ACT-03-06, MIFP-06-09, hep-th/0604107

# Flipped and Unflipped $SU(5)$ as Type IIA Flux Vacua

Ching-Ming Chen,[1] Tianjun Li,[2,3] and Dimitri V. Nanopoulos[1,4,5]

[1]*George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA*

[2]*Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08854, USA*

[3]*Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, P. R. China*

[4]*Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA*

[5]*Academy of Athens, Division of Natural Sciences, 28 Panepistimiou Avenue, Athens 10679, Greece*

(Dated: April 12, 2018)



## Abstract

On Type IIA orientifolds with flux compactifications in supersymmetric AdS vacua, we for the first time construct $SU(5)$ models with three anti-symmetric **10** representations and without symmetric **15** representations. We show that all the pairs of the anti-fundamental $\bar{\mathbf{5}}$ and fundamental **5** representations can obtain GUT/string-scale vector-like masses after the additional gauge symmetry breaking via supersymmetry preserving Higgs mechanism. Then we have exact three $\bar{\mathbf{5}}$, and no other chiral exotic particles that are charged under $SU(5)$ due to the non-abelian anomaly free condition. Moreover, we can break the $SU(5)$ gauge symmetry down to the SM gauge symmetry via D6-brane splitting, and solve the doublet-triplet splitting problem. Assuming that the extra one (or several) pair(s) of Higgs doublets and adjoint particles obtain GUT/string-scale masses via high-dimensional operators, we only have the MSSM in the observable sector below the GUT scale. Then the observed low energy gauge couplings can be generated via RGE running if we choose the suitable grand unified gauge coupling by adjusting the string scale. Furthermore, we construct the first flipped $SU(5)$ model with exact three **10**, and the first flipped $SU(5)$ model in which all the Yukawa couplings are allowed by the global $U(1)$ symmetries.

PACS numbers: 11.25.Mj, 11.25.Wx

## I. INTRODUCTION

The major challenge and lasting problem in string phenomenology is to construct realistic Standard-like string models with moduli stabilization. In the beginning, string model building was mainly concentrated on the weakly coupled heterotic string theory, and rather successful models like flipped $SU(5)$ [1, 2] in its stringy form were constructed [3]. Meanwhile, the first standard compactification of strong coupled heterotic string theory or M-theory on $S^1/Z_2$ was given in Ref. [4]. Due to the advent of D-branes in the second string revolution [5], we can construct consistent four-dimensional chiral models with non-abelian gauge symmetry on Type II orientifolds.

Type II orientifolds with intersecting D-branes have been extremely valuable in string model building during the last few years. The chiral fermions can arise from the intersections of D-branes in the internal space [6] with T-dual description in terms of magnetized D-branes [7]. In addition, a lot of non-supersymmetric three-family Standard-like models and grand unified models without Ramond-Ramond (RR) tadpole on Type IIA orientifolds with intersecting D6-branes were constructed [8, 9, 10]. However, there generically exist the uncancelled Neveu-Schwarz-Neveu-Schwarz (NSNS) tadpoles and the gauge hierarchy problem. To solve these two problems, the first quasi-realistic supersymmetric models have been constructed in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold with intersecting D6-branes [11]. Subsequently, supersymmetric Standard-like models, Pati-Salam models, $SU(5)$ models as well as flipped $SU(5)$ models have been constructed systematically [12, 13, 14, 15, 16, 17, 18], and their phenomenological consequences have been studied [19, 20]. Also, the supersymmetric constructions on other orientifolds were discussed as well [21]. There are two main constraints on supersymmetric model building: RR tadpole cancellation conditions and four-dimensional $N = 1$ supersymmetry conditions.

However, the moduli stabilization in open string and closed string sectors is still an open problem, although some of the complex structure parameters (in the Type IIA picture) and the dilaton field may be stabilized due to the gaugino condensations in the hidden sector in some models [20]. Another way to stabilize the compactification moduli fields is turning on the supergravity fluxes [22]. The point is that a supergravity potential can be generated, and the continuous moduli space of the string vacua in the four-dimensional effective theory can be lifted. On Type IIB orientifolds, the supergravity fluxes contribute large positive

D3-brane charges due to the Dirac quantization conditions, and then modify the global RR tadpole cancellation conditions significantly and imposes strong constraints on consistent model building [23, 24]. Thus, one can construct three-family and four-family Standard-like models [17, 25, 26, 27, 28, 29] if and only if one introduces magnetized D9-branes with large negative D3-brane charges in the hidden and observable sectors. By the way, it has been recently shown that if non-geometric fluxes and new Type IIB S-duality fluxes are introduced, they can contribute negative D-brane charges to the RR tadpole cancellation conditions in supersymmetric Minkowski vacua on Type IIB orientifolds [30, 31]. But, we will not consider it in this paper.

The techniques for consistent chiral flux compactifications on Type IIA orientifolds with intersecting D6-branes were developed recently [32, 33, 34, 35]. Interestingly enough, in supersymmetric AdS vacua, the metric, NSNS and RR fluxes can contribute negative D6-brane charges to all the RR tadpole cancellation conditions, *i. e.*, the RR tadpole cancellation conditions give no constraints on consistent model building [36]. Thus, the supersymmetric flux models on Type IIA orientifolds are mainly constrained by four-dimensional $N = 1$ supersymmetry conditions, and then we can construct rather realistic intersecting D6-brane models [36].

In this paper, we construct Grand Unified Theories (GUTs) such as $SU(5)$ and flipped $SU(5)$ models on Type IIA orientifolds with flux compactifications in supersymmetric AdS vacua. Although the up-type quark Yukawa couplings and down-type quark Yukawa couplings are forbidden respectively in the $SU(5)$ models and flipped $SU(5)$ models by the $U(1)$ symmetries, these models do have some interesting features, for example, the gauge coupling unification which generically can not be realized in the other models for D-brane constructions. However, in the previous $SU(5)$ model building in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z_2} \times \mathbb{Z_2})$ orientifold without fluxes, we can easily show that we cannot construct models with three anti-symmetric **10** representations and without symmetric **15** representations [13, 18]. In addition, for models with three anti-symmetric representations and some symmetric representations, the net number of anti-fundamental $\bar{\mathbf{5}}$ and fundamental **5** representations can not be three due to the non-abelian anomaly free conditions, *i. e.*, one does not have exact three families of the SM fermions (in our convention, we define the net number of vector-like particles $X$ and $\bar{X}$ as the number of $X$ minus the number of $\bar{X}$ where $X$ can be $\bar{\mathbf{5}}$ or **10**.). Moreover, in the previous flipped $SU(5)$ models [16, 17, 29], both the net number of **10** and

$\overline{\mathbf{10}}$ and the net number of $\overline{\mathbf{5}}$ and $\mathbf{5}$ are not three, and at least some Yukawa couplings are forbidden by the global $U(1)$ symmetries.

On Type IIA orientifolds with flux compactifications in supersymmetric AdS vacua, we construct the $SU(5)$ models with three anti-symmetric $\mathbf{10}$ representations and without symmetric $\mathbf{15}$ representations. Although the net number of the $\bar{\mathbf{5}}$ and $\mathbf{5}$ is three due to the non-Abelian anomaly free condition, the initial $\bar{\mathbf{5}}$ number $n_{\bar{\mathbf{5}}}$ is not three and the initial $\mathbf{5}$ number is $n_{\bar{\mathbf{5}}}-3$. We show that all the $\bar{\mathbf{5}}$ and $\mathbf{5}$ pairs can obtain GUT/string-scale vector-like masses after the extra gauge symmetry breaking via supersymmetry preserving Higgs mechanism. So, unlike the previous D-brane models, there are exact three $\bar{\mathbf{5}}$, and no chiral exotic particles that are charged under $SU(5)$. In addition, the $SU(5)$ gauge symmetry can be broken down to the Standard Model (SM) gauge symmetry via D6-brane splitting, and the doublet-triplet splitting problem can be solved. If the extra one (or several) pair(s) of Higgs doublets and adjoint particles can get GUT/string-scale masses via high-dimensional operators, we obtain the Minimal Supersymmetric Standard Model (MSSM) in the observable sector after we decouple the heavy particles around the GUT/string scale. Thus, choosing the suitable grand unified gauge coupling by adjusting the string scale, we can explain the observed low energy gauge couplings via renormalization group equation (RGE) running. However, how to generate the up-type quark Yukawa couplings, which are forbidden by the global $U(1)$ symmetry, deserves further study.

Furthermore, we consider the flipped $SU(5)$ models. In order to have at least one pair of Higgs fields $\mathbf{10}$ and $\overline{\mathbf{10}}$, we must have the symmetric representations, and then the net number of $\overline{\mathbf{5}}$ and $\mathbf{5}$ can not be three if the net number of $\mathbf{10}$ and $\overline{\mathbf{10}}$ is three due to the non-abelian anomaly free condition. For the first time, we construct the flipped $SU(5)$ model with exact three $\mathbf{10}$, and the flipped $SU(5)$ model in which all the Yukawa couplings are allowed by the global $U(1)$ symmetries. We will also comment on two more flipped $SU(5)$ models, and try to avoid as much extra matter as possible.

This paper is organized as follows. In Section II, we briefly review the intersecting D6-brane model building on Type IIA orientifolds with flux compactifications. We study the general conditions for three-family $SU(5)$ model building in Section III. And we discuss the $SU(5)$ and flipped $SU(5)$ models in Sections IV and V, respectively. Discussion and conclusions are given in Section VI.

## II. FLUX MODEL BUILDING ON TYPE IIA ORIENTIFOLDS

We briefly review the rules for the intersecting D6-brane model building in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with flux compactifications [33, 34]. Because the model building rules in Type IIA theory on $\mathbf{T^6}$ orientifold with flux compactifications are quite similar, we only explain the differences for simplicity.

### A. Type IIA Theory on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifold

We consider $\mathbf{T^6}$ to be a six-torus factorized as $\mathbf{T^6} = \mathbf{T^2} \times \mathbf{T^2} \times \mathbf{T^2}$ whose complex coordinates are $z_i$, $i = 1,\ 2,\ 3$ for the $i$-th two-torus, respectively. The $\theta$ and $\omega$ generators for the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ act on the complex coordinates as following

$$\begin{aligned} \theta :\ & (z_1, z_2, z_3) \to (-z_1, -z_2, z_3) \ , \\ \omega :\ & (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \ . \end{aligned} \tag{1}$$

We implement an orientifold projection $\Omega R$, where $\Omega$ is the world-sheet parity, and $R$ acts on the complex coordinates as

$$R : (z_1, z_2, z_3) \to (\overline{z}_1, \overline{z}_2, \overline{z}_3) \ . \tag{2}$$

Thus, we have four kinds of orientifold 6-planes (O6-planes) under the actions of $\Omega R$, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$, respectively. In addition, we introduce some stacks of D6-branes which wrap on the factorized three-cycles. There are two kinds of complex structures consistent with orientifold projection for a two-torus – rectangular and tilted [11, 37]. If we denote the homology classes of the three cycles wrapped by $a$ stack of $N_a$ D6-branes as $n_a^i[a_i] + m_a^i[b_i]$ and $n_a^i[a_i'] + m_a^i[b_i]$ with $[a_i'] = [a_i] + \frac{1}{2}[b_i]$ for the rectangular and tilted two-tori respectively, we can label a generic one cycle by $(n_a^i, l_a^i)$ in which $l_a^i \equiv m_a^i$ for a rectangular two-torus while $l_a^i \equiv 2\tilde{m}_a^i = 2m_a^i + n_a^i$ for a tilted two-torus [13]. For $a$ stack of $N_a$ D6-branes along the cycle $(n_a^i, l_a^i)$, we also need to include their $\Omega R$ images $N_{a'}$ with wrapping numbers $(n_a^i, -l_a^i)$. For the D6-branes on the top of O6-planes, we count them and their $\Omega R$ images independently. So, the homology three-cycles for $a$ stack of $N_a$ D6-branes and its orientifold image $a'$ are

$$[\Pi_a] = \prod_{i=1}^{3} \left(n_a^i[a_i] + 2^{-\beta_i} l_a^i [b_i]\right), \quad [\Pi_{a'}] = \prod_{i=1}^{3} \left(n_a^i[a_i] - 2^{-\beta_i} l_a^i [b_i]\right) \ , \tag{3}$$

where $\beta_i = 0$ if the $i$-th two-torus is rectangular and $\beta_i = 1$ if it is tilted. The homology three-cycles wrapped by the four O6-planes are

$$\Omega R : [\Pi_{\Omega R}] = 2^3 [a_1] \times [a_2] \times [a_3] \ , \tag{4}$$

$$\Omega R\omega : [\Pi_{\Omega R\omega}] = -2^{3-\beta_2-\beta_3} [a_1] \times [b_2] \times [b_3] \ , \tag{5}$$

$$\Omega R\theta\omega : [\Pi_{\Omega R\theta\omega}] = -2^{3-\beta_1-\beta_3} [b_1] \times [a_2] \times [b_3] \ , \tag{6}$$

$$\Omega R\theta : [\Pi_{\Omega R}] = -2^{3-\beta_1-\beta_2} [b_1] \times [b_2] \times [a_3] \ . \tag{7}$$

Therefore, the intersection numbers are

$$I_{ab} = [\Pi_a][\Pi_b] = 2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i - n_b^i l_a^i) \ , \tag{8}$$

$$I_{ab'} = [\Pi_a]\,[\Pi_{b'}] = -2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i + n_b^i l_a^i) \ , \tag{9}$$

$$I_{aa'} = [\Pi_a]\,[\Pi_{a'}] = -2^{3-k} \prod_{i=1}^{3} (n_a^i l_a^i) \ , \tag{10}$$

$$I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k} (-l_a^1 l_a^2 l_a^3 + l_a^1 n_a^2 n_a^3 + n_a^1 l_a^2 n_a^3 + n_a^1 n_a^2 l_a^3) \ , \tag{11}$$

where $[\Pi_{O6}] = [\Pi_{\Omega R}] + [\Pi_{\Omega R\omega}] + [\Pi_{\Omega R\theta\omega}] + [\Pi_{\Omega R\theta}]$ is the sum of O6-plane homology three-cycles wrapped by the four O6-planes, and $k = \beta_1 + \beta_2 + \beta_3$ is the total number of tilted two-tori.

