CTP-TAMU-35/90
ACT-6
CERN-TH.5715/90
April, 1990

CONSTRAINTS ON GRAND UNIFIED SUPERSTRING THEORIES

JOHN ELLIS,[(a)] JORGE L. LOPEZ[(b)] AND D. V. NANOPOULOS[(c)]

Center for Theoretical Physics, Department of Physics
Texas A&M University, College Station, TX 77843-4242, USA

and

Astroparticle Physics Group, Houston Advanced Research Center (HARC)
The Woodlands, TX 77381, USA

ABSTRACT

We evaluate some constraints on the construction of Grand Unified Superstring Theories (GUSTs) using higher level Kac-Moody algebras on the world-sheet. We demonstrate that in the free fermionic formulation of the heterotic string in four dimensions, there are no $SU(5), SO(10)$ or E_6 GUT models *at any level.* Even in more general formulations, an analysis of the basic GUST model-building constraints, including a realistic hidden gauge group, reveals that there are no E_6 models and any $SO(10)$ models can only exist at level 5. Also, any such $SU(5)$ models can exist only for levels $4 \leq k \leq 19$. These $SO(10)$ and $SU(5)$ models risk having many large, massless, phenomenogically troublesome representations. We also show that with a suitable hidden sector gauge group, it is possible to avoid free light fractionally charged particles, which are endemic to string derived models. We list all such groups and their representations for the flipped $SU(5) \times U(1)$ model. We conclude that a sufficiently binding hidden sector gauge group becomes a basic model-building constraint.

(a) Permanent address: CERN Theory Division, 1211 Geneva 23, Switzerland.
(b) Supported by an ICSC–World Laboratory Scholarship.
(c) Supported in part by DOE grant DE-AS05-81ER40039.

Although many particle theorists are convinced that string is the Theory of Everything, there is no consensus on the right road to four-dimensional experimental reality. At the level of string perturbation theory, there appears to exist a myriad of consistent vacua which have yet to be enumerated. Non-perturbative string effects may eventually give us grounds for favoring some subset of vacua (perhaps one single vacuum) over all the others, but purely theoretical criteria are not yet sufficient to fix the choice of string vacuum. Under these circumstances, one must appeal to phenomenological criteria to help select the appropriate vacuum.

One step in this choice is the decision whether to favor grand unification within the context of the low-energy effective field theories. The attractions of the GUT framework are well-known [1]: they include the prospect of observable baryon decay at a rate that is neither excessively rapid nor too slow to be detected, and a natural framework for obtaining very light neutrino masses. Both of these features are known to be problematic in a wide class of string vacua that do not contain a form of four-dimensional GUT gauge symmetry [2].

However, there is a well-known obstacle to the construction of GUTs in the string framework, namely their apparent reliance on adjoint and higher-dimensional Higgs representations [1]. For example, conventional $SU(5)$ needs an adjoint **24** Higgs representation to break the gauge symmetry down to the $SU(3) \times SU(2) \times U(1)$ of the Standard Model, and higher dimensional representations such as **50**, $\overline{\mathbf{50}}$ and **75** are often invoked [3] to split naturally the light Higgs doublets from their heavy triplet partners, whilst a **45** would be required to give realistic quark and lepton masses [4] unless one postulates nonrenormalizable interactions [5]. In $SO(10)$ GUTs, adjoint **45** and higher-dimensional **54**, **120**, **126**, and **210** representations are commonly introduced [6] to play analogous roles, and E_6 models often include **78**, **351**, and **351′** representations [7]. On the other hand, one GUT is known that avoids adjoints and higher-dimensional Higgs representations, namely flipped $SU(5) \times U(1)$ [8,9]. No representations larger than **10** and $\overline{\mathbf{10}}$ are required to break the GUT symmetry and split Higgs doublets and triplets naturally [9], whilst realistic quark and lepton mass matrices may be realized [10,11] by nonrenormalizable interactions if the GUT scale is one or two orders of magnitude below the string unification scale [12]. Also, the pattern of $SU(5) \times U(1)$ symmetry breaking to $SU(3) \times SU(2) \times U(1)$ is unique. This is to be contrasted with the above GUT models where a large number of degenerate vacua are possible leading to severe cosmological problems in the early Universe [13].

Adjoint and higher-dimensional representations are known [14] to be absent from the low-energy spectrum of string models whose gauge symmetry is underlain by a current algebra realized at the lowest Kac-Moody level $k = 1$ (as shown below). However, it is known [15] that they can in principle be obtained in string theories with higher-level Kac-Moody algebras: $k > 1$. So far no realistic string model with $k > 1$ has been constructed and this is known not to be an easy task [16]. As an aid to those contemplating such a task, we compile in this paper some of the constraints that such a Grand Unified Superstring Theory (GUST) must satisfy and point out some of the principal obstacles to its construction.

Another important constraint on models derived from the superstring is the absence of free light fractionally charged particles. These can in principle be confined by hidden sector gauge interactions, but this must be checked for each model. We characterize possible choices of the hidden sector for level-one flipped $SU(5) \times U(1)$ models. These are the only consistent GUSTs known at present.

We start our analysis of GUST model-building by compiling our phenomenological desiderata.

