CERN–TH.6481/92
CTP–TAMU–42/92
ACT–10/92

# Constraints From Gauge Coupling Unification On The Scale Of Supersymmetry Breaking



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ABSTRACT

We reanalyze precision LEP data and coupling constant unification in the minimal supersymmetric $SU(5)$ model including the evolution of the gaugino masses. We derive general bounds on the primordial gaugino supersymmetry-breaking mass-scale $m_{1/2}$ in terms of the various input parameters. The model cannot accommodate $m_{1/2} < 1\,\mathrm{TeV}$ for values of $\alpha_3(m_Z) < 0.115$, even for extreme $1-\sigma$ values of the other inputs. We emphasize the sensitivity of this type of calculations to the various input parameters.

CERN-TH.6481/92
CTP-TAMU-42/92
ACT-10/92
April 1992

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## 1. Introduction

Gauge coupling unification has always led to attractive predictions of Grand Unified Theories, GUTs [1]. Although qualitatively successful, minimal non-supersymmetric GUTs predict $\sin^2\theta_W(m_Z)$ about 8 to 5 percent lower than the present LEP value [2]. However, minimal supersymmetric GUTs predict $\sin^2\theta_W(m_Z)$ tantalizingly close to the LEP value [3]. Some authors have suggested that the amazingly close agreement of LEP data with minimal supersymmetric GUTs is evidence for supersymmetry at a scale near 1 TeV [4], in accord with theoretical prejudice based on naturalness arguments. Previously, we developed analytically one-loop expressions including new particle thresholds, and corrected these expressions to two-loop accuracy [5]. These expressions were then applied to to a specific model, minimal supersymmetric $SU(5)$ [6], in an attempt to bound the supersymmetry-breaking scale in the form of a universal primordial gaugino mass, $m_{1/2}$ [7].

We found that the data then available favored $m_{1/2} < 65\,\mathrm{GeV}$ or $m_{1/2} > 21\,\mathrm{TeV}$ at the one-standard-deviation $(1-\sigma)$ level. However, in those papers we did not include the dependences of the physical gaugino mass thresholds on the gauge couplings, which has recently been shown to have a significant impact on the favored range of $m_{1/2}$: the evolution of gaugino masses (EGM) effect [8].

In this letter, we correct and continue our previous work [7] on the derivation of rigorous bounds on $m_{1/2}$, incorporating this EGM effect and the latest available LEP data. Since the largest uncertainty in $m_{1/2}$ is now that due to $\alpha_3(m_Z)$, we present our results as bounds on $m_{1/2}$ as a function of $\alpha_3(m_Z)$ for specific values and plausible ranges of the other parameters: $\sin^2\theta_W(m_Z)$, $\alpha_{em}(m_Z)$, $m_t$, the Higgs superpotential mixing parameter $\mu$, and the Higgs mass $m_h$. Throughout this paper, $\sin^2\theta_W(m_Z)$ and all gauge couplings are defined using the $\overline{MS}$ scheme. Using extreme $1-\sigma$ values of the other inputs, we find that $m_{1/2} < 1\,\mathrm{TeV}$ can be obtained only for $\alpha_3(m_Z) > 0.115$. We show how to modify our results for different values of the inputs, noting that improved precision will most likely increase the minimum value of $\alpha_3(m_Z)$ compatible with $m_{1/2} < 1\,\mathrm{TeV}$.

## 2. Basic Formulae and LEP data

The prediction for $\sin^2\theta_W(m_Z)$ in minimal supersymmetric $SU(5)$ may be written as [7]

$$\sin^2\theta_W(m_Z) = 0.2 + \frac{7\alpha_{em}(m_Z)}{15\alpha_3(m_Z)} + 0.0029 + \delta_s(light) + \delta_s(heavy) + \delta_s(conv) \tag{2.1}$$

where 0.0029 corrects the analytic one-loop calculation to two-loop accuracy. The scheme conversion term $\delta_s(conv)$ is negligible. Because of constraints on the Higgs triplets mediating proton decay, the heavy particle threshold correction $\delta_s(heavy) > 0$ in minimal supersymmetric $SU(5)$ [7]. The contribution from light particle thresholds is:

$$\begin{aligned}\delta_s(light) =& \frac{\alpha_{em}(m_Z)}{20\pi}[-3ln(\frac{m_t}{m_Z}) + \frac{28}{3}ln(\frac{c_{\tilde{g}}m_{1/2}}{m_Z}) - \frac{32}{3}ln(\frac{c_{\tilde{w}}m_{1/2}}{m_Z}) \\ & - ln(\frac{m_h}{m_Z}) - 4ln(\frac{\mu}{m_Z}) + \frac{4}{3}f(m_{1/2}, y, w)]\end{aligned} \tag{2.2}$$

where

$$\begin{aligned}f(m_{1/2}, y, w) =& \frac{15}{8}ln(\frac{m_{1/2}\sqrt{c_{\tilde{q}} + y}}{m_Z}) - \frac{9}{4}ln(\frac{m_{1/2}\sqrt{c_{\tilde{e}_l} + y}}{m_Z}) \\ & + \frac{3}{2}ln(\frac{m_{1/2}\sqrt{c_{\tilde{e}_r} + y}}{m_Z}) - \frac{19}{48}ln(\frac{m_{1/2}\sqrt{c_{\tilde{q}} + y + w}}{m_Z}) \\ & - \frac{35}{48}ln(\frac{m_{1/2}\sqrt{c_{\tilde{q}} + y - w}}{m_Z})\end{aligned} \tag{2.3}$$

and $y \equiv (m_0/m_{1/2})^2$, where $m_0$ is a universal primordial supersymmetry-breaking spin-zero mass, $w$ was defined in [7] and the logarithms should be set to zero if the threshold is below $m_Z$.

The value of $\sin^2\theta_W(m_Z)$ is known from an analysis of precision data from LEP and elsewhere:

$$\sin^2\theta_W(m_Z) = 0.2327 \pm 0.0007 \quad [9] \tag{2.4}$$

The value of $\alpha_{em}(m_Z)$ is also well-known:

$$\alpha_{em}(m_Z) = \frac{1}{127.9 \pm 0.2} \quad [10][11] \tag{2.5}$$

However, a glance at Table 1 shows that there is a huge range in the experimental determinations of $\alpha_3(m_Z)$ [12]. Naively taking an equally-weighted mean of these eight different values and the uncertainty of the least uncertain experiment gives $\alpha_3(m_Z) = .116 \pm .005$. However, it is disturbing that the different experimental determinations of $\alpha_3(m_Z)$ do not overlap. Except for the DELPHI value, all the LEP measurements are higher than, and do not even overlap the other three lower-energy measurements. It is known that higher-order corrections to these LEP values are important, and preliminary attempts to sum the dominant contributions to the perturbative higher-order effects indicate that the LEP results could be shifted down to values nearer the other measurements [13].

We take an agnostic approach in this paper and derive bounds on $m_{1/2}$ for arbitrary values of $\alpha_3(m_Z)$, and await further progress in its determination.

## 3. Derivation of Bounds on $m_{1/2}$

In this section, we use the information of the previous section to derive bounds on $m_{1/2}$ for different values of $\alpha_3(m_Z)$, assuming $\delta_s(heavy) = 0$[2] and ignoring the negligible $\delta_s(conv)$.

Consider first a scenario where the winos are lighter than $m_Z$ but the gluinos are heavier than $m_Z$. The formula (2.1) then gives:

$$ln(\frac{m_{1/2}}{m_Z}) = \frac{1}{7}X - \frac{\pi}{\alpha_3} - c_{\tilde{g}} \tag{3.1}$$

where

$$\begin{aligned} X =& \frac{15\pi}{\alpha_{em}(m_Z)}(\sin^2\theta_W(m_Z) - .2029) \\ &+ \frac{9}{4}ln(\frac{m_t}{m_Z}) + 3ln(\frac{\mu}{m_Z}) - \frac{3}{4}ln(\frac{m_h}{m_Z}) - f(m_{1/2}, y, w) \end{aligned} \tag{3.2}$$

Now consider the case where both the gluinos and the winos are heavier than $m_Z$, in which case (2.1) gives:

$$ln(\frac{m_{1/2}}{m_Z}) = -X + \frac{7\pi}{\alpha_3} + 7c_{\tilde{g}} - 8c_{\tilde{w}} \tag{3.3}$$

To obtain the most generous bounds from (3.1) and (3.3), we would need to maximize $X$, minimize $c_{\tilde{g}}$, and maximize $c_{\tilde{w}}$.