For $a$ stack of $N_a$ D6-branes and its $\Omega R$ image, we have $U(N_a/2)$ gauge symmetry, while for $a$ stack of $N_a$ D6-branes and its $\Omega R$ image on the top of O6-plane, we obtain $USp(N_a)$ gauge symmetry. The general spectrum of D6-branes' intersecting at generic angles, which is valid for both rectangular and tilted two-tori, is given in Table I. The four-dimensional $N = 1$ supersymmetric models on Type IIA orientifolds with intersecting D6-branes are mainly constrained in two aspects: four-dimensional $N = 1$ supersymmetry conditions, and RR tadpole cancellation conditions.

| Sector | Representation |
|---|---|
| $aa$ | $U(N_a/2)$ vector multiplet and 3 adjoint chiral multiplets |
| $ab+ba$ | $I_{ab}$ $(\frac{N_a}{2}, \frac{\overline{N_b}}{2})$ chiral multiplets |
| $ab'+b'a$ | $I_{ab'}$ $(\frac{N_a}{2}, \frac{N_b}{2})$ chiral multiplets |
| $aa'+a'a$ | $\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{aO6})$ anti-symmetric chiral multiplets |
| | $\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO6})$ symmetric chiral multiplets |

TABLE I: The general spectrum for the intersecting D6-brane model building in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold with flux compactifications.

To simplify the notation, we define the following products of wrapping numbers

$$\begin{array}{rl} A_a \equiv -n_a^1 n_a^2 n_a^3, \quad B_a \equiv n_a^1 l_a^2 l_a^3, \quad C_a \equiv l_a^1 n_a^2 l_a^3, \quad D_a \equiv l_a^1 l_a^2 n_a^3, \\ \tilde{A}_a \equiv -l_a^1 l_a^2 l_a^3, \ \tilde{B}_a \equiv l_a^1 n_a^2 n_a^3, \ \tilde{C}_a \equiv n_a^1 l_a^2 n_a^3, \ \tilde{D}_a \equiv n_a^1 n_a^2 l_a^3. \end{array} \tag{12}$$

(1) *Four-Dimensional $N=1$ Supersymmetry Conditions*

The four-dimensional $N=1$ supersymmetry can be preserved by the orientation projection ($\Omega R$) if and only if the rotation angle of any D6-brane with respect to any O6-plane is an element of $SU(3)$ [6], *i. e.*, $\theta_1 + \theta_2 + \theta_3 = 0$ mod $2\pi$, where $\theta_i$ is the angle between the $D6$-brane and the O6-plane on the $i$-th two-torus. Then the supersymmetry conditions can be rewritten as [13]

$$x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a \ = \ 0 \ , \tag{13}$$

$$A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D \ < \ 0 \ , \tag{14}$$

where $x_A = \lambda,\ x_B = \lambda 2^{\beta_2+\beta3}/\chi_2\chi_3,\ x_C = \lambda 2^{\beta_1+\beta3}/\chi_1\chi_3$, and $x_D = \lambda 2^{\beta_1+\beta2}/\chi_1\chi_2$ in which $\chi_i = R_i^2/R_i^1$ are the complex structure parameters and $\lambda$ is a positive real number.

(2) *RR Tadpole Cancellation Conditions*

The total RR charges from the D6-branes and O6-planes and from the metric, NSNS, and RR fluxes must vanish since the RR field flux lines are conserved. With the filler branes on the top of the four O6-planes, we obtain the RR tadpole cancellation conditions [33, 34]:

$$2^k N^{(1)} - \sum_a N_a A_a + \frac{1}{2}(mh_0 + q_1 a_1 + q_2 a_2 + q_3 a_3) \; = \; 16 \; , \tag{15}$$

$$- 2^{\beta_1} N^{(2)} + \sum_a 2^{-\beta_2-\beta 3} N_a B_a + \frac{1}{2}(mh_1 - q_1 b_{11} - q_2 b_{21} - q_3 b_{31}) \; = \; -2^{4-\beta_2-\beta_3} \; , \tag{16}$$

$$- 2^{\beta_2} N^{(3)} + \sum_a 2^{-\beta_1-\beta_3} N_a C_a + \frac{1}{2}(mh_2 - q_1 b_{12} - q_2 b_{22} - q_3 b_{32}) \; = \; -2^{4-\beta_1-\beta_3} \; , \tag{17}$$

$$- 2^{\beta_3} N^{(4)} + \sum_a 2^{-\beta_1-\beta_2} N_a D_a + \frac{1}{2}(mh_3 - q_1 b_{13} - q_2 b_{23} - q_3 b_{33}) \; = \; -2^{4-\beta_1-\beta_2} \; , \tag{18}$$

where $2N^{(i)}$ are the number of filler branes wrapping along the $i$-th O6-plane which is defined in Table II. In addition, $a_i$ and $b_{ij}$ arise from the metric fluxes, $h_0$ and $h_i$ arise from the NSNS fluxes, and $m$ and $q_i$ arise from the RR fluxes. We consider these fluxes ($a_i$, $b_{ij}$, $h_0$, $h_i$, $m$ and $q_i$) quantized in units of 8 so that we can avoid the problems with flux Dirac quantization conditions.

TABLE II: Wrapping numbers of the four O6-planes.

| Orientifold Action | O6-Plane | $(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$ |
|---|---|---|
| $\Omega R$ | 1 | $(2^{\beta_1}, 0) \times (2^{\beta_2}, 0) \times (2^{\beta_3}, 0)$ |
| $\Omega R\omega$ | 2 | $(2^{\beta_1}, 0) \times (0, -2^{\beta_2}) \times (0, 2^{\beta_3})$ |
| $\Omega R\theta\omega$ | 3 | $(0, -2^{\beta_1}) \times (2^{\beta_2}, 0) \times (0, 2^{\beta_3})$ |
| $\Omega R\theta$ | 4 | $(0, -2^{\beta_1}) \times (0, 2^{\beta_2}) \times (2^{\beta_3}, 0)$ |

In this paper, we concentrate on the supersymmetric AdS vacua with metric, NSNS and RR fluxes [34]. For simplicity, we assume that the Kähler moduli $T_i$ satisfy $T_1 = T_2 = T_3$, then we obtain $q_1 = q_2 = q_3 \equiv q$ from the superpotential in [34]. To satisfy the Jacobi identities for metric fluxes, we consider the solution $a_i = a$, $b_{ii} = -b_i$, and $b_{ji} = b_i$ in which $j \neq i$ [34].

To have supersymmetric minima [34], we obtain that

$$3a\mathrm{Re}S \;=\; b_i \mathrm{Re}U_i \;, \quad \text{for} \;\; i = 1,\; 2,\; 3 \;, \tag{19}$$

where

$$\mathrm{Re}S \;\equiv\; \frac{e^{-\phi}}{\sqrt{\chi_1\chi_2\chi_3}} \;, \;\mathrm{Re}U_i \;\equiv\; e^{-\phi}\sqrt{\frac{\chi_j\chi_k}{\chi_i}} \;, \tag{20}$$

where $S$ and $U_i$ are respectively dilaton and complex structure moduli, $\phi$ is the four-dimensional T-duality invariant dilaton, and $i \neq j \neq k \neq i$. And then we have

$$b_1 = \frac{3a}{\chi_2\chi_3} \;, \; b_2 = \frac{3a}{\chi_1\chi_3} \;, \; b_3 = \frac{3a}{\chi_1\chi_2} \;. \tag{21}$$

Moreover, there are consistency conditions

$$3h_i a + h_0 b_i = 0 \;, \quad \text{for} \;\; i = 1,\; 2,\; 3 \;. \tag{22}$$

So we have

$$h_1 = -\frac{h_0}{\chi_2\chi_3} \;, \; h_2 = -\frac{h_0}{\chi_1\chi_3} \;, \; h_3 = -\frac{h_0}{\chi_1\chi_2} \;. \tag{23}$$

Thus, the RR tadpole cancellation conditions can be rewritten as following

$$2^k N^{(1)} - \sum_a N_a A_a + \frac{1}{2}(h_0 m + 3aq) \;=\; 16 \;, \tag{24}$$

$$-\,2^{\beta_1} N^{(2)} + \sum_a 2^{-\beta_2-\beta_3} N_a B_a - \frac{1}{2\chi_2\chi_3}(h_0 m + 3aq) \;=\; -2^{4-\beta_2-\beta_3} \;, \tag{25}$$

$$-\,2^{\beta_2} N^{(3)} + \sum_a 2^{-\beta_1-\beta_3} N_a C_a - \frac{1}{2\chi_1\chi_3}(h_0 m + 3aq) \;=\; -2^{4-\beta_1-\beta_3} \;, \tag{26}$$

$$-\,2^{\beta_3} N^{(4)} + \sum_a 2^{-\beta_1-\beta_2} N_a D_a - \frac{1}{2\chi_1\chi_2}(h_0 m + 3aq) \;=\; -2^{4-\beta_1-\beta_2} \;. \tag{27}$$

Therefore, if $(h_0 m + 3aq) < 0$, the supergravity fluxes contribute negative D6-brane charges to all the RR tadpole cancellation conditions, and then, the RR tadpole cancellation conditions give no constraints on the consistent model building because we can always introduce suitable supergravity fluxes and some stacks of D6-branes in the hidden sector to cancel the RR tadpoles. Also, if $(h_0 m + 3aq) = 0$, the supergravity fluxes do not contribute to any D6-brane charges, and then do not affect the RR tadpole cancellation conditions.

In addition, the Freed-Witten anomaly cancellation condition is [34]

$$- 2^{-k} h_0 \tilde{A}_a + 2^{-\beta_1} h_1 \tilde{B}_a + 2^{-\beta_2} h_2 \tilde{C}_a + 2^{-\beta_3} h_3 \tilde{D}_a = 0 \ . \tag{28}$$

It can be shown that if Eqs. (13), (19), and (22) are satisfied, the Freed-Witten anomaly is automatically cancelled. So, we will not consider the Freed-Witten anomaly in our model building.

Furthermore, in addition to the above RR tadpole cancellation conditions, the discrete D-brane RR charges classified by $\mathbb{Z}_2$ K-theory groups in the presence of orientifolds, which are subtle and invisible by the ordinary homology [25, 38], should also be taken into account [23]. The K-theory conditions for a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold are

$$\sum_a 2^{-k} \tilde{A}_a = \sum_a 2^{-\beta_1} \tilde{B}_a = \sum_a 2^{-\beta_2} \tilde{C}_a = \sum_a 2^{-\beta_3} \tilde{D}_a = 0 \bmod 4 \ . \tag{29}$$

### B. Type IIA Theory on $\mathbf{T^6}$ Orientifold

The intersecting D6-brane model building in Type IIA theory on $\mathbf{T^6}$ orientifold with flux compactifications is similar to that on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. For the model building rules in the previous subsection, we only need to make the following changes:

(1) For $a$ stack of $N_a$ D6-branes and its $\Omega R$ image, we have $U(N_a)$ gauge symmetry, while for $a$ stack of $N_a$ D6-branes and its $\Omega R$ image on the top of O6-plane, we obtain $USp(2N_a)$ gauge symmetry. Also, we present the general spectrum of D6-branes' intersecting at generic angles in Type IIA theory on $\mathbf{T^6}$ orientifold in Table III.

(2) We only have the $\Omega R$ O6-planes, so, $[\Pi_{O6}] = [\Pi_{\Omega R}]$ in Eq. (11), and the right-hand sides of Eqs. (16), (17) and (18) are zero.

(3) The metric, NSNS and RR fluxes ($a_i$, $b_{ij}$, $h_0$, $h_i$, $m$ and $q_i$) are quantized in units of 2.

