energy sparticle spectrum.

7.2 The Pati-Salam Model from $SO(10)$

We consider the following $SO(10)$ gauge symmetry breaking chain

$$
\begin{aligned}
SO(10) &\rightarrow SU(4)_C \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
&\rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \ . \qquad (7.35)
\end{aligned}
$$

Other symmetry breaking chains can be discussed similarly.

Let us explain our convention. We denote the gauge couplings for the $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$, and $SU(4)_C$ gauge symmetries as g_{2L}, g_{2R}, $\widetilde{g}_{B-L}$ (or traditional g_{B-L}), and g_4, respectively. We denote the gaugino masses for the $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$, and $SU(4)_C$ gauge symmetries as M_{2L}, M_{2R}, M_{B-L} and M_4, respectively. We denote the one-loop beta functions for the $SU(2)_L$, $SU(2)_R$, $U(1)_{B-L}$, and $SU(4)_C$ gauge symmetries as b_{2L}, b_{2R}, $\widetilde{b}_{B-L}$ and b_4, respectively. In addition, we denote the universal supersymmetry breaking scale as M_S, the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry breaking scale as M_{LR}, and the $SU(4)_C$ gauge symmetry breaking scale as M_{PS}. Also, we denote the $U(1)_{B-L}$ charge for the particle ϕ_i as $Y^{B-L}_{\phi_i}$.

The generator $U(1)_{B-L}$ in $SU(4)_C$ is

$$
\widetilde{g}^{B-L} T^{B-L} \ = \ g_{B-L} \mathrm{diag}\left(\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, -\frac{1}{2}\right) \ . \qquad (7.36)
$$

So we can obtain the normalization of g_{B-L} into $SU(4)_C$

$$
g_{B-L} = \sqrt{\frac{3}{2}} \widetilde{g}^{B-L} \ . \qquad (7.37)
$$

Neglecting the Yukawa coupling terms and trilinear soft terms, we obtain the RGEs for the scalar masses of the first two generations in the Pati-Salam model

$$
16\pi^2 \frac{dm^2_{\widetilde{F}^L_j}}{dt} = 4\pi \frac{d}{dt}\left[-\frac{15}{b_4} M_4^2 - \frac{6}{b_{2L}} M_{2L}^2\right] \ , \qquad (7.38)
$$

$$
16\pi^2 \frac{dm^2_{\widetilde{F}^{Rc}_j}}{dt} = 4\pi \frac{d}{dt}\left[-\frac{15}{b_4} M_4^2 - \frac{6}{b_{2R}} M_{2R}^2\right] \ , \qquad (7.39)
$$

which gives

$$
\frac{d}{dt}\left[4m^2_{\widetilde{F}^L_j} + \frac{15}{b_4} M_4^2 + \frac{6}{b_{2L}} M_{2L}^2\right] = 0 \ , \qquad (7.40)
$$

$$
\frac{d}{dt}\left[4m^2_{\widetilde{F}^{Rc}_j} + \frac{15}{b_4} M_4^2 + \frac{6}{b_{2R}} M_{2R}^2\right] = 0 \ . \qquad (7.41)
$$

The RGEs of the scalar masses for the first two generations in the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ model are

$$
16\pi^2 \frac{dm^2_{\widetilde{Q}_j}}{dt} = -\frac{32}{3} g_3^2 M_3^2 - 6 g_{2L}^2 M_{2L}^2 - \frac{1}{3} \widetilde{g}^2_{B-L} M^2_{B-L} + \frac{1}{2} \widetilde{g}^2_{B-L} S' \ , \qquad (7.42)
$$

$$16\pi^2 \frac{dm^2_{\widetilde{U}^c_j, \widetilde{D}^c_j}}{dt} = -\frac{32}{3} g_3^2 M_3^2 - 6 g_{2R}^2 M_{2R}^2 - \frac{1}{3} \widetilde{g}_{B-L}^2 M_{B-L}^2 - \frac{1}{2} \widetilde{g}_{B-L}^2 S' \ , \tag{7.43}$$

$$16\pi^2 \frac{dm^2_{\widetilde{L}_j}}{dt} = -6 g_{2L}^2 M_{2L}^2 - 3 \widetilde{g}_{B-L} M_{B-L}^2 - \frac{3}{2} \widetilde{g}_{B-L}^2 S' \ , \tag{7.44}$$

