

Measurement of the Bottom-Strange Meson Mixing Phase in the Full CDF Data Set

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We report a measurement of the bottom-strange meson mixing phase β_s using the time evolution of $B_s^0 \rightarrow J/\psi(\rightarrow \mu^+\mu^-)\phi(\rightarrow K^+K^-)$ decays in which the quark-flavor content of the bottom-strange meson is identified at production. This measurement uses the full data set of proton-antiproton collisions at $\sqrt{s} = 1.96$ TeV collected by the Collider Detector experiment at the Fermilab Tevatron, corresponding to 9.6 fb^{-1} of integrated luminosity. We report confidence regions in the two-dimensional space of β_s and the B_s^0 decay-width difference $\Delta\Gamma_s$, and measure $\beta_s \in [-\pi/2, -1.51] \cup [-0.06, 0.30] \cup [1.26, \pi/2]$ at the 68% confidence level, in agreement with the standard model expectation. Assuming the standard model value of β_s , we also determine $\Delta\Gamma_s = 0.068 \pm 0.026(\text{stat}) \pm 0.009(\text{syst}) \text{ ps}^{-1}$ and the mean B_s^0 lifetime, $\tau_s = 1.528 \pm 0.019(\text{stat}) \pm 0.009(\text{syst}) \text{ ps}$, which are consistent and competitive with determinations by other experiments.

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The noninvariance of the physics laws under the simultaneous transformations of parity and charge conjugation (CP violation) is accommodated in the standard model (SM) through the presence of a single irreducible complex phase in the weak-interaction couplings of quarks. A broad class of generic extensions of the SM is expected to naturally introduce additional sources of CP violation that should be observable, making CP -violation studies promising to search for experimental indications of new particles or interactions. Thus far, CP violation has been established in transitions of strange and bottom hadrons, with effects consistent with the SM interpretation [1–3]. Much less information is available for bottom-strange mesons, B_s^0 . Studies of $B_s^0-\bar{B}_s^0$ flavor oscillations are unique in that they probe the quark-mixing (Cabibbo-Kobayashi-Maskawa, CKM) matrix element V_{ts} , which directly enters the mixing amplitude.

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Large non-SM enhancements of the mixing amplitude are excluded by the precise determination of the oscillation frequency in 2006 [4]. However, non-SM particles or couplings involved in the mixing may also increase the size of the observed CP violation by enhancing the mixing phase $\beta_s = \arg[-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)]$ [5] with respect to the value expected from the CKM hierarchy, $\beta_s^{\text{SM}} \approx 0.02$ [2], henceforth referred to as ‘SM expectation’. A non-SM enhancement of β_s would also decrease the size of the decay-width difference between the light and heavy mass eigenstates of the B_s^0 meson, $\Delta\Gamma_s = \Gamma_L - \Gamma_H$. The values of the mixing phase and width difference are loosely constrained, and currently the subject of intense experimental activity. The analysis of the time evolution of $B_s^0 \rightarrow J/\psi\phi$ decays provides the most effective determination of β_s and $\Delta\Gamma_s$ [6]. Assuming negligible contributions from sub-leading decay amplitudes [7], the underlying $b \rightarrow c\bar{c}s$ quark transition is dominated by a single real amplitude, making β_s the sole CP -violating phase observable, through the interference between the amplitudes of decays occurring with and without oscillations.

The first determinations of β_s , by the CDF and D0 experiments, suggested a mild deviation from the SM expectation [8]. The interest in this measurement increased further recently, because of the 3.9σ departure from the SM expectation of the dimuon asymmetry observed by D0 in semileptonic decays of $B_{(s)}^0$ mesons [9], which is tightly correlated with β_s , if generated in the B_s^0 sector [5]. While updated measurements in $B_s^0 \rightarrow J/\psi\phi$ decays [10–13] showed increased consistency with the SM, more precise experimental information is needed for a conclusive interpretation.

In this Letter we report a measurement of β_s ; $\Delta\Gamma_s$; the mean lifetime of heavy and light B_s^0 mass eigenstates, $\tau_s = 2/(\Gamma_H + \Gamma_L)$; and the angular momentum composition of the signal sample using the final data set collected by the CDF experiment at the Tevatron proton-antiproton collider, corresponding to an integrated luminosity of 9.6 fb^{-1} . The analysis closely follows a previous measurement in a subset of the present data [10], and introduces an improved determination of the sample composition based on a new study of the K^+K^- and $J/\psi K^+K^-$ mass distributions.

