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Ghost-Free de Sitter Supergravities as Consistent Reductions of String and M-theory

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ABSTRACT

We study properties of supergravity theories with non-compact gaugings, and their higher-dimensional interpretations via consistent reductions on the inhomogeneous non-compact hyperboloidal spaces $\mathcal{H}^{p,q}$. The gauged supergravities are free of ghosts, despite the non-compactness of the gauge groups. We give a general discussion of the existence of stationary points in the scalar potentials of such supergravities. These are of interest since they can be associated with de Sitter vacuum configurations. We give explicit results for consistent reductions on $\mathcal{H}^{p,q}$ in various examples, derived from analytic continuation of previously-known consistent sphere reductions. In addition we also consider black hole and cosmological solutions, for specific examples of non-compact gaugings in $D = 4, 5, 7$.

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Contents

1 Introduction

Over the past few years important progress [1–6] has been made in understanding the full non-linear structure of certain Kaluza-Klein sphere reductions of string and M-theory, leading to gauged supergravities with maximal supersymmetry in lower dimensions. These reductions, also referred to as Pauli reductions [7], are consistent only for specific supergravity theories compactified on spheres of specific dimension [5,7]. In particular, they lead to gauged supergravity theories with anti-de Sitter vacua having a *negative cosmological constant*, or to dilatonic vacua corresponding to domain wall solutions with a potential of the type $-|\lambda|\, e^{\phi}$ [2].

On the other hand the origin of de-Sitter vacua arising from consistent reductions of string and M-theory is less well studied. It is known that there exist gauged supergravities with non-compact gauge groups, which can be obtained from the usual compact gaugings by means of appropriate analytic continuations. These were extensively studied in [8–10]. With the more recent advances in understanding the higher-dimensional origins of the compact gauged supergravities via consistent sphere reductions, it is therefore worthwhile re-examining the non-compact gaugings, with a view to studying their higher-dimensional origins from string or M-theory.

The essential features of the geometrical structures involved in the reductions to non-compact gauged supergravities can be summarised as follows. For the compact gauged theories, notably in $D = 7$, 5 and 4 dimensions, one makes reductions of $D = 11$ supergravity on S^4, type IIB supergravity on S^5 or $D = 11$ supergravity on S^7 respectively. In many cases, the general structure of the internal S^n metric is

$$ds^2 = T^{-1}_{AB}\, d\mu^A\, d\mu^B\,, \tag{1.1}$$

where T^{-1}_{AB} is a matrix of scalar fields, and μ^A are coordinates on $\mathbb{R}^{n+1}$, subject to the constraint

$$\delta_{AB}\, \mu^A\, \mu^B = 1 \tag{1.2}$$

which restricts the μ^A to lie on the sphere. In each case there exists a ground state where the lower-dimensional scalar fields vanish, corresponding to $T_{AB} = \delta_{AB}$, and the internal metric becomes that of the round sphere S^n, as

$$ds^2 = \delta_{AB}\, d\mu^A\, d\mu^B\,. \tag{1.3}$$

Here, of course, both (1.2) and (1.3) are invariant under $SO(n+1)$, and thus the internal manifold in the vacuum state has the $SO(n+1)$ isometry of the round sphere S^n.

In the reduction to the non-compact gauging, the constraint (1.2) is replaced by

$$\eta_{AB}\,\mu^A\,\mu^B = 1\,, \tag{1.4}$$

where $\eta_{AB} = \mathrm{diag}(1, 1, \ldots, 1, -1, -1, \ldots -1)$, with p eigenvalues $+1$, and $q = n+1-p$ eigenvalues -1. However, the "trivial" scalar configuration (which may, or may not, be a stationary point of the scalar potential) is still given by $T_{AB} = \delta_{AB}$. Thus the internal metric in the trivial-scalar configuration is given by

$$ds^2 = \delta_{AB}\,d\mu^A\,d\mu^B\,, \qquad \eta_{AB}\,\mu^A\,\mu^B = 1\,. \tag{1.5}$$

The first equation is invariant under $SO(p+q)$ while the second is invariant under $SO(p,q)$, and so the metric is invariant under the common subgroup, $SO(p)\times SO(q)$. The spaces described by these metrics are hyperboloidal, and are designated by $\mathcal{H}^{p,q}$ [8,10]. Note that the metrics are always positive definite. When p and q are both non-zero, the space $\mathcal{H}^{p,q}$ is *non-compact*, and furthermore it is *inhomogeneous* (i.e. it is not a coset space). Particular reductions of supergravity theories on the non-compact hyperboloidal spaces $\mathcal{H}^{p,q}$ yield theories in lower dimensions that may have vacuum solutions with positive cosmological constant. The scalar fields T_{AB} play an essential role in these theories, in ensuring that all the lower-dimensional gauge fields (as well as the scalars themselves) have standard positive-energy kinetic terms, despite the occurrence of non-compact gauge groups.[1]

It is worth emphasising the distinction between the nature of the reductions we are considering here, which involve the inhomogeneous non-compact spaces $\mathcal{H}^{p,q}$, and reductions involving non-compact spaces that have non-compact isometry groups. In our reductions, the fiducial metric defined by (1.5) necessarily has a *compact* isometry group, and this lies at the heart of why one obtains lower-dimensional theories with no ghost-like gauge fields associated with "wrong-sign" kinetic terms. The full non-compact $SO(p,q)$ gauge group is always spontaneously broken in any solution, and in fact the residual unbroken gauge group is always compact. By contrast, if one performs a reduction on a space with a *non-compact* isometry group, such as the homogeneous hyperbolic plane $\mathbf{H}^2 = SL(2,\mathbb{R})/O(2)$, there will always be ghost-like gauge fields associated with the non-compact generators of the isometry group. This is because, in a linearised analysis of small fluctuations around

[1] There are also reductions of the so-called *-theories [11] on the hyperboloidal spaces with positive definite metric (see e.g., [12] and references therein) may provide examples of stable de-Sitter vacua of anti-de Sitter vacua (see, e.g., [13] and references therein). Note however that in these cases the *-theory already suffers from ghost-fields and the non-linear Kaluza-Klein ansatz for the p-form field strengths involves complex values.

the vacuum solution, the Yang-Mills field strengths in the kinetic terms will be contracted with the indefinite-signature Cartan-Killing metric of the non-compact isometry group.[2]

If we consider the example of 2-dimensional non-compact spaces, the two contrasting situations can be illustrated by considering $\mathcal{H}^{2,1}$ and $\mathbf{H}^2$, defined by

$$\begin{aligned} \mathcal{H}^{2,1}: &\qquad ds^2 = d\mu_1^2 + d\mu_2^2 + d\mu_3^2\,, \qquad \mu_1^2 + \mu_2^2 - \mu_3^2 = 1\,, \\ \mathbf{H}^2: &\qquad d\tilde{s}^2 = d\mu_1^2 + d\mu_2^2 - d\mu_3^2\,, \qquad \mu_1^2 + \mu_2^2 - \mu_3^2 = 1\,. \end{aligned} \tag{1.6}$$

Both of these metrics have positive-definite signature, but $\mathcal{H}^{2,1}$ is inhomogeneous, with the isometry group $O(2)$, while $\mathbf{H}^2$ is homogeneous, with isometry group $O(2,1)$. The $\mathcal{H}^{2,1}$ metric provides a basis for reducing to give an $O(2,1)$ gauged ghost-free supergravity with (at most) a surviving $O(2)$ gauge group in the vacuum. By contrast, the $\mathbf{H}^2$ metric could yield an $O(2,1)$ gauged theory which would have indefinite-signature kinetic terms for the gauge fields, and a vacuum with a surviving $O(2,1)$ gauge group. (The question of whether one can find *consistent* reductions, for which the massive Kaluza-Klein towers can be set to zero, is a more subtle one. However, this is quite distinct from the present question of whether or not the kinetic terms for the gauge fields have the correct sign for ghost-freedom.)

The purpose of this paper is to analyse possible consistent reductions of string and M-theory on hyperboloidal spaces $\mathcal{H}^{p,q}$, and the properties of the vacuum solutions for the resulting theories in lower dimensions. The analysis is facilitated by the fact that we can obtain these reductions as analytic continuation of sphere reductions whose consistent non-linear Kaluza-Klein Ansätze have been extensively studied [1, 3, 4, 6].

Our discussions will focus on the bosonic sectors of the supergravity theories, and so we will not generally be explicitly addressing the important question of whether the reductions and truncations we study are also compatible with supersymmetry. However, the general results on non-compact gaugings in works such as [8, 9] show that the necessary supersymmetric completions of the bosonic sectors do indeed exist, and this provides compelling evidence to support the idea that our consistent reductions can be extended to include the fermionic sectors.

The paper is organised as follows. In section 2 we analyse the extrema of the scalar field potentials that generically arise in theories obtained from a reduction of string and M-theory on hyperboloidal spaces $\mathcal{H}^{p,q}$, and for completeness we analyse the extrema of the

[2]It should be stressed, therefore, that it is the signature of the Cartan-Killing metric of the isometry group, and *not* the signature of the metric on the internal space itself, that governs the signs of the gauge-field kinetic terms.

potentials arising from reductions on spheres S^{p-1}. At such extrema one can truncate the theory to fixed values of the scalar fields. In the case of more that one extremum (as is the case for compact reductions) one can also address the properties of renormalisation group flows (in the dual field theories) interpolating between such (non-positive) stationary points of the scalar potential. Note that potentials arising from hyperboloidal reductions always turn out to have positive definite extrema, while in the sphere reductions the potentials are negative at the extrema. In the case of S^{p-1} with $p \leq 3$ there is only one extremum, while for $p \geq 4$ there is a second extremum with a larger negative value of the potential.

In section 3 we discuss the non-linear Kaluza-Klein Ansätze for Pauli reductions on both compact (spheres S^{p+q-1}) and non-compact (hyperboloidal $\mathcal{H}^{p,q}$) spaces, by employing a description of scalar fields in terms of the vielbeine on the scalar coset manifold $SL(n, \mathbb{R})/SO(n)$ $(n = p + q)$. It should be emphasised that although the non-compact reductions are derived from the compact ones by analytic continuation, the internal manifolds $\mathcal{H}^{p,q}$ are inhomogeneous, even though the original spheres S^{p+q+1} are homogeneous spaces. The explicit example that is described in detail is that of $(p, q) = (4, 4)$, a reduction of 11-dimensional supergravity on S^7 and $\mathcal{H}^{4,4}$. Other consistent examples of sphere reductions can analogously be discussed in the context of hyperboloidal reductions as well.

In section 4 we focus on the study of de Sitter supergravity in four-dimensions, beginning with the $N = 4$ theory, obtained as a Pauli reduction of 11-dimensional supergravity on the hyperboloidal space $\mathcal{H}^{4,4}$ space. In particular, we employ an analytic continuation to derive this reduction from the corresponding consistent reduction on S^7. In addition, we discuss a truncation of this theory to $N = 2$ de Sitter supergravity, which makes contact with a recent result in the literature [14]. We also consider more general $N = 2$ theories in four dimensions, obtaining via analytic continuations examples corresponding to reductions on $\mathcal{H}^{4,4}$ and $\mathcal{H}^{6,2}$. In section 5, we consider examples of de Sitter type gauged supergravities in five and seven dimensions.

In section 6, we present examples of charged black hole and cosmological solutions for specific examples of non-compact gaugings in $D = 4, 5, 7$. These solutions are related to the corresponding multi-charged black holes of AdS gauged supergravities. Concluding remarks are given in section 7.

2 Extrema of Scalar Potentials

In this section, we shall focus our attention on scalar potentials of the form

$$V = 2T_{ij}\, T_{ij} - (T_{ii})^2\,, \tag{2.1}$$

where T_{ij} is a symmetric matrix of scalar fields. There are several examples in gauged supergravities where potentials of this type arise from dimensional reductions on spheres. Commonly, but not always, the scalar matrix is unimodular. To encompass also those cases where it is not restricted to be unimodular, it is useful to extract the determinant by defining

$$T_{ij} = \Phi\, \widetilde T_{ij}\,, \qquad \hbox{where} \quad \det(\widetilde T_{ij}) = 1\,, \tag{2.2}$$

and thus we have

$$V = \Phi^2\, \widetilde V\,, \qquad \widetilde V \equiv 2\widetilde T_{ij}\, \widetilde T_{ij} - (\widetilde T_{ii})^2\,, \tag{2.3}$$

Our study will begin by considering the stationary points of $\widetilde V$. One application of these results will be for discussing the circumstances under which one can perform a consistent truncation of the scalar fields in the associated gauged supergravity. In order to be able to truncate the theory to fixed values of the scalars $\widetilde T_{ij}$, the necessary condition, dictated by the equations of motion, is that these fixed values correspond to a stationary point of their potential.