Natural doublet-triplet splitting: The Weinberg-Salam Higgs doublets $h_2, \bar{h}_2$ required for breaking $SU(2)_L \times U(1)_Y$ symmetry must have mass parameters $\lesssim 100\,\text{GeV}$. In any GUT model these doublets come from multiplets containing color triplet fields $h_3, \bar{h}_3$ that couple to quark pairs and to antilepton-antiquark combinations. They could therefore mediate baryon-number-violating interactions, and consistency with the observed length of the proton life-time requires

$m_{h_3,\bar{h}_3} \gtrsim 10^{16}$ to 10^{17} GeV. The large $h_2 - h_3$ mass difference could be imposed by *fiat*, and would then not be renormalized in a theory with approximate supersymmetry, but one would prefer a natural mechanism for doublet-triplet splitting. A general philosophy for achieving this is to couple via a Yukawa interaction the GUT Higgs multiplets to another multiplet which contains a color triplet that mix *via* a Dirac mass term with $h_3, \bar{h}_3$, but no corresponding doublet to give a large mass to $h_2, \bar{h}_2$ — the missing partner mechanism [3].

The lowest dimensional such multiplets in an $SU(5)$ GUT are the **50** and $\overline{\mathbf{50}}$, which can couple to the **5** and $\bar{\mathbf{5}}$ representations *via* a self-adjoint **75** representation. The latter can also be used to break $SU(5)$ to $SU(3) \times SU(2) \times U(1)$, thereby obviating the need for an adjoint **24** of Higgs. If one wishes to use discrete symmetries to exclude all couplings that could invalidate this simple mechanism, more **75**, **50**, and $\overline{\mathbf{50}}$ representations are required. However, we obtain as our minimum requirement the presence of one of each of the **75**, **50**, and $\overline{\mathbf{50}}$ representations.

In the case of $SO(10)$ GUTs, the minimum representations containing a **50** of $SU(5)$ is a $\overline{\mathbf{126}}$, and the smallest containing a **75** of $SU(5)$ is a **210**. Natural mechanisms for doublet-triplet splitting in $SO(10)$ have indeed been proposed in Ref. [17], using a combination of the **126**, $\overline{\mathbf{126}}$, and **210** representations. We are not aware of any detailed studies of natural doublet-triplet splitting mechanisms in E_6 GUTs, but **351′**, $\overline{\mathbf{351'}}$, and **650** representations would be required at least.

Realistic fermion masses: It is well-known that the measured b and τ masses are consistent with equality at the GUT scale [18]. This occurs naturally in the minimal $SU(5)$ GUT with the Weinberg-Salam Higgs doublets contained in **5** and $\bar{\mathbf{5}}$ representations. However, $m_d/m_s \neq m_e/m_\mu$, which indicates the need to complicate the GUT Higgs structure. Since $\bar{\mathbf{5}} \times \mathbf{10} = \mathbf{5} + \overline{\mathbf{45}}$, the natural possibility would be to introduce **45** and $\overline{\mathbf{45}}$ Higgs representations, which give $m_q/m_l = 3$ before renormalization [4], and combine those with the **5** and $\bar{\mathbf{5}}$ Higgses to get a realistic mass spectrum. The corresponding choice in an

$SO(10)$ model would be a $\mathbf{126}+\overline{\mathbf{126}}$ pair of Higgs representations [6], and in E_6 a $\mathbf{351'}+\overline{\mathbf{351'}}$ pair of representations [7].

We should, however, point out that it is possible to avoid postulating these large representations for the quark and lepton mass matrices by advocating non-renormalizable interactions [5]. For example, in the $SU(5)$ case a $\bar{\mathbf{5}} \times \mathbf{10} \times \mathbf{45}$ coupling could in principle be replaced by $\bar{\mathbf{5}} \times \mathbf{10} \times \bar{\mathbf{5}} \times \mathbf{24}$ coupling. In point of fact, most of these large representations are already required for natural doublet-triplet mass splitting, as seen above.

Adequate hidden sector: Studies of supersymmetry breaking in string models invariably lead one to consider hidden sector gauge groups that become strong at an intermediate scale [19,20] . Some form of gaugino [19] and/or chiral multiplet [20] condensation then triggers spontaneous breaking of local supersymmetry. The most general constraint that one can impose is $\Lambda_h > \Lambda_{QCD}$, where Λ_h is the scale at which the hidden sector gauge group becomes strong. We recall that the beta function for an asymptotically-free gauge theory is maximal for gauge degrees of freedom only, and decreases with any matter field representations added in. Hence, confining $SU(n)$ hidden gauge groups with $n = 2,3$ will have to have very limited or no matter representations to be feasible. However, it is common for string-derived models to have a rich hidden matter spectrum. Thus, we require $SU(4)$ as a minimal hidden gauge group. A more restrictive constraint on the hidden gauge group relates to the confinement of fractionally charged particles, as discussed below.

The origin of spacetime gauge symmetries in string theory is in the Kac-Moody algebra on the world-sheet [21]. The so-called affine Kac-Moody algebra $\widehat{G}$ underlies the spacetime gauge symmetry G. Depending on the choice of world-sheet currents, these algebras can be realized at different levels parametrized by the positive integer k. All states in the theory fall into representations of this algebra. However, for a fixed level k, only a certain number of these representations are unitary and may appear in a sensible string model. The requirement that must be met is

$$\sum_{i=1}^{\mathrm{rank}\,G} n_i m_i \leq k, \tag{1}$$

where the n_i are the Dynkin labels of the highest weight of the representation in question and the m_i are fixed positive integers for a given G (see Table 1*). The conformal dimensions of the unitary representations r depend on the level k and are given by

$$h_r = \frac{C_r}{2k + C_A} \tag{2}$$

where C_r is the quadratic Casimir of r, and C_A that of the adjoint. For practical purposes it is more convenient to work with the index of the representation, l_r defined as

$$l_r = \frac{\dim r}{\dim G} C_r, \tag{3}$$

rather than with C_r. Conformal symmetry also implies that massless states have $h \leq 1$, while massive ones have $h > 1$. The final fact we need for our analysis is the contribution to the central charge from the level k Kac-Moody algebra,

$$c = \frac{k \dim G}{k + \tilde{h}}, \tag{4}$$

where $\tilde{h}$ is the dual Coexter number for the group G (see Table 1), and for simply-laced Lie groups $\tilde{h} = \frac{1}{2} C_A$.