The CDF II detector is a magnetic spectrometer surrounded by electromagnetic and hadronic calorimeters and muon detectors that has cylindrical geometry with forward-backward symmetry. Charged particle trajectories (tracks) are reconstructed using single- and double-sided silicon microstrip sensors arranged in seven cylindrical layers [14] and an open cell drift chamber with 96 layers of sense wires [15], all immersed in a 1.4 T axial magnetic field. The resolution on the momentum component transverse to the beam, p_T , is $\sigma_{p_T}/p_T^2 \approx 0.07\%$ (p_T in GeV/c), corresponding to a mass resolution of our B_s^0 signal of about $9 \text{ MeV}/c^2$. Muons with $p_T > 1.5 \text{ GeV}/c$

are detected in multiwire drift chambers [16]. A time-of-flight detector identifies charged particles with $p_T < 2 \text{ GeV}/c$ [17], complemented by the ionization-energy-loss measurement in the drift chamber at higher transverse momenta. The combined identification performance corresponds to a separation between charged kaons and pions of approximately two Gaussian standard deviations, nearly constant in the relevant momentum range. Events enriched in $J/\psi \rightarrow \mu^+\mu^-$ decays are recorded using a low- p_T dimuon online selection (trigger) that requires two oppositely-charged particles reconstructed in the drift chamber matched to muon chamber track segments, with a dimuon mass between 2.7 and 4.0 GeV/c^2 .

In the analysis, two tracks matched to muon pairs are required to be consistent with a $J/\psi \rightarrow \mu^+\mu^-$ decay, with dimuon mass $3.04 < m_{\mu\mu} < 3.14 \text{ GeV}/c^2$. These are combined with another pair of tracks consistent with a $\phi \rightarrow K^+K^-$ decay, $1.009 < m_{KK} < 1.028 \text{ GeV}/c^2$, in a kinematic fit to a common vertex. A dimuon mass constraint to the known J/ψ mass [1] improves the B_s^0 mass resolution. An artificial neural network (NN) classifier [10] combines multiple discriminating variables into a single quantity that statistically separates the signal from the dominant background from combinations of real J/ψ decays with random track pairs and a minor component of random four-track combinations (both collectively referred to as combinatorics). The NN is trained with simulated events for the signal and data from sidebands in B_s^0 mass, $[5.29, 5.31] \cup [5.42, 5.45] \text{ GeV}/c^2$, for the background. In decreasing order of discriminating power, the input variables to the NN include kinematic quantities, muon and hadron particle identification information, and vertex fit quality parameters.

Figure 1 shows the $J/\psi K^+K^-$ mass distribution from the final sample of candidates that pass an NN threshold chosen as to maximize the sensitivity to the measurement of β_s [10]. The distribution shows a signal of approximately 11 000 decays, above a fairly constant background dominated by the prompt combinatorial component, and smaller contributions from mis-reconstructed B decays.

We determine the quantities of interest using a fit to the time evolution of bottom-strange mesons. The differences in time evolution of states initially produced as a B_s^0 or \bar{B}_s^0 meson are included in the fit as well as the differences between decays that result in a CP -odd or CP -even combination of the $J/\psi\phi$ angular momenta. The proper decay time of a B_s^0 candidate is a fit observable calculated as $t = ML_{xy}/p_T$, where L_{xy} is the distance from the primary vertex to the B_s^0 decay vertex, projected onto the B_s^0 momentum in the plane transverse to the beam, \vec{p}_T ; and M is the known mass of the B_s^0 meson [1]. The proper decay-time uncertainty, σ_t , is calculated from the measurement uncertainties in L_{xy} . Because the B_s^0 meson has spin zero and J/ψ and ϕ have spin one, the $B_s^0 \rightarrow J/\psi\phi$ decay involves three possible angular momentum states of the $J/\psi\phi$ system. These are