$$16\pi^2 \frac{dm^2_{\widetilde{E}^c_j}}{dt} = -6 g_{2R}^2 M_{2R}^2 - 3 \widetilde{g}_{B-L} M_{B-L}^2 + \frac{3}{2} \widetilde{g}_{B-L}^2 S' \ , \tag{7.45}$$

where

$$S' = Tr[Y^{B-L}_{\phi_i} m^2(\phi_i)] \ . \tag{7.46}$$

We consider the following linear combination of the squared scalar masses

$$\begin{aligned}
& 16\pi^2 \frac{d}{dt} \left(m^2_{\widetilde{U}^c_j} + m^2_{\widetilde{E}^c_j} - 2 m^2_{\widetilde{Q}_j} \right) \\
= \ & 4\pi^2 \frac{d}{dt} \left[\frac{32}{3b_3} M_3^2 + \frac{12}{b_{2L}} M_{2L}^2 - \frac{20}{3b_1} M_1^2 \right] \quad \text{for} \ \ M_S < \mu < M_{LR} \\
= \ & 4\pi^2 \frac{d}{dt} \left[\frac{32}{3b_3} M_3^2 + \frac{12}{b_{2L}} M_{2L}^2 - \frac{12}{b_{2R}} M_{2R}^2 - \frac{8}{3\widetilde{b}_{B-L}} M_{B-L}^2 \right] \\
& \text{for} \ \ M_{LR} < \mu < M_{PS} \\
= \ & 4\pi^2 \frac{d}{dt} \left[\frac{12}{b_{2L}} M_{2L}^2 - \frac{12}{b_{2R}} M_{2R}^2 \right] \quad \text{for} \ \ M_{PS} < \mu < M_U \ .
\end{aligned} \tag{7.47}$$

From this, we obtain the scalar and gaugino mass relations which are exact from the GUT scale to the supersymmetry breaking scale at one loop

$$4 \left(m^2_{\widetilde{U}^c_j} + m^2_{\widetilde{E}^c_j} - 2 m^2_{\widetilde{Q}_j} \right) - \frac{32}{3b_3} M_3^2 - \frac{12}{b_{2L}} M_{2L}^2 + \frac{20}{3b_1} M_1^2 = C_o^1 \ , \tag{7.48}$$

$$\begin{aligned}
& 4 \left(m^2_{\widetilde{U}^c_j} + m^2_{\widetilde{E}^c_j} - 2 m^2_{\widetilde{Q}_j} \right) - \frac{32}{3b_3} M_3^2 - \frac{12}{b_{2L}} M_{2L}^2 + \frac{12}{b_{2R}} M_{2R}^2 \\
& + \frac{8}{3\widetilde{b}_{B-L}} M_{B-L}^2 = C_o^2 \ ,
\end{aligned} \tag{7.49}$$

$$4 \left(m^2_{\widetilde{U}^c_j} + m^2_{\widetilde{E}^c_j} - 2 m^2_{\widetilde{Q}_j} \right) - \frac{12}{b_{2L}} M_{2L}^2 + \frac{12}{b_{2R}} M_{2R}^2 = C_o^3 \ . \tag{7.50}$$

The differences between the constants C_o^1 and C_o^2 and between the constants C_o^2 and C_o^3 are the threshold contributions from the extra particles due to gauge symmetry breaking. Thus, the three constants can be determined by matching the threshold contributions at the symmetry breaking scales. The difference between C_o^2 and C_o^3 is

$$C_o^2 - C_o^3 = - \left(\frac{32}{3b_3} - \frac{8}{3\widetilde{b}_{B-L}} \right) \frac{M_3^2(\mu)}{g_3^2(\mu)} g_3^2(M_{PS}) \ , \tag{7.51}$$

which can be determined at the scale M_{PS}. At this $SU(3)_C \times U(1)_{B-L}$ unification scale, we have

$$\frac{M_3}{g_3^2} = \frac{M_{B-L}}{\widetilde{g}_{B-L}^2} = \frac{M_4}{g_4^2} \ . \tag{7.52}$$