We first note that in the heterotic string $c \leq 22$ and hence for gauge groups with $\dim G > 22$ there is an upper bound on the value of k,

$$k \leq k^{max} \equiv \left[\frac{22\tilde{h}}{\dim G - 22} \right], \tag{5}$$

where $[x]$ is the integer part of x. For the unitary, orthogonal, and exceptional groups these values are given in Table 2. For the common case in which $G =$

* In what follows we concentrate on the phenomenologically appealing simply-laced groups. The non-simply laced Lie groups, *i.e.*, $SO(2n+1), Sp(n), G_2$, and F_4 can be analyzed analogously.

$\otimes_i G_i^{(k_i)}$, the constraint $\sum_i c_i \leq 22$ gives yet stronger constraints on the k_i. Indeed, the additional requirement of a minimal hidden sector gauge group $G_{hidden} \supset SU(4)$ reduces the contribution to c from the observable gauge group to $c_{obs} \leq 19$ (recall that $\mathrm{rank}\, G \leq c \leq \dim G$), see Table 2. In Table 2 we also give the corresponding results for a possible "maximal" E_8 hidden gauge group. We should point out that the results in Table 2 assume the minimal contribution to c from the possible hidden gauge groups. This is the case at level 1. For hidden gauge groups at levels $k > 1$, the results in Table 2 (column labelled $k_{(b)}^{max}$) are further constrained. For example, with an $SU(4)_h$ at $k_4 > 1$, the $SU(5)$ and $SO(10)$ $k_{(b)}^{max}$ entries get reduced from 19 to 12 and from 5 to 4, respectively.

Next we obtain all unitary massless representations that can occur at level 1. For $SO(2n)$ groups we find using (1) that only the singlet, vector, spinor and conjugate spinor representations are allowed. Furthermore, these representations have conformal dimensions:* $h_{singlet} = 0$, $h_{vector} = 1/2$, $h_{spinor} = n/8$, and hence the spinor representation is massless for $n \leq 8$ only. For $SU(n)$ groups we find that at level 1 only the $n-1$ totally antisymmetric representations (plus the singlet) are unitary. These representations have dimension $\binom{n}{p}$, $1 \leq p \leq [n/2]$ and at level 1 are massless for† $h = p(n-p)/2n \leq 1$. The resulting unitary massless representations are collected in Table 3. For E_6 at level 1 we only find the singlet, **27**, and $\overline{\mathbf{27}}$ representations (all massless); for E_7 we find the singlet and **56** (both massless); and for E_8 only the singlet representation. From these results we infer that at level 1 there are no allowed adjoint representations.‡ Futhermore, we find a few massless representations of dimensionality higher than the respective adjoints, namely the **70** of $SU(8)$, the **84**, $\overline{\mathbf{84}}$ of $SU(9)$ and the **128** of $SO(16)$. These results have already been found in [14] where a restricted

* We use (2) and [22] $\dim G = n(2n-1)$, $C_A = 4n-4$, $l_{vector} = 2$, $l_{spinor} = 2^{n-3}$.

† This follows from (2) after using [22]: $\dim G = n^2 - 1$, $C_A = 2n$, and $l_r = \binom{n-2}{p-1}$.

‡ This remark pertains to the chiral supermultiplets, the gauge supermultiplets are always present and are represented by the Kac-Moody currents themselves. Adjoint matter representations of this kind can generally exist for $N > 1$ but are forbidden for $N = 1$ spacetime supersymmetry [14].

class of four-dimensional heterotic string models constructed in terms of free world-sheet fermions was considered.

We now turn to the analysis of each of the traditional GUT groups ($SU(5)$, $SO(10)$, and E_6) in the light of the phenomenological constraints advocated above.

SU(5): For the doublet-triplet splitting we need **50**, $\overline{\mathbf{50}}$, and **75** Higgs representations, and for a realistic fermion mass spectrum also the **45**, $\overline{\mathbf{45}}$. All these representations are unitary at level $k \geq 2$. At level k they have conformal dimensions

$$h_{\mathbf{45},\overline{\mathbf{45}}} = \frac{32}{5(k+5)}, \quad h_{\mathbf{50},\overline{\mathbf{50}}} = \frac{42}{5(k+5)}, \quad h_{\mathbf{75}} = \frac{8}{(k+5)}. \tag{6}$$

For these representations to be massless one needs: **45**,$\overline{\mathbf{45}}$, $k \geq 2$; **50**,$\overline{\mathbf{50}}$, $k \geq 4$; **75**, $k \geq 3$. We also note that at level 4 the following are all the unitary massless representations: **1**,**5**,$\overline{\mathbf{5}}$,**10**,$\overline{\mathbf{10}}$,**15**,**24**,**40**,$\overline{\mathbf{40}}$,**45**,$\overline{\mathbf{45}}$, **50**,$\overline{\mathbf{50}}$,**70**,**75**. Hence, *a priori* there will be several exotic representations in the spectrum.

SO(10): In this case we require the **126**, $\overline{\mathbf{126}}$, and **210** representations which are all unitary starting at level 2. Their conformal dimensions are given by

$$h_{\mathbf{126},\overline{\mathbf{126}}} = \frac{25}{2(k+8)}, \quad h_{\mathbf{210}} = \frac{12}{(k+8)}. \tag{7}$$

For these representations to be massless we would need: **126**,$\overline{\mathbf{126}}$, $k \geq 5$; **210**, $k \geq 4$. At level 5 the following are all the unitary massless representations available: **1**,**10**,**16**,$\overline{\mathbf{16}}$,**45**,**54**,**120**,**126**,$\overline{\mathbf{126}}$,**144**, $\overline{\mathbf{144}}$,**210**. Hence lots of exotic states would again be expected.