The scalar matrix $\widetilde T_{ij}$ is conveniently parameterised in terms of a scalar vielbein $\widetilde\Pi_A{}^i$, in terms of which one has

$$\widetilde T_{ij} = \widetilde\Pi_i^{-1\, A}\, \widetilde\Pi_j^{-1\, B}\, \eta_{AB}\,. \tag{2.4}$$

We shall consider the situation of scalars associated with a gauging of $SO(p,q)$, for which we shall have the $SO(p,q)$ invariant metric

$$\eta_{AB} = \hbox{diag}\,(+1,+1,\ldots,+1,-1,-1,\ldots,-1)\,, \tag{2.5}$$

with p plus signs and q minus signs. The vielbein $\widetilde\Pi_A{}^i$ parameterises the scalar coset manifold $SL(n,\mathbb{R})/SO(n)$, where $n=p+q$. Note that the denominator group $SO(p+q)$ is always compact regardless of whether the gauge group $SO(p,q)$ is compact or non-compact. Thus i,j are $SO(p+q)$ indices, raised and lowered with δ_{ij}, while A,B are $SO(p,q)$ indices, raised and lowered with η_{AB}.

In order to study the extrema of the potential $\widetilde V$, it is convenient to perform local transformations to diagonalise the scalar vielbein, implying that we can write

$$\tilde T_{ij} = \hbox{diag}\,(X_1, X_2,\ldots,X_p,-X_{\bar 1},-X_{\bar 2},\cdots,-X_{\bar q})\;, \tag{2.6}$$

where X_a and $X_{\bar{a}}$ are all positive, subject to the unimodularity constraint:

$$\prod_{a=1}^{p} X_a \prod_{\bar{a}=1}^{q} X_{\bar{a}} = 1\,. \tag{2.7}$$

Note that we must have $q = 2r$, where r is an integer, in order to have $\det(\widetilde{T}_{ij}) = +1$ with positive X_a and $X_{\bar{a}}$.

The potential $\widetilde{V}$ now takes the form:

$$\widetilde{V} = 2\sum_{a=1}^{p} X_a^2 + 2\sum_{\bar{a}=1}^{q} X_{\bar{a}}^2 - \Big(\sum_{a=1}^{p} X_a - \sum_{\bar{a}=1}^{q} X_{\bar{a}}\Big)^2\,. \tag{2.8}$$

Without loss of generality we may assume that $p \ge q$, since if q exceeded p we could simply redefine our notion of what is a time-like and what a space-like direction. (The overall sign of $\widetilde{T}_{ij}$ plays no role in the analysis.)

The discussion at this stage divides into two cases, depending on whether $q = 2r = 0$ (the compact case), or $q = 2r \ge 2$ (the non-compact case). We shall begin by considering the non-compact case.

2.1 Non-compact Case

In this subsection we shall consider non-compact cases, where $q = 2r \ge 2$.

The extrema of $\widetilde{V}$ can be determined by introducing a Lagrange multiplier to enforce the constraint (2.7), and defining

$$S = \widetilde{V} + \lambda\Big(\prod_{a=1}^{p} X_a \prod_{\bar{a}=1}^{q} X_{\bar{a}} - 1\Big)\,. \tag{2.9}$$

The equations following from requiring S to be stationary under the variations of the X_a and $X_{\bar{a}}$ imply

$$X_a^2 - 2\,\sigma_-\, X_a + \tfrac{1}{4}\lambda \;\;=\;\; 0\,,\quad a = 1,\cdots,p\,, \tag{2.10}$$

$$X_{\bar{a}}^2 + 2\,\sigma_-\, X_{\bar{a}} + \tfrac{1}{4}\lambda \;\;=\;\; 0\,,\quad b = 1,\cdots,q\,, \tag{2.11}$$

where $\sigma_- \equiv \frac{1}{4}(\sum_a X_a\, - \sum_{\bar{a}} X_{\bar{a}})$.

The solutions for X_a and $X_{\bar{b}}$ can in principle each have two values:

$$X_a = \sigma_- \pm \sqrt{\sigma_-^2 - \tfrac{1}{4}\lambda}\,,\qquad X_{\bar{a}} = -\sigma_- \pm \sqrt{\sigma_-^2 - \tfrac{1}{4}\lambda}\,. \tag{2.12}$$

However, it follows from (2.10) and (2.11) that

$$4(X_a + X_{\bar{a}}) + \lambda\,(X_a^{-1} + X_{\bar{a}}^{-1}) = 0 \tag{2.13}$$

for any a and $\bar{a}$, and so the positivity of the X_a and $X_{\bar{a}}$ implies that $\lambda < 0$. Consequently, the positivity of X_a and $X_{\bar{a}}$ implies that the plus signs must be chosen for both equations in (2.12). Thus we must have all X_a equal, $X_a \equiv X$, and all $X_{\bar{a}}$ equal, $X_{\bar{a}} \equiv \bar{X}$, at any valid stationary point. From Eqs.(2.12) it then follows we shall have

$$\lambda = -4X\,\bar{X}\,, \qquad (p-2)\,X = (q-2)\,\bar{X}\,. \tag{2.14}$$

There is therefore a special case in which $q = 2$ and hence also $p = 2$; otherwise, it must be that $q = 2r \geq 4$, and $p \geq 3$.

For $q = 2r \geq 4$, the explicit solution (2.12)), subject to the constraint (2.7), yields the result:

$$X_a = X = \left(\frac{q-2}{p-2}\right)^{\frac{q}{p+q}}\,, \qquad X_{\bar{a}} = \bar{X} = \left(\frac{p-2}{q-2}\right)^{\frac{p}{p+q}}\,, \tag{2.15}$$

and the potential at the extremum has a positive value:

$$\widetilde{V}_0 = 2\,(p+q)\,\left(\frac{q-2}{p-2}\right)^{\frac{q-p}{q+p}}\,. \tag{2.16}$$

Note that the extremum of $\tilde{V}$ always corresponds to the positive value of the potential. One can also prove that this extremum is always a *saddle point* of the potential. Note that this result is consistent with a general argument that non-compact reductions produce extrema of the scalar potential that have always tachyonic direction [15]. (See however [16] where the non-compact gauging produced an example of de Sitter vacuum that is a minimum of the potential.)

The special case when $p = q = 2$ leads to $X\,\bar{X} = 1$ and $\lambda = -4$ at the stationary point. The value of $X = \bar{X}^{-1}$ is undetermined, meaning there is a “flat direction,” and the potential on this line of stationary points is given by

$$\widetilde{V}_0 = 8\,. \tag{2.17}$$

Cases that arise in supergravities are associated with consistent Pauli reductions on spheres: S^2, S^3, S^4, S^5 or S^7 (or their non-compact versions where S^{p+q-1} is replaced by $\mathcal{H}^{p,q}$), and thus with

$$p+q = 3, 4, 5, 6, 8\,. \tag{2.18}$$

Given our findings above, namely that stationary points for $\widetilde{V}$ arise in the non-compact cases for $q = 2$ with $p = 2$, or $q = 2r \geq 4$ with $p \geq 3$, we see that stationary points will arise only for two non-compact gauge groups associated with consistent Pauli reductions on hyperboloidal spaces, namely $SO(2,2)$ and $SO(4,4)$. (Recall that we can always take

$p \geq q$.) These are associated respectively with the replacements of the following spheres by the corresponding hyperboloidal spaces:

$$S^3 \longrightarrow \mathcal{H}^{2,2}\,, \qquad S^7 \longrightarrow \mathcal{H}^{4,4}\,. \tag{2.19}$$

The first of these arises in the consistent Pauli reduction of type I ten-dimensional supergravity to give $N=2$ gauged supergravity in $D=7$; this reduction was first derived, for the compact choice S^3, in [17]. Its non-compact analogue, with the consistent reduction on $\mathcal{H}^{2,2}$, was recently studied in [18]; the resulting $N=2$, $SO(2,2)$ gauged seven-dimensional supergravity was used in order to obtain the Salam-Sezgin [19] $N=(1,0)$ gauged supergravity in six dimensions, by means of a further S^1 reduction and consistent chiral truncation [18].

The second case in (2.19) arises in the consistent S^7 reduction of eleven-dimensional supergravity [20]. The investigation of the truncations that can be made by reducing instead on $\mathcal{H}^{4,4}$ and then setting the scalar fields in $\widetilde{T}_{ij}$ to their fixed values, corresponding to the extrema of the potential $\tilde{V}$, will be studied in section 4 below.

2.2 Compact case

For completeness we shall also analyse the extrema of the potential arising from the compact cases, such as those arising in the case of consistent sphere reductions. These correspond to taking $q=0$ in the discussion of section 2.1. If a consistent reduction on the sphere S^{p-1} exists, it will give rise to a scalar potential with an $SO(p)$ symmetry. Again, we focus on the case where $\widetilde{T}_{ij}$ is unimodular. At the extrema of the potential $\tilde{V}$ will again a truncation of the the scalar fields $\tilde{T}_{ij}$ to the their fixed valued at these extrema.

Stationary points of the potential (2.8) (with $q=0$) will be governed by (2.10), where $\sigma_- \equiv \frac{1}{4}\sum X_a$. It follows from (2.10) that

$$2(p-2)\sum_a X_a = \lambda \sum_a X_a^{-1}\,, \tag{2.20}$$

and therefore that $\lambda \geq 0$ (since in order to have a non-trivial situation, we must certainly have $p \geq 2$). It then follows that the solutions of (2.10), namely

$$X_a = \sigma_- \pm \sqrt{\sigma_-^2 - \tfrac{1}{4}\lambda} \tag{2.21}$$

can be positive for either choice of sign. Thus in principle we can have

$$X_a = \alpha\,, \quad 1 \leq a \leq m\,; \qquad X_a = \beta\,, \quad m+1 \leq a \leq m+n\,, \tag{2.22}$$

where $\alpha \equiv \sigma_- + \sqrt{\sigma_-^2 - \frac{1}{4}\lambda}$, $\beta \equiv \sigma_- - \sqrt{\sigma_-^2 - \frac{1}{4}\lambda}$ and $m+n=p$.

There are now two possible sets of solutions. The first corresponds to taking all X_a equal, in which case without loss of generality we may take $n = 0$ and so $X_a = \alpha$ for all a. From the unimodularity of $\widetilde{T}_{ij}$ it then follows that $\alpha \equiv 1$ and hence we have

$$X_a = 1\,, \qquad \widetilde{V}_0 = -p\,(p-2)\,. \tag{2.23}$$

The second possibility, in which unequal values α and β occur for non-vanishing numbers m and $n = p - m$ of the X_a, implies that

$$(m-2)\,\alpha + (n-2)\,\beta = 0\,, \tag{2.24}$$

and hence positivity of the X_a's (i.e. $\alpha > 0$ and $\beta > 0$) implies that the only remaining solutions of (2.24) are those corresponding to $m = 1$ ($n = p-1 \geq 3$) or $n = 1$ ($m = p-1 \geq 3$). Choosing, without loss of generality, $m = 1$, we then find

$$X_1 = \alpha = (p-3)^{\frac{p-1}{p}}\,, \quad X_a = \beta = (p-3)^{-\frac{1}{p}}\,, \quad 2 \leq a \leq p\,, \quad \widetilde{V}_0 = -2p\,(p-3)^{\frac{p-2}{p}}\,. \tag{2.25}$$

Note that for $p < 4$, the only extremum of the potential is the "trivial" one with all $X_a = 1$. On the other hand for $p \geq 4$, the potential has two extrema with the property that the "trivial" one has always a less negative cosmological constant. In the context of the renormalization group flow (associated with the dual field theory) the flows start in the ultra-violet regime at the trivial minimum and run toward the non-trivial one in the infra-red regime.

3 Pauli Reductions on Hyperboloidal Spaces $\mathcal{H}^{p,q}$

In this section, we shall enumerate some examples of supergravities with non-compact gaugings that can be obtained by means of consistent Pauli reductions on the hyperboloidal spaces $\mathcal{H}^{p,q}$. These examples are in one-to-one correspondence with already known cases of supergravities with compact gaugings coming from consistent Pauli sphere reductions. The hyperboloidal reductions can in fact be obtained by making analytic continuations of the existing sphere reductions. An equivalent, and rather more elegant approach, is first to rewrite the sphere-reduction examples in a notation where the passage from the compact to non-compact internal space is accomplished merely by a replacement of a Euclidean-signature metric on the gauge group by an indefinite-signature metric. As a consequence, the non-linear Kaluza-Klein Ansätze, both in the compact and the non-compact cases involve only real values for the p-form field strengths, and positive definite metric in the internal space. In addition the resulting lower dimensional theories contain only fields with positive definite kinetic energy.