For mSUGRA with universal gaugino and scalar masses, we have

$$\frac{M_3}{g_3^2} = \frac{M_{B-L}}{\widetilde{g}_{B-L}^2} = \frac{M_{2L}}{g_{2L}^2} = \frac{M_{2R}}{g_{2R}^2} = \frac{M_4}{g_4^2} \ . \tag{7.53}$$

Thus, we can get the scalar and gaugino mass relations in supersymmetric Standard Models

$$\begin{aligned} & 4\left(m^2_{\widetilde{U}^c_j} + m^2_{\widetilde{E}^c_j} - 2m^2_{\widetilde{Q}_j}\right) - \frac{32}{3b_3}M_3^2 - \frac{12}{b_{2L}}M_2^2 + \frac{20}{3b_1}M_1^2 \\ = & \left(\frac{8}{3\widetilde{b}_{B-L}} - \frac{32}{3b_3}\right)\frac{M_3^2(\mu)}{g_3^4(\mu)}g_3^4(M_{PS}) + \frac{20}{3b_1}\frac{M_1^2(\mu)}{g_1^4(\mu)}g_1^4(M_{LR}) \\ - & \left(\frac{12}{b_{2R}}g_{2R}^4(M_{LR}) + \frac{8}{3\widetilde{b}_{B-L}}g_{B-L}^4(M_{LR})\right)\frac{M_3^2(\mu)}{g_3^4(\mu)} \ . \end{aligned} \tag{7.54}$$

If we know the low energy sparticle spectrum at the LHC and ILC and $g_1^2(M_{LR})$ from the RGE running, we can get the coefficients

$$c = \left(\frac{8}{3\widetilde{b}_{B-L}} - \frac{32}{b_3}\right)g_3^4(M_{PS}) - \left(\frac{12}{b_{2R}}g_{2R}^4(M_{LR}) + \frac{8}{3\widetilde{b}_{B-L}}g_{B-L}^4(M_{LR})\right) \ , \tag{7.55}$$

by fitting the experimental data.

For GmSUGRA with non-universal gaugino and scalar masses, we consider the Higgs field in the **210** representation whose singlet component $(\mathbf{1},\mathbf{1},\mathbf{1})$ acquires a VEV. To give mass to the gluino, we require that the universal gaugino mass be non-zero. From Eq. (5.5), we obtain

$$m^2_{\widetilde{E}^c_j} + m^2_{\widetilde{U}^c_j} - 2m^2_{\widetilde{Q}_j} = -\sqrt{2}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3} \ . \tag{7.56}$$

Thus, the constant combination in the supersymmetric Standard Model is

$$\begin{aligned} & 4\left(m^2_{\widetilde{U}^c_j} + m^2_{\widetilde{E}^c_j} - 2m^2_{\widetilde{Q}_j}\right) - \frac{32}{3b_3}M_3^2 - \frac{12}{b_{2L}}M_{2L}^2 + \frac{20}{3b_1}M_1^2 \\ = & -\sqrt{2}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M_*^3} + \left(\frac{8}{3\widetilde{b}_{B-L}} - \frac{32}{3b_3}\right)\frac{M_3^2(\mu)}{g_3^4(\mu)}g_3^4(M_{PS}) + \frac{20}{3b_1}\frac{M_1^2(\mu)}{g_1^4(\mu)}g_1^4(M_{LR}) \\ - & \left(\frac{12}{b_{2R}}g_{2R}^4(M_{LR})\frac{M_{2R}^2(\mu)}{g_{2R}^4(\mu)} + \frac{8}{3\widetilde{b}_{B-L}}g_{B-L}^4(M_{LR})\frac{M_3^2(\mu)}{g_3^4(\mu)}\right) \ . \end{aligned} \tag{7.57}$$

Therefore, the scalar and gaugino mass relations in mSUGRA are different from those in GmSUGRA. Moreover, we can break the $SO(10)$ gauge symmetry down to the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry by giving VEVs to the $(\mathbf{15},\mathbf{1},\mathbf{1})$ components of the Higgs field in the **45** and **210** representations under $SU(4)_C \times SU(2)_L \times SU(2)_R$. Because the discussions are similar, we will not present them here.