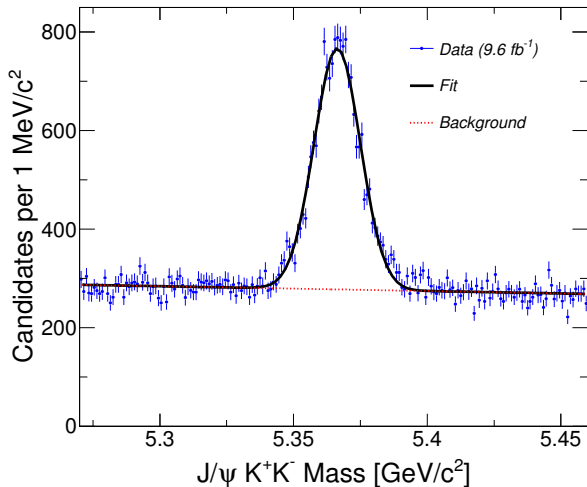


FIG. 1: (Color online) Distribution of $J/\psi K^+ K^-$ mass with fit projection overlaid.

combined into three polarization amplitudes, longitudinal polarization (A_0), and transverse polarization with the linear polarization vectors of the vector mesons parallel (A_{\parallel}) or perpendicular (A_{\perp}) to each other. The first two states are CP even, while the last state is CP odd. A CP -odd state can also be produced by a nonresonant $K^+ K^-$ pair or can originate from the decay of the spin-0 $f_0(980)$ meson, which results in another independent decay amplitude, the S -wave A_S .

To enhance the sensitivity to β_s , the time-evolution of the four decay amplitudes along with six interference terms is fitted simultaneously by exploiting differences in the distribution of the kaons' and muons' decay angles. The angles are parametrized in the transversity basis, $\vec{\rho} = (\cos \Theta, \Phi, \cos \Psi)$ [18], which allows a convenient separation of the CP -even and CP -odd terms in the likelihood. Reference [19] details the expression for the decay rate differential in the decay time and angles. The rate is a function of the physics parameters of interest, β_s , $\Delta\Gamma_s$, τ_s , and the decay amplitudes with their CP -conserving phases. For these we choose A_0 to be real and define the CP -conserving phases as $\delta_{\parallel} = \arg(A_{\parallel}/A_0)$, $\delta_{\perp} = \arg(A_{\perp}/A_0)$ and $\delta_S = \arg(A_S/A_0)$. The decay rate is also a function of the B_s^0 mixing frequency, which is a fit parameter constrained to the experimental value measured by CDF, $\Delta m_s = 17.77 \pm 0.12 \text{ ps}^{-1}$ [4].

The flavor of the meson at the time of production is inferred by two independent classes of flavor tagging algorithms [10], which exploit specific features of the incoherent production of $b\bar{b}$ quarks-pairs in $p\bar{p}$ collisions. Using flavor conservation of the strong interaction, the opposite-side flavor tag (OST) infers the signal production flavor from the decay products of the b hadron produced by the other b quark in the event by using the

charge of muons or electrons from semileptonic B decays or the net charge of the opposite-side jet. The same-side kaon tag (SSKT) deduces the signal production flavor by exploiting charge-flavor correlations of the neighboring kaons produced during its fragmentation. The fraction of candidates tagged by a combination of OST algorithms totals $\varepsilon_{\text{OST}} = (92.8 \pm 0.1)\%$. The probability of wrongly-tagging the meson, w_{OST} , is determined per event and calibrated using 82 000 $B^{\pm} \rightarrow J/\psi(\rightarrow \mu^+ \mu^-) K^{\pm}$ decays fully reconstructed in the same sample as the signal [20]. Because the B^{\pm} does not oscillate, the OST tag is compared with the actual flavor, known from the charge of the K^{\pm} meson. A single scale factor that matches the predicted mistag probability to the one observed in data is then determined to be 1.085 ± 0.035 . The observed averaged dilution, $\mathcal{D}_{\text{OST}} = 1 - 2w_{\text{OST}}$, equals $(12.3 \pm 0.6)\%$ including the scale factor, resulting in a tagging power of $\varepsilon_{\text{OST}} \mathcal{D}_{\text{OST}}^2 = (1.39 \pm 0.05)\%$. The SSKT algorithms tag a smaller fraction of candidates, $\varepsilon_{\text{SSKT}} = (52.2 \pm 0.7)\%$, with better precision. In the $B_s^0 \rightarrow J/\psi \phi$ sample an average dilution of $\mathcal{D}_{\text{SSKT}} = (25.9 \pm 5.4)\%$ is achieved including a 0.94 ± 0.20 scale factor obtained by measuring the B_s^0 oscillation amplitude in approximately 11 000 (1 850) $B_s^0 \rightarrow D_s^- \pi^+ (\pi^+ \pi^-)$ decays reconstructed in the data corresponding to the first 5.2 fb^{-1} [10]. The resulting SSKT tagging power is $\varepsilon_{\text{SSKT}} \mathcal{D}_{\text{SSKT}}^2 = (3.5 \pm 1.4)\%$. Higher instantaneous luminosity conditions in later data resulted in a reduced trigger efficiency for hadronic B_s^0 decays. Hence, the additional sample of $B_s^0 \rightarrow D_s^- \pi^+ (\pi^+ \pi^-)$ decays is too limited for a significant test of the SSKT performance. Because the SSKT calibration is known for early data only, we conservatively restrict its use to the events collected in that period. Simulation shows that this results in a degradation in β_s resolution not exceeding 15%.