A detailed enumeration of theories where a consistent Pauli sphere reduction is known to exist was given in [5]. These are examples of dimensional reductions on coset spaces, which in practice are usually spheres. If no fields were truncated out in the dimensional reduction, the process of dimensional reduction would necessarily be consistent, and one would end up with infinite towers of massive fields as well as a finite number of massless fields that included the metric and the gauge bosons of the isometry group of the internal coset space. Generically, it is inconsistent to set the infinite towers of massive fields to zero, because non-linear terms built from the massless fields that are retained will act as sources for the massive fields that one wants to set to zero. By a Pauli reduction we mean a reduction in which one *can*, exceptionally, consistently set all the massive fields to zero, with the set of lower-dimensional fields that are retained including the Yang-Mills gauge bosons associated with the entire isometry group of the internal coset manifold. Thus for a Pauli reduction on the sphere S^n, the retained lower-dimensional fields would include the Yang-Mills gauge fields for the group $SO(n+1)$. The success of a consistent Pauli reduction depends on remarkable "conspiracies" between properties of the internal coset space and properties of the theory one is reducing.

The list of consistent Pauli reductions presented in [5] comprised a number of examples with internal spaces S^n. The fields that are retained in the reduction include the metric, the gauge bosons $A^{ij}_{(1)}$ of $SO(n+1)$, and scalars described by a symmetric $(n+1)\times(n+1)$ matrix T_{ij}:

p-form	Dilaton	Higher-Dim	Lower-Dim.	Sphere	Gauge Group	Extra fields
$F_{(2)}$	Yes	Any D	$D-2$	S^2	$SO(3)$	None
$F_{(3)}$	Yes	Any D	$D-3$	S^3	$SO(4)$	$A_{(2)}$
$F_{(3)}$	Yes	Any D	3	S^{D-3}	$SO(D-2)$	None
$F_{(4)}$	No	11	7	S^4	$SO(5)$	$A^i_{(3)}$
$F_{(4)}$	No	11	4	S^7	$SO(8)$	$\phi_{[ijk\ell]+}$
$F_{(5)}={*F_{(5)}}$	No	10	5	S^5	$SO(6)$	None

Table 1: Consistent Pauli reductions on S^n, retaining $SO(n+1)$ gauge fields. The last column indicates what additional fields, beyond the metric, the gauge fields $A^{ij}_{(1)}$ and the scalars T_{ij}, are massless, and must therefore be included, in a consistent truncation. The Table is taken from [5].

The first row in Table 1 corresponds to Pauli reductions of an Einstein-Maxwell-dilaton system in D dimensions on S^2. The second and third rows correspond to Pauli reductions of the low-energy effective theory of the bosonic string in D dimensions on S^3 or S^{D-3}. The fourth and fifth rows correspond to the Pauli reduction of eleven-dimensional supergravity on S^4 or S^7, and the last row corresponds to the Pauli reduction of type IIB supergravity on S^5.

Each of these examples has its own particular features, and one cannot give a "universal" Pauli-reduction ansatz that encapsulates them all in a single set of formulae. In particular, the set of additional fields that might need to be retained in order to achieve a consistent reduction is highly theory-specific. For example, in the S^7 reduction of eleven-dimensional supergravity one must retain an additional massless 35 pseudo-scalar fields, as well as the 35 scalars described by T_{ij}. We shall not attempt, therefore to present general formulae, nor shall we present the known details in all the above cases. Rather, we shall take one of the Pauli reductions as an example, in order to show how the previously obtained reduction formulae can be straightforwardly modified to generalise from the compact case where the reduction is on S^n to the non-compact case where the reduction is on $\mathcal{H}^{p,q}$, where $n = p + q - 1$. It should then be clear how the analogous transitions are achieved in all the other examples.

We shall take for our example the consistent S^4 Pauli reduction of eleven-dimensional supergravity. The complete reduction ansatz was derived in [3]; it was re-expressed in a form that we shall adopt here in [17]. The reduction ansätze for the eleven-dimensional metric and 4-form filed strength are given by

$$d\hat{s}_{11}^2 = \Delta^{1/3}\, ds_7^2 + \frac{1}{g^2}\Delta^{-2/3}\, T^{-1}_{AB}\, D\mu^A\, D\mu^B\,, \tag{3.1}$$

$$\begin{aligned}\hat{F}_{(4)} = {}& \frac{1}{4!}\,\epsilon_{A_1\cdots A_5}\Big[-\frac{1}{g^3}U\,\Delta^{-2}\mu^{A_1}D\mu^{A_2}\wedge\cdots\wedge D\mu^{A_5}\\ &+\frac{4}{g^3}\Delta^{-2}\,T^{A_1}{}_B\,DT^{i_2}{}_C\,\mu^B\,\mu^C\,D\mu^{A_3}\wedge\cdots\wedge D\mu^{A_5}\\ &+\frac{6}{g^2}\Delta^{-1}F_{(2)}^{A_1A_2}\wedge D\mu^{A_3}\wedge D\mu^{A_4}\,T^{A_5}{}_B\,\mu^j B\Big] - T_{AB}\,{*S^A_{(3)}}\,\mu^B + \frac{1}{g}\,S_{(3)\,A}\wedge D\mu^A\,,\end{aligned} \tag{3.2}$$

where

$$\begin{gathered}U \equiv 2T_{AC}\,T_{BD}\,\eta^{CD}\,\mu^A\,\mu^B - \Delta\,T_{AB}\,\eta^{AB}\,,\qquad \Delta\equiv T_{AB}\,\mu^A\,\mu^B\,,\\ F_{(2)\,A}{}^B \equiv dA_{(1)\,A}{}^B + g\,A_{(1)\,A}{}^C\wedge A_{(1)\,C}{}^B\,,\qquad D\mu^A\equiv d\mu^A + g\,A^A_{(1)\,B}\,\mu^B\,,\\ DT_{AB}\equiv dT_{AB} + g\,A_{(1)\,A}{}^C\,T_{CB} + g\,A_{(1)\,B}{}^C\,T_{AC}\,,\qquad \mu^A\,\mu^B\,\eta_{AB}\equiv 1\,,\end{gathered} \tag{3.3}$$

where the symmetric matrix T_{AB}, which parameterises the scalar coset $SL(5,\mathbb{R})/SO(5)$, is unimodular.

Aside from a small change of notation, the only difference between the ansatz above and the one presented in [17] is that in the latter the gauge group was taken to be $SO(5)$, meaning that $\eta_{AB} = \delta_{AB}$, whereas here η_{AB} is allowed to have indefinite signature (p,q), $p+q=5$, corresponding to an $SO(p,q)$ gauging. Note that all $A, B, \ldots$ gauge-group indices are raised and lowered with η_{AB}.

Substituting the above ansatz into the equations of motion of eleven-dimensional supergravity, one finds that the lower-dimensional fields satisfy the equations of motion of $SO(p,q)$-gauged $N=4$ supergravity in seven dimensions, which follow from the Lagrangian

$$\begin{aligned}\mathcal{L}_7 \;=\;& R\,{*\mathbb{1}} - {*P^{ij}} \wedge P^{ij} - \tfrac{1}{4} T^{-1}_{AC}\, T^{-1}_{BD}\, {*F^{AB}_{(2)}} \wedge F^{CD}_{(2)} - \tfrac{1}{2} T_{AB}\, {*S^A_{(3)}} \wedge S^B_{(3)} \\ & + \frac{1}{2g} S^A_{(3)} \wedge DS^B_{(3)}\, \eta_{AB} - \frac{1}{8g} \epsilon_{AB_1\cdots B_4}\, S^A_{(3)} \wedge F^{B_1B_2}_{(2)} \wedge F^{B_3B_4}_{(2)} + \frac{1}{g}\Omega_{(7)} - V\, {*\mathbb{1}}\,,\end{aligned} \tag{3.4}$$

where

$$P_{ij} \equiv \Pi^{-1}{}_{(i}{}^A\, (\delta_A{}^B\, d + g\, A_{(1)\,A}{}^B) \Pi_B{}^k\, \delta_{j)k} \tag{3.5}$$

and the potential V is given by

$$V = \tfrac{1}{2} g^2 \Big(2T_{ij}\, T_{ij} - (T_{ii})^2 \Big)\,. \tag{3.6}$$

Note that T_{ij}, with $SO(5)_c$ indices, and T_{AB}, with $SO(p,q)_g$ indices, are given in terms of the scalar vielbein Π^i_A by

$$T_{ij} = \Pi^{-1}_i{}^A\, \Pi^{-1}_j{}^B\, \eta_{AB}\,, \qquad T^{AB} = \Pi^{-1}{}_i{}^A\, \Pi^{-1}{}_i{}^B\,. \tag{3.7}$$

The form of the Chern-Simons term $\Omega_{(7)}$, built from the Yang-Mills fields, can be found in [17].

The geometry of the "internal" manifold can easily be seen from the above expressions. From the metric reduction ansatz in (3.1), we see that if we consider the situation where the scalars and Yang-Mills fields are taken to be trivial, meaning in particular that $T_{AB} = \delta_{AB}$ (see (3.7)), we shall have

$$d\hat{s}^2_{11} = \Delta^{1/3}\, ds_7^2 + \frac{1}{g^2}\, \Delta^{2/3}\, \delta_{AB}\, d\mu^A\, d\mu^B\,, \tag{3.8}$$

where

$$\Delta = \delta_{AB}\, \mu^A\, \mu^B\,, \tag{3.9}$$

and, of course, $\eta_{AB}\, \mu^A\, \mu^B = 1$. Thus the internal metric here is positive definite, and its isometry group is the intersection of $SO(p+q) = SO(5)$, which leaves the Euclidean metric δ_{AB} invariant, and $SO(p,q)$, which leaves η_{AB} invariant. This intersection is $SO(p)\times SO(q)$.

We have presented the case of the consistent Pauli reductions of eleven-dimensional to $D=7$ as an explicit example. With appropriate changes, one can straightforwardly discuss all the other known consistent Pauli reductions.

One generic property of these reduction is the appearance of the scalar field potential, which has a universal form of the type (2.3).

4 de Sitter-type Supergravities in Four Dimensions

In this section, we shall study explicit examples of four-dimensional non-compact gauged supergravities, which can be obtained by a process of analytic continuation, and their associated embeddings in eleven dimensions via consistent reductions.

4.1 $N=4$ de Sitter gauged theory

For this construction, we shall begin with the $SO(4)$ gauged $N=4$ supergravity in four dimensions, and then perform an analytic continuation to a de Sitter type supergravity. By continuing the known consistent S^7 reduction from $D=11$, we shall show how the de Sitter supergravity arises as a consistent reduction on $\mathcal{H}^{4,4}$.

