

# Neutrino Mixing Predictions of a Minimal SO(10) Model with Suppressed Proton Decay

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## Abstract

During the past year, a minimal renormalizable supersymmetric SO(10) model has been proposed with the following properties: it predicts a naturally stable dark matter and neutrino mixing angles  $\theta_{\text{atm}}$  and  $\theta_{13}$  while at the same time accommodating CKM CP violation among quarks with no SUSY CP problem. Suppression of proton decay for all allowed values of  $\tan\beta$  strongly restricts the flavor structure of the model making it predictive for other processes as well. We discuss the following predictions of the model in this paper, e.g. down-type quark masses, and neutrino oscillation parameters,  $U_{e3}$ ,  $\delta_{\text{MNSP}}$ , which will be tested by long baseline experiments such as T2K and subsequent experiments using the neutrino beam from JPARC. We also calculate lepton flavor violation and the lepton asymmetry of the Universe in this model.

# 1 Introduction

The existence of non-zero neutrino masses appears to have considerably narrowed the choice of grand unified theories, with those based on the group  $SO(10)$  being preferred for several reasons.  $SO(10)$  is the minimal group with all the ingredients for a small neutrino mass since its **16**-dimensional spinor representation contains the right-handed neutrino,  $\nu_R$ , which is needed to implement the seesaw mechanism[1] along with the other fermions of the standard model. It also contains the  $B-L$  symmetry needed to keep the right-handed neutrino masses below the Planck scale and provides a group theoretic rationale for the belief that neutrinos most likely are Majorana particles. The unification of all the fermions into the **16**-plet also raises the hope that the number of parameters required to describe fermion masses and mixing will be less than those in the standard model, thus making the model predictive for neutrinos.

Initial attempts to realize this hope were made in a class of minimal  $SO(10)$  models with a single **10** and a single  $\overline{\mathbf{126}}$  Higgs multiplets[2, 3] which led to predictions for neutrino mixings as well as two of the masses without any extra symmetry assumptions. The gratifying result was the natural manner in which the large solar and atmospheric mixings arose in these models with predictions in gross agreement with current neutrino observations. The detailed predictions for the case where the Yukawa couplings are CP conserving are however away from the current central values of the neutrino parameters though still in agreement with observations at three  $\sigma$  level.

Encouraged by this initial success in understanding large neutrino mixings, attempts were made to study CP violation in this model[4, 5] by making the Yukawa couplings complex. It was found that fitting fermion masses and mixings forces the CKM phase to be in the second quadrant rather than in the first. This question has been reanalyzed in two recent papers: (1) using type II seesaw in these context of  $SO(10)$  models[6], if one allowed a very high value of the strange quark mass, the CKM phase could be in the first quadrant[7]; (2) in type I seesaw (and mixed case), it is shown that all the masses and mixings (within 99% CL) can be fit for a finely tuned range of parameters[8]. While one could take this to be an indication possibly of new physics contributions to CP violation, a more conservative point of view would be to demand that the model be extended to generate CKM CP violation and see whether its predictivity in the neutrino sector is still preserved. One way to achieve this is to go beyond this minimal Higgs structure by including the **120** Higgs field[9, 10, 11]. In particular, it was

shown in Ref.[9, 10] that the model not only accommodates CKM CP violation but it also solves the SUSY CP problem[9] and has the potential to solve the strong CP problem as well. This would make such models plausible candidates for a theory not only of neutrino masses but also of CP violating phenomena.

As is well known, GUT models generically lead to an unstable proton and therefore the lifetime of proton provides additional constraints on them. Several GUT models have already been severely constrained by the present experimental bounds on the proton lifetime[12, 13]. The question of proton decay in this class of SO(10) models has been under scrutiny in several papers[10, 14, 15]. In particular, in Ref.[10], we showed that requiring the suppression of proton decay for both small and large  $\tan\beta$  severely constrains the textures of the Yukawa couplings. In particular, **120** becomes a necessity for this to happen. The reason for this is that cancellation of different terms in the decay amplitudes for different proton decay processes are absent in this formalism. This makes the model very predictive. In this paper, we study these predictions.

In the process of suppressing the decay amplitude, the number of free parameters becomes less and the model becomes very predictive in the quark-lepton sector. For example, the down-type quark masses,  $|U_{e3}|$ ,  $\delta_{\text{MNSP}}$  etc, are predictions of this model. We also predict the lepton asymmetry which then get converted to baryon asymmetry via sphaleron process in the thermal leptogenesis scenario. Since the baryon asymmetry is determined quite accurately from the recent experimental data, it is important to calculate the prediction of this model. We can also predict the probability of muon type neutrino to electron type neutrino oscillation ( $P_{\nu_\mu \rightarrow \nu_e}$ ) as a function of distance and the energy of the neutrino beam. The future measurements of this probability at the T2K [16] and the subsequent experiments would shed more lights on this model.

This paper is organized as follows: The model and the Yukawa matrices in our model are discussed in section 2, and the natural realization of proton decay suppression and the preferable Yukawa structure are discussed in section 3. In section 4, the predictions of our model are presented and the impact of these solutions for T2K and lepton flavor violations are discussed. The lepton asymmetry is calculated and then it is converted into baryon asymmetry in section 5. Section 6 is for the conclusion.

## 2 Fermion Mass Matrices in a Minimal SO(10) Model

We first introduce the Yukawa interactions and the contents of Higgs fields in the SO(10) model. The Yukawa superpotential involves the couplings of **16**-dimensional matter spinors  $\psi_i$  ( $i$  denotes a generation index) with **10** ( $H$ ),  $\overline{\mathbf{126}}$  ( $\overline{\Delta}$ ), and **120** ( $D$ ) dimensional Higgs fields:

$$W_Y = \frac{1}{2}h_{ij}\psi_i\psi_jH + \frac{1}{2}f_{ij}\psi_i\psi_j\overline{\Delta} + \frac{1}{2}h'_{ij}\psi_i\psi_jD. \quad (1)$$

The Yukawa couplings,  $h$  and  $f$  are symmetric matrices and  $h'$  is an anti-symmetric matrix due to the SO(10) symmetry. The Higgs doublet fields not only exist in  $H$ ,  $\overline{\Delta}$ ,  $D$ , but also exist in other Higgs fields which are needed in the model. For example, a **210** Higgs field ( $\Phi$ ) is employed to break the SO(10) symmetry down to the standard model and  $\Phi$  contains Higgs doublet fields. One **126** Higgs multiplet  $\Delta$  is introduced as a vector-like pair of  $\overline{\Delta}$  and this field also contains a Higgs doublet. The VEV of this pair reduces the rank of SO(10) group and helps to keep supersymmetry unbroken down to the weak scale. Altogether, we have six pairs of Higgs doublets:  $\varphi_d = (H_d^{10}, D_d^1, D_d^2, \overline{\Delta}_d, \Delta_d, \Phi_d)$ ,  $\varphi_u = (H_u^{10}, D_u^1, D_u^2, \Delta_u, \overline{\Delta}_u, \Phi_u)$ , where superscripts 1, 2 of  $D_{u,d}$  stand for SU(4) singlet and adjoint pieces under the  $G_{422} = \text{SU}(4) \times \text{SU}(2) \times \text{SU}(2)$  decomposition. The mass term of the Higgs doublets is given as  $(\varphi_d)_a(M_D)_{ab}(\varphi_u)_b$ , and the expression of the matrix  $M_D$  is given in Ref.[17]. The mass matrix of the Higgs doublets is diagonalized by unitary matrices  $U$  and  $V$ :  $UM_DV^T = M_D^{\text{diag}}$ . We assume that the lightest Higgs pair (MSSM doublets) has masses of the order of the weak scale. The MSSM Higgs doublets are given as linear combinations:  $H_d = U_{1a}^*(\varphi_d)_a$ ,  $H_u = V_{1a}^*(\varphi_u)_a$ . Since we concentrate on the structure of Yukawa couplings, we do not describe the dynamical reason of the mass hierarchy in this paper.