For physically-relevant values of $w$ (those which give positive squared masses for the stop squarks), $f(m_{1/2}, y, w)$ is minimized at $w = -8(c_{\tilde{q}} + y)/27$. With this value of $w$, $f(y, w)$ has one extremum, a maximum, at $y = (c_{\tilde{l}_l}c_{\tilde{l}_r} + 2c_{\tilde{l}_l}c_{\tilde{q}} - c_{\tilde{l}_r}c_{\tilde{q}})/(3c_{\tilde{l}_l} - 2c_{\tilde{l}_r} - c_{\tilde{q}})$, and approaches -0.025 as y becomes very large. Since the values of the $c's$ that we encounter satisfy $c_{\tilde{q}} > 1 > c_{\tilde{l}_l}, c_{\tilde{l}_r}$, the minimum of $f(y, w)$ is indeed -0.025 for values of $y > 0$. We have searched numerically the region where some of the scalars contibuting to $f(m_{1/2}, y, w)$ are lighter than $m_Z$ to verify that it does not take smaller values in this region.

Taking the extreme $1-\sigma$ values $\sin^2\theta_W(m_Z) = 0.2334$, $m_t = 95\,\mathrm{GeV}$, $\alpha_{em}(m_Z) = 1/128.1$ and using a naturalness bound of 1 TeV on $\mu$ and $m_h$ gives a maximum numerical value for $X$ of 193.2. Note that the extreme values of $\sin^2\theta_W(m_Z)$ and $m_t$ are correlated, and that the contribution of the squarks and sleptons which is represented by $f(y, w)$ is negligible.

[2] If $\delta_s(heavy) > 0$ the bounds will be even tighter.

Estimating the values of $c_{\tilde{g}}$ and $c_{\tilde{w}}$ is more involved, since they depend on the unkown value of $m_{1/2}$ and an iterative procedure must be used [8]. At the one-loop

$$c_{\tilde{g}} = \frac{\alpha_3(c_{\tilde{g}} m_{1/2})}{\alpha_U} \quad c_{\tilde{w}} = \frac{\alpha_2(c_{\tilde{w}} m_{1/2})}{\alpha_U} \tag{3.4}$$

where $c_{\tilde{g}} m_{1/2}$ and $c_{\tilde{w}} m_{1/2}$ are the gluino and wino masses renormalized at $m_{\tilde{g}}, m_{\tilde{w}}$.

From the one-loop expression for renormalizing a coupling from $m_Z$ up to its gaugino threshold:

$$\alpha(m_{gaugino}) = \frac{\alpha(m_Z)}{1 - \frac{b}{2\pi} ln(\frac{\alpha(m_{gaugino}) m_{1/2}}{m_Z \alpha_U})} \tag{3.5}$$

We see that, for $b < 0$, $\alpha(m_{gaugino})$ increases with $b$. In order to minimize $c_{\tilde{g}}$, we want to use the minimum value of $b_3 = -7$ possible in the MSSM below the gluino threshold. Similarly, to maximize $c_{\tilde{w}}$ we use the maximum value of $b_2 = -1/3$ possible below the wino threshold.

Fitting the results of an analytic one-loop calculation to a numeric two-loop calculations for central values gives [7]:

$$\begin{aligned}\frac{1}{\alpha_U} =& \frac{3}{20\alpha_{em}(m_Z)} + \frac{1}{5\alpha_3(m_Z)} - 0.7 + \frac{1}{5\pi}[3ln(\frac{m_{\tilde{g}}}{m_Z}) + \frac{1}{8}ln(\frac{m_h}{m_Z}) + \frac{3}{8}ln(\frac{m_{\tilde{l}_l}}{m_Z}) \\ &+ \frac{3}{8}ln(\frac{m_{\tilde{l}_r}}{m_Z}) + \frac{17}{4}ln(\frac{m_{\tilde{q}}}{m_Z}) + \frac{83}{48}ln(\frac{m_t}{m_Z}) + \frac{1}{2}ln(\frac{m_{\tilde{w}}}{m_Z}) + \frac{1}{2}ln(\frac{\mu}{m_Z})]\end{aligned} \tag{3.6}$$

where the stop squarks have been taken degenerate with the other squarks. Thus, $\alpha_U$ decreases with the thresholds. Taking upper bounds of 165 GeV on the top mass [9] and 3 TeV on the other thresholds gives the range