$\mathbf{E_6}$: We require **351′**,$\overline{\mathbf{351'}}$ for realistic fermion mass matrices, and the **650** is the smallest replacement of the adjoint which might admit a natural doublet-triplet splitting mechanism. These are unitary at level $k \geq 2$ and have conformal dimensions

$$h_{\mathbf{351'},\overline{\mathbf{351'}}} = \frac{56}{3(k+12)}, \quad h_{\mathbf{650}} = \frac{18}{(k+12)}, \tag{8}$$

and hence will be massless for: **351′**,$\overline{\mathbf{351'}}$, $k \geq 7$; **650**, $k \geq 6$. Note that these values of k exceed the maximum allowed value for E_6 (see Table 2).

The above results are summarized in Table 4. We conclude that phenomenologically acceptable $SU(5)$ GUT models could be attained at levels $4 \leq k \leq 19(12)$, for a level $k_4 = 1\,(k_4 > 1)$ minimal hidden gauge group. $SO(10)$ GUT models with minimal hidden sector could only be built at level 5, and even so only if $k_4 = 1$. Higher-level minimal hidden sector gauge groups and additional hidden or observable gauge groups contributing $c > 1.7$ (*i.e.*, anything larger than one extra $U(1)$ or $SU(2)$ beyond $SO(10)\times SU(4)_h$), would eliminate the $SO(10)$ GUT possibility altogether. Both GUT models will likely have undesired massless exotic states. These states could strongly affect the running of the couplings to low energies and hence ruin the succesful GUT predictions for $\sin^2\theta_W$, m_b/m_τ, etc. Also, at such high levels a very large number of massive representations appear as well.* These will likely produce large threshold effects in the renormalization group equations of the gauge couplings at the string scale [23]. Note that an E_8 hidden sector gauge group, which is commonplace in Calabi-Yau compatifications of the heterotic string, would also (see Table 2) eliminate the $SO(10)$ GUT alternative and limit the $SU(5)$ GUT scenario to levels $4 \leq k \leq 7$. Finally, no phenomenologically acceptable E_6 GUT models appear to exist.

We end this section with a few remarks applicable to the restricted class of models built in the free-fermionic formulation [24]. In this formulation, gauge groups realized at levels $k > 1$ are constructed in terms of real fermions [15], and hence all such conformal fields must have dimensions which are multiples of $1/16$, *i.e.*, the minimal conformal dimension in the Ising model.† The conformal dimensions of the indispensable lowest-dimensional representations of $SU(5)$ at level $k > 1$,

$$h_{\mathbf{5},\bar{\mathbf{5}}} = \frac{12}{5(k+5)}, \quad h_{\mathbf{10},\overline{\mathbf{10}}} = \frac{18}{5(k+5)}, \tag{9}$$

* For $SU(5)$ at level 4 one gets massive representations up to the **1176**, and for $SO(10)$ at level 5 up to the **72765** representation.

† For gauge groups realized in terms of complex fermions, and hence at level 1 [15], the conformal dimensions of the fields involved are not restricted to be multiples of $1/16$ [25].

are not multiples of 1/16, and hence cannot exist by themselves. They must be supplemented by additional quantum numbers, thus making the gauge group not the traditional $SU(5)$. This is precisely what happens in the case of flipped $SU(5) \times U(1)$ and other $U(5)$ embeddings [26]. In the case of $SO(10)$, the conformal dimensions of the $\mathbf{16}$,$\overline{\mathbf{16}}$, and $\mathbf{45}$ at level $k > 1$ are

$$h_{\mathbf{16},\overline{\mathbf{16}}} = \frac{45}{8(k+8)}, \quad h_{\mathbf{45}} = \frac{8}{(k+8)}, \tag{10}$$

Bearing in mind that in this case $k^{max} = 7$, one can see that even though for $k = 2, 7$ one can have $\mathbf{16}$,$\overline{\mathbf{16}}$ representations, the adjoint is never allowed in this formalism. In E_6 we have

$$h_{\mathbf{27},\overline{\mathbf{27}}} = \frac{26}{3(k+12)}, \tag{11}$$

which is never a multiple of 1/16. Hence these GUT groups at levels $k > 1$ could only be viable if combined nontrivially with other degrees of freedom on the world-sheet. Clearly, flipped $SU(5) \times U(1)$ emerges at this point as the unchallenged candidate for the construction of a realistic unified model in the free fermionic formulation.

In general, an additional requirement that must be satisfied in a free fermionic model is that the total contribution from the Kac-Moody currents to the central charge must be an integer or half-integer. This imposes further constraints on the levels of the gauge groups that can appear in a particular model. This constraint is however trivially satisfied by level 1 simply-laced gauge groups which give $c = \mathrm{rank}\, G$. For example, in the flipped $SU(5) \times U(1)$ model of [11], the complete gauge group is $SU(5) \times U(1)^5 \times SO(10) \times SO(6)$, with all gauge groups realized at level 1. We get $c_{gauge} = 4 + 5 \cdot 1 + 5 + 3 = 17$ which complemented by 10 Ising models [11] gives $c_{tot} = 17 + \frac{1}{2} \cdot 10 = 22$.