The unbinned maximum likelihood fit uses 9 observables from each event to determine 32 parameters including β_s and $\Delta\Gamma$, other physics parameters such as B_s^0 lifetime, amplitudes and phases, and several other quantities, called *nuisance* parameters, such as tagging dilution scale factors. The fit uses the information of the reconstructed B_s^0 candidate mass and its uncertainty, m and σ_m ; the B_s^0 candidate proper decay time and its uncertainty, t and σ_t ; the three transversity angles, $\vec{\rho}$; and tag information, \mathcal{D} and ξ ; where \mathcal{D} is the event-specific dilution given by the mistag probability, and ξ is the tag decision. Both tagged and untagged events are used in the fit. The single-event likelihood is described in terms of signal, P_s , and background, P_b , probability density functions (density henceforth) as

$$\mathcal{L} \propto f_s P_s(m|\sigma_m) P_s(t, \vec{\rho}, \xi | \mathcal{D}, \sigma_t) P_s(\sigma_t) P_s(\mathcal{D}) + (1 - f_s) P_b(m) P_b(t|\sigma_t) P_b(\vec{\rho}) P_b(\sigma_t) P_b(\mathcal{D}), \quad (1)$$

where f_s is the fraction of signal events. The signal mass density, $P_s(m|\sigma_m)$, is parametrized as a single Gaus-

sian with a width determined independently for each candidate. The background mass density, $P_b(m)$, is parametrized as a straight line. The time and angular dependence of the signal, $P_s(t, \vec{\rho}, \xi, |\mathcal{D}, \sigma_t)$, for a single flavor tag are written in terms of two densities, P for B_s^0 and \bar{P} for \bar{B}_s^0 , as

$$\left(\frac{1 + \xi \mathcal{D}}{2} P(t, \vec{\rho} | \sigma_t) + \frac{1 - \xi \mathcal{D}}{2} \bar{P}(t, \vec{\rho} | \sigma_t) \right) \varepsilon(\vec{\rho}), \quad (2)$$

which is extended to the case of OST and SSKT independent flavor tags. Acceptance effects on the transversity angle distributions are modeled with an empirical three-dimensional joint probability density function extracted from simulation, $\varepsilon(\vec{\rho})$, whose largest excursions do not exceed 15%. The time and angular distributions for flavor-tagged B_s^0 (\bar{B}_s^0) decays, P (\bar{P}), are given by the normalized decay rate as functions of decay time and transversity angles of Ref. [19], assuming no direct CP violation in the decay. Building on previous measurements [21], we model the decay-time density for the background, $P_b(t | \sigma_t)$, with a δ -function at $t = 0$, one positive, and two negative exponential functions. All time-dependent terms are convolved with a proper time resolution function, modeled as a sum of two Gaussians with common mean and independent widths determined by the fit. The resulting decay-time resolution is equivalent to that of a Gaussian distribution with 90 fs standard deviation. The background angular probability density is determined from B_s^0 mass sideband events to factorize as $P_b(\vec{\rho}) = P_b(\cos \Theta) P_b(\Phi) P_b(\cos \Psi)$. The distributions of the decay-time uncertainty and the event-specific dilution differ for signal and background events, thus their densities are explicitly included in the likelihood. The probability density functions of the decay-time uncertainties, $P_s(\sigma_t)$ and $P_b(\sigma_t)$, are described with an empirical model from an independent fit to the data. The signal density, $P_s(\mathcal{D})$, is determined from binned background-subtracted signal distributions, while the background density, $P_b(\mathcal{D})$, is modeled from candidates in the signal sidebands. Potential sources of systematic uncertainties, associated with imprecisely known calibration factors of tagging dilutions, are taken into account by floating these factors in the fit within Gaussian constraints.