We use ‘‘ $Y$ -diagonal basis’’ (or SU(5) basis) to describe the standard model decomposition of the SO(10) representation[17, 18]. The above Yukawa interaction includes mass terms of the quark and lepton fields as follows:

$$\begin{aligned} W_Y^{\text{mass}} &= hH_d^{10}(qd^c + le^c) + hH_u^{10}(qu^c + l\nu^c) \\ &+ \frac{1}{\sqrt{3}}f\overline{\Delta}_d(qd^c - 3le^c) + \frac{1}{\sqrt{3}}f\overline{\Delta}_u(qu^c - 3l\nu^c) + \sqrt{2}f\nu^c\nu^c\overline{\Delta}_R + \sqrt{2}f\ell\ell\overline{\Delta}_L \\ &+ h'D_d^1(qd^c + le^c) + h'D_u^1(qu^c + l\nu^c) + \frac{1}{\sqrt{3}}h'D_d^2(qd^c - 3le^c) - \frac{1}{\sqrt{3}}h'D_u^2(qu^c - 3l\nu^c), \end{aligned} \quad (2)$$

where  $q, u^c, d^c, \ell, e^c, \nu^c$  are the quark and lepton fields for the standard model, which are all unified into one spinor representation of SO(10). The VEVs of the fields  $\overline{\Delta}_R : (\mathbf{1}, \mathbf{1}, 0)$  and

$\overline{\Delta}_L : (\mathbf{1}, \mathbf{3}, 1)$  give neutrino Majorana masses. We obtain the Yukawa coupling matrices for fermions as

$$Y_u = \bar{h} + r_2 \bar{f} + r_3 \bar{h}', \quad (3)$$

$$Y_d = r_1(\bar{h} + \bar{f} + \bar{h}'), \quad (4)$$

$$Y_e = r_1(\bar{h} - 3\bar{f} + c_e \bar{h}'), \quad (5)$$

$$Y_\nu = \bar{h} - 3r_2 \bar{f} + c_\nu \bar{h}', \quad (6)$$

where the subscripts  $u, d, e, \nu$  denotes for up-type quark, down-type quark, charged-lepton, and Dirac neutrino Yukawa couplings, respectively, and

$$\bar{h} = V_{11}h, \quad \bar{f} = U_{14}/(\sqrt{3}r_1)f, \quad \bar{h}' = (U_{12} + U_{13}/\sqrt{3})/r_1h', \quad (7)$$

$$r_1 = \frac{U_{11}}{V_{11}}, \quad r_2 = r_1 \frac{V_{15}}{U_{14}}, \quad r_3 = r_1 \frac{V_{12} - V_{13}/\sqrt{3}}{U_{12} + U_{13}/\sqrt{3}}, \quad (8)$$

$$c_e = \frac{U_{12} - \sqrt{3}U_{13}}{U_{12} + U_{13}/\sqrt{3}}, \quad c_\nu = r_1 \frac{V_{12} + \sqrt{3}V_{13}}{U_{12} + U_{13}/\sqrt{3}}. \quad (9)$$

The light neutrino mass is obtained as

$$m_\nu^{\text{light}} = M_L - M_\nu^D M_R^{-1} (M_\nu^D)^T, \quad (10)$$

where  $M_\nu^D = Y_\nu \langle H_u \rangle$ ,  $M_L = 2\sqrt{2}f \langle \overline{\Delta}_L \rangle$ , and  $M_R = 2\sqrt{2}f \langle \overline{\Delta}_R \rangle$ .

In this paper, we consider that the Yukawa coupling matrices for quarks and leptons are hermitian. The hermiticity of fermion mass matrices can be obtained in the assumption: The original Lagrangian has symmetry under charge conjugation in addition to the SO(10) gauge symmetry. In the  $Y$ -diagonal basis, the SO(10) vector representation in terms of  $X_a$  can be decomposed such that  $(X_1, X_3, X_5, X_7, X_9)$  transforms as  $\mathbf{5}$ -plet, and  $(X_2, X_4, X_6, X_8, X_{10})$  transforms as  $\overline{\mathbf{5}}$ -plet under SU(5)  $\times$  U(1) decomposition. We define the charge conjugation of the field written in the  $Y$ -diagonal basis,  $X_a \xrightarrow{\mathcal{C}} X_a^*$ . The conjugation of the higher rank tensor representation is also defined similarly using the  $Y$ -diagonal basis. Since the SO(10) symmetric Lagrangian has parity invariance as an internal symmetry (actually, it is D-parity), the theory now has CP symmetry. Imposing that Lagrangian is invariant under the CP conjugation, the Yukawa couplings,  $h_{ij}$ ,  $f_{ij}$  and  $h'_{ij}$  and all masses and couplings in the Higgs superpotential are all real. However, when SO(10) symmetry is broken down, the CP symmetry can be spontaneously broken by the VEV of  $\mathbf{45}$  Higgs field. Due to the non-existence of cubic terms for

**45** Higgs field, the VEV of **45** can be pure imaginary. Consequently, the mixing of the lightest Higgs doublets with the Higgs doublets present in **120** involves a pure imaginary coefficient which will make the fermion masses hermitian in this model[9]. This symmetry has wider implications. For instance, the model presents a solution to the SUSY CP problem and strong CP problem. It is possible to explain the quark masses and mixing angles and the neutrino sectors by using the above parameters[9].

### 3 Proton Decay and Flavor Structure

As mentioned above, it was shown recently by us[10] that the suppression of proton decay for all  $\tan\beta$  determines the flavor structure of the three matrices  $h, f, h'$  to a very narrow range. We review this argument in this section.

The proton decay is mediated by the colored Higgs triplets,  $\varphi_T + \varphi_{\overline{T}} : ((\mathbf{3}, \mathbf{1}, -1/3) + c.c.)$  and  $\varphi_C + \varphi_{\overline{C}} : ((\mathbf{3}, \mathbf{1}, -4/3) + c.c.)$ . These Higgs triplets appear in  $\mathbf{10} + \mathbf{120} + \mathbf{126} + \overline{\mathbf{126}} + \mathbf{210}$  multiplets. We generate both  $LLLL$  ( $C_L$ ) and  $RRRR$  ( $C_R$ ) operators:

$$-W_5 = \frac{1}{2} C_L^{ijkl} q_k q_l q_i \ell_j + C_R^{ijkl} e_k^c u_l^c u_i^c d_j^c. \quad (11)$$

These operators are obtained by integrating out the triplet Higgs fields,  $\varphi_{\overline{T}} = (H_{\overline{T}}, D_{\overline{T}}, D'_{\overline{T}}, \overline{\Delta}_{\overline{T}}, \Delta_{\overline{T}}, \Delta'_{\overline{T}}, \Phi_{\overline{T}})$  and  $\varphi_T = (H_T, D_T, D'_T, \Delta_T, \overline{\Delta}_T, \overline{\Delta}'_T, \Phi_T)$ . The fields with ‘‘ are decuplet, and the others are sextet or **15**-plet under SU(4) decomposition. The  $C_R$  operator has also contributions from the triplets,  $\varphi_{\overline{C}} = (D_{\overline{C}}, \Delta_{\overline{C}})$  and  $\varphi_C = (D_C, \overline{\Delta}_C)$ . The mass term of the Higgs triplets are given as  $(\varphi_{\overline{T}})_a (M_T)_{ab} (\varphi_T)_b + (\varphi_{\overline{C}})_a (M_C)_{ab} (\varphi_C)_b$ . The mass matrices,  $M_T$  and  $M_C$ , are  $7 \times 7$  and  $2 \times 2$  matrices respectively, and their explicit forms are given in the literature[17]. The Yukawa couplings which cause proton decay are written as

$$\begin{aligned} W_Y^{\text{trip.}} &= h H_{\overline{T}} (q\ell + u^c d^c) + h H_T \left( \frac{1}{2} qq + e^c u^c \right) + f \overline{\Delta}_{\overline{T}} (q\ell - u^c d^c) + f \overline{\Delta}_T \left( \frac{1}{2} qq - e^c u^c \right) \\ &+ \sqrt{2} f \overline{\Delta}'_{\overline{T}} e^c u^c + \sqrt{2} h' (D_{\overline{T}} u^c d^c + D'_{\overline{T}} q\ell - D_T e^c u^c + D'_T e^c u^c) \\ &+ 2f \overline{\Delta}_C d^c e^c + 2h' D_{\overline{C}} u^c u^c + 2h' D_C d^c e^c. \end{aligned} \quad (12)$$

The dimension five operators are written by the Yukawa couplings  $h, f$  and  $h'$  as follow:

$$C_L^{ijkl} = c h_{ij} h_{kl} + x_1 f_{ij} f_{kl} + x_2 h_{ij} f_{kl} + x_3 f_{ij} h_{kl} + x_4 h'_{ij} h_{kl} + x_5 h'_{ij} f_{kl}, \quad (13)$$

$$\begin{aligned} C_R^{ijkl} &= c h_{ij} h_{kl} + y_1 f_{ij} f_{kl} + y_2 h_{ij} f_{kl} + y_3 f_{ij} h_{kl} + y_4 h'_{ij} h_{kl} + y_5 h'_{ij} f_{kl} \\ &+ y_6 h_{ij} h'_{kl} + y_7 f_{ij} h'_{kl} + y_8 h'_{ij} h'_{kl} + y_9 h'_{il} f_{jk} + y_{10} h'_{il} h'_{jk}. \end{aligned} \quad (14)$$

The coefficient  $c$  is given as  $c = (M_T^{-1})_{11}$ , and the other coefficients  $x_i, y_i$  are also given by the components of  $M_T^{-1}$  or  $M_C^{-1}$ . Note that the  $H_T$  and  $\bar{\Delta}_T$  have opposite D-parity and we get  $y_3 = -x_3$ . The  $y_9$  and  $y_{10}$  terms are generated by  $\varphi_C + \varphi_{\bar{C}}$ .