To begin, let us consider the bosonic sector of the four-dimensional $N=4$ gauged $SO(4)$ supergravity. In the notation of [4], the bosonic Lagrangian may be written as

$$\begin{aligned}\mathcal{L}_4 \;\; = \;\; & R\,{*\mathbb{1}} - \tfrac12 {*d\phi}\wedge d\phi - \tfrac12 e^{2\phi}\,{*d\chi}\wedge d\chi - V\,{*\mathbb{1}} \\ & -\tfrac12 e^{-\phi}\,{*F^i_{(2)}}\wedge F^i_{(2)} - \tfrac12 \frac{e^{\phi}}{1+\chi^2\,e^{2\phi}}\,{*\widetilde F^i_{(2)}}\wedge \widetilde F^i_{(2)}\,, \\ & -\tfrac12\chi\,F^i_{(2)}\wedge F^i_{(2)} + \tfrac12\frac{\chi\,e^{2\phi}}{1+\chi^2\,e^{2\phi}}\,\widetilde F^i_{(2)}\wedge\widetilde F^i_{(2)}\,,\end{aligned} \tag{4.1}$$

where the potential V is

$$V = -2g^2\,(4+2\cosh\phi+\chi^2\,e^{\phi})\,, \tag{4.2}$$

and

$$F^i_{(2)} = dA^i_{(1)} + \tfrac12 g\,\epsilon_{ijk}\,A^j_{(1)}\wedge A^k_{(1)}\,, \qquad \widetilde F^i_{(2)} = d\widetilde A^i_{(1)} + \tfrac12 g\,\epsilon_{ijk}\,\widetilde A^j_{(1)}\wedge\widetilde A^k_{(1)}\,. \tag{4.3}$$

We now perform the following continuations:

$$g\longrightarrow -\mathrm{i}\,g\,,\quad A^i_{(1)}\longrightarrow \mathrm{i}\,A^i_{(1)}\,,\qquad \widetilde A^i_{(1)}\longrightarrow \mathrm{i}\,\widetilde A^i_{(1)}\,,\quad \phi\longrightarrow\phi+\mathrm{i}\,\pi\,. \tag{4.4}$$

After doing this, the Lagrangian (4.1) retains the same form

$$\mathcal{L}_4 \;\; = \;\; R\,{*\mathbb{1}} - \tfrac12 {*d\phi}\wedge d\phi - \tfrac12 e^{2\phi}\,{*d\chi}\wedge d\chi - V\,{*\mathbb{1}}$$

$$-\tfrac{1}{2}e^{-\phi}\, {*F^i_{(2)}}\wedge F^i_{(2)} - \tfrac{1}{2}\frac{e^{\phi}}{1+\chi^2\, e^{2\phi}}\, {*\widetilde{F}^i_{(2)}}\wedge \widetilde{F}^i_{(2)}\,, \qquad (4.5)$$

$$-\tfrac{1}{2}\chi\, F^i_{(2)}\wedge F^i_{(2)} + \tfrac{1}{2}\frac{\chi\, e^{2\phi}}{1+\chi^2\, e^{2\phi}}\, \widetilde{F}^i_{(2)}\wedge \widetilde{F}^i_{(2)}\,,$$

except that now the potential V in (4.2) is replaced by

$$V = 2g^2\,(4 - 2\cosh\phi - \chi^2\, e^{\phi})\,, \qquad (4.6)$$

The $SU(2)\times SU(2)$ Yang-Mills fields are still given by the same expressions (4.3). Note that the kinetic terms for all fields retain their conventional signs, and thus the theory is still ghost-free. One can see very clearly in this example that it is the presence of the couplings of the scalar fields in the Yang-Mills kinetic terms that allows their signs to remain the standard ones despite the continuations $A_{(1)} \longrightarrow \mathrm{i}\, A_{(1)}$, owing to the compensating sign changes induced by the continuation $\phi \longrightarrow \phi + \mathrm{i}\,\pi$. However, the scalar potential, which in the original compact form (4.2) had a minimum at $(\phi = 0, \chi = 0)$ with $V_0 = -12g^2$, now has a minimum at $(\phi = 0, \chi = 0)$ with $V_0 = +4g^2$.

It should be noted that after the analytic continuations the gauge group continues to be the compact group $SO(4) \sim SU(2)\times SU(2)$. If we had performed an analogous analytic continuation on the full $N = 8$ gauged $SO(8)$ supergravity of de Wit and Nicolai [20], we would have obtained the non-compact gauging with $SO(4,4)$. This would be subject to a spontaneous symmetry breaking to its compact $SO(4)\times SO(4)$ subgroup, and in fact the gauge fields that are retained in the truncated $N = 4$ theory that we are considering here reside entirely within one of these $SO(4)$ factors, and hence only a compact gauge group is seen here.

The embedding of this de Sitter-type $N = 4$ gauged supergravity in $D = 11$ can be seen by performing the corresponding analytic continuations in the S^7 reduction formulae obtained in [4]. From the formulae in section 2 of [4], we see that after implementing the continuations (4.4) on the four-dimensional fields, we should also make the continuation $\xi \longrightarrow \mathrm{i}\,\xi$ on the "azimuthal" coordinate of the description of S^7 as a foliation of $S^3\times S^3$

surfaces. This results in the consistent reduction ansatz[3]

$$d\hat{s}_{11}^2 = \Delta^{\frac{2}{3}}\, ds_4^2 + 2g^{-2}\,\Delta^{\frac{2}{3}}\, d\xi^2 + \tfrac{1}{2} g^{-2}\,\Delta^{\frac{2}{3}}\,\Big[\frac{c^2}{c^2\,X^2+s^2}\,\sum_i (h^i)^2 + \frac{s^2}{s^2\,\widetilde{X}^2+c^2}\,\sum_i (\tilde{h}^i)^2\Big]\,, \quad (4.7)$$

where

$$\begin{aligned} &X \equiv e^{\frac{1}{2}\phi}\,, \qquad \widetilde{X} \equiv X^{-1}\, q\,, \qquad q^2 \equiv 1 + \chi^2\, X^4\,,\\ &\Delta \equiv \Big[(c^2\,X^2+s^2)(s^2\,\widetilde{X}^2+c^2)\Big]^{\frac{1}{2}}\,,\\ &c \equiv \cosh\xi\,, \qquad s \equiv \sinh\xi\,,\\ &h^i \equiv \sigma_i - g\, A^i_{(1)}\,, \qquad \tilde{h}^i \equiv \tilde{\sigma}_i - g\,\widetilde{A}^i_{(1)}\,. \end{aligned} \quad (4.8)$$

The three quantities σ_i are left-invariant 1-forms on $S^3 = SU(2)$, and the three $\tilde{\sigma}_i$ are left-invariant 1-forms on a second S^3. They satisfy

$$d\sigma_i = -\tfrac{1}{2}\epsilon_{ijk}\,\sigma_j\wedge\sigma_k\,, \qquad d\tilde{\sigma}_i = -\tfrac{1}{2}\epsilon_{ijk}\,\tilde{\sigma}_j\wedge\tilde{\sigma}_k\,. \quad (4.9)$$

The reduction ansatz for $\hat{F}_{(4)}$ given in [4] becomes

$$\hat{F}_{(4)} = -g\,\sqrt{2}\,U\,\epsilon_{(4)} - \frac{4s\,c}{g\,\sqrt{2}}\,X^{-1}\,{*dX}\wedge d\xi + \frac{\sqrt{2}s\,c}{g}\,\chi\,X^4\,{*d\chi}\wedge d\xi + \hat{F}'_{(4)} + \hat{F}''_{(4)}\,, \quad (4.10)$$

where

$$U = -X^2\,c^2 + \widetilde{X}^2\,s^2 + 2\,, \quad (4.11)$$

and $\hat{F}'_{(4)} = d\hat{A}'_{(3)}$, with

$$\hat{A}'_{(3)} = f\,\epsilon_{(3)} + \tilde{f}\,\tilde{\epsilon}_3\,, \quad (4.12)$$

where $\epsilon_{(3)} = \frac{1}{6}\epsilon_{ijk}\,h^i\wedge h^j\wedge h^k$ and $\tilde{\epsilon}_{(3)} = \frac{1}{6}\epsilon_{ijk}\,\tilde{h}^i\wedge\tilde{h}^j\wedge\tilde{h}^k$. The functions f and $\tilde{f}$ are given by

$$\begin{aligned} f &= \frac{1}{2\sqrt{2}}\,g^{-3}\,c^4\chi\,X^2\,(c^2\,X^2+s^2)^{-1}\,,\\ \tilde{f} &= \frac{1}{2\sqrt{2}}\,g^{-3}\,s^4\,\chi\,X^2\,(s^2\,\widetilde{X}^2+c^2)^{-1}\,. \end{aligned} \quad (4.13)$$

[3]Note that when performing the analytic continuation of the expressions in [4], the sign of the entire eleven-dimensional metric reverses, owing to a sign change of the quantity Δ^2 defined there. This must be compensated by using the "trombone" scaling symmetry of the eleven-dimensional theory, under which $\hat{g}_{MN} \longrightarrow \lambda^2\,\hat{g}_{MN}$, $\hat{A}_{MNP} \longrightarrow \lambda^3\,\hat{A}_{MNP}$, $\hat{\psi}_M \longrightarrow \lambda\,\hat{\psi}_M$. This is a symmetry of the $D=11$ equations of motion, corresponding to a homogeneous constant scaling of the action. Specifically, we shall take $\lambda = -\mathrm{i}$. As we shall see below, the associated imaginary rescaling of the antisymmetric tensor is precisely what is needed in order to obtain a real expression after the continuations.

The terms in $\hat{F}''_{(4)}$ comprise those involving the $SU(2)\times SU(2)$ Yang-Mills field strengths $F^i_{(2)}$ and $\widetilde{F}^i_{(2)}$. These are given by

$$\begin{aligned}\hat{F}''_{(4)} &= \tfrac{1}{\sqrt{2}}\,g^{-2}\,X^{-2}\,(-s\,c\,d\xi\wedge h^i+\tfrac14 c^2\,\epsilon_{ijk}\,h^j\wedge h^k)\wedge({*F^i_{(2)}}-\chi\,X^2\,F^i_{(2)})\\ &\quad+\tfrac{1}{\sqrt{2}}\,g^{-2}\,\widetilde{X}^{-2}\,(s\,c\,d\xi\wedge\tilde{h}^i-\tfrac14 s^2\,\epsilon_{ijk}\,\tilde{h}^j\wedge\tilde{h}^k)\wedge({*\widetilde{F}^i_{(2)}}+\chi\,X^2\,\widetilde{F}^i_{(2)})\,.\end{aligned}\tag{4.14}$$

Note that in obtaining these real expressions for $\hat{F}_{(4)}$, we made use of the overall rescaling $\hat{A}_{MNP}\longrightarrow \mathrm{i}\,\hat{A}_{MNP}$ that we discussed in the previous footnote.

It is instructive to look at the nature of the internal 7-metric in the "ground state" where $\phi=\chi=0=A^i_{(1)}=\widetilde{A}^i_{(1)}$. From (4.7) we see that we shall have $d\hat{s}_{11}^2=\Delta^{2/3}\,ds_4^2+2\Delta^{-1/3}\,ds_7^2$ with $\Delta=\cosh 2\xi$ and

$$ds_7^2=\cosh(2\xi)\,d\xi^2+\tfrac14\cosh^2\xi\,\sigma_i^2+\tfrac14\sinh^2\xi\,\tilde{\sigma}_i^2\,.\tag{4.15}$$

This is precisely the standard "undistorted" positive-definite metric on the the hyperboloid $\mathcal{H}^{4,4}$. This can be seen by expressing the coordinates $\mu^i=(\mu_a,\mu_{\bar a})$ on $\mathbb{R}^8$, subject to the hyperboloidal constraint $\mu_a\,\mu_a-\mu_{\bar a}\,\mu_{\bar a}=1$ as

$$\mu_a=u_a\,\cosh\xi\,,\qquad \mu_{\bar a}=v_{\bar a}\,\sinh\xi\,,\tag{4.16}$$

where $u_a\,u_a=1$ and $v_{\bar a}\,v_{\bar a}=1$ define two 3-spheres, and substituting into the positive definite metric $ds^2=d\mu_a\,d\mu_a+d\mu_{\bar a}\,d\mu_{\bar a}$ on $\mathbb{R}^8$.

The theory that we have obtained here as a consistent reduction is the bosonic sector of an $N=4$ de Sitter-type supergravity. This can be consistently truncated to $N=3$, by setting the two sets of $SU(2)$ gauge fields equal, and at the same time setting $\phi=\chi=0$. In order to keep a canonical normalisation for the remaining $SU(2)$ Yang-Mills fields, we should also send $A^i_{(1)}\longrightarrow\frac{1}{\sqrt{2}}\,A^i_{(1)}$, $g\longrightarrow\sqrt{2}\,g$. Upon doing so, we obtain the bosonic Lagrangian

$$\mathcal{L}_4=R\,{*\mathbb{1}}-\tfrac12{*F^i_{(2)}}\wedge F^i_{(2)}-8g^2\,{*\mathbb{1}}\,,\tag{4.17}$$

where $F^i_{(2)}=dA^i_{(1)}+mg\,\epsilon_{ijk}\,A^j_{(1)}\wedge A^k_{(1)}$. This bosonic sector of the truncated $N=3$ de Sitter supergravity is precisely the one that was obtained recently in [14], together with its embedding in eleven-dimensional supergravity. In that work, the embedding of the theory was derived from scratch. It is interesting that by obtaining the theory as the $N=3$ truncation of the larger $N=4$ theory, we can make use of previous results in the literature [4] in order to establish the reduction procedure from $D=11$. However if one truncates to $N=3$ *before* looking at the embedding in $D=11$, the absence of the scalar fields precludes one from implementing the analytic continuation in (4.4) that allowed us to perform a continuation of the $N = 4$ S^7 reduction of [4].