(4) To have three families of the SM fermions, we obtain that at least one of the three two-tori is tilted. Thus, the right-hand side of Eq. (29) is 0 mod 2 for the K-theory conditions in our model building [39].

| Sector | Representation |
|---|---|
| $aa$ | $U(N_a)$ vector multiplet and 3 adjoint chiral multiplets |
| $ab+ba$ | $I_{ab}$ $(N_a, \overline{N_b})$ chiral multiplets |
| $ab'+b'a$ | $I_{ab'}$ $(N_a, N_b)$ chiral multiplets |
| $aa'+a'a$ | $\frac{1}{2}(I_{aa'}+I_{aO6})$ anti-symmetric chiral multiplets |
| | $\frac{1}{2}(I_{aa'}-I_{aO6})$ symmetric chiral multiplets |

TABLE III: The general spectrum for the intersecting D6-brane model building in Type IIA theory on $\mathbf{T^6}$ orientifold with flux compactifications, in particular, $I_{aO6} = [\Pi_a][\Pi_{O6}] = -2^{3-k} l_a^1 l_a^2 l_a^3$.

## III. GENERAL CONDITIONS FOR THREE-FAMILY $SU(5)$ MODEL BUILDING

We would like to construct three-family $SU(5)$ models. First, we consider the Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2}\times\mathbb{Z}_\mathbf{2})$ orientifold. Let us denote $SU(5)$ stack of D6-branes as $a$ stack. The numbers of the anti-symmetric representations $n_{\mathbf{10}}$ and symmetric representations $n_{\mathbf{15}}$ are

$$n_{\mathbf{10}} \;=\; 2^{1-k}\left[(1-2A_a)\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a\right]~, \tag{30}$$

$$n_{\mathbf{15}} \;=\; -2^{1-k}\left[(1+2A_a)\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a\right]~. \tag{31}$$

Because we require $n_{\mathbf{15}}=0$, we have

$$\tilde{B}_a + \tilde{C}_a + \tilde{D}_a \;=\; -(1+2A_a)\tilde{A}_a~. \tag{32}$$

Then we obtain

$$n_{\mathbf{10}} \;=\; -2^{3-k}A_a\tilde{A}_a~. \tag{33}$$

Therefore, $k=3$, *i. e.*, all three two-tori must be tilted. There are four possibilities for $A_a$ and $\tilde{A}_a$:

(1) $A_a = 1$ and $\tilde{A}_a = -3$

In this case, we have

$$\tilde{B}_a + \tilde{C}_a + \tilde{D}_a \;=\; 9~. \tag{34}$$

It is easy to show that there is no solution.

(2) $A_a = -1$ and $\tilde{A}_a = 3$

In this case, we have

$$\tilde{B}_a + \tilde{C}_a + \tilde{D}_a \ = \ 3 \ . \tag{35}$$

Up to T-duality and permutations of three different two-tori, there is one and only one possibility for the wrapping numbers for $SU(5)$ stacks of D6-branes

$$(1,3) \times (1,1) \times (1,-1) \ . \tag{36}$$

(3) $A_a = 3$ and $\tilde{A}_a = -1$

In this case, we have

$$\tilde{B}_a + \tilde{C}_a + \tilde{D}_a \ = \ 7 \ . \tag{37}$$

Up to T-duality and permutations of three different two-tori, there is one and only one possibility for the wrapping numbers for $SU(5)$ stacks of D6-branes

$$(-3,1) \times (-1,1) \times (-1,1) \ . \tag{38}$$

(4) $A_a = -3$ and $\tilde{A}_a = 1$

In this case, we have

$$\tilde{B}_a + \tilde{C}_a + \tilde{D}_a \ = \ 5 \ . \tag{39}$$

Up to T-duality and permutations of three different two-tori, there is one and only one possibility for the wrapping numbers for $SU(5)$ stacks of D6-branes

$$(3,-1) \times (1,1) \times (1,1) \ . \tag{40}$$

Second, let us consider Type IIA $\mathbf{T^6}$ orientifold. The numbers of the anti-symmetric and symmetric representations are

$$n_{\mathbf{10}} \ = \ 2^{2-k}(1-A_a)\tilde{A}_a \ , \ n_{\mathbf{15}} \ = \ -\, 2^{2-k}(1+A_a)\tilde{A}_a \ . \tag{41}$$

Because we require $n_{\mathbf{15}} = 0$, we have

$$A_a = -1 \ . \tag{42}$$

Then we obtain

$$n_{\mathbf{10}} \;=\; 2^{3-k}\tilde{A}_a \;. \tag{43}$$

Therefore, we have $k = 3$ and $\tilde{A}_a = 3$. Up to T-duality and permutations of three different two-tori, there are four possibilities for the wrapping numbers for $SU(5)$ stacks of D6-branes

$$(1,-3) \times (1,1) \times (1,1) \;; \tag{44}$$

$$(1,3) \times (1,1) \times (1,-1) \;; \tag{45}$$

$$(1,3) \times (-1,1) \times (-1,1) \;; \tag{46}$$

$$(-1,3) \times (1,1) \times (-1,1) \;. \tag{47}$$

## IV. $SU(5)$ MODELS

In the previous $SU(5)$ model building in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold without fluxes, one can easily show that one can not construct the models with three anti-symmetric representations and without symmetric representations [13, 18]. And then, for the models with three anti-symmetric representations and some symmetric representations, the net number of $\bar{\mathbf{5}}$ and $\mathbf{5}$ can not be three due to the non-abelian anomaly free conditions, *i. e.*, one does not have exact three families of the SM fermions [13, 18].

In this Section, we will present $SU(5)$ models with three anti-symmetric $\mathbf{10}$ representations and without symmetric $\mathbf{15}$ representations. Although the net number of $\bar{\mathbf{5}}$ and $\mathbf{5}$ is three due to the non-Abelian anomaly free condition, the initial $\bar{\mathbf{5}}$ number is not three. For a concrete model, we will show that after the additional gauge symmetry breaking via supersymmetry preserving Higgs mechanism, the $\bar{\mathbf{5}}$ and $\mathbf{5}$ pairs can form the massive vector-like particles with masses around the GUT/string scale. Then we will have exact three $\bar{\mathbf{5}}$ and no $\mathbf{5}$. Moreover, we can break the $SU(5)$ gauge symmetry down to the SM gauge symmetry via D6-brane splitting, and solve the doublet-triplet splitting problem. If the extra one pair of Higgs doublets and adjoint particles can obtain GUT/string-scale masses via high-dimensional operators, we only have the MSSM in the observable sector below the GUT scale. And then we can explain the observed low energy gauge couplings. We also briefly comment on two more models where the phenomenological discussions are similar.

### A. Model SU(5)-I

We present the D6-brane configurations and intersection numbers for the Model SU(5)-I in Table IV, and its particle spectrum in the observable and Higgs sectors in Table V. The wrapping numbers for $SU(5)$ stack of D6-branes are equivalent to these in Eq. (44) by using sign equivalent principle and interchanging the first and third two-tori [14].

| stk | $N$ | $(n_1,l_1)(n_2,l_2)(n_3,l_3)$ | A | S | $b$ | $b'$ | $c$ | $c'$ | $d$ | $d'$ | $e$ | $e'$ | $f$ | $f'$ | $g$ | $g'$ | $O6$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a$ | 5 | ( 1, 1) (-1,-1) (-1, 3) | 3 | 0 | 0(2) | -3 | -3 | 0(6) | 3 | 0(0) | 0(3) | 0(1) | 1 | - | -3 | - | 3 |
| $b$ | 1 | ( 0, 2) ( 1,-3) ( 1,-3) | -9 | 9 | - | - | -9 | 0(3) | -3 | 0(2) | 3 | 0(1) | 2 | - | 0(3) | - | -18 |
| $c$ | 1 | ( 1,-1) ( 1, 3) ( 2, 0) | 0 | 0 | - | - | - | - | 0(6) | 3 | -3 | 0(3) | -2 | - | 0(1) | - | 0(3) |
| $d$ | 1 | (-1, 1) ( 1,-1) (-1,-3) | -3 | 0 | - | - | - | - | - | - | 0(1) | 0(3) | -1 | - | 3 | - | -3 |
| $e$ | 1 | (-1,-1) ( 0, 2) ( 1, 3) | 3 | -3 | - | - | - | - | - | - | - | - | 0(1) | - | 0(3) | - | 6 |
| $f$ | 6 | ( 2, 0) ( 0,-2) ( 0, 2) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - | - |
| $g$ | 4 | ( 0,-2) ( 0, 2) ( 2, 0) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - | - |
| $O6$ | 2 | ( 2, 0) ( 2, 0) ( 2, 0) | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |

TABLE IV: D6-brane configurations and intersection numbers for the Model SU(5)-I on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^4 \times USp(12) \times USp(8) \times USp(4)$, and the complex structure parameters are $\chi_1 = 2\sqrt{3/5}$, $\chi_2 = 2\sqrt{1/15}$, and $\chi_3 = 2\sqrt{15}/9$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $a = 8$, and $m = 2$.

The four global $U(1)$'s from the additional $U(1)$ gauge symmetry breaking due to the Green-Schwarz mechanism are

$$\begin{aligned} U(1)_1 &= 5U(1)_a + 2U(1)_b - 2U(1)_c - U(1)_d \ , \\ U(1)_2 &= 5U(1)_a + 6U(1)_c - U(1)_d - 2U(1)_e \ , \\ U(1)_3 &= -15U(1)_a + 3U(1)_d \ , \\ U(1)_4 &= 15U(1)_a - 18U(1)_b - 3U(1)_d + 6U(1)_e \ . \end{aligned} \tag{48}$$

And the anomaly-free $U(1)$ is

$$U(1)_{free} = U(1)_a + U(1)_b + U(1)_c + 5U(1)_d + 3U(1)_e \ . \tag{49}$$

In this model, we have three $\mathbf{10}$ representations, thirty $\bar{\mathbf{5}}$ representations, and twenty-seven $\mathbf{5}$ representations for $SU(5)$. Then, the net number of $\bar{\mathbf{5}}$ and $\mathbf{5}$ is three. So, the key question is whether we can give the GUT/string-scale vector-like masses to twenty-seven pairs of $\bar{\mathbf{5}}$ and $\mathbf{5}$.

Let us discuss how to decouple the vector-like particles via supersymmetry preserving

| Rep. | Multi. | $U(1)_a$ | $U(1)_b$ | $U(1)_c$ | $U(1)_d$ | $U(1)_e$ | $U(1)_1$ | $U(1)_2$ | $U(1)_3$ | $U(1)_4$ | $U(1)_{free}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $(\mathbf{10},\mathbf{1})$ | 3 | 2 | 0 | 0 | 0 | 0 | 10 | 10 | -30 | 30 | 2 |
| $(\bar{\mathbf{5}}_a,\bar{\mathbf{1}}_b)$ | 3 | -1 | -1 | 0 | 0 | 0 | -7 | -5 | 15 | -15 | -2 |
| $(\bar{\mathbf{1}}_b,\mathbf{1}_c)$ | 3 | 0 | -1 | 1 | 0 | 0 | -4 | 6 | 0 | 18 | 0 |
| $(\mathbf{5}_a,\bar{\mathbf{1}}_b)^\star$ | 2 | 1 | -1 | 0 | 0 | 0 | 3 | 5 | -15 | 33 | 0 |
| $(\bar{\mathbf{5}}_a,\mathbf{1}_b)^\star$ | 2 | -1 | 1 | 0 | 0 | 0 | -3 | -5 | 15 | -33 | 0 |
| $(\bar{\mathbf{5}}_a,\mathbf{1}_c)$ | 3 | -1 | 0 | 1 | 0 | 0 | -7 | 1 | 15 | -15 | 0 |
| $(\mathbf{5}_a,\bar{\mathbf{1}}_d)$ | 3 | 1 | 0 | 0 | -1 | 0 | 6 | 6 | -18 | 18 | -4 |
| $(\mathbf{5}_a,\mathbf{12}_f)$ | 1 | 1 | 0 | 0 | 0 | 0 | 5 | 5 | -15 | 15 | 1 |
| $(\bar{\mathbf{5}}_a,\mathbf{8}_g)$ | 3 | -1 | 0 | 0 | 0 | 0 | -5 | -5 | 15 | -15 | -1 |
| $(\mathbf{5}_a,\mathbf{4}_{O6})$ | 3 | 1 | 0 | 0 | 0 | 0 | 5 | 5 | -15 | 15 | 1 |
| $(\mathbf{1}_e,\mathbf{4}_{O6})$ | 6 | 0 | 0 | 0 | 0 | 1 | 0 | -2 | 0 | 6 | 3 |
| $(\bar{\mathbf{1}}_b,\mathbf{1}_c)$ | 6 | 0 | -1 | 1 | 0 | 0 | -4 | 6 | 0 | 18 | 0 |
| $(\bar{\mathbf{1}}_b,\mathbf{1}_d)$ | 3 | 0 | -1 | 0 | 1 | 0 | -3 | -1 | 3 | 15 | 4 |
| $(\mathbf{1}_b,\bar{\mathbf{1}}_e)$ | 3 | 0 | 1 | 0 | 0 | -1 | 2 | 2 | 0 | -24 | -2 |
| $(\mathbf{1}_c,\mathbf{1}_d)$ | 3 | 0 | 0 | 1 | 1 | 0 | -3 | 5 | 3 | -3 | 6 |
| $(\bar{\mathbf{1}}_c,\mathbf{1}_e)$ | 3 | 0 | 0 | -1 | 0 | 1 | 2 | -8 | 0 | 6 | 2 |
| $(\mathbf{12}_f,\mathbf{8}_g)^\star$ | 4+4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $(\mathbf{8}_g,\mathbf{4}_{O6})^\star$ | 4+4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $(\mathbf{1}_c,\bar{\mathbf{1}}_d)^\star$ | 6 | 0 | 0 | 1 | -1 | 0 | -1 | 7 | -3 | 3 | -4 |
| $(\bar{\mathbf{1}}_c,\mathbf{1}_d)^\star$ | 6 | 0 | 0 | -1 | 1 | 0 | 1 | -7 | 3 | -3 | 4 |
| $(\mathbf{1}_e,\mathbf{12}_f)^\star$ | 1 | 0 | 0 | 0 | 0 | 1 | 0 | -2 | 0 | 6 | 3 |
| $(\bar{\mathbf{1}}_e,\mathbf{12}_f)^\star$ | 1 | 0 | 0 | 0 | 0 | -1 | 0 | 2 | 0 | -6 | -3 |
| $(\mathbf{1}_e,\mathbf{8}_g)^\star$ | 1 | 0 | 0 | 0 | 0 | 1 | 0 | -2 | 0 | 6 | 3 |
| $(\bar{\mathbf{1}}_e,\mathbf{8}_g)^\star$ | 1 | 0 | 0 | 0 | 0 | -1 | 0 | 2 | 0 | -6 | -3 |
| Additional chiral and non-chiral Matter | | | | | | | | | | | |