$$\frac{3}{20\alpha_{em}} + \frac{1}{5\alpha_3(m_Z)} - 0.7 < \frac{1}{\alpha_U} < \frac{3}{20\alpha_{em}} + \frac{1}{5\alpha_3(m_Z)} + 1.4 \tag{3.7}$$

for the coupling at the unification scale. Numerically, we find a slight variation of the solutions of (3.1) and (3.3) over this range of $\alpha_U$, with both values increasing with $\alpha_U$. Therefore, we use the maximum value in (3.1) and the minimum value in (3.3).

Finally, we use the central value of $\alpha_{em}(m_Z)$ to calculate $c_{\tilde{w}}$ and $\alpha_U$, neglecting the small variations from the uncertainty in $\alpha_{em}(m_Z)$.

## 4. Results and Conclusions

Fig. 1 shows on the left the triangular region excluded by the calculation of the previous section, which is based on the formulae of Ref. [7] with the EGM effect incorporated as suggested in Ref. [8]. The upper solid line is obtained by iterating (3.3), describing a scenario where $m_{\tilde{w}} > m_Z$, and the lower solid line is obtained by iterating (3.1), describing a scenario where $m_{\tilde{w}} < m_Z$. The area between these two solid lines is excluded. An approximate lower bound of $m_{1/2} > 45\,\mathrm{GeV}$ from CDF gluino searches and an upper naturalness bound $m_{1/2} < 1\,\mathrm{TeV}$ are represented as dashed lines. To have $m_{1/2} < 1\,\mathrm{TeV}$, $\alpha_3(m_Z) > 0.115$ is needed. Note that the $J/\Psi$, $\Upsilon$ and deep inelastic values of $\alpha_3(m_Z)$ are below this lower bound! If the lower bound on $m_{1/2}$ can be pushed to $60\,\mathrm{GeV}$, $m_{1/2} > 1\,\mathrm{TeV}$ will then require $\alpha_3(m_Z) > 0.116$.

Table 2 shows the value of $X$ for different values of the inputs within their $1-\sigma$ range. Since our results use extreme $1-\sigma$ values to maximize $X$, $X$ can only decrease if these errors are reduced , giving an even tighter lower bound on $\alpha_3(m_Z)$. For reference, we also show in fig. 1 the bounds from $X = 189.4$ corresponding to central input values as a dashed line, and the minimum value of $X = 185.4$ as a dotted line. Note these bounds are very sensitive to the values of the input parameters. Since the various approximations which have been made all loosen the bound on $\alpha_3(m_Z)$, the bounds would become stricter if our hypotheses could be tightened. Whilst the minimal supersymmetric $SU(5)$ GUT model cannot be ruled out on the basis of coupling constant unification, things are becoming very tight. Explaining the discrepancy between the different determinations of $\alpha_3(m_Z)$ is crucial. If one believes the high values, or even the average values of $\alpha_3(m_Z)$, the model is in good shape. However, if higher-order corrections [13] do actually bring the high values of $\alpha_3(m_Z)$ down to match the low values, then coupling constant unification cannot be achieved in the minimal model at the $1-\sigma$ level.