Up to now we have been considering constraints on GUT observable gauge groups. We now turn our attention to constraints on possible hidden sector gauge groups. Above we have outlined the minimal requirements for such a group in the light of supersymmetry breaking. Here we consider further constraints based on the phenomenological need to have no free light fractionally charged particles. This discussion can be made most efficiently in the language of "simple currents" [27] of the Kac-Moody algebra underlying the gauge group. In the Standard Model, the charge quantization condition can be summarized as follows [28]. For any state transforming under $SU(3)\times SU(2)\times U(1)$ as (R,r,Y), we must have

$$\alpha_J=\frac{1}{3}t_3(R)+\frac{1}{2}t_2(r)+Y\in\mathbf{Z}, \tag{12}$$

where t_n is the n-ality* of the respective $SU(n)$ representation. The Standard Model fermionic content,

$$\begin{aligned}&\begin{pmatrix}u\\d\end{pmatrix}_L:(\mathbf{3},\mathbf{2},\frac{1}{6});\quad u^c:(\overline{\mathbf{3}},\mathbf{1},-\frac{2}{3});\quad d^c:(\overline{\mathbf{3}},\mathbf{1},\frac{1}{3});\\&\begin{pmatrix}\nu\\e\end{pmatrix}_L:(\mathbf{1},\mathbf{2},-\frac{1}{2});\quad e^c:(\mathbf{1},\mathbf{1},1),\end{aligned} \tag{13}$$

can be easily seen to obey rule (12). This rule states that any fractionally charged particle must have nontrivial transformation properties under $SU(3)_C$ such that physical color singlet states are integrally charged. For example, the quark doublet has $\frac{1}{2}t_2+Y=2/3$ and hence we demand that the quarks have $\frac{1}{3}t_3=1/3$, *i.e.*, are in the fundamental representation of $SU(3)_C$. This way physical color singlets (*i.e.*, hadrons) are guaranteed to have integral charge.

In the context of $SU(n)$ level k_n Kac-Moody algebras one can find [27] some primary fields which have very simple fusion rules with all the other primary fields in the conformal field theory. These are called simple currents $J_n^{(i_n)}$, $i_n=0,1,\ldots,(n-1)$ and have conformal dimension $h_J=i_nk_n(n-i_n)/2n$. The monodromy (*i.e.*, charge under J) with some field Φ transforming under the $\mathbf{R}$

* For a $SU(n)$ representation $\mathbf{R}$ with Dynkin labels $\{a_k\}$, the n-ality is given by [22] $t_n(\mathbf{R})=\sum_{k=1}^{n-1}ka_k\,(mod\,n)$. For $SU(2)$, integer (half-integer) spin representations have $t_2=0\,(1)$. In general we have $t_n(\mathbf{n})=-t_n(\overline{\mathbf{n}})=1$; $t_n(\mathbf{n(n-1)/2})=2$; $t_n(\mathbf{1})=t_n(\mathbf{n^2-1})=0$.

representation of $SU(n)$ is $\alpha_J = i_n t_n(\mathbf{R})/n$. Similarly, level k_m $SO(2m)$ Kac-Moody algebras possess simple currents $J_m^{(j_m)}$, $j_m = 0,1,2,3$, with conformal dimensions: 0, $j_m = 0$; $\frac{1}{2}k_m$, $j_m = 2$; $\frac{m}{8}k_m$, $j_m = 1,3$. The charges of the fields under these currents are: $j_m\frac{c}{4}$, for $m = odd$; and $\tilde{c}_{j_m}$, for $m = even$. Here c is the conjugacy class label of the representation, and $\tilde{c}_{j_m}$ is a similar representation-dependent quantity [29], with $\tilde{c}_{j_m=0} = 0$.

If the simple current J has integer monodromy with all fields Φ, then J will also belong to the conformal field theory, and in order not to violate modular invariance J must have integral conformal dimension, *i.e.*, $h_J \in \mathbf{Z}$ [27,28]. In the Standard Model, the simple current $J_{SM} = (J_3^{(1)}, J_2^{(1)}, J_1)$ has this property due to (12), and hence [28]

$$h_J = \frac{1}{3}k_3 + \frac{1}{4}k_2 + \frac{1}{4}k_1 \in \mathbf{Z} \Leftrightarrow 4k_3 + 3k_2 + 3k_1 = 0\,(mod\,12). \tag{14}$$

The $U(1)$ current J_1 has arbitrary Kac-Moody level and has been defined as having $h_{J_1} = q^2/k_1$, with $q(J_1) = k_1/2$, hence $h_{J_1} = k_1/4$. The value of k_1 can be fixed in cases where the $U(1)$ is embedded in a larger group, but is arbitrary otherwise. Recalling that at levels $k > 1$ the string gauge coupling constant is rescaled by $\sqrt{k}$, *i.e.*, $g \to g/\sqrt{k}$, we find

$$\sin^2\theta_W = \frac{k_2}{k_1 + k_2}, \tag{15}$$

at the string scale. For level 1 $SU(3)$ and $SU(2)$ Kac-Moody algebras we then get

$$k_1 = \frac{5}{3}, \frac{17}{3}, \frac{29}{3}, \cdots, \tag{16}$$

$$\sin^2\theta_W = \frac{3}{8}, \frac{3}{20}, \frac{3}{32}, \cdots. \tag{17}$$

The choice $k_1 = 5/3$ gives the succesful $SU(5)$ prediction for $\sin^2\theta_W$. However, in this case the gauge group extends to $SU(5)$ [28] (since $h_J = 1$) and there is no way to break this group down to the Standard Model. For this reason, in Ref. [28] it was argued that one either had to construct higher-level Kac-Moody algebras for the Standard Model gauge groups, or do away with the charge quantization rule (12).