TABLE V: The particle spectrum in the observable and Higgs sectors in the Model SU(5)-I with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like matter.

Higgs mechanism. We have the following superpotential from three-point functions:

$$
\begin{aligned}
W_3 \ =\ & y^A_{ijk}(\bar{\mathbf{5}}_a, \mathbf{1}_c)_i(\mathbf{5}_a, \bar{\mathbf{1}}_d)_j(\bar{\mathbf{1}}_c, \mathbf{1}_d)_k + y^B_{ik}(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i(\mathbf{5}_a, \mathbf{12}_f)(\mathbf{12}_f, \mathbf{8}_g)_k \\
& + y^C_{ijk}(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i(\mathbf{5}_a, \mathbf{4}_{O6})_j(\mathbf{8}_g, \mathbf{4}_{O6})_k \ . \qquad (50)
\end{aligned}
$$

After the Higgs fields $(\bar{\mathbf{1}}_c, \mathbf{1}_d)_k$ obtain vacuum expectation values (VEVs), we can give the vector-like masses to three pairs of $(\bar{\mathbf{5}}_a, \mathbf{1}_c)_i$ and $(\mathbf{5}_a, \bar{\mathbf{1}}_d)_j$ because the $(\bar{\mathbf{5}}_a, \mathbf{1}_c)_i$, $(\mathbf{5}_a, \bar{\mathbf{1}}_d)_j$ and three of six $(\bar{\mathbf{1}}_c, \mathbf{1}_d)_k$ arises from the intersections on the third two-torus. In addition, after the Higgs fields $(\mathbf{12}_f, \mathbf{8}_g)_k$ and $(\mathbf{8}_g, \mathbf{4}_{O6})_k$ obtain VEVs, we can give vector-like masses to eight pairs of $\bar{\mathbf{5}}$ and $\mathbf{5}$ in $(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i$ and $(\mathbf{5}_a, \mathbf{12}_f)$, and to four pairs of $\bar{\mathbf{5}}$ and $\mathbf{5}$ in $(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i$ and $(\mathbf{5}_a, \mathbf{4}_{O6})_j$, respectively.

To further give vector-like masses to additional twelve pairs of $\bar{\mathbf{5}}$ and $\mathbf{5}$, we introduce the following superpotential from four-point functions:

$$
\begin{aligned}
W_3 \ =\ & y^D_{ik}(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i(\mathbf{5}_a, \mathbf{12}_f)(\mathbf{1}_e, \mathbf{12}_f)(\bar{\mathbf{1}}_e, \mathbf{8}_g)_k + y^{D\prime}_{ik}(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i(\mathbf{5}_a, \mathbf{12}_f)(\bar{\mathbf{1}}_e, \mathbf{12}_f)(\mathbf{1}_e, \mathbf{8}_g)_k \\
& + y^E_{ijkl}(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i(\mathbf{5}_a, \mathbf{4}_{O6})_j(\bar{\mathbf{1}}_e, \mathbf{8}_g)_k(\mathbf{1}_e, \mathbf{4}_{O6})_l \ . \qquad (51)
\end{aligned}
$$

We point out that $(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i$, $(\mathbf{5}_a, \mathbf{4}_{O6})_j$, $(\bar{\mathbf{1}}_e, \mathbf{8}_g)_k$, and three of six $(\mathbf{1}_e, \mathbf{4}_{O6})_l$ arise from the intersections on the third two-torus. If we give VEVs to the Higgs fields $(\mathbf{1}_e, \mathbf{12}_f)$, $(\bar{\mathbf{1}}_e, \mathbf{8}_g)_k$, $(\bar{\mathbf{1}}_e, \mathbf{12}_f)$, $(\mathbf{1}_e, \mathbf{8}_g)_k$, and $(\mathbf{1}_e, \mathbf{4}_{O6})_l$, we can generate the vector-like masses for the rest twelve pairs of $\bar{\mathbf{5}}$ and $\mathbf{5}$ in $(\bar{\mathbf{5}}_a, \mathbf{8}_g)_i$ and $(\mathbf{5}_a, \mathbf{12}_f)/(\mathbf{5}_a, \mathbf{4}_{O6})_j$. Therefore, we only have three $\bar{\mathbf{5}}$ and do not have $\mathbf{5}$ after the Higgs mechanism at the GUT/string scale. Note that there are three ($U(1)_e$ symmetric) singlets with charge $-\mathbf{2}$ and six $(\mathbf{1}_e, \mathbf{4}_{O6})_l$ with charge $+\mathbf{1}$ under the $U(1)_e$ gauge symmetry, we obtain that the D-flatness for $U(1)_e$ gauge symmetry can be preserved if we give VEVs to these singlets. And the D-flatness for other broken gauge symmetries can be preserved because all the other relevant Higgs particles are vector-like. Also, it is obvious that we have the F-flatness for above superpotential. Thus, the Higgs mechanism can preserve supersymmetry.

To break the $SU(5)$ gauge symmetry down to the SM gauge symmetry, we split the $a$ stack of D6-branes into $a_3$ and $a_2$ stacks with respectively 3 and 2 D6-branes. To have the vector-like MSSM Higgs doublets, we assume that the $a_2$ and $b$ stacks of D6-branes are parallel and on the top of each other on the third two-torus. Then we obtain two pairs of vector-like Higgs doublets $(\mathbf{2}_{a_2}, \bar{\mathbf{1}}_b)_j$ and $(\bar{\mathbf{2}}_{a_2}, \mathbf{1}_b)_j$ ($j = 1, 2$) with quantum numbers $(\mathbf{1}, \mathbf{2}, \mathbf{1/2})$ and $(\mathbf{1}, \bar{\mathbf{2}}, -\mathbf{1/2})$ under $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry. We also assume that the $a_3$ and $b$ stacks of D6-branes are not on the top of each other on the third two-torus. So, the vector-like triplets will obtain the masses around the string scale, and the doublet-triplet splitting problem is solved. Therefore, below the GUT scale, we have SM gauge symmetry, three families of the SM fermions, two pairs of Higgs doublets, and three adjoint particles for each gauge symmetry in the observable sector.

Suppose that one pair of the Higgs doublets and adjoint particles obtain the GUT/string-scale vector-like masses via high-dimensional operators, we only have the MSSM below the GUT scale. And then, if we choose the suitable grand unified gauge coupling by adjusting the string scale $M_S$, the observed low energy gauge couplings can be generated via RGE running. Let us discuss the gauge coupling and the string scale. For a generic stack $\sigma$ of D6-branes, its gauge coupling at the string scale is [40]

$$(g_{YM}^{\sigma})^2 = \frac{\sqrt{8\pi} M_s}{M_{Pl}} \frac{1}{\prod_{i=1}^{3} \sqrt{\left(n_{\sigma}^i\right)^2 \chi_i^{-1} + \left(2^{-\beta_i} l_{\sigma}^i\right)^2 \chi_i}} \ . \tag{52}$$

So, the $SU(5)$ gauge coupling $g_{YM}^a$ at the string scale is

$$(g_{YM}^a)^2 \ = \ \frac{(375\pi^2)^{1/4}}{4} \frac{M_S}{M_{Pl}} \ \simeq \ \frac{2M_S}{M_{Pl}} \ . \tag{53}$$

Thus, we can have the suitable grand unified gauge coupling $g_{YM}^a$ by adjusting the string scale. As an example, to have the MSSM unified gauge coupling $g_{MSSM}$ which is about $1/\sqrt{2}$, we choose $M_S \simeq M_{Pl}/4$ which is close to the string scale in the heterotic string theory.

### 1. Comment on Other Models

We present the D6-brane configurations and intersection numbers for the Model SU(5)-II on Type IIA $\mathbf{T^6}$ orientifold and the Model SU(5)-III on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold in Tables VI and VII, respectively. The wrapping numbers for $SU(5)$ stack of D6-branes in the Model SU(5)-II are the same as these in the Model SU(5)-I, and the wrapping numbers for $SU(5)$ stack of D6-branes in the Model SU(5)-III are given in Eq. (36). Similar to the Model SU(5)-I, we have three **10** representations, and three $\bar{\mathbf{5}}$ representations after the additional gauge symmetry breaking by the supersymmetry preserving Higgs mechanism. If the extra Higgs doublets and adjoint particles obtain GUT/string-scale masses via high-dimensional operators, we can explain the observed low energy gauge couplings via RGE running.

| stk | $N$ | $(n_1,l_1)(n_2,l_2)(n_3,l_3)$ | A | S | $b$ | $b'$ | $c$ | $c'$ | $d$ | $d'$ | $e$ | $e'$ | $f$ | $f'$ | $g$ | $g'$ | $h$ | $h'$ | $i$ | $i'$ | $O6$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a$ | 5 | ( 1, 1) (-1,-1) (-1, 3) | 3 | 0 | -3 | 0(6) | -2 | -2 | 4 | 0(1) | 5 | 0(2) | 12 | 0(9) | -5 | -8 | 1 | - | -3 | - | 3 |
| $b$ | 1 | ( 1,-1) ( 1, 3) ( 2, 0) | 0 | 0 | - | - | 3 | 0(1) | -2 | 2 | 0(16) | 2 | -6 | 3 | -2 | 1 | -2 | - | 0(1) | - | 0(3) |
| $c$ | 1 | ( 0, 2) ( 1,-3) ( 1,-1) | -3 | 3 | - | - | - | - | -3 | 0(6) | -1 | 8 | -10 | -8 | 2 | -1 | 2 | - | 0(1) | - | -6 |
| $d$ | 1 | (-3, 1) ( 1,-1) (-1,-1) | -2 | -1 | - | - | - | - | - | - | 2 | -10 | 0(2) | -5 | 5 | 8 | -2 | - | 3 | - | -1 |
| $e$ | 1 | ( 1,-1) ( 3,-7) ( 0, 2) | -7 | 7 | - | - | - | - | - | - | - | - | -14 | -35 | 18 | 9 | 0(3) | - | 6 | - | -14 |
| $f$ | 1 | ( 2, 0) (-1, 1) (-7,-3) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | 3 | 24 | 0(7) | - | 6 | - | 0(3) |
| $g$ | 1 | ( 1,-3) ( 0, 2) ( 3, 1) | 3 | -3 | - | - | - | - | - | - | - | - | - | - | - | - | 0(9) | - | 0(1) | - | 6 |
| $h$ | 4 | ( 2, 0) ( 0,-2) ( 0, 2) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| $i$ | 4 | ( 0,-2) ( 0, 2) ( 2, 0) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| $O6$ | 2 | ( 2, 0) ( 2, 0) ( 2, 0) | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |

TABLE VI: D6-brane configurations and intersection numbers for the Model SU(5)-II on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^6 \times USp(8) \times USp(8) \times USp(4)$, and the complex structure parameters are $\chi_1 = 6/\sqrt{7}$, $\chi_2 = 2/\sqrt{7}$, and $\chi_3 = 2\sqrt{7}/3$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -12(3q+2)$, $a = 24$, and $m = 2$.

