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$SU(2)$ Reduction of Six-dimensional $(1,0)$ Supergravity

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ABSTRACT

We obtain a gauged supergravity theory in three dimensions with eight real supersymmetries by means of a Scherk-Schwarz reduction of pure $N = (1,0)$ supergravity in six dimension on the $SU(2)$ group manifold. The $SU(2)$ Yang-Mills fields in the model propagate, since they have an ordinary kinetic term in addition to Chern-Simons couplings. The other propagating degrees of freedom consist of a dilaton, five scalars which parameterise the coset $SL(3,R)/SO(3)$, three vector fields in the adjoint of $SU(2)$, and twelve spin $\frac{1}{2}$ fermions. The model admits an AdS_3 vacuum solution. We also show how a charged black hole solution can be obtained, by performing a dimensional reduction of the rotating self-dual string of six-dimensional $(1,0)$ supergravity.

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Contents

1 Introduction

The importance of supergravities in diverse dimensions in the context of string theory has been widely appreciated for a long time. However, the role of gauged supergravities has been appreciated only relatively recent, and especially it has become more evident with developments in the AdS/CFT correspondence. Supergravities are referred to as "gauged" when their R-symmetry group, or any subgroup thereof, is gauged. Those which have played role in the AdS/CFT correspondence admit AdS vacuum solutions, but not all gauged supergravities admit AdS vacua (see, for example, [1]), and not all supergravities which admit AdS vacua are gauged (see, for example, [2]). We shall refer to those gauged supergravities that do admit AdS vacua as "AdS supergravities".

In this paper, we shall fill a gap in our knowledge of AdS supergravities by constructing one in three dimensions with eight real supersymmetries. We shall do this by means of a Scherk-Schwarz reduction [3] of the chiral $N = (1,0)$ supergravity in $D = 6$. The main motivation for our work is to make progress in finding the still elusive supergravity theory that is expected to arise in the $\mathrm{AdS}_3 \times S_3$ compactification of $(2,0)$ supergravity theory [4, 5], which in turn, emerges from type IIB string theory reduced on $K3$ [6, 7]. As a first step in this direction, here we construct the AdS_3 supergravity that is pertinent to the $\mathrm{AdS}_3 \times S^3$ compactification of pure $N = (1,0)$ supergravity in $D = 6$ [8], which can also be embedded in heterotic string compactifying on K3. We do so by employing a Scherk-Schwarz reduction on the $SU(2)$ group manifold. Before commenting further on our results, let us review some facts about gauged and AdS supergravity theories that can be obtained from consistent Kaluza-Klein reductions, which will clarify the rationale behind our having chosen the Scherk-Schwarz reduction scheme here.

Many examples of gauged supergravities can be obtained by consistent Kaluza-Klein sphere reduction of higher-dimensional ungauged supergravities.[1] Notable cases include the S^5 reduction of type IIB supergravity and the S^7 [9] and S^4 [10, 11] reductions of eleven-dimensional supergravity; these give rise to the maximal gauged supergravities in $D = 5$,

[1]A *consistent* Kaluza-Klein reduction is defined here as one where all the gauge bosons of the isometry group G of the compactifying manifold are retained in a truncation keeping only a finite number of the lower-dimensional fields, with the essential requirement that setting the truncated fields to zero must be consistent with their own equations of motion. Put another way, the reduction ansatz is consistent if all the higher-dimensional equations of motion are satisfied as a consequence of the equations of motion for the retained lower-dimensional fields. It is only in very exceptional cases that such consistent Kaluza-Klein reductions on compactifying spaces other than tori are possible.

$D = 7$ and $D = 4$ respectively.[2] The reductions to maximal supergravities are extremely complicated, especially in the S^5 and S^7 cases. Several examples giving gauged supergravities with lesser supersymmetry have been worked out in complete detail (at least in their bosonic sectors). These include the half-maximal supersymmetric gauged supergravities in $D = 7$, 6, 5 and 4 dimensions [12, 13, 14, 15].

The fact that the above Kaluza-Klein sphere reductions are consistent is quite remarkable, and there is no general understanding of why they work. The consistency depends crucially on "conspiracies" between contributions from the metric and the other fields in the higher-dimensional theories. One might suspect from the above examples that supersymmetry plays a crucial role, but in fact this is somewhat misleading. There are examples where supersymmetric theories cannot be consistently reduced on spheres, and there are examples where non-supersymmetric theories *can* be consistently reduced on spheres.