The proton decay operators can be written conveniently by diagonalizing the Higgs triplet mass matrix  $M_T$  by two unitary matrices,  $X$  and  $Y$ , as  $XM_T Y^T = \text{diag}(M_1, M_2, \dots, M_7)$ ,

$$C_L^{ijkl} = \sum_a \frac{1}{M_a} (X_{a1}h + X_{a4}f + \sqrt{2}X_{a3}h')_{ij} (Y_{a1}h + Y_{a5}f)_{kl}, \quad (15)$$

$$C_R^{ijkl} = \sum_a \frac{1}{M_a} (X_{a1}h - X_{a4}f + \sqrt{2}X_{a2}h')_{ij} (Y_{a1}h - (Y_{a5} - \sqrt{2}Y_{a6})f + \sqrt{2}(Y_{a3} - Y_{a2})h')_{kl} \\ + (y_9, y_{10} \text{ terms}). \quad (16)$$

In the SU(5) limit, only one of the colored triplets is much lighter than the others, i.e.  $M_1 \ll M_a$  ( $a \neq 1$ ), and we can obtain the following relations for the diagonalizing matrices from the explicit form of the Higgs mass matrices in Ref.[17, 18]:  $U_{11} = X_{11}$ ,  $V_{11} = Y_{11}$ ,  $U_{14} = X_{14} = 0$ ,  $V_{15} : Y_{15} : Y_{16} = \sqrt{3} : 1 : \sqrt{2}$ ,  $U_{12} : U_{13} : X_{12} : X_{13} = V_{12} : V_{13} : Y_{12} : Y_{13} = 1 : \sqrt{3} : \sqrt{2} : \sqrt{2}$ . Using the above relations, we get

$$r_2 \rightarrow \infty, \quad \bar{f} \rightarrow 0, \quad r_3 = 0, \quad c_e = -1, \quad c_\nu = 2r_1 V_{12}/U_{12}, \quad (17)$$

for the Yukawa matrices in Eqs.(3-6) and thus, as expected, we get the SU(5) relations,  $Y_u = Y_u^T$ ,  $Y_d = Y_e^T$ , and the dimension five proton decay operators can be written in terms of the Yukawa couplings as  $C_L^{ijkl} \simeq C_R^{ijkl} \simeq (Y_d)_{ij}(Y_u)_{kl}/M_1$  and we know that proton decay is not suppressed at this limit and this limit is also not good to satisfy the quark-lepton masses.

The proton decay amplitude can be written as  $A = \alpha_2 \beta_p / (4\pi M_T m_{SUSY}) \tilde{A}$ , where

$$\tilde{A} = c\tilde{A}_{hh} + x_1\tilde{A}_{ff} + x_2\tilde{A}_{hf} + x_3\tilde{A}_{fh} + x_4\tilde{A}_{h'h} + x_5\tilde{A}_{h'f}. \quad (18)$$

The coefficients  $c$  and  $x_i$  are given in Eq.(13), and there are also similar  $C_R$  contributions. To satisfy the current nucleon decay bounds, we need  $|\tilde{A}_{p \rightarrow K\bar{\nu}}| \lesssim 10^{-8}$ ,  $|\tilde{A}_{n \rightarrow \pi\bar{\nu}}| \lesssim 2 \cdot 10^{-8}$  and  $|\tilde{A}_{n \rightarrow K\bar{\nu}}| \lesssim 5 \cdot 10^{-8}$  if the colored Higgsino mass is  $2 \cdot 10^{16}$  GeV, and squark and wino masses are around 1 TeV and 250 GeV, respectively. Instead of inducing any cancellation among different terms in the amplitude, we will try to suppress the individual contributions.

One way to suppress the decay amplitude is by demanding cancellation among different terms, a strategy common in the literature. In order to achieve that, we need a cancellation among  $h, f$  and  $h'$  to have small couplings for first and second generations in the expressions in

Eqs.(13,14). However, we also need cancellation among the same couplings to generate the large mass hierarchy among the quark masses and in general, the coefficients  $r_2, r_3$  in up-type Yukawa matrix and  $x_i, y_i$  in proton decay operators are unrelated. Further, the  $\overline{\mathbf{126}}$  Higgs contribution has opposite signatures ( $y_3 = -x_3$ ) for  $C_L$  and  $C_R$ . Therefore, the cancellation required to obtain small Yukawa coupling for  $Y_u$  by tuning  $r_2\bar{f}$  can not simultaneously suppress both  $C_L$  and  $C_R$  operators by tuning  $X_{14}$  in Eqs.(15, 16). Moreover, the  $\mathbf{120}$  contribution to the proton decay amplitude has vanishing contribution to the  $kl$  part of  $C_L^{ijkl}$  due to antisymmetric  $h'$ . Thus, if the cancellation in  $Y_u$  requires a tuning of  $r_3h'$ , the  $C_L$  operator can not be suppressed.

The best way to avoid the cancellation is to choose smaller values of  $r_{2,3}$ . Let us start with  $r_{2,3} \simeq 0$ . In this case, the  $C_L$  can be written as  $C_L^{ijkl} \propto (Y_u + \gamma h')_{ij}(Y_u)_{kl}$  and in the operator  $C_R^{ijkl}$ ,  $kl$  part is also related to  $Y_u$ . This will correspond to the case where  $X_{14}, Y_{15} \sim 0$ . The  $RRRR$  contribution to  $p \rightarrow K\bar{\nu}_\tau$  mode is suppressed compared to the minimal SU(5) model by a suppression factor  $\lambda_u/\lambda_d \sim 1/100$  for  $\tan\beta \sim 50$ . Similarly, since the  $kl$  part of  $C_L$  are also related to the  $Y_u$  instead of  $Y_d$ , the  $LLLL$  contribution to the  $p \rightarrow K\bar{\nu}$  is also suppressed even for  $\tan\beta \sim 50$ , compared to the SU(5) model (since  $\lambda_c/\lambda_s \sim 1/5$ ). However, these suppressions are not enough to satisfy the current experimental bound.

In order to satisfy the bounds naturally, we need  $\tilde{A}_{hh} \lesssim 5 \cdot 10^{-8}$  in the expression, Eq.(18). If  $\tilde{A}_{hh} \gtrsim 10^{-7}$ , we need to tune  $x_i$  and  $y_i$  for every decay mode to cancel  $\tilde{A}_{hh}$ , which is unnatural. The  $\tilde{A}_{hh}$  depends on the magnitudes of the elements from the [1,2] block of  $\bar{h}$  which is specifically determined from the fit to the up-type Yukawa coupling as a function of  $r_2$  and  $r_3$ . In the case  $r_{2,3} \sim O(1)$ , the suppression of up- and charm-quark Yukawa couplings are acquired by fine-tuning of  $r_{2,3}$ , and thus the elements from [1,2] block of  $\bar{h}$  are of the order of down- and strange-quark Yukawa couplings. In that case, we find  $\tilde{A}_{hh} \sim 10^{-4}$  which requires a very high level of fine-tuning for all the decay modes. In the case  $r_2, r_3 \sim 0$  for generical fits, the Yukawa coupling  $\bar{h}$  is close to  $Y_u$  in the  $Y_u$ -diagonal basis by definition. However, even in the case, the  $\tilde{A}_{hh}$  is of the order of  $10^{-7}$  for generical fits and further tuning among the coefficients  $x_i$  and  $y_i$  is needed to satisfy the current experimental data. We therefore need a specific type of Yukawa texture to suppress the proton decay rate. To suppress  $\tilde{A}_{hh}$ , the elements  $\bar{h}_{11}$  and  $\bar{h}_{22}$  (in  $\bar{h}$ -diagonal basis) are needed to be suppressed rather than the up- and charm-quark Yukawa couplings, respectively. As a result, we need Yukawa texture to be  $\bar{h} \simeq \text{diag}(\sim 0, \sim 0, O(1))$ .