In view of the above difficulties in constructing higher-level observable sectors, we now present a generalization of the charge quantization rule that allows level 1 Kac-Moody algebras without jeopardizing the succesful prediction for $\sin^2\theta_W$. As a pilot example, we consider the level-one flipped $SU(5)\times U(1)$ model which has electric charge operator $Q=T_3-\frac{1}{5}Y+\frac{2}{5}\widetilde{Y}$, where $U(1)_{\widetilde{Y}}$ is the $U(1)$ outside $SU(5)$. The charge quantization condition is now extended to include the extra $U(1)_{\widetilde{Y}}$ *plus* a contribution from a hidden sector $SU(n)_h^{k_n}\times SO(2m)_h^{k_m}$ gauge group. That is, we require that any fractionally charged particle transform nontrivially under $SU(3)_C$ or $SU(n)_h\times SO(2m)_h$ such that physical singlet states under these groups have integral charges. Clearly, this can be generalized to multiple non-abelian gauge groups. For a particle transforming under $SU(5)\times U(1)_{\widetilde{Y}}\times SU(n)_h\times SO(2m)_h\supset SU(3)_C\times SU(2)_L\times U(1)_Y\times U(1)_{\widetilde{Y}}\times SU(n)_h\times SO(2m)_h$ as $(\mathbf{R},\mathbf{r},Y,\widetilde{Y},\mathbf{R}_n,\mathbf{R}_m)$ we write

$$\alpha_J=\frac{1}{3}t_3(\mathbf{R})+\frac{1}{2}t_2(\mathbf{r})-\frac{1}{5}Y+\frac{2}{5}\widetilde{Y}+\frac{i_n}{n}t_n(\mathbf{R}_n)+\begin{cases} j_m\, c(\mathbf{R}_m)/4; & m=odd\\ \tilde{c}_{j_m}(\mathbf{R}_m); & m=even\end{cases}\in\mathbf{Z}. \tag{18}$$

We now explore the consequences of enforcing this rule on the spectrum of a string-derived model. This will constrain the possible forms of the charge quantization condition (*i.e.*, values of i_n and j_m) and the hidden gauge groups to which it can be applied. We do this by considering the simple current $J_{FP5}=(J_3^{(1)},J_2^{(1)},J_1,J_{\tilde{1}},\,J_n^{(i_n)},J_m^{(j_m)})$, which due to (18) must have integral conformal dimension,

$$h_J=\frac{1}{3}k_3+\frac{1}{4}k_2+\frac{1}{100}(k_1+4k_{\tilde{1}})+\frac{i_nk_n(n-i_n)}{2n}+\begin{cases}0, & j_m=0\\ \frac{1}{2}k_m, & j_m=2\\ \frac{m}{8}k_m, & j_m=1,3\end{cases}\in\mathbf{Z}. \tag{19}$$

We take $k_3=k_2=1$, and $k_1=5/3$ as required by $SU(5)$. Also, $k_{\tilde{1}}=10$ is fixed by the embedding of $SU(5) \times U(1)_{\tilde{Y}}$ in $SO(10)$ in the usual way. We can now explore the possible hidden gauge groups, their representations and allowed levels compatible with α_J, $h_J \in \mathbf{Z}$. Note that by construction these representations will have $t_3 = t_2 = Y = 0$, and hence $Q = \frac{2}{5}\tilde{Y}$. With the above values the conditions to be satisfied reduce to

$$\alpha_J = Q + \frac{i_n t_n}{n} + \begin{cases} j_m\, c/4, & m = odd \\ \tilde{c}_{j_m}, & m = even \end{cases} \in \mathbf{Z}, \tag{20}$$

$$h_J = 1 + \frac{i_n k_n (n - i_n)}{2n} + \begin{cases} 0, & j_m = 0 \\ \frac{1}{2} k_m, & j_m = 2 \\ \frac{m}{8} k_m, & j_m = 1,3 \end{cases} \in \mathbf{Z}. \tag{21}$$

Note that to avoid symmetry enhancement to $SO(10)$ we must not have $i_n = j_m = 0$, *i.e.*, $h_J = 1$. We start with (21). The number of solutions to this Diophantine equation is very large in general. However, one can search for solutions with $k_n = k_m = 1$, which could most easily be implemented in a fermionic construction. Furthermore, the requirement $c(SU(5) \times U(1)) + c(SU(n)^{k_n=1}) + c(SO(2m)^{k_m=1}) \leq 22$ gives $n + m \leq 18$. In this way we get a manageable number of solutions. For $\mathrm{rank}(G_{hidden}) \leq 8$ the solutions are

$$SU(4)_{i_4=2} \times SO(10)_{j_5=2}, \tag{22}$$

$$SU(2)_{i_4=1} \times SO(12)_{j_6=1,3}, \tag{23}$$

$$SU(4)_{i_4=1,3} \times SO(10)_{j_5=1,3}. \tag{24}$$

We have thus obtained the allowed charge quantization conditions (*i.e.*, values of i_n and j_m) and the hidden-sector gauge groups which can exist in a string-derived model at level 1.

One can now use the constraint (20) to determine the allowed representations and their electric charges. The results for the phenomenologically relevant $SU(4)$ case are given in Table 5. We should point out that the hidden sector massless spectrum obtained in Ref. [11] in an analysis of a superstring derived flipped

$SU(5) \times U(1)$ level 1 model, falls *precisely* within the $SU(4)_{i_4=2} \times SO(10)_{j_5=2}$ case. A detailed study of the phenomenological consequences of these states is in preparation [30].