In fact a more universal characterisation of when a theory admits a consistent sphere reduction can be given by first studying the global symmetry of the theory when it is instead Kaluza-Klein reduced on a torus of the same dimension as the sphere. This was discussed in depth in [16]. The key point is that a generic theory reduced on T^n has a $GL(n, \mathbb{R})$ global symmetry, and so its maximal compact subgroup is $SO(n)$. By contrast, the theory that one would obtain by instead reducing on S^n would have an $SO(n+1)$ gauge group, and so by sending the gauge-coupling to zero there would have to be at least an $SO(n+1)$ compact subgroup in the resulting global symmetry group, contradicting the previous observation that generically the maximal compact subgroup is only $SO(n)$. Put another way, if there were a consistent S^n reduction then one would have to be able to gauge an $SO(n + 1)$ subgroup of the global symmetry group of the T^n reduction, and for a generic theory the toroidal reduction does not yield a large enough global symmetry group.

The only way in which a reduction on S^n could be consistent is therefore if there is actually an enhanced global symmetry group in the reduction on T^n. This occurs only if there is some "conspiracy" between the contributions from the T^n reduction of metric and the other fields in the theory. Such conspiracies are indeed sometimes seen in supergravity reductions (including the toroidal reductions of type IIB and eleven-dimensional supergravity), and it is precisely these conspiracies that also allow the consistent sphere reductions to

[2] To be precise, the consistency of the S^5 reduction has never been proven, and the reduction ansatz for the S^7 example has not been fully explicitly exhibited. The explicit reduction ansatz for the S^4 case has been given,and its consistency has been proven, modulo the assumption that the inclusion of quartic fermion terms will not upset results established at lower order in fermions. In all cases, compelling circumstantial evidence implying the consistency of the reductions has been found.

work. However, as was shown in [16], there exist also examples of purely bosonic theories that also admit consistent sphere reductions; a notable set of cases is provided by the S^3 or S^{D-3} reductions of the bosonic string effective action in any dimension D.

Since there exist $\mathrm{AdS}_3 \times S^3$ supersymmetric vacua of the six-dimensional ungauged supergravities, it is natural to enquire whether there might exist associated consistent S^3 reductions. An extension of the arguments presented in [16] suggests that such consistent reductions are not possible.[3] On the other hand, one would expect that there should exist an AdS_3 gauged supergravity, with a higher-dimensional origin, which would play an important role in the $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence. This would then be analogous to the examples in $4 \le D \le 8$.

Indeed there exists an alternative way of performing a consistent Kaluza-Klein reduction on S^3, exploiting the fact that it is isomorphic to the group manifold $SU(2)$. This reduction, known as Scherk-Schwarz reduction [3], has the merit that it is guaranteed to be consistent when applied to any theory at all. It does, however, give rise only to the gauge fields of $SU(2)$, rather than the $SO(4) \sim SU(2) \times SU(2)$ that would have arisen had there existed a consistent sphere reduction of the kind we were describing above. In this paper, we shall implement the Scherk-Schwarz procedure for the case of an S^3 reduction of the $N = (1, 0)$ chiral supergravity in $D = 6$. This gives rise to a gauged supergravity in three dimensions with an $SU(2)$ gauge group that admits an AdS_3 vacuum solution.

The gauged supergravity obtained here bears similarities to maximal AdS supergravities in $D = 5$ [17, 18], $D = 6$ [19] and $D = 7$ [20]. After we present our results, we shall comment on these similarities in the concluding section. It is worth pointing out here, however, that our AdS_3 supergravity differs from other gauged supergravities that have been constructed so far in three dimensions [21, 22, 23, 24, 25, 26], in which the Yang-Mills fields are non-propagating since they belong to the supergravity multiplet and are described solely by a Chern-Simons term. Our model corresponds to a fusion of non-propagating Poincaré supergravity [27, 28] with propagating fields originating from the reduction of the graviton and 2-form field of $N = (1, 0)$ supergravity in $D = 6$. In particular, the bosonic field content consists of a dilaton; five scalars which parameterise the coset $SL(3, R)/SO(3)$; the gauge fields of $SU(2) \sim SO(3)$, all of which originate from the six-dimensional metric; and three vector fields which originate from the six-dimensional 2-form potential. Thus altogether