Further truncations to lesser supersymmetry are also possible. One can, for example, consider a truncation to $N = 1$, in which one retains just a Maxwell multiplet as well as the supergravity multiplet. In the bosonic sector, the Lagrangian is obtained from (4.17) by retaining just a $U(1)$ gauge field, and so one has

$$\mathcal{L}_4 = R\,{*\mathbb{1}} - \tfrac{1}{2}{*F_{(2)}} \wedge F_{(2)} - 8g^2\,{*\mathbb{1}}\,, \tag{4.18}$$

In fact it is presumably the case that this is the bosonic sector of the axially-gauged $N = 1$ de Sitter supergravity constructed in [21]. (Since we have not explicitly studied the fermionic sector here this remains conjectural at this stage, but the existence of the non-compact gauged $N = 8$ supergravities discussed in [8, 9], and of the axially-gauged $N = 1$ supergravity obtained in [21], lend credence to the conjecture.) In particular, this implies that any solution of four-dimensional Einstein-Maxwell gravity with a positive cosmological constant can be embedded in the de Sitter supergravity of [21], and hence, via our consistent reduction, it can be lifted to a solution in eleven-dimensional supergravity. Examples of such four-dimensional solutions include the cosmological multi back hole solutions of [22].

4.2 $N = 2$ de Sitter gauged theories

In this section, we shall consider $N = 2$ de Sitter supergravities obtained by starting with the four-dimensional $N = 2$ supergravity with $U(1)^4$ gauging, whose consistent embedding in $D = 11$ supergravity was discussed in [1]. For simplicity, we shall follow [1] and omit the three axionic scalar fields that form part of the supergravity theory. Their inclusion in the four-dimensional theory itself is straightforward, and we refer the reader to Appendix B of [1] for a discussion of the details. Including the axions in the consistent reduction from $D = 11$ is a more difficult problem, and we shall not attempt to address that here.

With the axions omitted, the bosonic sector of the four-dimensional Lagrangian for the $U(1)^4$ gauged theory is given by

$$e^{-1}\,\mathcal{L} = R - \tfrac{1}{2}(\partial\vec{\varphi})^2 - \tfrac{1}{4}\sum_{i=1}^{4} X_i^{-2}\,(F^i)^2 - V\,, \tag{4.19}$$

where $\vec{\varphi} = (\varphi_1, \varphi_2, \varphi_3)$, the scalar potential is given by

$$\begin{aligned} V &= -4g^2 \sum_{i<j} X_i\,X_j \\ &= -8g^2\,(\cosh\varphi_1 + \cosh\varphi_2 + \cosh\varphi_3)\,, \end{aligned} \tag{4.20}$$

and

$$X_1 = e^{-\frac{1}{2}(\varphi_1+\varphi_2+\varphi_3)}\,,\qquad X_2 = e^{-\frac{1}{2}(\varphi_1-\varphi_2-\varphi_3)}\,,$$
$$X_3 = e^{-\frac{1}{2}(-\varphi_1+\varphi_2-\varphi_3)}\,,\qquad X_4 = e^{-\frac{1}{2}(-\varphi_1-\varphi_2+\varphi_3)}\,. \tag{4.21}$$

The embedding of the $U(1)^4$ gauged theory in $D = 11$ supergravity was constructed in [1]; it involves a consistent Pauli-type reduction on S^7, and is given by

$$d\hat{s}_{11}^2 = \Delta^{2/3}\, ds_4^2 + g^{-2}\,\Delta^{-1/3}\sum_{i=1}^4 X_i^{-1}\Big(d\mu_i^2 + \mu_i^2\,(d\phi_i + g\,A^i)^2\Big)\,, \tag{4.22}$$

$$\begin{aligned}\hat{F}_{(4)} = \sum_{i=1}^4 \Big(&2g\,(X_i^2\,\mu_i^2 - \Delta\,X_i)\,\epsilon_{(4)} + \frac{1}{2g}\,X_i^{-1}\,{*dX_i}\wedge d(i\mu_i^2)i \\ &-\frac{1}{2g^2}\,X_i^{-2}\,d(\mu_i^2)\wedge(d\phi_i + g\,A^i)\wedge{*F^i}\Big)\,,\end{aligned} \tag{4.23}$$

where

$$\Delta = \sum_{i=1}^4 X_i\,\mu_i^2\,,\qquad \sum_{i=1}^4 \mu_i^2 = 1\,, \tag{4.24}$$

$\epsilon_{(4)}$ is the volume form in the four-dimensional metric ds_4^2, and $*$ denotes Hodge dualisation in the four-dimensional metric. Note that the round 7-sphere arises when the scalars are trivial ($X_i = 1$), and is described in terms of the four constrained coordinates μ_i and the four azimuthal angles ϕ_i by

$$d\Omega_7^2 = \sum_{i=1}^4 (d\mu_i^2 + \mu_i^2\,d\phi_i^2)\,. \tag{4.25}$$

We shall now describe two inequivalent analytic continuations, one of which corresponds to a replacement of S^7 by $\mathcal{H}^{4,4}$, and the other to a replacement of S^7 by $\mathcal{H}^{6,2}$. The consistent reductions in these cases, obtained by appropriate analytic continuations of the complete S^7 reduction of de Wit and Nicolai [20], would yield $N = 8$ supergravities with the non-compact gaugings $SO(4,4)$ and $SO(6,2)$ respectively. In our case, where we start with the restricted $N = 2$ gauged theory of supergravity coupled to three vector multiplets, we are retaining only the $U(1)^4$ gauge field in the Cartan subgroup of $SO(8)$. After the analytic continuations, in each case we will still have $U(1)^4$ gauge fields; these are in the compact Cartan subgroups of $SO(4,4)$ and $SO(6,2)$ respectively. We shall therefore refer to the two analytically continued theories as the $SO(4,4)$ and $SO(6,2)$ cases respectively, even though our truncations retain only $U(1)^4$ gauge fields.

In accordance with our general results in section 2, the former theory will have a scalar potential with a stationary point, whilst the latter will not.

4.2.1 The $SO(4,4)$ case

To perform the analytic continuations in this case, we take

$$\varphi_1 \longrightarrow \varphi_1 + \mathrm{i}\,\pi\,, \qquad A^i \longrightarrow \mathrm{i}\,A^i\,, \qquad g \longrightarrow -\mathrm{i}\,g\,, \tag{4.26}$$

with φ_2 and φ_3 left unchanged. This implies that we shall have

$$X_1 \longrightarrow -\mathrm{i}\,X_1\,, \quad X_2 \longrightarrow -\mathrm{i}\,X_2\,, \quad X_3 \longrightarrow \mathrm{i}\,X_3\,, \quad X_4 \longrightarrow \mathrm{i}\,X_4\,. \tag{4.27}$$

The Lagrangian (4.19) will therefore retain the identical form, except that now the potential (4.21) will be replaced by

$$V = 8g^2\,(\cosh\varphi_2 + \cosh\varphi_3 - \cosh\varphi_1)\,. \tag{4.28}$$

Turning now to the embedding in eleven-dimensional supergravity, we make the corresponding continuations

$$\mu_3 \longrightarrow -\mathrm{i}\,\mu_3\,, \qquad \mu_4 \longrightarrow -\mathrm{i}\,\mu_4\,, \tag{4.29}$$

while leaving μ_1 and μ_2 unchanged. This implies that we shall have $\Delta \longrightarrow -\mathrm{i}\,\Delta$, for which we may define the cube root so that

$$\Delta^{1/3} \longrightarrow \mathrm{i}\,\Delta^{1/3}\,. \tag{4.30}$$

Finally, we perform a "trombone" rescaling $d\hat{s}_{11}^2 \longrightarrow \lambda^2\,d\hat{s}_{11}^2$, $\hat{A}_{(3)} \longrightarrow \lambda^3\,\hat{A}_{(3)}$ with $\lambda = \mathrm{i}$. We therefore arrive at the metric and field-strength reductions

$$\begin{aligned} d\hat{s}_{11}^2 &= \Delta^{2/3}\,ds_4^2 + g^{-2}\,\Delta^{-1/3}\sum_{i=1}^4 X_i^{-1}\Big(d\mu_i^2 + \mu_i^2\,(d\phi_i + g\,A^i)^2\Big)\,, && (4.31)\\ \hat{F}_{(4)} &= \sum_{i=1}^4 \eta_i\,\Big(2g\,(X_i^2\,\mu_i^2 - \Delta\,X_i)\,\epsilon_{(4)} + \frac{1}{2g}\,X_i^{-1}\,{*dX_i}\wedge d(i\mu_i^2)i \\ &\qquad -\frac{1}{2g^2}\,X_i^{-2}\,d(\mu_i^2)\wedge(d\phi_i + g\,A^i)\wedge{*F^i}\Big)\,, && (4.32)\end{aligned}$$

where

$$\begin{aligned} \Delta &= \sum_{i=1}^4 X_i\,\mu_i^2\,, \qquad \mu_1^2 + \mu_2^2 - \mu_3^2 - \mu_4^2 = 1\,,\\ \eta_i &= (1,1,-1,-1)\,. && (4.33)\end{aligned}$$

Note that if the scalar fields are taken to be trivial ($X_i = 1$), the internal space has the positive-definite metric on $\mathcal{H}^{4,4}$ given by

$$ds_7^2 = \sum_{i=1}^4 (d\mu_i^2 + \mu_i^2\,d\phi_i^2)\,, \tag{4.34}$$

where the μ_i coordinates are subject to the constraint given in (4.33).

4.2.2 The $SO(6,2)$ case

Our analytic continuation in this case is taken to be

$$\begin{aligned} &\varphi_1 \longrightarrow \varphi_1 + \frac{\mathrm{i}\,\pi}{2}\,, \qquad \varphi_2 \longrightarrow \varphi_2 + \frac{\mathrm{i}\,\pi}{2}\,, \qquad \varphi_3 \longrightarrow \varphi_3 - \frac{\mathrm{i}\,\pi}{2}\,, \\ &A^i \longrightarrow e^{-\frac{\mathrm{i}}{4}\pi}\, A^i \qquad g \longrightarrow e^{\frac{\mathrm{i}}{4}\pi}\, g\,, \end{aligned} \tag{4.35}$$

which implies

$$X_1 \longrightarrow e^{-\frac{\mathrm{i}}{4}\pi}\, X_1\,, \quad X_2 \longrightarrow e^{-\frac{\mathrm{i}}{4}\pi}\, X_2\,, \quad X_3 \longrightarrow e^{-\frac{\mathrm{i}}{4}\pi}\, X_3\,, \quad X_4 \longrightarrow e^{\frac{3\mathrm{i}}{4}\pi}\, X_4\,. \tag{4.36}$$

This leaves the form of the Lagrangian (4.19) unchanged except that now the scalar potential (4.21) is replaced by

$$V = 8g^2\,(\sinh\varphi_1 + \sinh\varphi_2 - \sinh\varphi_3)\,. \tag{4.37}$$

This has no stationary points, and it is unbounded from above and below.

For the embedding in eleven-dimensional supergravity, we make the corresponding continuation

$$\mu_4 \longrightarrow e^{-\frac{\mathrm{i}}{2}\pi}\,\mu_4\,, \tag{4.38}$$

while leaving μ_1, μ_2 and μ_3 unchanged. This means that Δ defined in (4.24) will be changed according to $\Delta \longrightarrow \exp(-\frac{\mathrm{i}}{4}\pi)\,\Delta$, for which we shall have the replacement in the cube root:

$$\Delta^{1/3} \longrightarrow e^{-\frac{3\mathrm{i}}{4}\pi}\,\Delta^{1/3}\,. \tag{4.39}$$

Finally, we perform a trombone rescaling $d\hat{s}_{11}^2 \longrightarrow \lambda^2\, d\hat{s}_{11}^2$, $\hat{A}_{(3)} \longrightarrow \lambda^3\,\hat{A}_{(3)}$ of the eleven dimensional fields with $\lambda = \exp(-\frac{\mathrm{i}}{4}\pi)$. This leads to the following expressions for the metric and field strength reductions:

$$d\hat{s}_{11}^2 = \Delta^{2/3}\, ds_4^2 + g^{-2}\,\Delta^{-1/3}\sum_{i=1}^4 X_i^{-1}\Big(d\mu_i^2 + \mu_i^2\,(d\phi_i + g\,A^i)^2\Big)\,, \tag{4.40}$$

$$\begin{aligned} \hat{F}_{(4)} = \sum_{i=1}^4 \eta_i \Big(&- 2g\,(X_i^2\,\mu_i^2 - \Delta\,X_i)\,\epsilon_{(4)} - \frac{1}{2g}\,X_i^{-1} {*dX_i} \wedge d(\mu_i^2) \\ &+ \frac{1}{2g^2}\,X_i^{-2}\,d(\mu_i^2) \wedge (d\phi_i + g\,A^i) \wedge {*F^i}\Big)\,, \end{aligned} \tag{4.41}$$

where

$$\begin{aligned} \Delta &= \sum_{i=1}^4 X_i\,\mu_i^2\,, \qquad \mu_1^2 + \mu_2^2 + \mu_3^2 - \mu_4^2 = 1\,, \\ \eta_i &= (1,1,1,-1)\,. \end{aligned} \tag{4.42}$$

Note that if the scalar fields are taken to be trivial ($X_i = 1$), the internal space has the positive-definite metric on $\mathcal{H}^{6,2}$ given by

$$ds_7^2 = \sum_{i=1}^{4}(d\mu_i^2 + \mu_i^2\, d\phi_i^2)\,, \tag{4.43}$$

where the μ_i coordinates are subject to the constraint given in (4.42). Note also that, in accordance with our discussion of the existence of stationary points in section 2, the scalar potential given in (4.37) for this $\mathcal{H}^{6,2}$ reduction has no extrema.