It is well known that constructing a GUST is not very easy. No examples exist at level 1, except for flipped $SU(5) \times U(1)$. We have indicated in this paper some of the obstacles to constructing higher-level superstring GUTs, which may not be insuperable. On the other hand, it has been shown that level-1 models have in general problems with charge quantization. The flipped $SU(5) \times U(1)$ model avoids this trap by confining the fractionally charged particles, and we have shown in this paper how this solution to the confinement constraint could be generalized. It remains to be seen whether other viable superstring GUTs will survive the model-building constraints outlined in this paper.

Acknowledgement: We would like to thank A. Schellekens and I. Antoniadis informative discussions. J.L.L. would like to thank L. Durand and D. Reiss for useful discussions at the early stages of this work.

REFERENCES

1. For a review see *e.g.*, P. Langacker, Phys. Rep. **72** (1981) 185; C. Kounnas, A. Masiero, D. V. Nanopoulos, and K. Olive, *Grand Unification with and without Supersymmetry and Cosmological Implications*, (World Scientific, 1984).
2. For a review, see L.E. Ibáñez, CERN Preprint TH. 5405/89 (1989) and references therein.
3. A. Masiero, D. V. Nanopoulos, K. Tamvakis, and D. Yanagida, Phys. Lett. B **115** (1982) 380; B. Grinstein, Nucl. Phys. B **206** (1982) 387.
4. H. Georgi and C. Jarlskog, Phys. Lett. B **86** (1979) 297.
5. J. Ellis and M. K. Gaillard, Phys. Lett. B **88** (1979) 315; D. V. Nanopoulos and M. Srednicki, Phys. Lett. B **124** (1983) 37.
6. H. Georgi and D. V. Nanopoulos, Nucl. Phys. B **155** (1979) 52.
7. R. Barbieri and D. V. Nanopoulos, Phys. Lett. B **91** (1980) 369.
8. S. Barr, Phys. Lett. B **112** (1982) 219; J. Derendinger, J. Kim, and D. V. Nanopoulos, Phys. Lett. B **139** (1984) 170.
9. I. Antoniadis, J. Ellis, J. Hagelin, and D. V. Nanopoulos, Phys. Lett. B **194** (1987) 231.
10. I. Antoniadis, J. Ellis, J. Hagelin, and D. V. Nanopoulos, Phys. Lett. B **205** (1988) 459; Phys. Lett. B **208** (1988) 209.
11. I. Antoniadis, J. Ellis, J. Hagelin, and D. V. Nanopoulos, Phys. Lett. B **231** (1989) 65.
12. J. Ellis, J. Hagelin, S. Kelley, and D. V. Nanopoulos, Nucl. Phys. B **311** (1989) 1.
13. T. Hübsch, S. Meljanac, S. Pallua, and G. Ross, Phys. Lett. B **161** (1985) 122.

14. H. Dreiner, J. Lopez, D. V. Nanopoulos, and D. Reiss, Phys. Lett. B **216** (1989) 283; University of Wisconsin report MAD/TH/89-2 (unpublished).

15. D. Lewellen, SLAC preprint SLAC-PUB-5023.

16. A. Font, L. Ibáñez, and F. Quevedo, CERN preprint CERN-TH.5666/90.

17. J. Pulido and Maalampi, Phys. Lett. B **133** (1983) 197; J. Pulido, Phys. Lett. B **150** (1985) 163; K. Tamvakis, Phys. Lett. B **201** (1988) 95.

18. A. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B **135** (1978) 66; D. V. Nanopoulos and D. Ross, Nucl. Phys. B **157** (1979) 273; Phys. Lett. B **108** (1982) 351; Phys. Lett. B **118** (1982) 99.

19. J. Derendinger, L. Ibáñez, and H. Nilles, Phys. Lett. B **155** (1985) 65; M. Dine, R. Rohm, N. Seiberg, and E. Witten, Phys. Lett. B **156** (1985) 55.

20. I. Antoniadis, J. Ellis, A. Lahanas, and D. V. Nanopoulos, CERN preprint CERN-TH. 5604/89.

21. For a review see *e.g.*, P. Goddard and D. Olive, Int. J. Mod. Phys. A **1** (1986) 303.

22. See *e.g.*, R. Slansky, Phys. Rep. **79** (1981) 1.

23. V. Kaplunovsky, Nucl. Phys. B **307** (1988) 145.

24. I. Antoniadis, C. Bachas, and C. Kounnas, Nucl. Phys. B **289** (1987) 87; I. Antoniadis and C. Bachas, Nucl. Phys. B **298** (1988) 586; H. Kawai, D.C. Lewellen, and S.H.-H. Tye, Phys. Rev. Lett. **57** (1986) 1832; Phys. Rev. D **34** (1986) 3794; Nucl. Phys. B **288** (1987) 1; R. Bluhm, L. Dolan, and P. Goddard, Nucl. Phys. B **309** (1988) 330; H. Dreiner, J. L. Lopez, D. V. Nanopoulos, and D. Reiss, Nucl. Phys. B **320** (1989) 401.

25. S. Kalara, J. L. Lopez, and D. V. Nanopoulos, Texas A & M University preprint CTP-TAMU-34/90.

26. I. Antoniadis and C. Bachas in Ref. [24].

27. A. Schellekens and S. Yankielowicz, Nucl. Phys. B **327** (1989) 673; CERN preprint CERN-TH.5622/90.

28. A. Schellekens, Phys. Lett. B **237** (1990) 363.

29. A. Schellekens, private communication.

30. J. Ellis, J. Lopez, and D. V. Nanopoulos, in preparation.

Table 1: List of values of the m_i used in (1) for the simply-laced Lie algebras. The dual Coexter number $\bar{h}$ is also given. Note that $\bar{h} = 1 + \sum_i m_i$.