7.3 The Flipped $SU(5) \times U(1)_X$ Model from $SO(10)$

In the flipped $SU(5) \times U(1)_X$ model with $SO(10)$ origin, we have two-step gauge coupling unification: the $SU(3)_C \times SU(2)_L$ gauge symmetry is unified at the scale M_{23}, and then the $SU(5) \times U(1)_X$ gauge symmetry is unified at the scale M_U. At the M_{23} scale, we have the following gauge coupling relation

$$\frac{1}{{g_1}^2} = \frac{24}{25}\frac{1}{g_{1X}^2} + \frac{1}{25}\frac{1}{g_5^2} \ , \tag{7.58}$$

where g_{1X} and g_5 are the gauge couplings for the $U(1)_X$ and $SU(5)$ gauge symmetries, respectively.

Our conventions in this section are as follows. We denote the gaugino masses for the $U(1)_X$ and $SU(5)$ gauge symmetries as M_{1X} and M_5, respectively. We denote the one-loop beta functions for the $U(1)_X$ and $SU(5)$ gauge symmetries as b_{1X} and b_5, respectively. Also, we denote the $U(1)_X$ charge for the particle ϕ_i as $Y^X_{\phi_i}$. With this notation, the RGEs for the scalar masses of the first two generations from the scale M_{23} to M_U are

$$16\pi^2 \frac{dm^2_{\widetilde{F}_j}}{dt} = -\frac{144}{5} g_5^2 M_5^2 - \frac{1}{5} g_{1X}^2 M_{1X}^2 + \frac{1}{20} g_{1X}^2 \tilde{S} \ , \tag{7.59}$$

$$16\pi^2 \frac{dm^2_{\widetilde{\bar{f}}_j}}{dt} = -\frac{96}{5} g_5^2 M_5^2 - \frac{9}{5} g_{1X}^2 M_{1X}^2 - \frac{3}{20} g_{1X}^2 \tilde{S} \ , \tag{7.60}$$

$$16\pi^2 \frac{dm^2_{\widetilde{\bar{l}}_j}}{dt} = -5 g_{1X}^2 M_{1X}^2 + \frac{1}{4} g_{1X}^2 \tilde{S} \ , \tag{7.61}$$

where $\tilde{S}$ is

$$\tilde{S} = Tr[Y^X_{\phi_i} m^2(\phi_i)] \ . \tag{7.62}$$

We consider the following scalar and gaugino mass relation

$$16\pi^2 \frac{d}{dt}\left(m^2_{\widetilde{E}^c_j} + m^2_{\widetilde{U}^c_j} - 2m^2_{\widetilde{Q}_j}\right) \tag{7.63}$$

$$= 4\pi^2 \frac{d}{dt}\left[\frac{32}{3b_3} M_3^2 + \frac{12}{b_2} M_2^2 - \frac{20}{3b_1} M_1^2\right] \quad \text{for} \quad M_S < \mu < M_{23} \tag{7.64}$$

$$= 4\pi^2 \frac{d}{dt}\left[\frac{192}{5b_5} M_5^2 - \frac{32}{5b_{1X}} M_{1X}^2\right] \quad \text{for} \quad M_{23} < \mu < M_U \ . \tag{7.65}$$

In the mSUGRA with universal gaugino and scalar masses, we have at the scale M_U

$$4\left(m^2_{\widetilde{U}^c_j} + m^2_{\widetilde{E}^c_j} - 2m^2_{\widetilde{Q}_j}\right) - \left(\frac{192}{5b_5} M_5^2 - \frac{32}{5b_{1X}} M_{1X}^2\right) \tag{7.66}$$

$$= -\left(\frac{192}{5b_5} - \frac{32}{5b_{1X}}\right) \frac{M_3^2(\mu)}{g_3^4(\mu)} g_3^4(M_U) \ . \tag{7.67}$$