We conclude this paper with a brief explanation of our attitude towards these detailed estimates of the supersymmetry-breaking scale in the minimal supersymmetric $SU(5)$ model. We regard these calculations as illustrative, relevant for developing the technology needed to calculate this scale in any explicit model, and indicating how far one can get on the basis of our present experimental knowledge using the theoretical tools available, but we do not take the minimal supersymmetric $SU(5)$ model as seriously as do some of our colleagues. Even in a purely field-theoretical context, this model has to be modified if the Higgs doublets and triplets are to be split in a natural way, and possible difficulties with too-rapid proton decay are to be avoided [14]. More fundamentally, one cannot nowadays be blind to the promise and successes of string theory in unifying quantum gravity with the other particle interactions. It is known [15] that the adjoint Higgs multiplet needed in the minimal supersymmetric $SU(5)$ model cannot be obtained from string theories in which gauge charges appear at the Kac-Moody level 1, which is the formulation used in all models constructed so far. Among existing string models, the one closest in philosophy to traditional $SU(5)$ is the flipped $SU(5)$ model [16], which also incorporates natural Higgs doublet- triplet splitting. This model offers the possibility that $\sin^2\theta_W(m_Z)$ could be smaller than in minimal supersymmetric $SU(5)$, which is not inconsistent with the data, particularly if the lower quoted values of $\alpha_3(m_Z)$ turn out to be correct. In this and many other string-derived models the value of the string unification scale at which all the gauge couplings appear to become equal is calculated [17] to be significantly higher than the grand unification scale calculated in minimal supersymmetric $SU(5)$. Within the context of flipped $SU(5)$, this reinforces the suggestion that $\sin^2\theta_W(m_Z)$ might be decreased. More generally, this scale discrepancy suggests the existence of additional particles beyond those in the Standard Model or its minimal GUT extension. One example, the String Inspired Standard Model (SISM), was proposed in ref. [18]. It would be desirable to carry out in this model and others derived from string calculations as detailed as those described in this and our previous papers, but such a study would take us beyond the scope of this note. The reader should keep in mind, though, the observation that analyses of precision LEP data can be used to test string ideas, as well as the GUT ideas discussed in the bulk of this paper. Yet more motivation, as if our experimental colleagues needed any, to carry on compressing the LEP error bars, in particular for $\alpha_3(m_Z)$.

**Acknowledgements**

The work of D.V.N. is partially supported by DOE grant DE-FG05-91-ER-40633.

**Figure Caption**

The allowed ranges of $m_{1/2}$ for different values of $\alpha_3(m_Z)$ are shown, in the context of minimal supersymmetric $SU(5)$. The value of $m_{1/2}$ for the central values of the other inputs is displayed as a dashed line. The minimal value of $\alpha_3(m_Z)$ assuming $1-\sigma$ variations of the other inputs is shown as a solid line, and the maximal $1-\sigma$ value of $\alpha_3(m_Z)$ as a dotted line. For any given value of $\alpha_3(m_Z)$, values of $m_{1/2}$ to the left of the solid line, or to the right of the dotted line, are accordingly disfavoured. Also shown as horizontal dashed lines are a lower bound $m_{1/2} > 45$ GeV from CDF gluino searches, and an upper bound $m_{1/2} < 1$ TeV motivated by naturalness. It can be seen that $m_{1/2} < 1$ TeV is favoured only if $\alpha_3(m_Z) > 0.115$.

| Experiment | Central Value | Error |
|---|---|---|
| ALEPH jets | 0.125 | $\pm 0.005$ |
| DELPHI jets | 0.113 | $\pm 0.007$ |
| L3 jets | 0.125 | $\pm 0.009$ |
| OPAL jets | 0.122 | $\pm 0.006$ |
| OPAL $\tau$ | 0.123 | $\pm 0.007$ |
| $J/\Psi$ | 0.108 | $\pm 0.005$ |
| $\Upsilon$ | 0.109 | $\pm 0.005$ |
| Deep Inelastic | 0.109 | $\pm 0.005$ |
| Average | 0.116 | $\pm 0.005$ |

*Table 1* - Experimental Values of $\alpha_3(m_Z)$.

| Input Values | $\sin^2\theta_W(m_Z)$ | $m_t$ | $\alpha_{em}(m_Z)$ | $m_h$ | $\mu$ | $X$ |
|---|---|---|---|---|---|---|
| $X_{max}$ | 0.2334 | 95 GeV | 1/128.1 | 1 TeV | 1 TeV | 193.2 |
| central $\sin^2\theta_W(m_Z)$,$m_t$ | 0.2327 | 130 GeV | 1/128.1 | 1 TeV | 1 TeV | 189.7 |
| central $\alpha_{em}(m_Z)$ | 0.2334 | 95 GeV | 1/127.9 | 1 TeV | 1 TeV | 192.9 |
| $X_{central}$ | 0.2327 | 130 GeV | 1/127.9 | 1 TeV | 1 TeV | 189.4 |
| $X_{min}$ | 0.2320 | 160 GeV | 1/127.7 | 1 TeV | 1 TeV | 185.4 |
| $X_{max}$: $\mu, m_h < 500$ GeV | 0.2334 | 95 GeV | 1/128.1 | 500 GeV | 500 GeV | 190.6 |

*Table 2* - Sensitivity of X to various inputs:

$\mu, m_h < 1$ TeV unless otherwise stated.

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