| stk | $N$ | $(n_1,l_1)$ $(n_2,l_2)$ $(n_3,l_3)$ | A | S | $b$ | $b'$ | $c$ | $c'$ | $d$ | $d'$ | $e$ | $e'$ | $f$ | $f'$ | $g$ | $g'$ | $h$ | $h'$ | $i$ | $i'$ | $j$ | $j'$ | $k$ | $k'$ | $O6^2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a$ | 10 | ( 1, 3) ( 1, 1) ( 1,-1) | 3 | 0 | -3 | 0(2) | -2 | -2 | 3 | 0 | -40 | -7 | 15 | 0 | 30 | -6 | 5 | -16 | -10 | -10 | 5 | 6 | 3 | 8 | 3 |
| $b$ | 2 | ( 1,-3) ( 1, 3) ( 2, 0) | 0 | 0 | - | - | 3 | 0(1) | 0 | 3 | 0 | 0 | 0 | 27 | -27 | 0 | 0 | 0 | 3 | 24 | 8 | 10 | -8 | -10 | -6 |
| $c$ | 2 | ( 0, 2) ( 1,-3) ( 3,-1) | 0 | 0 | - | - | - | - | -2 | 2 | -40 | 3 | -24 | 3 | 0 | 6 | -3 | -24 | 0 | 0 | -3 | 6 | 15 | 6 | 6 |
| $d$ | 2 | ( 1,-3) ( 1,-1) ( 1, 1) | -3 | 0 | - | - | - | - | - | - | 7 | 40 | 0 | -15 | 6 | -30 | 16 | -5 | 10 | 10 | 0 | -5 | -8 | -3 | -3 |
| $e$ | 2 | (-3,11)(-3,-11)( 2, 0) | 0 | 0 | - | - | - | - | - | - | - | - | -33 | 330 | 330 | 33 | 0 | 0 | 54 | 243 | 70 | -35 | -70 | -128 | -66 |
| $f$ | 2 | ( 1,-3) (-2, 0) (-7,-3) | -9 | 9 | - | - | - | - | - | - | - | - | - | - | 0 | 189 | 168 | -21 | 0 | 147 | 14 | -35 | -98 | -49 | -42 |
| $g$ | 2 | ( 2, 0) ( 1,-3) ( 7, 3) | -9 | 9 | - | - | - | - | - | - | - | - | - | - | - | - | -21 | 168 | 0 | -42 | -49 | -98 | -35 | 24 | 0 |
| $h$ | 2 | (-3,-7) (-3, 7) ( 2, 0) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 189 | 0 | -56 | -70 | 56 | 70 | 42 |
| $i$ | 2 | ( 0, 2) (-3,-7) ( 7, 3) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | -35 | 24 | -49 | -98 | -42 |
| $j$ | 2 | (-1, 7) (-1, 1) ( 0, 2) | -3 | 3 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 0 | 0 | 0 |
| $k$ | 2 | ( 1, 1) ( 1, 7) ( 0,-2) | -3 | 3 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 0 |
| $O6^2$ | 8 | ( 2, 0) ( 0,-2) ( 0, 2) | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |

TABLE VII: D6-brane configurations and intersection numbers for the Model SU(5)-III on Type IIA $\mathbf{T^6}/(\mathbb{Z_2}\times\mathbb{Z_2})$ orientifold. The complete gauge symmetry is $U(5)\times U(1)^{10}\times USp(8)$, and the complex structure parameters are $\chi_1 = 2/\sqrt{7}$, $\chi_2 = 2/\sqrt{7}$, and $\chi_3 = 2\sqrt{7}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+8)$, $a = 32$, and $m = 8$.

## V. FLIPPED $SU(5)$ MODELS

In the previous flipped $SU(5) \times U(1)_X$ model building [16, 17, 29], the net number of $\mathbf{10}$ and $\overline{\mathbf{10}}$ and the net number of $\overline{\mathbf{5}}$ and $\mathbf{5}$ are not three, and at least some Yukawa couplings (for example, the down-type quark Yukawa couplings) are forbidden by the global $U(1)$ symmetries. So, we would like to construct the better model. In order to obtain at least one pair of Higgs fields $\mathbf{10}$ and $\overline{\mathbf{10}}$, we must have the symmetric representation, and then the net number of $\overline{\mathbf{5}}$ and $\mathbf{5}$ can not be three if the net number of $\mathbf{10}$ and $\overline{\mathbf{10}}$ is three. For the first time, we will present the flipped $SU(5)$ model with exact three $\mathbf{10}$, and the model in which all the Yukawa couplings in the superpotential are allowed by the global $U(1)$ symmetries. We will also comment on two more flipped $SU(5)$ models, and try to avoid as much extra matter as possible.

### A. Basic Flipped $SU(5)$ Phenomenology

In a flipped $SU(5) \times U(1)_X$ [1, 2, 3] unified model, the electric charge generator $Q$ is only partially embedded in $SU(5)$, *i.e.*, $Q = T_3 - \frac{1}{5}Y' + \frac{2}{5}\tilde{Y}$, where $Y'$ is the $U(1)$ internal $SU(5)$ and $\tilde{Y}$ is the external $U(1)_X$ factor. Essentially, this means that the photon is 'shared' between $SU(5)$ and $U(1)_X$. The SM fermions plus the right-handed neutrino states reside within the representations $\bar{\mathbf{5}}$, $\mathbf{10}$, and $\mathbf{1}$ of $SU(5)$, which are collectively equivalent to a spinor $\mathbf{16}$ of $SO(10)$. The quark and lepton assignments are flipped by $u^c_L \leftrightarrow d^c_L$ and $\nu^c_L \leftrightarrow e^c_L$ relative to a conventional $SU(5)$ GUT embedding:

$$\bar{f}_{\bar{\mathbf{5}},-\mathbf{\frac{3}{2}}} = \begin{pmatrix} u^c_1 \\ u^c_2 \\ u^c_3 \\ e \\ \nu_e \end{pmatrix}_L \; ; \quad F_{\mathbf{10},\mathbf{\frac{1}{2}}} = \left( \begin{pmatrix} u \\ d \end{pmatrix}_L \; d^c_L \;\; \nu^c_L \right) ; \quad l_{\mathbf{1},\mathbf{\frac{5}{2}}} = e^c_L \; . \tag{54}$$

In particular this results in the $\mathbf{10}$ containing a neutral component with the same quantum numbers as $\nu^c_L$. So we can spontaneously break the GUT gauge symmetry by using a pair of $\mathbf{10}$ and $\overline{\mathbf{10}}$ of superheavy Higgs where the neutral components receive a large VEV, $\langle \nu^c_H \rangle =$

$\langle \bar{\nu}^c_H \rangle$,

$$H_{\mathbf{10},\frac{1}{2}} = \{Q_H,\ d^c_H,\ \nu^c_H\}\ ; \quad \bar{H}_{\overline{\mathbf{10}},-\frac{1}{2}} = \{Q_{\bar{H}},\ d^c_{\bar{H}},\ \nu^c_{\bar{H}}\}\ . \tag{55}$$

The spontaneous breaking of electroweak gauge symmetry is generated by the Higgs doublets $H_2$ and $\bar{H}_{\bar{2}}$

$$h_{\mathbf{5},-\mathbf{1}} = \{H_2, H_3\}\ ; \quad \bar{h}_{\bar{\mathbf{5}},\mathbf{1}} = \left\{\bar{H}_{\bar{2}}, \bar{H}_{\bar{3}}\right\}\ . \tag{56}$$

The flipped $SU(5)$ models have two very nice features which are generally not found in typical unified models: (i) a natural solution to the doublet ($H_2$)-triplet($H_3$) splitting problem of the electroweak Higgs pentaplets $h$ and $\bar{h}$ through the trilinear couplings of the Higgs fields: $H_{\mathbf{10}} \cdot H_{\mathbf{10}} \cdot h_{\mathbf{5}} \to \langle \nu^c_H \rangle\, d^c_H H_3$; (ii) an automatic see-saw mechanism that provide heavy right-handed neutrino mass through the coupling to singlet fields $\phi$, $F_{\mathbf{10}} \cdot \bar{H}_{\overline{\mathbf{10}}} \cdot \phi \to \left\langle \nu^c_{\bar{H}} \right\rangle \nu^c \phi$.

The generic superpotential $W$ for a flipped $SU(5)$ model is

$$\lambda_1 FFh + \lambda_2 F\bar{f}\bar{h} + \lambda_3 \bar{f} l^c h + \lambda_4 F\bar{H}\phi + \lambda_5 HHh + \lambda_6 \bar{H}\bar{H}\bar{h} + \cdots \in W\ , \tag{57}$$

where the first three terms provide masses for the quarks and leptons, the fourth is responsible for the heavy right-handed neutrino masses, and the last two terms are responsible for the doublet-triplet splitting mechanism [3].

### B. Model FSU(5)-I

We first construct the Model FSU(5)-I on Type IIA $\mathbf{T^6}$ orientifold which have exact three **10** representations. The D6-brane configurations and intersection numbers for the Model FSU(5)-I are given in Table VIII, and its particle spectrum in the observable sector is given in Table IX.

The $U(1)_X$ in flipped $SU(5) \times U(1)_X$ gauge symmetry is

$$\begin{aligned} U(1)_X \ =\ & \frac{1}{2}(U(1)_a - 5U(1)_b + 5U(1)_c + 5U(1)_d - 5U(1)_e \\ & +5U(1)_f + 5U(1)_g + 5U(1)_h)\ . \end{aligned} \tag{58}$$

The other massless $U(1)$'s are:

$$\begin{aligned} U(1)_U \ =\ & 5U(1)_a - 25U(1)_b + 25U(1)_c + 25U(1)_d + 107U(1)_e \\ & +25U(1)_f - 19U(1)_g + 25U(1)_h\ , \end{aligned} \tag{59}$$

| stk | $N$ | $(n_1,l_1)(n_2,l_2)(n_3,l_3)$ | A | S | $b$ | $b'$ | $c$ | $c'$ | $d$ | $d'$ | $e$ | $e'$ | $f$ | $f'$ | $g$ | $g'$ | $h$ | $h'$ | $i$ | $i'$ | $j$ | $j'$ | $k$ | $k'$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a$ | 5 | ( 0, 1) (-1,-1) ( 1, 3) | 3 | -3 | -3 | 0(1) | 3 | 0 | 0 | 0 | 0 | 3 | 2 | 1 | 9 | 18 | -2 | -1 | -1 | - | 0 | - | 0 | - |
| $b$ | 1 | ( 1,-1) ( 0, 2) ( 1, 3) | -6 | 6 | - | - | -6 | 0 | -1 | 2 | 0 | -6 | -4 | 0 | -18 | -36 | 0 | 2 | 0 | - | 2 | - | 0 | - |
| $c$ | 1 | ( 1, 1) ( 1,-1) ( 2, 0) | 0 | 0 | - | - | - | - | 1 | -2 | 0 | 0 | 0 | -2 | 0 | 0 | 4 | 0 | 2 | - | -2 | - | 0 | - |
| $d$ | 1 | ( 0, 1) ( 1,-3) ( 1,-1) | -3 | 3 | - | - | - | - | - | - | -2 | -1 | 0 | -3 | -15 | -12 | 0 | 3 | 1 | - | 0 | - | 0 | - |
| $e$ | 1 | ( 1,-1) ( 1, 1) ( 2, 0) | 0 | 0 | - | - | - | - | - | - | - | - | 2 | 0 | 0 | 0 | 0 | -4 | -2 | - | 2 | - | 0 | - |
| $f$ | 1 | ( 1, 1) ( 2, 0) ( 1,-1) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | -2 | 1 | 3 | 0 | 2 | - | 0 | - | -2 | - |
| $g$ | 1 | ( 3,-1) ( 3, 1) ( 2, 0) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - | 20 | -32 | -6 | - | 6 | - | 0 | - |
| $h$ | 1 | (-1, 1) (-1, 3) ( 0, 2) | -6 | 6 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | 0 | - | 0 | - | 2 | - |
| $i$ | 3 | ( 1, 0) ( 0,-2) ( 0, 2) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| $j$ | 1 | ( 0,-1) ( 2, 0) ( 0, 2) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| $k$ | 2 | ( 0,-1) ( 0, 2) ( 2, 0) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |

TABLE VIII: D6-brane configurations and intersection numbers for the Model FSU(5)-I on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^7 \times USp(6) \times USp(2) \times USp(4)$, and the complex structure parameters are $\chi_1 = 1/\sqrt{3}$, $\chi_2 = 2/\sqrt{3}$, and $\chi_3 = 2/\sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -12(3q+2)$, $a = 16$, and $m = 2$.

$$U(1)_V \; = \; U(1)_c - 2U(1)_d + U(1)_e + U(1)_f + U(1)_h \;, \tag{60}$$

$$U(1)_W \; = \; 4U(1)_b - 6U(1)_d - 10U(1)_e - U(1)_f + 2U(1)_g - U(1)_h \;. \tag{61}$$