The likelihood function shows two equivalent global maxima, corresponding to the solutions with positive and negative value of $\Delta\Gamma_s$, and additional local maxima generated by approximate symmetries [19]. Multiple solutions make the estimation of parameters and their uncertainties challenging with limited sample size. If β_s is fixed to its SM value, the fit shows unbiased estimates and Gaussian uncertainties for $\Delta\Gamma_s$, τ_s , polarization am-

plitudes, and the phase δ_\perp , yielding

$$\begin{aligned} \tau_s &= 1.528 \pm 0.019(\text{stat}) \pm 0.009(\text{syst}) \text{ ps}, \\ \Delta\Gamma_s &= 0.068 \pm 0.026(\text{stat}) \pm 0.009(\text{syst}) \text{ ps}^{-1}, \\ |A_0|^2 &= 0.512 \pm 0.012(\text{stat}) \pm 0.018(\text{syst}), \\ |A_\parallel|^2 &= 0.229 \pm 0.010(\text{stat}) \pm 0.014(\text{syst}), \\ \delta_\perp &= 2.79 \pm 0.53(\text{stat}) \pm 0.15(\text{syst}). \end{aligned}$$

The correlation between τ_s and $\Delta\Gamma_s$ is 0.52. We do not report a measurement of δ_\parallel . The fit determines $\delta_\parallel \approx \pi$, but the estimate is biased and its uncertainty is non-Gaussian because the likelihood symmetry under the $\delta_\parallel \rightarrow 2\pi - \delta_\parallel$ transformation [19] results in multiple maxima in the vicinity of $\delta_\parallel = \pi$. Systematic uncertainties include mismodeling of the signal mass model, decay-time resolution, acceptance description, and angular distribution of the background; an 8% contamination by $B^0 \rightarrow J/\psi K^*(892)^0$ and $B^0 \rightarrow J/\psi K^+ \pi^-$ decays misreconstructed as $B_s^0 \rightarrow J/\psi \phi$ decays; and silicon detector misalignment. For each source, uncertainties are determined by comparing the fit results from simulated samples in which the systematic effect is introduced in the model and samples simulated according to the default model. The uncertainty on the $\Delta\Gamma_s$ measurement is dominated by the mismodeling of the background decay time. The largest contribution to the uncertainty on τ_s is the effect of silicon detector misalignment. The angular acceptance model dominates the systematic uncertainties on the amplitudes.

If β_s is free to float in the fit, tests in statistical trials show that the maximum likelihood estimate is biased for the parameters of interest, and the biases depend on the true values of the parameters. Hence, we determine confidence regions in the β_s and $(\beta_s, \Delta\Gamma_s)$ spaces by using a profile-likelihood ratio statistic as a χ^2 variable and considering all other likelihood variables as nuisance parameters. The profile-likelihood ratio distributions observed in simulations deviate from the expected χ^2 distribution, yielding confidence regions that contain the true values of the parameters with lower probability than the nominal confidence level. In addition, the profile-likelihood ratio distribution depends on the true values of the unknown nuisance parameters. We use a large number of statistical trials to derive the profile-likelihood ratio distribution of our data. The effect of nuisance parameters is accounted for by randomly sampling their 30-dimensional space within 5σ of their estimates in data and using the most conservative of the resulting profile-likelihood ratio distributions to derive the final confidence regions. This procedure ensures that the confidence regions have nominal statistical coverage whatever the configuration of nuisance parameters values and increases the size of the β_s confidence interval by about 40%. We determine the confidence level for 32×48 evenly spaced points in $\beta_s \in [-\pi/2, \pi/2]$ and $\Delta\Gamma_s \in [-0.3, 0.3] \text{ ps}^{-1}$ and smoothly