Once  $\bar{h}$  is fixed, the other matrices  $\bar{f}$  and  $\bar{h}'$  are almost determined as

$$\bar{f} \simeq \begin{pmatrix} \sim 0 & \sim 0 & \lambda^3 \\ \sim 0 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix}, \quad \bar{h}' \simeq i \begin{pmatrix} 0 & \lambda^3 & \lambda^3 \\ -\lambda^3 & 0 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 0 \end{pmatrix}, \quad (19)$$

where  $\lambda \sim 0.2$ . The correct charm mass is generated by  $r_2 m_s / m_b \simeq \lambda_c$  ( $|r_2| \simeq 0.1 - 0.15$ ), and down-quark mass and Cabibbo angle  $\theta_C$  are generated by  $\bar{h}'_{12}$  with  $m_d / m_s \simeq \sin^2 \theta_C$ . Then we need  $\bar{f}_{11} \lesssim O(\lambda^6)$ ,  $\bar{f}_{12} \lesssim O(\lambda^4)$  and  $r_3 \lesssim O(\lambda^2)$  to obtain proper size of up-quark and electron masses. We have  $r_1 \simeq m_b / m_t \tan \beta$ . In the basis where  $Y_u$  is diagonal,  $\tilde{A}_{hh}$  in this texture is not completely zero but can become much smaller than  $10^{-8}$ .

We show one example for numerical fit for  $\tan \beta (M_Z) = 50$  :  $\bar{h} = \text{diag}(0, 0, 0.638)$ ,

$$\bar{f} \simeq \begin{pmatrix} 0 & -0.00044 & 0.00208 \\ -0.00044 & 0.00945 & 0.0101 \\ 0.00208 & 0.0101 & 0.0071 \end{pmatrix}, \quad \bar{h}' \simeq i \begin{pmatrix} 0 & -0.0022 & 0.00046 \\ 0.0022 & 0 & 0.0181 \\ -0.00046 & -0.0181 & 0 \end{pmatrix}, \quad (20)$$

$r_1 = 0.966$ ,  $r_2 = 0.135$ ,  $r_3 = 0$ ,  $|c_e| = 0.987$ .

After we suppress the  $\tilde{A}_{hh}$ , we also need to examine the contribution of the other components e.g.  $\tilde{A}_{ff, hf, fh, h'f, h'h, \dots}$ . Their coefficients,  $x_i, y_i$ , involve the colored Higgs mixings, which can be suppressed by our choice of the vacuum expectation values and the Higgs couplings. According to our numerical studies, some of the mixing angles must be less than about a few percent in the case of  $\tan \beta \sim 50$  to suppress the decay. However, the mixing angles can become larger as  $\tan \beta$  becomes smaller. In the above example of numerical fit,  $p \rightarrow K \bar{\nu}_\mu$  and  $p \rightarrow K \bar{\nu}_\tau$  modes are dominant for  $LLLL$  operator, and for  $RRRR$  operator, respectively. The  $\tilde{A}_{hh}$  for  $p \rightarrow K \bar{\nu}_\mu$  mode is  $\sim 2 \cdot 10^{-11}$ . The amplitudes for other components are  $10^{-8} - 10^{-6}$ .

We have seen that the suppression of proton decay requires  $r_3 \simeq 0$ , which is same as SU(5) condition for **120** Higgs coupling, and  $r_2 \simeq 0.1 - 0.15$ , which however is not a SU(5) condition. The second condition, as well as suppression of colored Higgs mixing, is implemented by requiring  $U_{14} \gg V_{15}, X_{14}, Y_{15}$ . We note that if  $U \simeq V$  is satisfied (which is like a up-down symmetry and therefore  $\tan \beta \simeq 50$ ), the  $r_3 \simeq 0$  condition also satisfies the SU(5)-like condition

$$c_e \simeq -1, \quad c_\nu \simeq 2, \quad (21)$$

for **120** Higgs coupling in  $Y_e$  and  $Y_\nu$ , though these conditions are not required to be satisfied to fit the fermion masses or to suppress the proton decay.

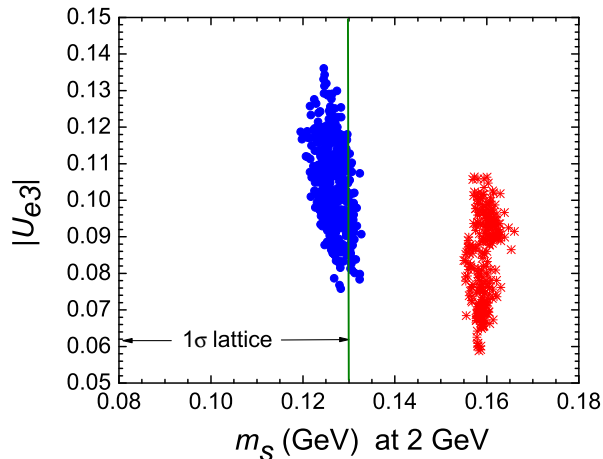


Figure 1:  $|U_{e3}|$  is plotted as a function of  $m_s$ . The two regions are described in the text.

## 4 The Model Predictions

Now we are ready to discuss the predictions of the model. The number of parameters in the models is 17: 3( $h$ ), 6( $f$ ), 3( $h'$ ) and 5 Higgs parameters ( $r_{1,2,3}$ ,  $c_{e,\nu}$ ). (We are working in the diagonal  $h$  basis). Now in the process of explaining the proton decay, some of the parameters are redundant to fit masses and mixings. We choose  $\bar{h}_{11,22} = 0$  and  $r_3 = 0$ . The  $r_3$  can be  $O(\lambda^2)$  in our Yukawa texture, but such a value of  $r_3$  gives only a small correction to the CKM mixings. We use a very small  $f_{11}$ , but even in the case  $\bar{f}_{11} = 0$ , the obtained Yukawa texture can fit all masses and mixings. We make it a free parameter since it affects the leptonic fitting. Since we will be working type II seesaw (actually, the Yukawa texture of  $\bar{f}$  is compatible to bi-maximal mixings),  $c_\nu$  is redundant in fitting fermion masses and mixings. This reduces the number of parameters to 13. In order to fix the remaining 13 parameters, we use the up-type quark masses, charged lepton masses, the CKM angles and the phase, the ratio of the squared of neutrino mass differences ( $\Delta m_{\text{sol}}^2/\Delta m_A^2$ ), and the bi-maximal mixings as input parameters. Consequently, the down-type quark masses,  $U_{e3}$  and  $\delta_{\text{MNSP}}$  etc are the predictions of this model.

We itemize below the essential features of our predictions.

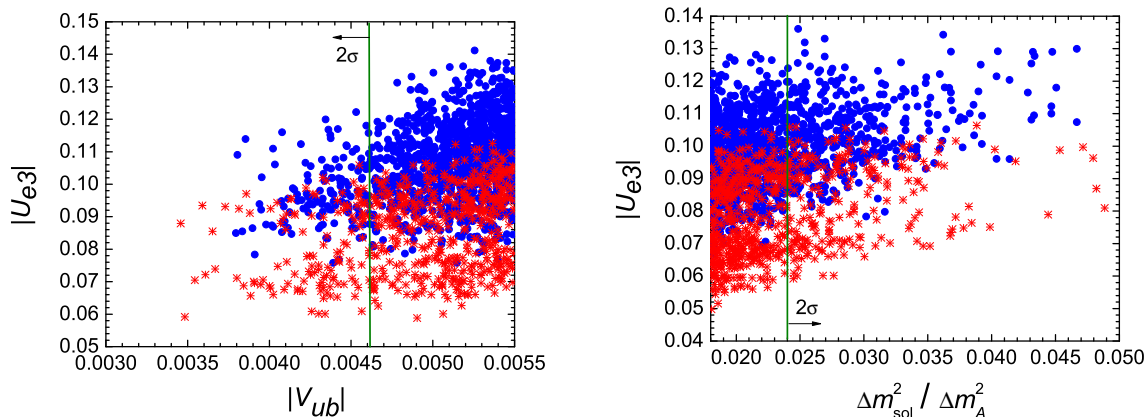


Figure 2:  $|U_{e3}|$  is plotted as a function of  $V_{ub}$  and  $\Delta m_{\text{sol}}^2 / \Delta m_A^2$ .

#### 4.1 Strange Quark Mass

In order to see how the down quark masses and in particular the strange quark mass gets constrained in our model, we first note from Eqs.(4,5,19) we get  $\det M_e \simeq |c_e|^2 \det M_d$ , when  $\bar{f}_{11} \ll \lambda^5$  and thus, the down quark and the charged lepton mass matrix leads to the Georgi-Jarskog (GJ) relation naturally when the SU(5) relation ( $c_e = -1$ ) is satisfied in **120** Higgs mixing.