And the four global $U(1)$'s are

$$\begin{aligned} U(1)_1 \; &= \; -5U(1)_a + 2U(1)_c + U(1)_d - 2U(1)_e + 2U(1)_f - 6U(1)_g \;, \\ U(1)_2 \; &= \; 2U(1)_b - 2U(1)_c + 2U(1)_e + 6U(1)_g \;, \\ U(1)_3 \; &= \; -2U(1)_f + 2U(1)_h \;, \\ U(1)_4 \; &= \; 15U(1)_a - 6U(1)_b - 3U(1)_d - 6U(1)_h \;. \end{aligned} \tag{62}$$

| Rep. | Multi. | $U(1)_a$ | $U(1)_b$ | $U(1)_c$ | $U(1)_d$ | $U(1)_e$ | $U(1)_f$ | $U(1)_g$ | $U(1)_h$ | $U(1)_X$ | $U(1)_1$ | $U(1)_2$ | $U(1)_3$ | $U(1)_4$ | $U(1)_U$ | $U(1)_V$ | $U(1)_W$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(\mathbf{10},\mathbf{1})$ | 3 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -10 | 0 | 0 | 30 | 10 | 0 | 0 |
| $(\bar{\mathbf{5}}_a,\mathbf{1}_b)$ | 3 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -3 | 5 | 2 | 0 | -21 | -30 | 0 | 4 |
| $(\mathbf{1}_d,\mathbf{1}_h)$ | 3 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 5 | 1 | 0 | 2 | -9 | 50 | -1 | -7 |
| $(\mathbf{10},\mathbf{1})$ | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -10 | 0 | 0 | 30 | 10 | 0 | 0 |
| $(\overline{\mathbf{10}},\mathbf{1})$ | 1 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 10 | 0 | 0 | -30 | -10 | 0 | 0 |
| $(\mathbf{5}_a,\mathbf{1}_b)^\star$ | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -2 | -5 | 2 | 0 | 9 | -20 | 0 | 4 |
| $(\bar{\mathbf{5}}_a,\bar{\mathbf{1}}_b)^\star$ | 1 | -1 | -1 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 5 | -2 | 0 | -9 | 20 | 0 | -4 |
| $(\mathbf{1}_c,\bar{\mathbf{1}}_h)$ | 4 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | -1 | 0 | 2 | -2 | 0 | 6 | 0 | 0 | 1 |
| $(\overline{\mathbf{15}},\mathbf{1})$ | 3 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 10 | 0 | 0 | -30 | -10 | 0 | 0 |
| $(\mathbf{10},\mathbf{1})$ | 2 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -10 | 0 | 0 | 30 | 10 | 0 | 0 |
| $(\overline{\mathbf{10}},\mathbf{1})$ | 2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 10 | 0 | 0 | -30 | -10 | 0 | 0 |
| Additional chiral and non-chiral Matter | | | | | | | | | | | | | | | | | |

TABLE IX: The particle spectrum in the observable sector in the Model FSU(5)-I, with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like matter.

### C. Model FSU(5)-II

We construct the Model FSU(5)-II on Type IIA $\mathbf{T^6}$ orientifold in which unlike the previous flipped $SU(5)$ model building [16, 17, 29], all the Yukawa couplings are allowed by the global $U(1)$ symmetries. The D6-brane configurations and intersection numbers for the Model FSU(5)-II are given in Tables X and XI, and its particle spectrum in the observable sector is given in Table XII.

The $U(1)_X$ gauge symmetry is

$$\begin{aligned} U(1)_X \;=\; & \frac{1}{2}(U(1)_a - 5U(1)_b + 5U(1)_c + 5U(1)_d + 5U(1)_e + 5U(1)_f + 5U(1)_g \\ & + 5U(1)_h + 5U(1)_i - 5U(1)_j - 5U(1)_k - 5U(1)_l) \;. \end{aligned} \quad (63)$$

| stk | $N$ | $(n_1,l_1)$ $(n_2,l_2)$ $(n_3,l_3)$ | A | S | $b$ | $b'$ | $c$ | $c'$ | $d$ | $d'$ | $e$ | $e'$ | $f$ | $f'$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a$ | 5 | ( 0, 1) (-1,-1) ( 3, 1) | 2 | -2 | -3 | -6 | 0(1014) | 0(864) | 0(242) | 0(392) | 0(6) | 0(0) | -3 | 0(3) |
| $b$ | 1 | ( 1, 3) ( 1, 3) ( 0,-1) | 18 | -18 | - | - | -114 | 111 | -200 | -425 | -6 | 3 | 0(24) | 0(6) |
| $c$ | 1 | ( 0, 1) (25,-1) ( 3,-25) | -50 | 50 | - | - | - | - | 0(197192) | 0(193442) | 0(864) | (1014) | 36 | 39 |
| $d$ | 1 | ( 0, 1)(-3,-25) (25, 1) | 50 | -50 | - | - | - | - | - | - | 0(392) | 0(242) | -250 | 275 |
| $e$ | 1 | ( 0, 1) ( 1,-1) ( 3, 1) | -2 | 2 | - | - | - | - | - | - | - | - | 0 | 3 |
| $f$ | 1 | ( 1,-9) ( 1,-1) ( 0, 1) | -18 | 18 | - | - | - | - | - | - | - | - | - | - |
| $g$ | 1 | ( 1, 0) ( 3,-1) ( 3, 1) | 0 | 0 | - | - | - | - | - | - | - | - | - | - |
| $h$ | 1 | ( 1, 0) ( 3, 1) ( 3,-1) | 0 | 0 | - | - | - | - | - | - | - | - | - | - |
| $i$ | 1 | ( 1, 1) ( 1, 9) ( 0,-1) | 18 | -18 | - | - | - | - | - | - | - | - | - | - |
| $j$ | 1 | ( 1,-1) ( 1,-9) ( 0, 1) | -18 | 18 | - | - | - | - | - | - | - | - | - | - |
| $k$ | 1 | ( 1,-1) (27, 1) ( 1, 0) | 0 | 0 | - | - | - | - | - | - | - | - | - | - |
| $l$ | 1 | ( 1, 1) (27,-1) ( 1, 0) | 0 | 0 | - | - | - | - | - | - | - | - | - | - |
| $O6$ | 8 | ( 1, 0) ( 2, 0) ( 1, 0) | - | - | - | - | - | - | - | - | - | - | - | - |

TABLE X: D6-brane configurations and intersection numbers (Part 1) for the Model FSU(5)-II on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^{11} \times USp(16)$, and the complex structure parameters are $\chi_1 = \sqrt{3}/27$, $\chi_2 = 2\sqrt{3}$, and $\chi_3 = \sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -6(q+2)$, $a = 24$, and $m = 12$.

The four global $U(1)$'s are:

$$
\begin{aligned}
U(1)_1 &= -15U(1)_a + 75U(1)_c - 75U(1)_d + 3U(1)_e - 27U(1)_k + 27U(1)_l \ , \\
U(1)_2 &= -3U(1)_g + 3U(1)_h + U(1)_k - U(1)_l \ , \\
U(1)_3 &= -U(1)_b + U(1)_f + 3U(1)_g - 3U(1)_h - U(1)_i + U(1)_j \ , \\
U(1)_4 &= 5U(1)_a + 9U(1)_b - 25U(1)_c + 25U(1)_d - U(1)_e - 9U(1)_f + 9U(1)_i - 9U(1)_j \ . \quad (64)
\end{aligned}
$$

There are seven other massless $U(1)$'s. As an example, we present two of them:

$$
\begin{aligned}
U(1)_V &= U(1)_b - U(1)_f + 2U(1)_g + 2U(1)_h - 2U(1)_i \ , \\
U(1)_W &= -36U(1)_b - 27U(1)_c + 36U(1)_f + 4U(1)_g + 29U(1)_h - 3U(1)_i + 75U(1)_l \ . \quad (65)
\end{aligned}
$$

This is the first trial flipped $SU(5)$ model where all the Yukawa couplings in superpotential in Eq. (57) are allowed by the global $U(1)$'s from the Green-Schwarz mechanism. To make

| stk | $N$ | $(n_1,l_1)$ $(n_2,l_2)$ $(n_3,l_3)$ | $g$ | $g'$ | $h$ | $h'$ | $i$ | $i'$ | $j$ | $j'$ | $k$ | $k'$ | $l$ | $l'$ | $O6$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a$ | 5 | ( 0, 1) (-1,-1) ( 3, 1) | 0 | 6 | 6 | 0 | -12 | -15 | -15 | -12 | 13 | 14 | 14 | 13 | 1 |
| $b$ | 1 | ( 1, 3) ( 1, 3) ( 0,-1) | 45 | 36 | 36 | 45 | 0 | 0 | 0 | 0 | 160 | 82 | 82 | 160 | 9 |
| $c$ | 1 | ( 0, 1) (25,-1) ( 3,-25) | 858 | -1008 | -1008 | 858 | 339 | 336 | 0 | 0 | 160 | 82 | 82 | 160 | -25 |
| $d$ | 1 | ( 0, 1) (-3,-25) (25, 1) | -858 | 1008 | 1008 | -858 | -25 | -650 | -650 | -25 | 336 | 339 | 339 | 336 | 25 |
| $e$ | 1 | ( 0, 1) ( 1,-1) ( 3, 1) | -6 | 0 | 0 | -6 | 15 | 12 | 12 | 15 | -14 | -13 | -13 | -14 | -1 |
| $f$ | 1 | ( 1,-9) ( 1,-1) ( 0, 1) | -27 | -54 | -54 | -27 | 0 | 0 | 0 | 0 | -112 | -130 | -130 | -112 | -9 |
| $g$ | 1 | ( 1, 0) ( 3,-1) ( 3, 1) | - | - | 0 | 0 | -42 | 39 | 39 | -42 | 15 | -12 | -12 | 15 | 0 |
| $h$ | 1 | ( 1, 0) ( 3, 1) ( 3,-1) | - | - | - | - | -39 | 42 | 42 | -39 | 12 | -15 | -15 | 12 | 0 |
| $i$ | 1 | ( 1, 1) ( 1, 9) ( 0,-1) | - | - | - | - | - | - | 0 | 0 | 242 | 0 | 0 | 242 | 0 |
| $j$ | 1 | ( 1,-1) ( 1,-9) ( 0, 1) | - | - | - | - | - | - | - | - | 0 | -242 | -242 | 0 | -9 |
| $k$ | 1 | ( 1,-1) (27, 1) ( 1, 0) | - | - | - | - | - | - | - | - | - | - | 0 | 0 | 0 |
| $l$ | 1 | ( 1, 1) (27,-1) ( 1, 0) | - | - | - | - | - | - | - | - | - | - | - | - | 0 |
| $O6$ | 8 | ( 1, 0) ( 2, 0) ( 1, 0) | - | - | - | - | - | - | - | - | - | - | - | - | - |

TABLE XI: D6-brane configurations and intersection numbers (Part 2) for the Model FSU(5)-II on Type IIA $\mathbf{T^6}$ orientifold.

the terms like $FFh$ or $HHh$ to be neutral under the global $U(1)$ symmetries, we need to set the Higgs pentaplet $h$ from the intersection between the $N = 5$ stack and a stack with large wrapping numbers (by a factor of 25 due to the flipped $SU(5)$ structure) and therefore we can not avoid extremely large exotic matter in the spectrum. In this model the Yukawa terms are:

$$\begin{aligned}
FFh &\rightarrow (\mathbf{10},\mathbf{1})(\mathbf{10},\mathbf{1})(\mathbf{5}_a,\overline{\mathbf{1}}_d) \ , \\
F\bar{f}\bar{h}' &\rightarrow (\mathbf{10},\mathbf{1})(\overline{\mathbf{5}}_a,\mathbf{1}_b)(\overline{\mathbf{5}}_a,\mathbf{1}_f) \ , \\
\bar{f}l^c h &\rightarrow (\overline{\mathbf{5}}_a,\mathbf{1}_b)(\overline{\mathbf{1}}_b,\mathbf{1}_d)(\mathbf{5}_a,\overline{\mathbf{1}}_d) \ , \\
F\bar{H}\phi &\rightarrow (\mathbf{10},\mathbf{1})(\overline{\mathbf{10}},\mathbf{1})(\mathbf{1}_b,\mathbf{1}_f) \ , \\
HHh &\rightarrow (\mathbf{10},\mathbf{1})(\mathbf{10},\mathbf{1})(\mathbf{5}_a,\overline{\mathbf{1}}_d) \ , \\
\bar{H}\bar{H}\bar{h} &\rightarrow (\overline{\mathbf{10}},\mathbf{1})(\overline{\mathbf{10}},\mathbf{1})(\overline{\mathbf{5}}_a,\mathbf{1}_d) \ .
\end{aligned} \tag{66}$$