5 Non-compact Gauged Supergravities in Higher Dimension

In this section we discuss two further examples associated with non-compact gauged supergravities, in five and seven dimensions. Our starting points are the $N = 2$ gauged theories whose consistent reductions from type IIB on S^5 and $D = 11$ supergravity on S^4 respectively were discussed in [1]. The five-dimensional theory has $U(1)^3$ gauge fields in the Cartan subgroup of the $SO(6)$ isometry of S^5, while the seven-dimensional theory has $U(1)^2$ gauge fields in the Cartan subgroup of the $SO(5)$ isometry of S^4.

5.1 Five-dimensional $N = 2$ gauged supergravity

In [1], the consistent Pauli reduction that yields $N = 2$ gauged supergravity coupled to two vector multiplets was given. The gauge fields lie in the $U(1)^3$ Cartan subgroup of the full $SO(6)$ gauge group of the $N = 8$ theory. The Lagrangian for the bosonic sector of the $N = 2$ theory is given by

$$e^{-1}\,\mathcal{L}_5 = R - \tfrac{1}{2}(\partial\varphi_1)^2 - \tfrac{1}{2}(\partial\varphi_2)^2 - \tfrac{1}{4}\sum_i X_i^{-2}\,(F^i)^2 + \tfrac{1}{4}\epsilon^{\mu\nu\rho\sigma\lambda}\,F^1_{\mu\nu}\,F^2_{\rho\sigma}\,A^3_\lambda - V\,, \tag{5.1}$$

where the scalar potential is given by

$$V = -4g^2\sum_{i=1}^{3} X_i^{-1}\,, \tag{5.2}$$

and we define

$$X_1 = e^{-\frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2}\,,\qquad X_2 = e^{-\frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2}\,,\qquad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}\,. \tag{5.3}$$

The embedding in the type IIB ten-dimensional theory involves a consistent Pauli reduction on S^5, described by [1]

$$d\hat{s}_{10}^2 \;=\; \Delta^{1/2}\,ds_5^2 + \frac{1}{g^2}\,\Delta^{-1/2}\sum_{i=1}^{3} X_i^{-1}\Big(d\mu_i^2 + \mu_i^2\,(d\phi_i + g\,A^i)^2\Big)\,,$$

$$
\begin{aligned}
\hat{G}_{(5)} &= \sum_{i=1}^{3}\Big(2g(X_i^2\,\mu_i^2 - \Delta\,X_i)\,\epsilon_{(5)} - \frac{1}{2g}\,X_i^{-1}\,{*dX_i}\wedge d(\mu_i^2) \\
&\quad + \frac{1}{2g^2}\,X_i^{-2}\,d(\mu_i^2)\wedge(d\phi_i + g\,A^i_{(1)})\wedge {*F^i_{(2)}}\Big)\,,
\end{aligned} \tag{5.4}
$$

where the self-dual 5-form is given by $\hat{F}_{(5)} = \hat{G}_5 + \hat{*}\hat{G}_5$, and

$$
\Delta = \sum_{i=1}^{3} X_i\,\mu_i^2\,,\qquad \sum_{i=1}^{3}\mu_i^2 = 1\,. \tag{5.5}
$$

One can perform an analytic continuation of the full S^5 reduction to describe an $N=8$ non-compact $SO(4,2)$ gauged supergravity. In the $N=2$ truncation considered here, the retained $U(1)^3$ gauge fields reside in the Cartan subgroup. The appropriate analytic continuation is achieved by sending

$$
\varphi_1 \longrightarrow \varphi_1 - \tfrac{2\mathrm{i}}{\sqrt{6}}\,\pi\,,\qquad A^i \longrightarrow e^{\frac{\mathrm{i}}{3}\pi}\,A^i\,,\qquad g \longrightarrow e^{-\frac{\mathrm{i}}{3}\pi}\,g\,, \tag{5.6}
$$

with φ_2 unchanged. Under these continuations, we shall have

$$
X_1 \longrightarrow e^{\frac{\mathrm{i}}{3}\pi}\,X_1\,,\qquad X_2 \longrightarrow e^{\frac{\mathrm{i}}{3}\pi}\,X_2\,,\qquad X_3 \longrightarrow e^{-\frac{2\mathrm{i}}{3}\pi}\,X_3\,. \tag{5.7}
$$

The Lagrangian (5.1) becomes

$$
e^{-1}\,\mathcal{L}_5 = R - \tfrac12(\partial\varphi_1)^2 - \tfrac12(\partial\varphi_2)^2 - \tfrac14\sum_i X_i^{-2}\,(F^i)^2 - \tfrac14\epsilon^{\mu\nu\rho\sigma\lambda}\,F^1_{\mu\nu}\,F^2_{\rho\sigma}\,A^3_\lambda - V\,, \tag{5.8}
$$

(i.e. the sign of the Chern-Simons term is reversed) with the scalar potential (5.2) being replaced by

$$
\begin{aligned}
V &= 4g^2\,(X_1^{-1} + X_2^{-1} - X_3^{-1})\,,\\
&= 4g^2\,(2e^{\frac{1}{\sqrt6}\varphi_1}\,\cosh\tfrac{1}{\sqrt2}\varphi_2 - e^{-\frac{2}{\sqrt2}\varphi_1})\,.
\end{aligned} \tag{5.9}
$$

It is easily seen that indeed, as expected from the discussion in section (2), this potential has no stationary points. It is unbounded from above and below.

In the description of the embedding in the type IIB theory, we must make corresponding continuation

$$
\mu_3 \longrightarrow e^{\frac{\mathrm{i}}{2}\pi}\,\mu_3\,, \tag{5.10}
$$

leaving μ_1 and μ_2 unchanged, implying that we shall have

$$
\Delta \longrightarrow e^{\frac{\mathrm{i}}{3}\pi}\,\Delta\,. \tag{5.11}
$$

The type IIB theory has a "trombone" symmetry under which we perform the rescalings $d\hat{s}_{10}^2 \longrightarrow \lambda^2\, d\hat{s}_{10}^2$, and $\hat{F}_{(5)} \longrightarrow \lambda^4\, \hat{F}_{(5)}$. If we take $\lambda = \exp(-\frac{\rm i}{12}\pi)$, the we finally arrive at the reduction

$$\begin{aligned}
d\hat{s}_{10}^2 &= \Delta^{1/2}\, ds_5^2 + \frac{1}{g^2}\,\Delta^{-1/2}\,\sum_{i=1}^3 X_i^{-1}\,\Big(d\mu_i^2 + \mu_i^2\,(d\phi_i + g\,A^i)^2\Big)\,,\\
\hat{G}_{(5)} &= \sum_{i=1}^3 \eta_i\,\Big(2g(X_i^2\,\mu_i^2 - \Delta\,X_i)\,\epsilon_{(5)} - \frac{1}{2g}\,X_i^{-1}\,{*dX_i}\wedge d(\mu_i^2)\\
&\quad + \frac{1}{2g^2}\,X_i^{-2}\,d(\mu_i^2)\wedge(d\phi_i + g\,A_{(1)}^i)\wedge{*F_{(2)}^i}\Big)\,,
\end{aligned} \tag{5.12}$$

where

$$\begin{aligned}
\Delta &= \sum_{i=1}^3 X_i\,\mu_i^2\,, \qquad \mu_1^2 + \mu_2^2 - \mu_3^2 = 1\,,\\
\eta_i &= (1, 1, -1)\,.
\end{aligned} \tag{5.13}$$

This describes a reduction on the non-compact hyperboloid $\mathcal{H}^{4,2}$, and the supergravity we have obtained here is the $N = 2$ truncation of the $N = 8$ non-compact gauged $SO(4, 2)$ supergravity in five dimensions.

5.2 Seven-dimensional $N = 2$ gauged supergravity

The expression for a consistent S^4 reduction that yields the $U(1)^2$ gauged $N = 2$ supergravity was given in [1]. The gauge fields lie in the Cartan subgroup of $SO(5)$. The bosonic Lagrangian is given by

$$e^{-1}\mathcal{L}_7 = R - \tfrac12(\partial\vec{\varphi})^2 - \tfrac14\sum_{i=1}^2 X_i^{-2}\,(F_{(2)}^i)^2 - V\,, \tag{5.14}$$

where the scalar potential V is given by

$$V = -g^2\left(4X_1X_2 + 2X_1^{-1}\,X_2^{-2} + 2X_2^{-1}\,X_1^{-2} + \tfrac12 - (X_1X_2)^{-4}\right)\,, \tag{5.15}$$

and

$$X_1 = e^{-\frac{1}{\sqrt{2}}\varphi_1 + \frac{1}{\sqrt{10}}\varphi_2}\,, \qquad X_2 = e^{\frac{1}{\sqrt{2}}\varphi_1 + \frac{1}{\sqrt{10}}\varphi_2}\,. \tag{5.16}$$

The theory is obtained via a consistent Pauli reduction on S^4 [1], with

$$d\hat{s}_{11}^2 = \widetilde{\Delta}^{1/3}\,ds_7^2 + g^{-2}\,\widetilde{\Delta}^{-2/3}\,\Big(X_0^{-1}\,d\mu_0^2 + \sum_{i=1}^2 X_i^{-1}\,(d\mu_i^2 + \mu_i^2\,(d\phi_i + g\,A_{(1)}^i)^2)\Big) \tag{5.17}$$

$$\begin{aligned}
\hat{*}\hat{F}_{(4)} &= 2g\,\sum_{\alpha=0}^2\Big(X_\alpha^2\,\mu_\alpha^2 - \widetilde{\Delta}\,X_\alpha\Big)\,\epsilon_{(7)} + g\,\widetilde{\Delta}\,X_0\,\epsilon_{(7)} + \frac{1}{2g}\,\sum_{\alpha=0}^2 X_\alpha^{-1}\,{*dX_\alpha}\wedge d(\mu_\alpha^2)\\
&\quad + \frac{1}{2g^2}\,\sum_{i=1}^2 X_i^{-2}\,d(\mu_i^2)\wedge(d\phi_i + g\,A_{(1)}^i)\wedge{*F_{(2)}^i}\,,
\end{aligned} \tag{5.18}$$

where the auxiliary variable X_0 is defined by $X_0 \equiv (X_1 X_2)^{-2}$, and we have

$$\Delta = \sum_{\alpha=0}^{2} X_\alpha\, \mu_\alpha^2\,, \qquad \sum_{\alpha=0}^{2} \mu_\alpha^2 = 1\,. \tag{5.19}$$

An analytic continuation to a de Sitter like supergravity is given by sending

$$\varphi_1 \longrightarrow \varphi_1 + \tfrac{\mathrm{i}}{\sqrt{2}}\pi\,, \qquad \varphi_2 \longrightarrow \varphi_2 + \tfrac{\mathrm{i}}{\sqrt{10}}\pi\,, \qquad A^1 \longrightarrow e^{\frac{3\mathrm{i}}{5}\pi}\, A^i\,, \qquad g \longrightarrow e^{-\frac{3\mathrm{i}}{5}\pi}\, g\,. \tag{5.20}$$

These imply that we shall have

$$X_0 \longrightarrow e^{-\frac{2\mathrm{i}}{5}\pi}\, X_0\,, \qquad X_1 \longrightarrow e^{-\frac{2\mathrm{i}}{5}\pi}\, X_1\,, \qquad X_2 \longrightarrow e^{\frac{3\mathrm{i}}{5}\pi}\, X_2\,. \tag{5.21}$$

Under this continuation, the Lagrangian is still given by (3.4), except that now the scalar potential (5.15) is replaced by

$$\begin{aligned} V &= g^2\,(4X_1\, X_2 - 2X_1^{-1}\, X_2^{-2} + 2X_1^{-2}\, X_2^{-1} + \tfrac{1}{2}(X_1\, X_2)^{-2})\,, \\ &= g^2\,(4e^{-\frac{3}{\sqrt{10}}\varphi_2}\, \sinh \tfrac{1}{\sqrt{2}}\varphi_1 + 4e^{\frac{2}{\sqrt{10}}\varphi_2} + \tfrac{1}{2}e^{-\frac{4}{\sqrt{10}}\varphi_2})\,. \end{aligned} \tag{5.22}$$

One can easily see that, as expected, this potential has no stationary points. It is unbounded from above and below.