The strange quark mass is predicted since no cancellation happens to derive  $V_{cb}$  (due to  $r_2, r_3 \ll 1$  and the hermiticity condition). The predicted value of strange quark mass has two separate regions, roughly  $m_s \sim 1/3 m_\mu (1 \pm O(\lambda^2))$ . In our numerical calculations, the negative sign corresponds to a strange quark mass:  $\bar{m}_s(\mu = 2 \text{ GeV}) \sim 120 - 130 \text{ MeV}$ . (The strange quark mass at 1 GeV is obtained by multiplying the strange mass at 2 GeV with 1.35.) This strange mass value is in the range of lattice derived value,  $\bar{m}_s(\mu = 2 \text{ GeV}) = (105 \pm 25) \text{ MeV}$ [19]. The GJ relation is realized for this result and thus the fitting of the lattice result supports  $|c_e| \simeq 1$  (when  $\bar{f}_{11} \ll \lambda^5$ ). If we use the positive signature in the rough estimation, we find the following value of the strange quark mass,  $\bar{m}_s(\mu = 2 \text{ GeV}) \sim 155 - 165 \text{ MeV}$  which is allowed by the QCD calculation[19] (In this case, non-zero values of  $\bar{f}_{11}$  are needed to make  $|c_e| = 1$ ). In Figure 1, we plot  $|U_{e3}|$  against the strange quark mass. We see the two regions corresponding to two different values of strange quark masses (corresponding to two different signs). We also note that the larger values of strange mass prefers lower values of  $U_{e3}$ . We use  $\alpha_s(M_Z) = 0.118$  and a 3-loop QCD and 1-loop QED running to evolve the strange mass from 2 GeV scale to

the electroweak scale[20].

Our prediction of strange-down quark mass ratio is  $m_s/m_d \simeq 17 - 18, 19 - 20.5$ . The non-lattice value of the ratio is  $m_s/m_d = 18.9 \pm 0.8$ [21].

The  $m_u/m_d$  ratio is not a prediction of this model since  $m_u$  is an input. To obtain a large atmospheric mixing,  $\bar{f}_{33}$  needs to be suppressed, and thus bottom-tau Yukawa coupling needs to be unified within several percent. In order to achieve such a situation, large  $\tan \beta \sim 50$  is needed (or  $\tan \beta \sim 2$ ). Thus, the bottom mass is predicted. However, this prediction is lost since it gets a sizable contribution from the soft SUSY breaking terms for large  $\tan \beta$ .

## 4.2 $|U_{e3}|$

Since there is no cancellation, we get the following stable approximate relation for  $U_{e3}$ :

$$|U_{e3}|^2 \approx \frac{\tan^2 \theta_{\text{sol}}}{1 - \tan^4 \theta_{\text{sol}}} \Delta m_{\text{sol}}^2 / \Delta m_A^2. \quad (22)$$

We also have the following relation since  $U_{e3}$  is related to the ratio  $\bar{f}_{13}$  and  $\bar{f}_{23}$ :

$$|U_{e3}| \approx \frac{1}{\sqrt{2}} \left| \frac{V_{ub}}{V_{cb}} \right|. \quad (23)$$

These relations are obtained since  $\bar{f}_{12}$  is small compared to  $\bar{f}_{13}$ . Actually, when  $\bar{f}_{12} \simeq \bar{f}_{13}$ ,  $U_{e3}$  can be canceled to be zero without relating to the solar mixing angle and  $V_{ub}$ . In our proton decay suppressed texture, the  $\bar{f}_{12}$  is suppressed due to the assumption  $\bar{h}_{11} \ll \lambda_u$ , and thus the  $|U_{e3}|$  prediction can be stable.

The  $|U_{e3}|$  is bounded from below through the mass squared ratio and solar mixing angle and has an upper limit due to the experimental bound on  $|V_{ub}| = (3.67 \pm 0.47) \times 10^{-3}$ . In Figure 2, we plot  $|U_{e3}|$  as a function  $V_{ub}$  and  $\Delta m_{\text{sol}}^2 / \Delta m_A^2$ . We find that for  $|V_{ub}| < 0.0045$  and  $\Delta m_{\text{sol}}^2 / \Delta m_A^2 > 0.022$ , the  $|U_{e3}|$  is bounded to be  $0.06 - 0.11$ . We plot  $|U_{e3}|$  as a function  $|c_e|$  in Figure 3. The value of  $c_e = -1$  (corresponding to the SU(5) condition for **120**) gives rise to  $|U_{e3}| \sim 0.1$ . Both large and small strange quark masses are allowed for  $c_e = -1$ .

## 4.3 MNSP Phase

The MNSP phase is also stable in this model and is given by the approximate expression:

$$\sin \delta_{\text{MNSP}} \sim \frac{1}{\sqrt{2}} \frac{\sin \theta_{12}^e}{\sin \theta_{13}^e} \sin \left( \tan^{-1} \frac{c_e \bar{h}'_{12}}{3 \bar{f}_{12}} \right), \quad (24)$$

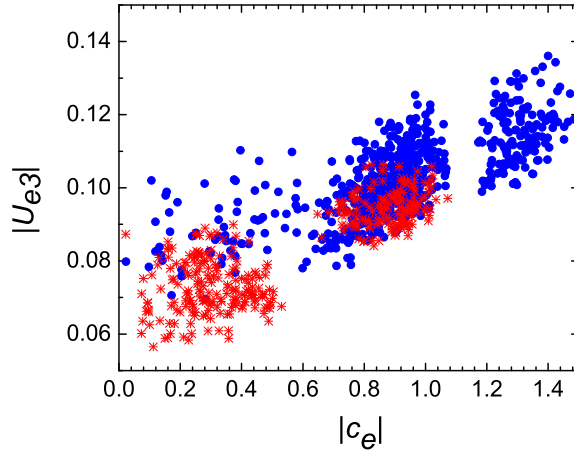


Figure 3:  $|U_{e3}|$  is plotted as a function of  $c_e$ .

where  $\theta_{12}^e$  and  $\theta_{13}^\nu$  are mixing angles in the diagonalizing matrix of  $Y_e$  and neutrino mass matrix, respectively. Approximately,  $\sin \theta_{13}^\nu \simeq |U_{e3}|$ . We plot  $|U_{e3}|$  as a function of MNSP phase  $\delta_{\text{MNSP}}$  in Figure 4. We find that the  $\delta_{\text{MNSP}}$  lies in the 2nd or 4th quadrant if  $c_e < 0$ . The 1st and the 3rd quadrants are absent since the solar mixing angle becomes small due to a cancellation between  $\theta_{12}^\nu$  and  $\theta_{12}^e$ . In Figure 4, we plot  $|c_e|$  as a function of  $\delta_{\text{MNSP}}$ . We see that  $c_e = -1$  (SU(5) relation of **120** Higgs mixing, and also with a favorable value by the GJ relation), generates  $\sin \delta_{\text{MNSP}} \simeq \pm(0.5 - 0.7)$ . To obtain this result, we assume that type II contribution is dominant.

The location of  $\delta_{\text{MNSP}}$  in the 2nd or 4th quadrant has impact on the probability of  $\nu_\mu$  to  $\nu_e$  oscillation ( $P_{\nu_\mu \rightarrow \nu_e}$ ) which will be measured at the T2K experiment and at the newly proposed Tokai-to-Korea experiment[22]. This probability depends on sine and cosine of  $\delta_{\text{MNSP}}$ , distance ( $L$ ), energy of the neutrino beam, mass squared differences ( $\Delta m_{13}^2$ ,  $\Delta m_{12}^2$ ), 3 mixing angles, and matter density. We take the energy of the beam is 0.7 GeV and the values of mass squared differences:  $\Delta m_{13}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ ,  $\Delta m_{12}^2 = 8 \times 10^{-5} \text{ eV}^2$  and  $U_{e3} = 0.1$ . In Figure 5, we plot the probability as a function of distance. The probability for  $\delta = 330^\circ$  is about 1.8 times bigger compared to the probability for  $\delta = 135^\circ$  when the beam arrives at Kamioka from Tokai ( $L = 295 \text{ km}$ ). The difference is magnified much more if we have a detector installed at Korea ( $L = 1000 \text{ km}$ ). Also we notice that a peak in the distribution appears at T2K and at the Tokai-to-Korea experiment. The location of the peak however will change if we change the

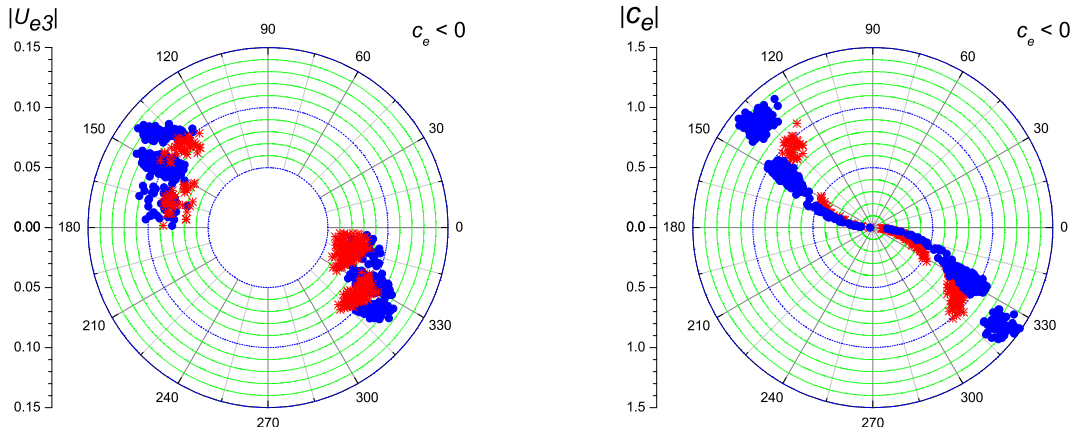


Figure 4:  $|U_{e3}|$  and  $c_e$  is shown as a function of  $\delta_{\text{MNSP}}$  for  $c_e < 0$ .

above parameters.