So we get the scalar and gaugino mass relation in supersymmetric Standard Models

$$4\left(m^2_{\widetilde{U}^c_j} + m^2_{\widetilde{E}^c_j} - 2m^2_{\widetilde{Q}_j}\right) - \frac{32}{3b_3} M_3^2 - \frac{12}{b_2} M_2^2 + \frac{20}{3b_1} M_1^2$$

$$= -\left(\frac{32}{3b_3} + \frac{12}{b_2} - \frac{192}{5b_5} + \frac{32}{5b_{1X}}\right)\frac{M_3^2(\mu)}{g_3^4(\mu)} g_3^4(M_{23}) + \frac{20}{3b_1}\frac{M_1^2(\mu)}{g_1^4(\mu)} g_1^4(M_{23})$$
$$-\left(\frac{192}{5b_5} - \frac{32}{5b_{1X}}\right)\frac{M_3^2(\mu)}{g_3^4(\mu)} g_3^4(M_U) \ . \tag{7.68}$$

In GmSUGRA with non-universal gaugino and scalar masses, we consider the Higgs field in the **210** representation whose singlet component $(\mathbf{1}, \mathbf{0})$ acquires a VEV. For non-universal gaugino masses, we consider the mass ratios in Refs. [17, 18, 19, 20, 21, 22, 23]

$$\frac{M_5}{-1} = \frac{M_{1X}}{4} \ . \tag{7.69}$$

With Eq. (5.21), we obtain at the scale M_U

$$4\left(m^2_{\widetilde{U}^c_j} + m^2_{\widetilde{E}^c_j} - 2m^2_{\widetilde{Q}_j}\right) - \left(\frac{192}{5b_5}M_5^2 - \frac{32}{5b_{1X}}M_{1X}^2\right) \tag{7.70}$$
$$= \frac{16}{\sqrt{5}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M^3} - \left(\frac{192}{5b_5} - \frac{512}{5b_{1X}}\right)\frac{M_3^2(\mu)}{g_3^4(\mu)} g_3^4(M_U) \ . \tag{7.71}$$

Thus, we obtain the scalar and gaugino mass relation in supersymmetric Standard Models

$$4\left(m^2_{\widetilde{U}^c_j} + m^2_{\widetilde{E}^c_j} - 2m^2_{\widetilde{Q}_j}\right) - \frac{32}{3b_3}M_3^2 - \frac{12}{b_2}M_2^2 + \frac{20}{3b_1}M_1^2$$
$$= \frac{16}{\sqrt{5}}\beta'_{\mathbf{210}}\frac{v|F_T|^2}{M^3} - \left(\frac{192}{5b_5} - \frac{512}{5b_{1X}}\right)\frac{M_3^2(\mu)}{g_3^4(\mu)} g_3^4(M_U)$$
$$-\left(\frac{32}{3b_3} + \frac{12}{b_2} - \frac{192}{5b_5} + \frac{512}{5b_{1X}}\right)\frac{M_3^2(\mu)}{g_3^4(\mu)} g_3^4(M_{23}) + \frac{20}{3b_1}\frac{M_1^2(\mu)}{g_1^4(\mu)} g_1^4(M_{23}) \ . \tag{7.72}$$

Therefore, the dependence on $M_3^2(\mu)/g_3^4(\mu)$ in mSUGRA is indeed different from that in GmSUGRA. Other gauge symmetry breaking chains can be discussed similarly.

8. Conclusions

In the GmSUGRA scenario with the high-dimensional operators containing the GUT Higgs fields, we systematically studied the supersymmetry breaking scalar masses, SM fermion Yukawa coupling terms, and trilinear soft terms in the $SU(5)$ model with GUT Higgs fields in the **24** and **75** representations, and in the $SO(10)$ model where the gauge symmetry is broken down to the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry, $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge symmetry, George-Glashow $SU(5) \times U(1)'$ gauge symmetry, flipped $SU(5) \times U(1)_X$ gauge symmetry, and $SU(3)_C \times SU(2)_L \times U(1)_1 \times U(1)_2$ gauge symmetry. In addition, we considered the scalar and gaugino mass relations, which can be preserved from the GUT scale to the electroweak scale under one-loop RGE running, in the $SU(5)$ model, the Pati-Salam model and the flipped $SU(5) \times U(1)_X$ model arising from the $SO(10)$ model. With such relations, we may distinguish the mSUGRA and GmSUGRA scenarios if we can measure the supersymmetric particle spectrum at the LHC and ILC. Thus, it provides us with another important window of opportunity at the Planck scale.

Note added: after our paper was submitted, we noticed the paper [44], which also studies the RGE invariants in the supersymmetric Standard Models.

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