The continuation of the embedding in eleven-dimensional supergravity is obtained by making the corresponding continuation,

$$\mu_0 \longrightarrow \mu_0\,, \qquad \mu_1 \longrightarrow \mu_1\,, \qquad \mu_2 \longrightarrow e^{-\frac{\mathrm{i}}{2}\pi}\, \mu_2\,. \tag{5.23}$$

This implies that we will have $\Delta \longrightarrow \exp(-\frac{2\mathrm{i}}{5}\pi)\,\Delta$. Using the trombone rescaling symmetry $d\hat{s}_{11}^2 \longrightarrow \lambda^2\, d\hat{s}_{11}^2$, $\hat{*}\hat{F}_{(4)} \longrightarrow \lambda^6\, \hat{*}\hat{F}_{(4)}$, with $\lambda = \exp(\frac{\mathrm{i}}{15}\pi)$, we find that the reduction (5.17), (5.18) becomes

$$d\hat{s}_{11}^2 = \Delta^{1/3}\, ds_7^2 + g^{-2}\,\Delta^{-2/3}\Big(X_0^{-1}\, d\mu_0^2 + \sum_{i=1}^{2} X_i^{-1}\,(d\mu_i^2 + \mu_i^2\,(d\phi_i + g\, A_{(1)}^i)^2)\Big)\,, \tag{5.24}$$

$$\begin{aligned} \hat{*}\hat{F}_{(4)} &= -2g\sum_{\alpha=0}^{2} \eta_\alpha\, X_\alpha^2\, \mu_\alpha^2\, \epsilon_{(7)} + g\,\Delta\,(X_0 + 2X_1 - 2X_2)\,\epsilon_{(7)} - \frac{1}{2g}\sum_{\alpha=0}^{2} \eta_\alpha\, X_\alpha^{-1} *dX_\alpha \wedge d(\mu_\alpha^2) \\ &\quad - \frac{1}{2g^2}\sum_{i=1}^{2} \eta_i\, X_i^{-2}\, d(\mu_i^2) \wedge (d\phi_i + g\, A_{(1)}^i) \wedge *F_{(2)}^i\,, \end{aligned} \tag{5.25}$$

where

$$\begin{aligned} \Delta &= \sum_{\alpha=0}^{2} X_\alpha\, \mu_\alpha^2\,, \qquad \mu_0^2 + \mu_1^2 - \mu_2^2 = 1\,, \\ \eta_\alpha &= (1, 1, -1)\,. \end{aligned} \tag{5.26}$$

The internal metric lives on the hyperboloidal space $\mathcal{H}^{3,2}$. In a full consistent reduction on this space, obtained by analytically continuing the consistent S^4 reduction [3] that gives the $N = 4$ gauged $SO(5)$ theory in seven dimensions, one would obtain the $N = 4$ non-compact $SO(3, 2)$ gauged supergravity. Our $N = 2$ truncation retains just the $U(1)^2$ gauge field in the Cartan subgroup of $SO(3, 2)$.

6 Black Hole and Cosmological de Sitter Solutions

In this section, we shall derive charged black hole and cosmological solutions for specific supergravities with non-compact gaugings in $D = 4$, 5 and 7. In the examples considered, only the charges residing in the Abelian subgroup of $SO(p, 2r)$ are turned on, yielding solutions that can all be described within an $N = 2$ truncation. Specifically the equations of motion for such multiply-charged solutions can be solved in the case of the $N = 2$ truncation of these supergravities in $D = 4$, $D = 5$ and $D = 7$. These solutions are related to the AdS charged black hole solutions of the $N = 2$ truncations of the respective $SO(8)$, $SO(6)$ and $SO(5)$ gauged supergravities.

6.1 Three-charge solutions of $D = 5$, $N = 2$ gauged $SO(2, 4)$ and $SO(4, 2)$ supergravities

These solutions are closely related to the AdS black hole solutions of the $N = 2$ truncation of five-dimensional $SO(6)$ gauged supergravity, coupled to the three Abelian vector supermultiplets. The latter solutions were derived in [23], and are of the form

$$\begin{aligned} ds_5^2 &= -(H_1 H_2 H_3)^{-2/3}\, f\, dt^2 + (H_1 H_2 H_3)^{1/3}\, (f^{-1}\, dr^2 + r^2 d\Omega_{3,k}^2)\ , \\ X_i &= H_i^{-1}\, (H_1 H_2 H_3)^{1/3}\ , \qquad A^i_{(1)} = \sqrt{k}\, (1 - H_i^{-1})\, \coth(\sqrt{k}\, \beta_i)\, dt\, , \end{aligned} \tag{6.1}$$

where

$$f = k - \frac{\mu}{r^2} + 4g^2\, r^2\, (H_1 H_2 H_3)\, , \tag{6.2}$$

and the harmonic functions H_i are given by

$$H_i = 1 + \frac{\mu\, \sinh^2(\sqrt{k}\, \beta_i)}{k\, r^2}\ . \tag{6.3}$$

Here, k can be 1, 0 or -1, corresponding to the cases where the foliations in the transverse space have the metric $d\Omega_{3,k}^2$ on the unit S^3, T^3 or $\mathbf{H}^3$, where $\mathbf{H}^3$ is the unit hyperbolic 3-space of constant negative curvature. Note that in order to satisfy the Einstein equations of motion, the constants c_i in the harmonic functions $H_i = c_i + O(1/r^2)$ in [23] were taken to be 1. This ensured that the contribution from the scalars X_i to the right-hand side of the Einstein equations was compatible with the metric contribution in the Einstein equations (see section 3.2 of [23] for details).

For the $N=2$ truncation of the $SO(2,4)$ and $SO(4,2)$ supergravities with the potential of the form (5.2), one can apply the analysis of [23] in a straightforward way. The equations of motion are solved with the same ansatz (6.1)-(6.2) for the metric, scalars and gauge fields, except that now the harmonic functions take the form

$$H_i = \eta_i + \frac{\mu \, \sinh^2(\sqrt{k}\,\beta_i)}{k\, r^2}\,, \qquad i = 1,\, 2,\, 3\,, \tag{6.4}$$

where: $\eta_i = (1,\,-1,\,-1)$ and $\eta_i = (1,\,1,\,-1)$ for $SO(2,4)$ and $SO(4,2)$ supergravities, respectively. Note that the integration constants η_i are determined by the Einstein equation $R^r_r + 2\,R^\theta_\theta = -2\,V$. (See also section 3.2 of [23].) The conditions $H_i \geq 0$ ensure that the metric remains real, and that the scalar fields are in the physical regime $X_i \geq 0$. This requires $\mu \geq 0$, and it constrains the radial coordinate r to lie only in a restricted range:

$$\mu \geq 0\,, \qquad 0 \leq r^2 \leq \min\left(\frac{\mu \sinh^2(\sqrt{k}\beta_2)}{k}\,, \frac{\mu \sinh^2(\sqrt{k}\beta_3)}{k}\right)\,, \tag{6.5}$$

or

$$\mu \geq 0\,, \qquad 0 \leq r^2 \leq \frac{\mu \sinh^2(\sqrt{k}\beta_3)}{k}\,, \tag{6.6}$$

for $SO(2,4)$ or $SO(4,2)$ gauged supergravity respectively. Note that the horizon(s) are determined by the zeros of the function f. Their location depends on the values of the parameters μ, g and β_i. (For a related discussion of horizons for AdS charged black holes in $SO(6)$ gauged supergravity, see section 4.3 of [23].) The solution has a curvature singularity both on the lower and upper limit of the r coordinate range (6.5) (for $SO(2,4)$ supergravity) and (6.6) (for $SO(4,2)$ supergravity).

The analytic continuation:

$$t \to \mathrm{i}\, t\,, \qquad r \to \mathrm{i}\, r\,, \qquad \theta \to \mathrm{i}\,\theta \tag{6.7}$$

yields a set of solutions with $\mu \leq 0$, for which the r coordinate is restricted to a range:

$$\mu \leq 0\,, \qquad 0 \leq r^2 \leq \min\left(\frac{|\mu| \sinh^2(\sqrt{k}\beta_2)}{k}\,, \frac{|\mu| \sinh^2(\sqrt{k}\beta_3)}{k}\right)\,, \tag{6.8}$$

or

$$\mu \leq 0\,, \qquad 0 \leq r^2 \leq \frac{|\mu| \sinh^2(\sqrt{k}\beta_3)}{k}\,, \tag{6.9}$$

for $SO(2,4)$ or $SO(4,2)$ gauged supergravity respectively.

Owing to the analytic continuation performed in(6.7), the solution with $k = +1$ corresponds to a cosmological solution on $\mathbf{H}^3$ with a "big crunch" singularity at the upper boundary of the time coordinate r, and a cosmological horizon at a zero of function $f = 0$. On the other hand the analytic continuation of the solution with $k = -1$ corresponds to a black hole solution with S^3 sections, and a naked singularity on both boundaries of the radial coordinate r.

While the Ansätze for these solutions bear similarities to the Ansätze for the $SO(6)$ AdS black hole solutions, the former are highly singular. This can be attributed to the fact that the potential (5.2) does not have an extremum, and is unbounded from above and below.

The limit $\mu \to 0$ and $\beta_i \to \infty$, keeping $\mu\, e^{2\beta_i} = 2\, q_i$ finite, leads to supersymmetric solutions. For this class of solution one can perform an analytic continuation $g \to \mathrm{i}\, g$ and formally solve the Killing spinor equations with the new ansatz for the metric:[4]

$$\begin{aligned} ds_5^2 &= -(H_1H_2H_3)^{-2/3}\, dt^2 + (H_1H_2H_3)^{1/3}\, (dr^2 + r^2 d\Omega^2_{3,k=+1})\,, \\ X_i &= H_i^{-1}(H_1H_2H_3)^{1/3}\,, \qquad A^i_{(1)} = \sqrt{k}\,(1 - H_i^{-1})\, dt\,, \qquad , \end{aligned} \tag{6.10}$$

with new harmonic functions:

$$H_i = 2\eta_i g\, t + \frac{q_i}{r^2}\,, \tag{6.11}$$

where $\eta_i = (1,\, -1,\, -1)$ and $\eta_i = (1,\, 1,\, -1)$ are solutions for $SO(4,2)$ and $SO(2,4)$ supergravities respectively. (Note that the analytic continuation $g \to \mathrm{i}\, g$ changes the overall sign of the gauged supergravity potential, and thus interchanges the $SO(4,2)$ and $SO(2,4)$ potentials.) This form of solutions allows for multi-centered black hole solutions, i.e.:

$$H_i = 2\eta_i g\, t + \sum_{j=1}^{N} \frac{q_{ij}}{(\vec{r}_i - \vec{r})^2}\,. \tag{6.12}$$

Again, the positivity of the harmonic functions H_i constrains the allowed range of the t and r coordinates.

The solution (6.10) can be obtained (see [24]) from the supersymmetric limit ($\mu \to 0$, $\beta_i \to \infty$, $\mu e^{2\beta_i} = 2\, q_i$) of the solution (6.1), by first performing the analytic continuation $g \to \mathrm{i}\, g$ and the coordinate transformation

$$r = r' \sqrt{2g\, t'}, \qquad \frac{dt'}{2g\, t'} = dt + F(r)dr, \qquad F(r) = \frac{-2g\, r \prod_{i=1}^{3} H_i(r)}{1 - 4g^2\, r^2 \prod_{i=1}^{3} H_i(r)}\,, \tag{6.13}$$

and then dropping the "primes".

[4] See Ref. [13,24] for details. These solutions are analogues of four-dimensional charged de-Sitter solutions first discussed by Kastor and Traschen [22].