## 5 Leptogenesis

Let us consider the calculation of baryon asymmetry of the Universe. As we will see below, a lepton asymmetry is generated by CP violating decays of right-handed neutrinos out of equilibrium[23]. The sphaleron processes[24] convert the lepton asymmetry to a baryon asymmetry[25]. Our goal is to see if the recent experimental value of the baryon-to-photon ratio, i.e. [26]

$$\eta_B = (6.3 \pm 0.3) \times 10^{-10}, \quad (25)$$

can be reproduced in our model.

The baryon-to-photon ratio is obtained as

$$\eta_B = -a \frac{1}{f_*^0} \kappa \epsilon_1, \quad (26)$$

where  $a$  is a sphaleron conversion factor  $a \simeq 0.35$ , and  $1/f_*^0$  is coming from the fact that the photon number density increases as the degrees of freedom present at the epoch of leptogenesis annihilate and is given by  $f_*^0 = g^*/g^0$ , where  $g^*$  is the number of degrees of freedom at  $T = M_{N_1}$  (mass of the lightest right-handed neutrino) and  $g^0 = 3.91$ , giving  $1/f_*^0 \simeq 0.016$ . The  $\epsilon_1$  is the amount of CP violating lepton asymmetry via the lightest right-handed neutrino, which is

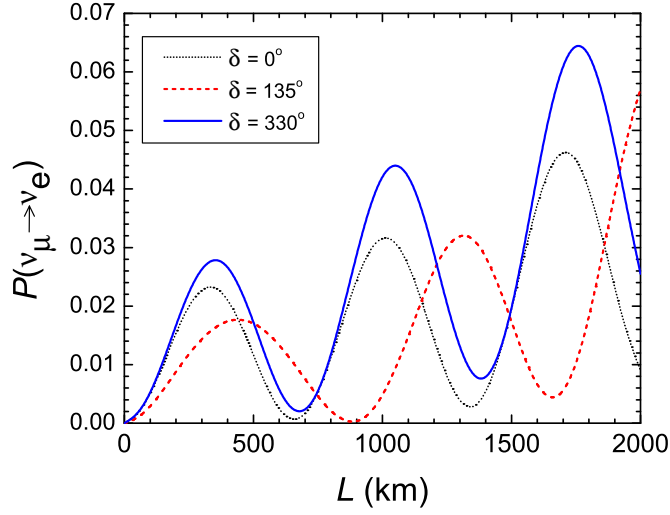


Figure 5: The probability  $P_{\nu_\mu \rightarrow \nu_e}$  as a function of  $L(\text{km})$ .

defined as

$$\epsilon_1 = \sum_i \frac{\Gamma(N_1 \rightarrow \ell_i H_u^*) - \Gamma(N_1 \rightarrow \bar{\ell}_i H_u)}{\Gamma(N_1 \rightarrow \ell_i H_u^*) + \Gamma(N_1 \rightarrow \bar{\ell}_i H_u)}. \quad (27)$$

The  $\kappa$  is an efficiency factor, which does not related on the CP violation of lepton asymmetry and parameterizes the effects of scattering and decay processes. In the thermal leptogenesis scenario, the  $\kappa$  is a calculable number by solving Boltzman equation. The  $\kappa$  is a function of an effective neutrino mass,  $\tilde{m}_1$ , which defined as

$$\tilde{m}_1 = \frac{[(\hat{M}_\nu^D)^\dagger \hat{M}_\nu^D]_{11}}{M_{N_1}}, \quad (28)$$

where  $\hat{M}_\nu^D$  is the Dirac neutrino mass matrix in the basis where right-handed Majorana mass is diagonal with real and positive eigenvalues,  $M_{N_i}$ . The Ref.[27] shows that the  $\kappa$  for  $10^{-2}$  eV  $< \tilde{m}_1 < 10^3$  eV is approximately given as

$$\kappa \simeq 0.3 \left( \frac{10^{-3} \text{eV}}{\tilde{m}_1} \right) \left( \ln \frac{\tilde{m}_1}{10^{-3} \text{eV}} \right)^{-0.6}, \quad (29)$$

and in Ref.[25], the  $\kappa$  is given by a power law as

$$\kappa = (2 \pm 1) \times 10^{-2} \left( \frac{0.01 \text{ eV}}{\tilde{m}_1} \right)^{1.1 \pm 0.1}. \quad (30)$$

Now let us see the prediction of our model. As we have seen, flavor structure has been already fixed in our model, and thus the  $\epsilon_1$  and  $\kappa$  can be calculated as a function of the

lightest right-handed neutrino mass and  $SU(2)_L$  triplet Higgs mass. For example, since the  $[(\hat{M}_\nu^D)^\dagger \hat{M}_\nu^D]_{11}$  is almost fixed in our model, the effective neutrino mass is a function of the lightest right-handed neutrino,  $M_{N_1}$ . As a result, the efficiency factor  $\kappa$  depends on only  $M_{N_1}$ . Consequently, the recent experimental data, Eq.(25), gives us a prediction to the mass scale.

Let us discuss the possible contributions to the CP violating lepton asymmetry in our model. The decay amplitude of the lightest right-handed neutrino in one-loop involves the right-handed neutrinos and  $SU(2)_L$  triplet Higgs[28] present in **126** +  $\overline{\mathbf{126}}$  and **54** (if there is one). The right-handed neutrino loop contribution is obtained as

$$\epsilon_1^N = -\frac{1}{8\pi} \sum_{i=2,3} \frac{\text{Im} [(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{1i}^2]}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}} F \left( \frac{M_{N_i}^2}{M_{N_1}^2} \right), \quad (31)$$

where  $\hat{Y}_\nu$  is the Dirac neutrino Yukawa coupling in the basis where right-handed Majorana mass is diagonal with real and positive eigenvalues,  $M_{N_i}$ . Since the expression of the  $\epsilon_1^N$  does not depend on the overall scale of right-handed Majorana neutrino mass, the value of  $\epsilon_1^N$  is almost determined in our model. The value is proportional to  $c_\nu$ , which is a coefficient of **120** Higgs contribution to Dirac neutrino Yukawa coupling, Eq.(6). Our prediction is [29]

$$|\epsilon_1^N| = (1 - 4) \times 10^{-4} (c_\nu/2). \quad (32)$$

Suppose that this  $\epsilon_1^N$  dominate the lepton asymmetry  $\epsilon_1$ , the value of  $\kappa$  is determined to be  $\kappa \simeq (0.25 - 1) \times 10^{-3} (2/c_\nu)$  to satisfy the current experimental data. The values of  $\kappa$  correspond to the effective neutrino mass,  $\tilde{m}_1 = (0.1 - 0.4) (c_\nu/2)$  eV, and we obtain the lightest right-handed Majorana mass is  $M_{N_1} = (0.4 - 1) \times 10^{13} (2/c_\nu)$  GeV. Using the relations,  $f = \sqrt{3}r_1/U_{14}\bar{f}$ ,  $M_R = 2\sqrt{2}fv_R$ , we obtain the corresponding value of VEV is  $v_R \simeq U_{14} (2/c_\nu) (1 - 2.5) \times 10^{15}$  GeV.

We also have the triplet Higgs loop contribution,  $\epsilon_1^\Delta$ , and the contribution may disturb the above prediction of lightest right-handed neutrino mass. In fact, as discussed in Ref.[28, 30], this triplet Higgs loop contribution may dominate  $\epsilon_1$  naively when we consider type II seesaw. However, the amount of lepton asymmetry depends on the flavor structure, and such naive estimation may not be correct. Actually, in our model, the  $\epsilon_1^\Delta$  can be calculated as a function of lightest right-handed Majorana mass, and the triplet mass. To see this, let us see how we can realize type II seesaw.