Because of the structure of Green-Schwarz mechanism in D-brane construction, to cancel the global $U(1)$'s charges for all the Yukawa couplings we expect a mixture state of Higgs

| Rep. | Multi. | $U(1)_X$ | $U(1)_1$ | $U(1)_2$ | $U(1)_3$ | $U(1)_4$ | $U(1)_V$ | $U(1)_W$ | $\cdots$ |
|---|---|---|---|---|---|---|---|---|---|
| $(\mathbf{10},\mathbf{1})$ | 3 | 1 | -30 | 0 | 0 | 10 | 0 | 0 | $\cdots$ |
| $(\bar{\mathbf{5}}_a,\mathbf{1}_b)$ | 3 | -3 | 15 | 0 | -1 | 4 | 1 | -36 | $\cdots$ |
| $(\bar{\mathbf{1}}_b,\mathbf{1}_d)$ | 3 | 5 | -75 | 0 | 1 | 16 | -1 | 36 | $\cdots$ |
| $(\mathbf{10},\mathbf{1})$ | 1 | 1 | -30 | 0 | 0 | 10 | 0 | 0 | $\cdots$ |
| $(\overline{\mathbf{10}},\mathbf{1})$ | 1 | -1 | 30 | 0 | 0 | -10 | 0 | 0 | $\cdots$ |
| $(\mathbf{5}_a,\bar{\mathbf{1}}_d)^\star$ | 1 | -2 | 60 | 0 | 0 | -20 | 0 | 0 | $\cdots$ |
| $\bar{h}_x$ $((\bar{\mathbf{5}}_a,\mathbf{1}_d)^\star/(\bar{\mathbf{5}}_a,\mathbf{1}_f)^\star)$ | 1 | 2 | -60/15 | 0 | 0/1 | 20/-14 | 0/-1 | 0/36 | $\cdots$ |
| $(\mathbf{1}_b,\mathbf{1}_f)$ | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | $\cdots$ |
| $(\overline{\mathbf{15}},\mathbf{1})$ | 2 | -1 | 30 | 0 | 0 | -10 | 0 | 0 | $\cdots$ |
| $(\overline{\mathbf{10}},\mathbf{1})$ | 1 | -1 | 30 | 0 | 0 | -10 | 0 | 0 | $\cdots$ |
| Additional chiral and non-chiral Matter | | | | | | | | | |

TABLE XII: The particle spectrum in the observable sector in the Model FSU(5)-II, with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like matter.

pentaplet $\bar{h}_x = c\bar{h}' + s\bar{h}$ where $\bar{h}'$ is from $F\bar{f}\bar{h}'$ and $\bar{h}$ is from $\bar{H}\bar{H}\bar{h}$. However, we may reintroduce the doublet-triplet splitting problem.

### D. Model FSU(5)-III

We present the D6-brane configurations and intersection numbers for the Model FSU(5)-III in Table XIII, and its particle spectrum in the observable sector in Table XIV.

The $U(1)_X$ gauge symmetry is

$$U(1)_X = \frac{1}{2}(U(1)_a - 5U(1)_b + 5U(1)_c - 5U(1)_d + 5U(1)_e + 5U(1)_f + 5U(1)_g + 5U(1)_h) \ . \quad (67)$$

| stk | $N$ | $(n_1,l_1)(n_2,l_2)(n_3,l_3)$ | A | S | $b$ | $b'$ | $c$ | $c'$ | $d$ | $d'$ | $e$ | $e'$ | $f$ | $f'$ | $g$ | $g'$ | $h$ | $h'$ | $O6$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a$ | 5 | ( 0, 1) (-1,-1) ( 3, 1) | 2 | -2 | -3 | 0(3) | 0(6) | 0(0) | -3 | -6 | 5 | 4 | 1 | 2 | 0 | 0 | 0 | 0 | 1 |
| $b$ | 1 | (-1,-3) ( 1,-1) ( 0, 1) | -6 | 6 | - | - | 0(3) | 3 | 0 | 0 | -24 | 0 | 12 | -12 | 12 | -9 | -9 | 12 | -3 |
| $c$ | 1 | ( 0, 1) ( 1,-1) ( 3,-1) | -2 | 2 | - | - | - | - | 6 | 3 | -4 | -5 | -2 | -1 | 0 | 0 | 0 | 0 | -1 |
| $d$ | 1 | ( 1, 1) ( 1, 3) ( 0,-1) | 6 | -6 | - | - | - | - | - | - | -28 | 52 | 40 | -40 | 30 | -33 | -33 | 30 | 3 |
| $e$ | 1 | ( 1, 3) ( 9,-1) ( 1, 0) | 0 | 0 | - | - | - | - | - | - | - | - | 0 | 0 | -56 | 7 | 7 | -56 | 0 |
| $f$ | 1 | ( 1,-9) ( 3, 1) ( 1, 0) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | 14 | 35 | 35 | 14 | 0 |
| $g$ | 1 | ( 0, 1) ( 7, 1) (-3,-7) | 14 | -14 | - | - | - | - | - | - | - | - | - | - | - | - | 0 | 0 | 7 |
| $h$ | 1 | ( 0, 1) ( 7,-1) ( 3,-7) | -14 | 14 | - | - | - | - | - | - | - | - | - | - | - | - | - | - | -7 |
| $O6$ | 8 | ( 1, 0) ( 2, 0) ( 1, 0) | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - | - |

TABLE XIII: D6-brane configurations and intersection numbers for the Model FSU(5)-III on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $U(5) \times U(1)^7 \times USp(16)$, and the complex structure parameters are $\chi_1 = 1/9$, $\chi_2 = 6$, and $\chi_3 = 1$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -6(3q+2)$, $a = 12$, and $m = 2$.

The four global $U(1)$'s are

$$
\begin{aligned}
U(1)_1 &= -15U(1)_a + 3U(1)_c + 27U(1)_e - 27U(1)_f - 21U(1)_g + 21U(1)_h \ , \\
U(1)_2 &= -U(1)_e + U(1)_f \ , \\
U(1)_3 &= U(1)_b - U(1)_d \ , \\
U(1)_4 &= 5U(1)_a - 3U(1)_b - U(1)_c + 3U(1)_d + 7U(1)_g - 7U(1)_h \ .
\end{aligned} \tag{68}
$$

And the other massless $U(1)$'s are:

$$
\begin{aligned}
U(1)_U &= U(1)_e + U(1)_f - U(1)_g - U(1)_h \ , \\
U(1)_V &= -10U(1)_a - 50U(1)_c + 13U(1)_e + 13U(1)_f + 13U(1)_g + 13U(1)_h \ , \\
U(1)_W &= 35U(1)_a - 7U(1)_c - 13U(1)_g + 13U(1)_h \ .
\end{aligned} \tag{69}
$$

| Rep. | Multi. | $U(1)_a$ | $U(1)_b$ | $U(1)_c$ | $U(1)_d$ | $U(1)_e$ | $U(1)_f$ | $U(1)_g$ | $U(1)_h$ | $U(1)_X$ | $U(1)_1$ | $U(1)_2$ | $U(1)_3$ | $U(1)_4$ | $U(1)_U$ | $U(1)_V$ | $U(1)_W$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(\mathbf{10}, \mathbf{1})$ | 3 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -30 | 0 | 0 | 10 | 0 | -20 | 70 |
| $(\bar{\mathbf{5}}_a, \mathbf{1}_b)$ | 3 | -1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -3 | 15 | 0 | 1 | -8 | 0 | -20 | 70 |
| $(\mathbf{1}_c, \bar{\mathbf{1}}_d)$ | 3 | 0 | 0 | 1 | -1 | 0 | 0 | 0 | 0 | 5 | 3 | 0 | 1 | -4 | 0 | -50 | -7 |
| $(\mathbf{10}, \mathbf{1})$ | 1 | 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | -30 | 0 | 0 | 10 | 0 | -20 | 70 |
| $(\overline{\mathbf{10}}, \mathbf{1})$ | 1 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 30 | 0 | 0 | -10 | 0 | 20 | -70 |
| $(\mathbf{5}_a, \bar{\mathbf{1}}_c)^\star$ | 1 | 1 | 0 | -1 | 0 | 0 | 0 | 0 | 0 | -2 | -18 | 0 | 0 | 6 | 0 | 40 | 42 |
| $(\bar{\mathbf{5}}_a, \mathbf{1}_c)^\star$ | 1 | -1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 2 | 18 | 0 | 0 | -6 | 0 | -40 | -42 |
| $(\bar{\mathbf{1}}_c, \mathbf{1}_e)$ | 4 | 0 | 0 | -1 | 0 | 1 | 0 | 0 | 0 | 0 | 24 | -1 | 0 | 1 | 1 | 63 | 7 |
| $(\overline{\mathbf{15}}, \mathbf{1})$ | 2 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 30 | 0 | 0 | -10 | 0 | 20 | -70 |
| $(\overline{\mathbf{10}}, \mathbf{1})$ | 1 | -2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 30 | 0 | 0 | -10 | 0 | 20 | -70 |
| Additional chiral and non-chiral Matter | | | | | | | | | | | | | | | | | |

TABLE XIV: The particle spectrum in the observable sector in the Model FSU(5)-III, with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like matter.

### E. Model FSU(5)-IV

We present the D6-brane configurations and intersection numbers for the Model FSU(5)-IV on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold in Tables XV and XVI, and its particle spectrum in the observable sector in Table XVII.

The $U(1)_X$ gauge symmetry is

$$\begin{aligned} U(1)_X \; &= \; \frac{1}{2}(U(1)_a - 5U(1)_b + 5U(1)_c + 5U(1)_d + 5U(1)_e + 5U(1)_f - 5U(1)_g \\ &\quad -5U(1)_h + 5U(1)_i - 5U(1)_j - 5U(1)_k) \; . \end{aligned} \tag{70}$$

And the four global $U(1)$'s are

$$\begin{aligned} U(1)_1 \; &= \; 6U(1)_c - 90U(1)_e - 18U(1)_g + 48U(1)_i - 6U(1)_j - 12U(1)_k \; , \\ U(1)_2 \; &= \; 2U(1)_b - 2U(1)_c + 30U(1)_e - 12U(1)_f + 14U(1)_h \; , \\ U(1)_3 \; &= \; -10U(1)_a - 2U(1)_d + 12U(1)_f + 6U(1)_g + 2U(1)_j \; , \\ U(1)_4 \; &= \; 30U(1)_a - 6U(1)_b + 6U(1)_d - 42U(1)_h - 48U(1)_i + 12U(1)_k \; . \end{aligned} \tag{71}$$

| stk | $N$ | $(n_1,l_1)(n_2,l_2)(n_3,l_3)$ | A | S | $b$ | $b'$ | $c$ | $c'$ | $d$ | $d'$ | $e$ | $e'$ | $f$ | $f'$ | $g$ | $g'$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a$ | 10 | ( 1, 3) ( 1, 1) ( 0,-1) | 2 | -2 | -3 | 0(1) | 0(2) | 0(3) | 0 | 0 | 24 | 24 | 12 | -6 | 6 | -3 |
| $b$ | 2 | ( 1,-3) ( 0,-1) (-1, 1) | -2 | 2 | - | - | -3 | 0(1) | 2 | -1 | -15 | -60 | -12 | 6 | 12 | 12 |
| $c$ | 2 | ( 1, 3) (-1, 1) (-1, 0) | 2 | -2 | - | - | - | - | 4 | 4 | 0 | 0 | 18 | 36 | -18 | -9 |
| $d$ | 2 | ( 1, 1) (-1,-3) ( 0, 1) | 2 | -2 | - | - | - | - | - | - | 84 | -36 | 6 | 0 | 15 | -12 |
| $e$ | 2 | (-5, 9) (-5,-3) ( 1, 0) | -30 | 30 | - | - | - | - | - | - | - | - | -486 | -324 | 162 | 243 |
| $f$ | 2 | ( 2, 0) (-1, 3) (-1,-3) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | 0 | -162 |
| $g$ | 2 | ( 1,-9) (-1, 0) (-1,-3) | -6 | 6 | - | - | - | - | - | - | - | - | - | - | - | - |
| $h$ | 2 | ( 1,-7) ( 0, 1) ( 7,-3) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - |
| $i$ | 2 | ( 0, 2) ( 4,-3) ( 3,-4) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - |
| $j$ | 2 | ( 1,-3) (-1, 0) (-1,-1) | -2 | 2 | - | - | - | - | - | - | - | - | - | - | - | - |
| $k$ | 2 | ( 0, 2) (-3,-1) ( 1, 3) | 0 | 0 | - | - | - | - | - | - | - | - | - | - | - | - |
| $O6^1$ | 10 | ( 2, 0) ( 1, 0) ( 1, 0) | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| $O6^2$ | 10 | ( 2, 0) ( 0,-1) ( 0, 1) | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| $O6^3$ | 8 | ( 0,-2) ( 1, 0) ( 0, 1) | - | - | - | - | - | - | - | - | - | - | - | - | - | - |
| $O6^4$ | 2 | ( 0,-2) ( 0, 1) ( 1, 0) | - | - | - | - | - | - | - | - | - | - | - | - | - | - |

TABLE XV: D6-brane configurations and intersection numbers (Part 1) for the Model FSU(5)-IV on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2}\times\mathbb{Z}_\mathbf{2})$ orientifold. The complete gauge symmetry is $U(5)\times U(1)^{10}\times USp(10)^2\times USp(8)\times USp(2)$, and the complex structure parameters are $\chi_1 = 2/3$, $\chi_2 = 1$, and $\chi_3 = 1$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -2(3q+8)$, $a = 16$, and $m = 8$.