6.2 Four-charge solutions of $D=4$, $N=2$ gauged $SO(4,4)$, $SO(2,6)$ and $SO(6,2)$ supergravities

In the $N=2$ truncation of four-dimensional $SO(4,4)$, $SO(2,6)$ and $SO(6,2)$ gauged supergravities, one can again find four-charge solutions, whose ansatz is closely related to the four-charge solutions of $SO(8)$ supergravity [25, 26]:

$$\begin{aligned} ds_4^2 &= (H_1H_2H_3H_4)^{-1/2}(-f\,dt^2) + (H_1H_2H_3H_4)^{1/2}(f^{-1}\,dr^2 + r^2 d\Omega_{2,\,k}^2) \\ X_i &= H_i^{-1}(H_1H_2H_3H_4)^{1/4}\,, \qquad A_{(1)}^i = \sqrt{k}\,(1-H_i^{-1})\,\coth(\sqrt{k}\,\beta_i)\,dt\,, \end{aligned} \tag{6.14}$$

with

$$f = k - \frac{\mu}{r} + 4g^2 r^2 H_1H_2H_3H_4\,, \tag{6.15}$$

and the harmonic functions

$$H_i = \eta_i + \frac{\mu \sinh^2(\sqrt{k}\beta_i)}{kr}\,, \qquad i = 1, \cdots, 4\,. \tag{6.16}$$

Here, one takes $\eta_i = (1,\,1,\,-1,\,-1)$, $\eta_i = (1,\,-1,\,-1,\,-1)$ and $\eta_i = (1,\,1,\,1,\,-1)$ for the $SO(4,4)$, $SO(2,6)$ and $SO(6,2)$ gauged supergravities respectively. The constraints on the integration constants η_i are again imposed by the Einstein equations.

The conditions $X_i \geq 0$ imply positivity of harmonic functions H_i, and they constrain the parameter and the range of the radial coordinate:

$$\mu > 0\,, \qquad 0 \leq r \leq \min\left(\frac{\mu\sinh^2(\sqrt{k}\beta_3)}{k}\,, \frac{\mu\sinh^2(\sqrt{k}\beta_4)}{k}\right)\,, \tag{6.17}$$

$$\mu > 0\,, \qquad 0 \leq r \leq \min\left(\frac{\mu\sinh^2(\sqrt{k}\beta_2)}{k}\,, \frac{\mu\sinh^2(\sqrt{k}\beta_3)}{k}\,, \frac{\mu\sinh^2(\sqrt{k}\beta_4)}{k}\right)\,, \tag{6.18}$$

$$\mu > 0\,, \qquad 0 \leq r \leq \frac{\mu\sinh^2(\sqrt{k}\beta_4)}{k}\,, \tag{6.19}$$

for the $SO(4,4)$, $SO(2,6)$ and $SO(6,2)$ gauged supergravities respectively. These solutions have naked singularities at both boundaries. Depending on the values of the g, μ and β_i parameters, the solutions can have horizons at zeros of the function f. There is also a mirror region with $\mu < 0$ and $r < 0$.

After performing the analytic continuation $\mu \to \mathrm{i}\,\mu$, $r \to \mathrm{i}\,r$, $t \to \mathrm{i}\,t$ and $\theta \to \mathrm{i}\,\theta$ of the solutions with $k=1$, one obtains cosmological solutions on $\mathbf{H}^2$ that are analogous to those in the five-dimensional case; they can have a a "big crunch" singularity at the upper bound of the time coordinate r, and a cosmological horizon at a zero of the function f, whose location is determined by the value of the parameters.

In spite of the fact that the potential has a (tachyonic) extremum, these solutions are singular, owing to the unbounded nature of the potential.

The limit $\mu \to 0$ and $\beta_i \to \infty$, keeping $\mu e^{2\beta_i} = 2\, q_i$ finite, leads to the supersymmetric solutions, in a fashion analogous to the $D = 5$ case we discussed previously. In the present case, an analytic continuation $g \to \mathrm{i}\, g$ allows for a class of solutions that take the form (see [13]):

$$
\begin{aligned}
ds_5^2 &= -(H_1 H_2 H_3 H_4)^{-1/2}\, dt^2 + (H_1 H_2 H_3 H_4)^{1/2}\, (dr^2 + r^2 d\Omega_{3,k=+1}^2)\,, \\
X_i &= H_i^{-1} (H_1 H_2 H_3 H_4)^{1/4}\,, \qquad A^i_{(1)} = \sqrt{k}\, (1 - H_i^{-1})\, dt\,,
\end{aligned}
\tag{6.20}
$$

with

$$
H_i = 2\, \eta_i g\, t + \frac{q_i}{r}\,. \tag{6.21}
$$

Here $\eta_i = (1,\, 1,\, -1,\, -1)$, $\eta_i = (1,\, -1,\, -1,\, -1)$ and $\eta_i = (1,\, 1,\, 1,\, -1)$ for $SO(4,4)$, $SO(6,2)$ and $SO(2,6)$ gauged supergravities, respectively. (Again the analytic continuation $g \to \mathrm{i}\, g$ changes the overall sign of the gauged supergravity potential and thus interchanges the $SO(6,2)$ potential with the $SO(2,6)$ one.) These solutions also allow for multi-centered black holes, i.e.

$$
H_i = 2\, \eta_i g\, t + \sum_{j=1}^{N} \frac{q_{ij}}{|\vec{r}_i - \vec{r}|}\,. \tag{6.22}
$$

Again, the positivity of the harmonic functions H_i constrains the allowed range of the t and r coordinates.

These solutions are different from the four-charge de Sitter solutions of stable de Sitter vacua, discussed in [13]. The latter can be obtained by the analytic continuation $g \to \mathrm{i}g$ of the BPS four-charge black hole solutions in $SO(8)$ gauged supergravity. They correspond to the four-charge solutions (6.20)-(6.22) with $\eta_i = (1,\, 1\, 1,\, 1)$ and asymptote at late times to the stable de Sitter vacuum. When one identifies the four charges $q_i = q$ $(i = 1, \cdots 4)$ the scalars become constant $(X_i = 1)$ and the solution becomes the de Sitter Reissner Nordström black hole of Kastor and Traschen [22] of the (stable) de Sitter vacuum.

In the case of $SO(4,4)$ gauge supergravity the potential also has a tachyonic de Sitter extremum. However, the solutions (6.20)-(6.22) with $\eta_i = (1,\, 1,\, -1,\, -1)$ do not admit a limit of the charge parameters q_i for which the scalars could become constant $(X_i = 1)$, and thus these solution cannot describe charged de Sitter black hole solutions in this tachyonic extremum.

6.3 Two-charge solutions of $D = 7$, $N = 2$ gauged $SO(1,4)$ and $SO(3,2)$ supergravities

Our last examples arise in the $N = 2$ truncation of seven-dimensional $SO(1,4)$ and $SO(3,2)$ supergravities. These solutions are closely related to the AdS black hole solutions of the $N = 2$ truncation of seven-dimensional gauged $SO(5)$ supergravity. The solutions here are of the form

$$\begin{aligned} ds_7^2 &= -(H_1 H_2)^{-4/5}\, f\, dt^2 + (H_1 H_2)^{1/5}\, (f^{-1}\, dr^2 + r^2 d\Omega_{5,k}^2)\ , \qquad (6.23)\\ X_i &= H_i^{-1}(H_1 H_2)^{1/2}\,, (i = 1,\, 2)\,, \qquad X_0 = (X_1 X_2)^{-2}\,,\\ A_{(1)}^i &= \sqrt{k}\,(1 - H_i^{-1})\, \coth(\sqrt{k}\,\beta_i)\, dt\,, \qquad i = 1, 2\,, \qquad (6.24) \end{aligned}$$

with

$$f = k - \frac{\mu}{r^4} + 4g^2\, r^2\, (H_1 H_2)\,, \tag{6.25}$$

and harmonic functions given by

$$H_i = \eta_i + \frac{\mu\, \sinh^2(\sqrt{k}\,\beta_i)}{k\, r^4}\,. \tag{6.26}$$

Here, $\eta_i = (-1,\, -1)$ and $\eta_i = (1,\, -1)$ for the $SO(1,4)$ and $SO(3,2)$ gauged supergravities respectively. The positivity of the X_i is ensured by requiring the harmonic functions H_i to be positive, which is achieved by the restrictions

$$\mu \ge 0\,, \qquad 0 \le r^4 \le \min\left(\frac{\mu \sinh^2(\sqrt{k}\beta_1)}{k}\,,\, \frac{\mu \sinh^2(\sqrt{k}\beta_2)}{k}\right)\,, \tag{6.27}$$

or

$$\mu \ge 0\,, \qquad 0 \le r^4 \le \frac{\mu \sinh^2(\sqrt{k}\beta_2)}{k}\,, \tag{6.28}$$

respectively. The analytic continuation

$$t \to \mathrm{i}\, t\,, \qquad r \to \mathrm{i}\, r\,, \qquad \theta \to \mathrm{i}\, \theta \tag{6.29}$$

yields a cosmological solution with $\mu \le 0$, and the time coordinate r is constrained to the same region as above.

Taking the supersymmetric limit and analytic continuation $g \to \mathrm{i}\, g$, one also obtains analogous "de Sitter" solutions:

$$\begin{aligned} ds_7^2 &= -(H_1 H_2)^{-4/5}\, dt^2 + (H_1 H_2)^{1/5}\, (dr^2 + r^2 d\Omega_{5,k=+1}^2)\ , \qquad (6.30)\\ X_i &= H_i^{-1}(H_1 H_2)^{1/2}\,, \qquad A_{(1)}^i = \sqrt{k}\,(1 - H_i^{-1})\, \beta_i)\, dt\,, \qquad i = 1, 2\,, \end{aligned}$$

with the harmonic functions

$$H_i = 2\,\eta_i g\, t + \frac{q_i}{r^4}\,. \tag{6.31}$$

We conclude this section with a comment about another solution associated with the $N = 2$ truncation of seven-dimensional supergravity, with the $SO(2,2)$ gauging. The domain wall solution [27] of the $N = 2$ supergravity for the $SO(4)$ gauging is given by

$$ds_7^2 = e^{2A}\, dx^\mu\, dx^\nu \eta_{\mu\nu} + e^{8A}\, dy^2\,, \qquad e^{-\frac{3}{\sqrt{10}}\phi} = H\,, \tag{6.32}$$

with

$$e^{-4\,A} = \frac{2g}{5(H^{-1/3})'}\,, \qquad H = e^{-\frac{3}{\sqrt{10}}\phi_0} + q|y|\,, \tag{6.33}$$

where a prime denotes a derivative with respect to y. The analytic continuations $g \to \mathrm{i}\, g$ and $\{t,\, y,\, q\} \to \mathrm{i}\,\{t,\, y,\, q\}$ turn this into a cosmological solution of the de Sitter gauged supergravity.

7 Conclusions

An elegant feature of gauged $SO(p,q)$ $(q = 2r)$ supergravity theories is that they can be obtained from gauged $SO(p+q)$ supergravity theories by means of straightforward analytic continuations. The consistent Pauli reductions of M-theory and string theory on spheres S^{p+q-1}, yielding gauged $SO(p+q)$ supergravities, have been studied extensively in the literature. This has allowed us to explore in a straightforward way the corresponding consistent Pauli reductions on the hyperboloidal spaces $\mathcal{H}^{p,q}$, thus yielding supergravity theories with non-compact gauge groups.

We provided a general analysis of the extrema of the unimodular part of the scalar potential for supergravities with $SO(p,q)$ gaugings. It turns out that only for seven-dimensional supergravity with an $SO(2,2)$ gauging, and four-dimensional supergravity with an $SO(4,4)$ gauging, does one obtain stable extrema of the unimodular part of the scalar potential. In the seven-dimensional case the potential still depends on a "volume" scalar, thus yielding (cosmological) solutions with a running scalar and a positive potential contribution. On the other hand, the four-dimensional case has an extremum of the scalar potential corresponding to a de Sitter vacuum, which, however, is a *saddle point*. This result is consistent with the analysis of the $N = 2$ gauged supergravity potentials with general non-compact isometries [15].

Interestingly, we also found that $D = 4$ gauged $SO(4,4)$, $SO(2,6)$ and $SO(6,2)$ supergravity, $D = 5$ gauged $SO(2,4)$ and $SO(4,6)$ supergravity and $D = 7$ gauged $SO(1,4)$

and $SO(3,2)$ supergravity admit Abelian charged black hole (and cosmological) solutions in their $N=2$ truncations, whose structures are closely related to the corresponding gauged $SO(p+q)$ supergravity black holes. However, the solutions obtained here are highly singular, in consequence of the unbounded nature of the scalar potentials.

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