In order to satisfy the type II VEV magnitude for  $v_L \equiv \langle \overline{\Delta}_L \rangle$ , one **54** Higgs is needed[31]. There are then two pairs of  $SU(5)$  submultiplet **15**s in the theory (**15** Higgs of  $SU(5)$  is the



one that contains the triplet that leads to the type II seesaw). For the triplet VEV term to dominate, one of these two pairs must have a mass around a scale of  $10^{13}$  GeV or so. It was shown in Ref.[31] that a linear combination of the SU(5) **15** sub-multiplets in **54** and **126** can have a mass around  $10^{13}$  GeV without conflicting with the symmetry breaking and coupling unification and the other one becomes closer to the SO(10) breaking scale.

We can write down the superpotential involving the triplets and their interactions (for simplicity, we omit the terms from **210** Higgs):

$$W = \lambda_1 H_u H_u \Delta_E + \lambda_2 v_R \Delta_E \bar{\Delta}_L + \bar{\lambda}_2 v_R \bar{\Delta}_E \Delta_L + Y_\Delta L L \bar{\Delta}_L + M_\Delta \Delta_L \bar{\Delta}_L + M_E \Delta_E \bar{\Delta}_E, \quad (33)$$

where  $Y_\Delta = 2\sqrt{2}f$ , and  $\Delta_E$  is a triplet in **54** Higgs. The VEV is given as

$$v_L \simeq \lambda_1 s_\Delta c_\Delta \frac{v_u^2}{M_{\Delta_1}}, \quad (34)$$

where  $s_\Delta, c_\Delta$  are mixing (sin and cos) of SU(2)<sub>L</sub> triplets,

$$s_\Delta c_\Delta \simeq \frac{\lambda_2 v_R}{M_{\Delta_2}}. \quad (35)$$

We denote  $M_{\Delta_{1,2}}$  as eigenmasses of SU(2)<sub>L</sub> triplets and we assume  $M_{\Delta_1} \ll M_{\Delta_2}$  to make type II dominant and  $M_\Delta$  to be around GUT scale. Note that  $M_{\Delta_1}$  is a free parameter even if we fix the  $v_L$ . However, assuming that the Higgs couplings are less than  $O(1)$ , the  $M_{\Delta_1}$  should be less than around  $10^{13}$  GeV.

The lepton asymmetry via triplets in the loop is given in the Ref.[28, 30],

$$\begin{aligned} \epsilon_1^\Delta &= -\frac{3}{8\pi} \lambda_1 c_\Delta s_\Delta \frac{M_{N_1}}{M_\Delta} \frac{\text{Im}(Y_\nu^\dagger Y_\Delta Y_\nu^*)_{11}}{(Y_\nu^\dagger Y_\nu)_{11}} G(y) \\ &= -\frac{3}{8\pi} \frac{M_{N_1}}{v_u^2} \frac{\text{Im}(Y_\nu^\dagger m_\nu^{\text{II}} Y_\nu^*)_{11}}{(Y_\nu^\dagger Y_\nu)_{11}} G(y), \end{aligned} \quad (36)$$

where  $G(y) = y \ln \frac{y+1}{y}$  and  $y = M_{\Delta_1}^2 / M_{N_1}^2$ . Our prediction of the triplet loop contribution is

$$|\epsilon_1^\Delta| = (2.5 - 6) \times 10^{-4} (c_\nu/2) \left( \frac{M_{N_1}}{10^{13} \text{ GeV}} \right) \left( \frac{m_{\nu_3}^{\text{II}}}{0.05 \text{ eV}} \right) G(y). \quad (37)$$

We note that the function  $G(y)$  is always smaller than 1. If the triplet mass  $M_{\Delta_1}$  is 10% compared to the lightest right-handed Majorana mass,  $M_{N_1}$ , we obtain  $G(y) \simeq 0.046$  and the triplet loop contribution  $\epsilon_1^\Delta$  can be negligible compared to the right-handed neutrino loop contribution  $\epsilon_1^N$ . When  $M_{\Delta_1}$  and  $M_{N_1}$  are comparable, then the both triplet and right-handed neutrino loops contribute to the lepton asymmetry.

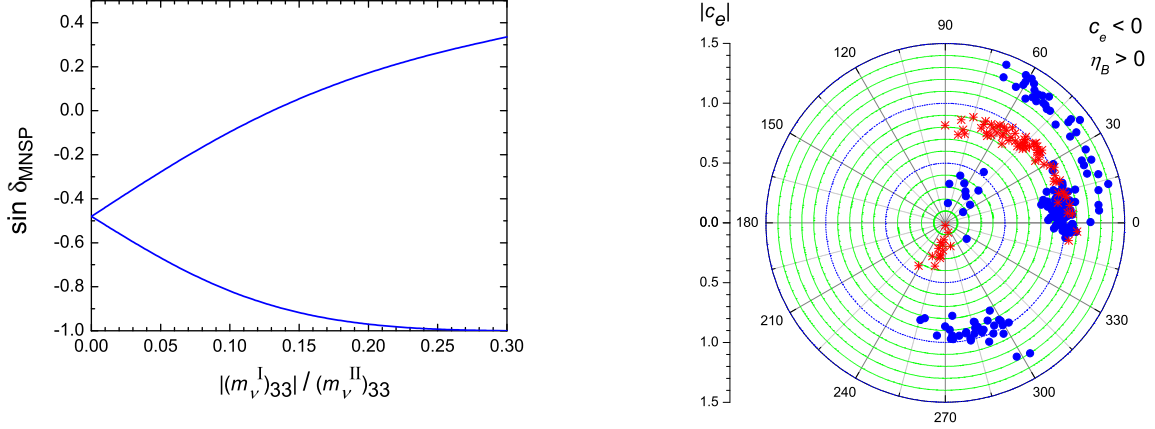


Figure 6:  $\sin \delta_{\text{MNSP}}$  is plotted as a function of the ratio of the type I contribution to the type II contribution ( $|(m_{\nu}^{\text{I}})_{33}|/(m_{\nu}^{\text{II}})_{33}$ ) in the heaviest neutrino mass [Left]. The  $|c_e|$  is plotted as a function of  $\delta_{\text{MNSP}}$  for  $\eta_B > 0$  and [Right] for  $v_R \simeq U_{14}(2/c_{\nu}) (1 - 2.5) \times 10^{15} \text{ GeV}$ .

It is possible to check the predictions, Eqs.(32,37), using the Yukawa couplings shown as an example in section 3. the Dirac neutrino coupling in the above basis with  $c_{\nu} = 2$  (which is a favorable value as we noted in the last paragraph in section 3) is given by:

$$\hat{Y}_{\nu} = \begin{pmatrix} 0.002 & 0.003 \exp(-1.54 i) & 0.0026 \exp(-0.344 i) \\ -0.0167 & 0.021 \exp(-1.53 i) & 0.025 \exp(3.37 i) \\ -0.229 & 0.417 \exp(4.70 i) & 0.422 \exp(-3.019 i) \end{pmatrix}. \quad (38)$$

The Majorana neutrino coupling matrix which gives the right-handed neutrino masses is proportional to the  $f$  coupling,

$$\hat{f} = \text{diag.}(0.00108, 0.00303, 0.0185). \quad (39)$$

If there is no huge cancellation between  $\epsilon_1^N$  and  $\epsilon_1^{\Delta}$ , the resulting lepton asymmetry  $\epsilon_1$  needs to be  $O(10^{-4})$  and consequently, the efficiency factor  $\kappa$  is  $O(10^{-3})$  and the lightest right-handed Majorana mass,  $M_{N_1}$ , is  $10^{13} \text{ GeV}$ . Note that we also have solutions where the Majorana mass scale is larger. In this case, since  $\kappa$  becomes larger, the lepton asymmetry  $\epsilon_1$  needs to be canceled between  $\epsilon_1^N$  and  $\epsilon_1^{\Delta}$  by choosing the triplet mass.

We note that the triplet Higgs decay also can produce lepton asymmetry by its own decay, but this contribution vanishes when the Hermitian structure of Yukawa coupling is assumed.