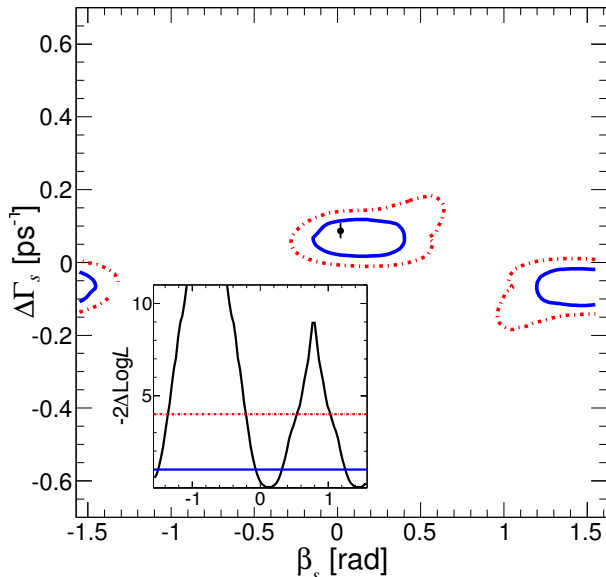


FIG. 2: (Color online) Confidence regions at the 68% (solid) and 95% C.L. (dashed) in the $(\beta_s, \Delta\Gamma_s)$ plane (main panel). The standard model prediction is shown as a circle with error bars [22]. The inset shows the coverage-corrected profile-likelihood ratio as a function of β_s , in which $\Delta\Gamma_s$ is treated as all other nuisance parameters.

interpolate between them to obtain a continuous region (Fig. 2). Assuming the standard model values for β_s and $\Delta\Gamma_s$, the probability to observe a profile-likelihood ratio equal to or higher than observed in data is 54%. By treating $\Delta\Gamma_s$ as a nuisance parameter, we also obtain $\beta_s \in [-\pi/2, -1.51] \cup [-0.06, 0.30] \cup [1.26, \pi/2]$ at the 68% C.L., and $\beta_s \in [-\pi/2, -1.36] \cup [-0.21, 0.53] \cup [1.04, \pi/2]$ at the 95% C.L. The fraction of S -wave in the K^+K^- mass range 1.009–1.028 GeV/ c^2 is determined from the angular information to be consistent with zero with $\mathcal{O}(2\%)$ uncertainty, which is in agreement with our previous determination [10] and the LHCb and ATLAS results [12, 13], and inconsistent with the D0 determination [11]. An auxiliary simultaneous fit of the K^+K^- and $J/\psi K^+K^-$ mass distributions [23] that includes the full resonance structure of the $B^0 \rightarrow J/\psi K^+\pi^-$ decay [24] is performed. The K^+K^- mass is fit in a range enlarged to 0.988–1.2 GeV/ c^2 using a relativistic Breit-Wigner distribution for the ϕ meson, the shape suggested in Ref. [25] for the $f_0(980)$ meson, and an empiric shape determined from data for the combinatorial background. In the 1.009–1.028 GeV/ c^2 mass range, this fit determines a $(0.8 \pm 0.2(\text{stat}))\%$ K^+K^- S -wave contribution in agreement with the central fit, and a contamination from misidentified B^0 decays of $(8.0 \pm 0.2(\text{stat}))\%$, which is significantly larger than the 1–2% values typically derived assuming only P -wave B^0 decays [10, 11]. If neglected, this

additional B^0 component could mimic a larger K^+K^- S -wave than present.

In summary we report the final CDF results on the B_s^0 mixing phase and decay width difference from the time-evolution of flavor-tagged $B_s^0 \rightarrow J/\psi\phi$ decays reconstructed in the full Tevatron Run II data set. This analysis improves and supersedes the previous CDF measurement obtained in a subset of the present data [10]. Considering $\Delta\Gamma_s$ as a nuisance parameter, and using the recent determination of the sign of $\Delta\Gamma_s$ [26], we find $-0.06 < \beta_s < 0.30$ at the 68% C.L. Assuming a SM value for β_s , we also report precise measurements of decay-width difference, $\Delta\Gamma_s = 0.068 \pm 0.026(\text{stat}) \pm 0.009(\text{syst})$ ps $^{-1}$, and mean B_s^0 lifetime, $\tau_s = 1.528 \pm 0.019(\text{stat}) \pm 0.009(\text{syst})$ ps. All results are consistent with expectations and with determinations of the same quantities from other experiments [11–13], and significantly improve the knowledge of the phenomenology on CP violation in B_s^0 mixing.

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