Lie Group	m_i	$\bar{h}$
$SO(2n)$	$(1,2,2,\ldots,2,1,1)$	$2n-2$
$SU(n)$	$(1,1,\ldots,1)$	n
E_6	$(1,2,3,2,1,2)$	12
E_7	$(2,3,4,3,2,1,2)$	18
E_8	$(2,3,4,5,6,4,2,3)$	30

Table 2: Maximum values of k compatible with $c \leq 22$ for the simply-laced Kac-Moody algebras. The different values of k^{max} apply when either (a) no hidden sector constraint is imposed, or (b) a minimal $SU(4)_h$ hidden sector is required, or (c) an E_8 hidden sector exists.

$SU(n)$	$k^{max}_{(a)}$	$k^{max}_{(b)}$	$k^{max}_{(c)}$	$SO(2n)$	$k^{max}_{(a)}$	$k^{max}_{(b)}$	$k^{max}_{(c)}$	Exceptional	$k^{max}_{(a)}$	$k^{max}_{(b)}$	$k^{max}_{(c)}$
5	55	19	7	10	7	5	3	E_6	4	3	2
6	10	7	4	12	5	4	2	E_7	3	3	2
7	5	4	2	14	3	3	2	E_8	2	2	1
8	4	3	2	16	3	2	1				
9	3	2	1	18–20	2	2	1				
10–11	2	2	1	22	2	1	1				
12	2	1	1	24–28	1	1	1				
13–15	1	1	1	30–38	1	1	–				
16–20	1	1	–	40–44	1	–	–				
21–23	1	–	–								

Table 3: Unitary, massless representations at level 1 for $SU(n)$ groups.

$SU(n)$	Representation
$n = 2$	$\mathbf{1}, \mathbf{2}$
$n = 3$	$\mathbf{1}, \mathbf{3}, \overline{\mathbf{3}}$
$n = 4$	$\mathbf{1}, \mathbf{4}, \overline{\mathbf{4}}, \mathbf{6}$
$n = 5$	$\mathbf{1}, \mathbf{5}, \overline{\mathbf{5}}, \mathbf{10}, \overline{\mathbf{10}}$
$n = 6$	$\mathbf{1}, \mathbf{6}, \overline{\mathbf{6}}, \mathbf{15}, \overline{\mathbf{15}}, \mathbf{20}$
$n = 7$	$\mathbf{1}, \mathbf{7}, \overline{\mathbf{7}}, \mathbf{21}, \overline{\mathbf{21}}, \mathbf{35}, \overline{\mathbf{35}}$
$n = 8$	$\mathbf{1}, \mathbf{8}, \overline{\mathbf{8}}, \mathbf{28}, \overline{\mathbf{28}}, \mathbf{56}, \overline{\mathbf{56}}, \mathbf{70}$
$n = 9$	$\mathbf{1}, \mathbf{9}, \overline{\mathbf{9}}, \mathbf{36}, \overline{\mathbf{36}}, \mathbf{84}, \overline{\mathbf{84}}$
$10 \leq n \leq 23$	$\mathbf{1}, \mathbf{n}, \overline{\mathbf{n}}, \mathbf{n(n-1)/2}, \overline{\mathbf{n(n-1)/2}}$

Table 4: Summary of phenomenological constraints on the level of the Kac-Moody algebras for the various GUT groups. The different values of k^{max} have the same meaning as in Table 2. The values in parenthesis under column $k^{max}_{(b)}$ refer to the case in which $SU(4)_h$ is realized at level $k_4 > 1$.

GUT	$k^{max}_{(a)}$	$k^{max}_{(b)}$	$k^{max}_{(c)}$	Adjoints	2–3 splitting	fermion masses	Exotica
$SU(5)$	55	19(12)	7	$k \geq 2$	$k \geq 4$	$k \geq 2$	yes
$SO(10)$	7	5(4)	3	$k \geq 2$	$k \geq 5$	$k \geq 5$	yes
E_6	4	3	2	$k \geq 2$	$k \geq 7$	$k \geq 7$	yes

Table 5: Allowed electric charges for the flipped $SU(5) \times U(1)$ model with level 1 hidden sector $SU(4) \times SO(10)$. Here t_4 is the quadrality of the $SU(4)$ representation and c is the conjugacy class of the $SO(10)$ representation. The notation $\frac{1}{n}\mathbf{Z}$ indicates that the charges are odd multiples of $\frac{1}{n}$.

t_4	c	$Q_{i_4=j_5=2}$	$Q_{i_4,j_5=1,3}$	$SU(4) \times SO(10)$ Reps.
$0,2$	$0,2$	$\mathbf{Z}$	$\mathbf{Z}$	$\{\mathbf{1},\mathbf{6},\mathbf{15},\ldots\} \times \{\mathbf{1},\mathbf{10},\mathbf{45},\ldots\}$
±1	$0,2$	$\frac{1}{2}\mathbf{Z}$	$\frac{1}{4}\mathbf{Z}$	$\{\mathbf{4},\overline{\mathbf{4}},\ldots\} \times \{\mathbf{1},\mathbf{10},\mathbf{45},\ldots\}$
$0,2$	±1	$\frac{1}{2}\mathbf{Z}$	$\frac{1}{4}\mathbf{Z}$	$\{\mathbf{1},\mathbf{6},\mathbf{15},\ldots\} \times \{\mathbf{16},\overline{\mathbf{16}},\ldots\}$
±1	±1	$\mathbf{Z}$	$\frac{1}{2}\mathbf{Z}$	$\{\mathbf{4},\overline{\mathbf{4}},\ldots\} \times \{\mathbf{16},\overline{\mathbf{16}},\ldots\}$