When  $M_{N_1} \sim 10^{13} \text{ GeV}$ , the type I contribution can disturb the prediction of oscillation parameters obtained in the previous section. We find that the  $|U_{e3}|$  prediction is not changed

much, but the prediction of the MNSP phase is modified due to the phase of type I contribution. As shown in figure 6, the  $\sin \delta_{\text{MNSP}}$  splits from the type II prediction when type I contribution increases (However, the correct neutrino fit in our model can not be obtained by pure type I contribution). In figure 6, we show the  $\sin \delta_{\text{MNSP}}$  as a function of the ratio of the type I contribution to the type II contribution ( $|(m_\nu^{\text{I}})_{33}|/(m_\nu^{\text{II}})_{33}$ ) to the heaviest neutrino mass. We use the  $\bar{h}$ ,  $\bar{f}$  and  $\bar{h}'$  values presented in section 3 (we use  $c_e \simeq -1$  and  $c_\nu = 2$ ) and vary  $v_R/U_{14}$  continuously. In figure 6, we also show the MNSP phase for the case of positive  $\eta_B$  dominated by the contribution given in Eq.(31). When  $v_R/U_{14} \gtrsim 2 \times 10^{16}$  GeV, type I contribution can be negligible and the MNSP phase lies around  $135^\circ - 150^\circ$ ,  $315^\circ - 330^\circ$  (when  $c_e \simeq -1$ ) as we have seen in the previous section. In this scenario of large  $v_R$ , the generated lepton asymmetry may be an order of magnitude bigger compared to the observation since the efficiency factor  $\kappa$  becomes larger. However, the left-handed triplet  $\Delta_L$  contribution in the loop can produce a 10% level cancellation and generate the right amount of lepton asymmetry. The location of the MNSP phase, which can be observed at T2K and the subsequent experiments as mentioned before, can be a probe to distinguish the scenario.

In our discussion so far, we did not concern ourselves with the nature of SUSY breaking mechanism. As is well known, gravitino production puts an upper bound of about  $10^9$  GeV on the reheating temperature  $T_R$  after inflation when the gravitino mass is in the range,  $100 \text{ GeV} \lesssim m_{3/2} \lesssim 1 \text{ TeV}$ [32]. Since in our model, the lightest right-handed neutrino which is supposed to be responsible for leptogenesis has a mass of  $10^{13}$  GeV or so, clearly the thermal leptogenesis picture outlined in this section will not work in this case. The possibilities to make the thermal leptogenesis scenario available are to consider a light gravitino such as  $m_{3/2} < 16 \text{ eV}$ [33], late time entropy production in a gauge mediation model[34], quasi thermal picture[35]. Otherwise, one can consider a non-thermal leptogenesis where the inflaton plays a key role[36]. We do not give details of the implications of such a choice for our SO(10) model except to note that it does not affect the fermion sector of the model.

## 6 Lepton Flavor Violation

We now discuss the lepton flavor violating processes e.g.  $\mu \rightarrow e\gamma$ ,  $\tau \rightarrow \mu\gamma$  etc. The operator for  $l_i \rightarrow l_j + \gamma$  is:

$$\mathcal{L}_{l_i \rightarrow l_j \gamma} = \frac{ie}{2m_l} \bar{l}_j \sigma^{\mu\nu} q_\nu (a_l P_L + a_r P_R) l_i \cdot A_\mu + h.c. \quad (40)$$

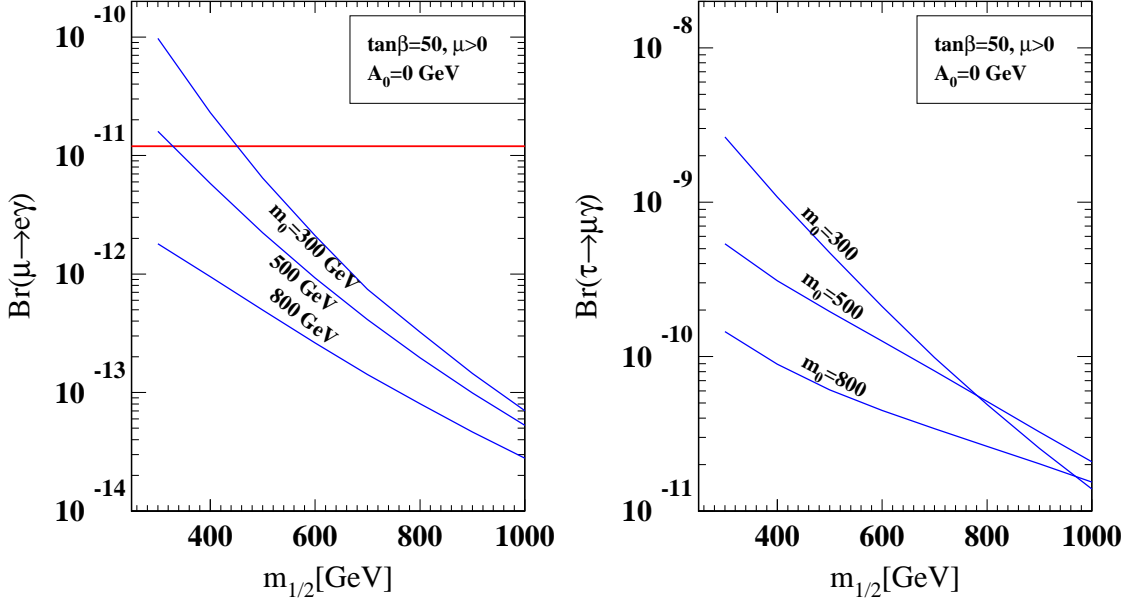


Figure 7:  $\text{BR}[\mu \rightarrow e + \gamma]$  and  $\text{BR}[\tau \rightarrow \mu + \gamma]$  are plotted as a function of  $m_{1/2}$  for different values of  $m_0$ .

where  $P_{L,R} \equiv (1 \mp \gamma_5)/2$  and  $\sigma^{\mu\nu} \equiv \frac{i}{2} [\gamma^\mu, \gamma^\nu]$ . The decay width for  $l_i \rightarrow l_j + \gamma$  can be written as:

$$\Gamma(l_i \rightarrow l_j + \gamma) = \frac{m_\mu e^2}{64\pi} (|a_l|^2 + |a_r|^2). \quad (41)$$

Then the branching ratio is obtained by multiplying this decay width with the lifetime of the  $l_i$  lepton. The supersymmetric contributions include the neutralino and chargino diagrams.

We work in the basis where the charged lepton masses are diagonal at the highest scale of the theory. We use the mSUGRA boundary conditions, i.e.  $m_0$ : universal scalar mass,  $m_{1/2}$ : universal gaugino mass,  $A_0$ : universal trilinear mass term,  $\tan\beta$ , sign of  $\mu$  to calculate the BRs. Since we have  $\lambda_b \simeq \lambda_\tau$ , the masses of the Higgs are not unified with the universal scalar mass for the sfermions for realistic parameter space after including the finite SUSY one loop correction to the b-quark mass. We plot  $\text{BR}[\mu \rightarrow e + \gamma]$  and  $\text{BR}[\tau \rightarrow \mu + \gamma]$  as a function of  $m_{1/2}$  for different values of  $m_0$  in Figure 7, and we find that the  $\text{BR}[\mu \rightarrow e + \gamma]$  can be quite large.

## 7 Conclusion

The suppression of proton decay is a major challenge for the grand unified models. We have constructed a minimal SO(10) model which can suppress the proton decay naturally. The model has **10**,  $\overline{\mathbf{126}}$  and **120** Higgs multiplets to generate the fermion masses. The CP symmetry of the model keeps the fermion masses hermitian at the grand unified scale and the SUSY CP problem is also under control. The neutrino masses are generated by type II seesaw. The model has 13 parameters to fit quark and lepton masses and mixings, and thus it gives predictions for down-type quark masses,  $|U_{e3}|$  and  $\delta_{\text{MNSP}}$ . The prediction for  $|U_{e3}|$  is  $0.06 - 0.11$  with the upper limit being imposed by  $V_{ub}$  and the lower limit by the mass squared ratio ( $\Delta m_{\text{sol}}^2/\Delta m_A^2$ ) and the solar mixing angle. The strange quark mass can be small (lattice calculated size) or large (QCD calculated size). The phase  $\delta_{\text{MNSP}}$  is in the 2nd or 4th quadrant when the SU(5)-like condition is satisfied in the 120 coupling. This feature has distinctive impact on the probability of  $\nu_\mu$  to  $\nu_e$  oscillation ( $P_{\nu_\mu \rightarrow \nu_e}$ ) which will be measured at the T2K and subsequent experiments. The lepton asymmetry generated in the decay of the right-handed neutrinos in this model produces the observed amount of baryon asymmetry. Depending on the magnitude of the symmetry breaking scale  $v_R$ , the observed baryon asymmetry predict the  $\delta_{\text{MNSP}}$  phase in different quadrants. The lepton flavor violating  $\text{Br}[\mu \rightarrow e + \gamma]$  can be large in this model.

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