There are six other massless $U(1)$'s. As an example, we present two of them:

$$\begin{aligned} U(1)_U &= -10U(1)_b + U(1)_c - U(1)_e - 2U(1)_f + 4U(1)_g + 2U(1)_h + 2U(1)_k \ , \\ U(1)_V &= 125U(1)_b - 80U(1)_c + 26U(1)_e - 85U(1)_h + 47U(1)_i - 47U(1)_k \ . \end{aligned} \tag{72}$$

## VI. DISCUSSION AND CONCLUSIONS

On Type IIA orientifolds with flux compactifications in supersymmetric AdS vacua, we for the first time constructed the exact three-family $SU(5)$ models. In these models, we have three $\mathbf{10}$ representations, and obtain three $\overline{\mathbf{5}}$ representations after the additional gauge symmetry breaking via supersymmetry preserving Higgs mechanism. So, there are exact

| stk | $N$ | $(n_1,l_1)(n_2,l_2)(n_3,l_3)$ | $h$ | $h'$ | $i$ | $i'$ | $j$ | $j'$ | $k$ | $k'$ | $O6^1$ | $O6^2$ | $O6^3$ | $O6^4$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $a$ | 10 | ( 1, 3) ( 1, 1) ( 0,-1) | -35 | -14 | -21 | 3 | 3 | 0 | 2 | -4 | 3 | 0 | 0 | -1 |
| $b$ | 2 | ( 1,-3) ( 0,-1) (-1, 1) | 0 | 0 | 4 | 28 | 0 | 0 | 12 | 6 | -3 | 0 | 1 | -3 |
| $c$ | 2 | ( 1, 3) (-1, 1) (-1, 0) | 15 | -6 | -4 | -28 | -3 | 0 | -12 | -6 | 0 | 3 | -1 | 0 |
| $d$ | 2 | ( 1, 1) (-1,-3) ( 0, 1) | -28 | -21 | -45 | 27 | 6 | -3 | 8 | -10 | 3 | 0 | 3 | -1 |
| $e$ | 2 | (-5, 9) (-5,-3) ( 1, 0) | 195 | -330 | 540 | -60 | 9 | -36 | 60 | 210 | 0 | -45 | 15 | 0 |
| $f$ | 2 | ( 2, 0) (-1, 3) (-1,-3) | 168 | 126 | -234 | 150 | 18 | -36 | 0 | -96 | 0 | 0 | -6 | 6 |
| $g$ | 2 | ( 1,-9) (-1, 0) (-1,-3) | -24 | 144 | 39 | 15 | 0 | 0 | 0 | 6 | 0 | -9 | 0 | 3 |
| $h$ | 2 | ( 1,-7) ( 0, 1) ( 7,-3) | - | - | 76 | 0 | -20 | 20 | 72 | 54 | -21 | 0 | 7 | 0 |
| $i$ | 2 | ( 0, 2) ( 4,-3) ( 3,-4) | - | - | - | - | -21 | -3 | 0 | 0 | -24 | 24 | 0 | 0 |
| $j$ | 2 | ( 1,-3) (-1, 0) (-1,-1) | - | - | - | - | - | - | -2 | 4 | 0 | -3 | 0 | 1 |
| $k$ | 2 | ( 0, 2) (-3,-1) ( 1, 3) | - | - | - | - | - | - | - | - | 6 | -6 | 0 | 0 |
| $O6^1$ | 10 | ( 2, 0) ( 1, 0) ( 1, 0) | - | - | - | - | - | - | - | - | - | - | - | - |
| $O6^2$ | 10 | ( 2, 0) ( 0,-1) ( 0, 1) | - | - | - | - | - | - | - | - | - | - | - | - |
| $O6^3$ | 8 | ( 0,-2) ( 1, 0) ( 0, 1) | - | - | - | - | - | - | - | - | - | - | - | - |
| $O6^4$ | 2 | ( 0,-2) ( 0, 1) ( 1, 0) | - | - | - | - | - | - | - | - | - | - | - | - |

TABLE XVI: D6-brane configurations and intersection numbers (Part 2) for the Model FSU(5)-IV on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold.

three families of the SM fermions, and no chiral exotic particles that are charged under $SU(5)$. In addition, we can break the $SU(5)$ gauge symmetry down to the SM gauge symmetry via D6-brane splitting, and solve the doublet-triplet splitting problem. If the extra one (or several) pair(s) of Higgs doublets and adjoint particles obtain GUT/string scale masses via high-dimensional operators, we only have the MSSM in the observable sector below the GUT scale. Choosing suitable grand unified gauge coupling by adjusting the string scale, we can explain the observed low energy gauge couplings via RGE running. However, how to generate the up-type quark Yukawa couplings, which are forbidden by the global $U(1)$ symmetry, deserves further study.

Furthermore, we considered the flipped $SU(5)$ models. In order to have at least one pair of Higgs fields $\mathbf{10}$ and $\overline{\mathbf{10}}$, we must have the symmetric representations, and then the net number of $\overline{\mathbf{5}}$ and $\mathbf{5}$ can not be three if the net number of $\mathbf{10}$ and $\overline{\mathbf{10}}$ is three due to the non-abelian anomaly free condition. We constructed the first model with three $\mathbf{10}$

| Rep. | Multi. | $U(1)_X$ | $U(1)_1$ | $U(1)_2$ | $U(1)_3$ | $U(1)_4$ | $U(1)_U$ | $U(1)_V$ | $\cdots$ |
|---|---|---|---|---|---|---|---|---|---|
| $(\mathbf{10},\mathbf{1})$ | 3 | 1 | 0 | 0 | -20 | 60 | 0 | 0 | $\cdots$ |
| $(\bar{\mathbf{5}}_a,\mathbf{1}_b)$ | 3 | -3 | 0 | 2 | 10 | -36 | -10 | 125 | $\cdots$ |
| $(\bar{\mathbf{1}}_b,\mathbf{1}_c)$ | 3 | 5 | 6 | -4 | 0 | 6 | 11 | -205 | $\cdots$ |
| $(\mathbf{10},\mathbf{1})$ | 1 | 1 | 0 | 0 | -20 | 60 | 0 | 0 | $\cdots$ |
| $(\overline{\mathbf{10}},\mathbf{1})$ | 1 | -1 | 0 | 0 | 20 | -60 | 0 | 0 | $\cdots$ |
| $(\mathbf{5}_a,\mathbf{1}_b)^\star$ | 1 | -2 | 0 | 2 | -10 | 24 | -10 | 125 | $\cdots$ |
| $(\bar{\mathbf{5}}_a,\bar{\mathbf{1}}_b)$ | 1 | 2 | 0 | -2 | 10 | -24 | 10 | -125 | $\cdots$ |
| $(\mathbf{1}_c,\bar{\mathbf{1}}_d)$ | 4 | 0 | 6 | -2 | 2 | -6 | 1 | -80 | $\cdots$ |
| $(\overline{\mathbf{15}},\mathbf{1})$ | 2 | -1 | 0 | 0 | 20 | -60 | 0 | 0 | $\cdots$ |
| $(\overline{\mathbf{10}},\mathbf{1})$ | 1 | -1 | 0 | 0 | 20 | -60 | 0 | 0 | $\cdots$ |
| Additional chiral and non-chiral Matter | | | | | | | | | |

TABLE XVII: The particle spectrum in the observable sector in the Model FSU(5)-IV, with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations indicate vector-like matter.

representations, and the first model where all the Yukawa couplings are allowed by the global $U(1)$ symmetries.

**Acknowledgments**

T.L. would like to thank R. Blumenhagen for helpful discussions. The research of T.L. was supported by DOE grant DE-FG02-96ER40959, and the research of D.V.N. was supported by DOE grant DE-FG03-95-Er-40917.

---

[1] S. M. Barr, Phys. Lett. B **112**, 219 (1982); Phys. Rev. D **40**, 2457 (1989).

[2] J. P. Derendinger, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B **139**, 170 (1984).

[3] I. Antoniadis, J. R. Ellis, J. S. Hagelin and D. V. Nanopoulos, Phys. Lett. B **194** (1987) 231; Phys. Lett. B **205** (1988) 459; Phys. Lett. B **208** (1988) 209 [Addendum-ibid. B **213** (1988)

562]; Phys. Lett. B **231** (1989) 65; J. L. Lopez, D. V. Nanopoulos and K. J. Yuan, Nucl. Phys. B **399**, 654 (1993).

[4] T. Li, J. L. Lopez and D. V. Nanopoulos, Phys. Rev. D **56**, 2602 (1997).

[5] J. Polchinski and E. Witten, Nucl. Phys. B **460**, 525 (1996).

[6] M. Berkooz, M. R. Douglas and R. G. Leigh, Nucl. Phys. B **480** (1996) 265.

[7] C. Bachas, hep-th/9503030.

[8] R. Blumenhagen, L. Goerlich, B. Kors and D. Lust, JHEP **0010**, 006 (2000).

[9] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, Phys. Lett. B **489**, 223 (2000).

[10] R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, hep-th/0502005, and the references therein.

[11] M. Cvetič, G. Shiu and A. M. Uranga, Phys. Rev. Lett. **87**, 201801 (2001); Nucl. Phys. B **615**, 3 (2001).

[12] M. Cvetič and I. Papadimitriou, Phys. Rev. D **67**, 126006 (2003).

[13] M. Cvetič, I. Papadimitriou and G. Shiu, Nucl. Phys. B **659**, 193 (2003) [Erratum-ibid. B **696**, 298 (2004)].

[14] M. Cvetič, T. Li and T. Liu, Nucl. Phys. B **698**, 163 (2004).

[15] M. Cvetič, P. Langacker, T. Li and T. Liu, Nucl. Phys. B **709**, 241 (2005).

[16] C.-M. Chen, G. V. Kraniotis, V. E. Mayes, D. V. Nanopoulos and J. W. Walker, Phys. Lett. B **611**, 156 (2005); Phys. Lett. B **625**, 96 (2005).

[17] C.-M. Chen, T. Li and D. V. Nanopoulos, Nucl. Phys. B **732**, 224 (2006).

[18] F. Gmeiner and M. Stein, hep-th/0603019.

[19] M. Cvetič, P. Langacker and G. Shiu, Phys. Rev. D **66**, 066004 (2002); Nucl. Phys. B **642**, 139 (2002).

[20] M. Cvetič, P. Langacker and J. Wang, Phys. Rev. D **68**, 046002 (2003).

[21] R. Blumenhagen, L. Görlich and T. Ott, JHEP **0301**, 021 (2003); G. Honecker, Nucl. Phys. **B666**, 175 (2003); G. Honecker and T. Ott, Phys. Rev. D **70**, 126010 (2004) [Erratum-ibid. D **71**, 069902 (2005)].

[22] S. Gukov, C. Vafa and E. Witten, Nucl. Phys. B **584**, 69 (2000) [Erratum-ibid. B **608**, 477 (2001)].

[23] J. F. G. Cascales and A. M. Uranga, JHEP **0305**, 011 (2003).

[24] R. Blumenhagen, D. Lüst and T. R. Taylor, Nucl. Phys. B **663**, 319 (2003).

[25] F. Marchesano and G. Shiu, Phys. Rev. D **71**, 011701 (2005); JHEP **0411**, 041 (2004).

[26] M. Cvetič and T. Liu, Phys. Lett. B **610**, 122 (2005).

[27] M. Cvetič, T. Li and T. Liu, Phys. Rev. D **71**, 106008 (2005).

[28] J. Kumar and J. D. Wells, hep-th/0506252.

[29] C. M. Chen, V. E. Mayes and D. V. Nanopoulos, Phys. Lett. B **633**, 618 (2006).

[30] G. Aldazabal, P. G. Camara, A. Font and L. E. Ibanez, hep-th/0602089.

[31] G. Villadoro and F. Zwirner, hep-th/0602120.

[32] T. W. Grimm and J. Louis, Nucl. Phys. B **718**, 153 (2005).

[33] G. Villadoro and F. Zwirner, JHEP **0506**, 047 (2005).

[34] P. G. Camara, A. Font and L. E. Ibanez, hep-th/0506066.

[35] P. G. Camara, hep-th/0512239.

[36] C. M. Chen, T. Li and D. V. Nanopoulos, Nucl. Phys. B **740**, 79 (2006).

[37] R. Blumenhagen, B. Körs and D. Lüst, JHEP **0102** (2001) 030.

[38] E. Witten, JHEP **9812**, 019 (1998).

[39] F. G. Marchesano Buznego, hep-th/0307252.

[40] T. Li and T. Liu, Phys. Lett. B **573**, 193 (2003).