[3] It is possible, however, to perform a different consistent S^3 reduction that results in a gauged three-dimensional supergravity which does not admit an AdS_3 vacuum solution, but instead a domain wall [16]. From the six-dimensional point of view, this solution is the near-horizon limit of a purely electric or purely magnetic string.

there are twelve bosonic degrees of freedom, and by supersymmetry, twelve fermionic ones. This is the same count of degrees of freedom that one finds in a toroidal compactification of pure $N = (1,0)$ supergravity in $D = 6$. We shall compare this with the massless Kaluza-Klein spectra of the $\mathrm{AdS}_3 \times S^3$ compactified $(1,0)$ and $N = (2,0)$ supergravities in the concluding section.

In this paper we shall also elaborate on the structure of the scalar potential that arises in our model, describe the $U(1)$ truncation of the model, and we shall show how a charged black hole solution can be obtained by performing a dimensional reduction of the rotating self-dual string in the six-dimensional $(1,0)$ supergravity.

The organisation of the paper is as follows. In section 2 we begin with a review of the Scherk-Schwarz S^3 reduction of a D-dimensional metric, deriving expressions for the Ricci tensor in $(D-3)$ dimensions. We then specialise to the case of six-dimensional $N = (1,0)$ supergravity, deriving the expressions for the reduction of the self-dual 3-form field, and hence for the complete reduction of the bosonic sector of the theory. We show how the potential for the $GL(3,\mathbb{R})/SO(3)$ scalars can be expressed in terms of a superpotential, and we derive a consistent truncation in which the $SU(2)$ Yang-Mills fields are reduced to $U(1)$.

In section 3 we extend the construction of the three-dimensional gauged supergravity by obtaining the supersymmetry transformation rules for the fermionic fields. In section 4 we consider some specific solutions of the three-dimensional supergravity, and their six-dimensional interpretation in terms of rotating self-dual strings. The paper closes with conclusions and speculations in section 5.

2 The bosonic sector

The bosonic sector of the $(1,0)$ six-dimensional supergravity theory comprises the metric tensor $\hat g_{MN}$ and a 2-form potential $\hat B_{(2)}$ whose field strength $\hat H_{(3)} = d\hat B_{(2)}$ is self-dual. The six-dimensional bosonic equations of motion are

$$\hat R_{MN} = \hat H_{MPQ}\, \hat H_N{}^{PQ}\,, \qquad d\hat{*}\hat H_{(3)} = 0\,, \qquad \hat{*}\hat H_{(3)} = \hat H_{(3)}\,. \tag{1}$$

We shall reduce the six-dimensional theory to three dimensions by compactifying on a 3-sphere, viewed as the group manifold $SU(2)$. The Kaluza-Klein reduction scheme that we employ will be the group manifold reduction of Scherk and Schwarz [3], in which a truncation to the set of all fields invariant under the left action G_L of the total isometry group $G_L \times G_R$ acting on the group manifold G. This truncation is guaranteed to be a consistent one, since on group-theoretic grounds non-linear products of the retained G_L-singlet fields cannot act as sources for the discarded G_L-non-singlet fields.

2.1 Reduction of the metric

It is convenient to introduce the left-invariant $SU(2)$ 1-forms σ^α, which satisfy the Maurer-Cartan algebra

$$d\sigma^\alpha = -\tfrac{1}{2} f^\alpha{}_{\beta\gamma}\, \sigma^\beta \wedge \sigma^\gamma\,, \tag{2}$$

where $f^\alpha{}_{\beta\gamma}$ are the $SU(2)$ structure constants. For now we shall consider the case where we reduce on $SU(2)$ from $(n+3)$ to n dimensions. The Kaluza-Klein metric reduction ansatz will then be given by[4]

$$d\hat{s}^2 = e^{2\alpha\,\phi}\, ds^2 + \frac{4}{g^2}\, e^{2\beta\,\phi}\, h_{\alpha\beta}\, \nu^\alpha\, \nu^\beta\,, \tag{3}$$

where ϕ is the "breathing-mode" scalar, $h_{\alpha\beta}$ denotes the remaining n-dimensional scalar fields (with the symmetric tensor $h_{\alpha\beta}$ being unimodular), and ν^α is given by

$$\nu^\alpha \equiv \sigma^\alpha - g\, A^\alpha\,. \tag{4}$$

Here A^α denotes the $SU(2)$ Yang-Mills potentials corresponding to the right-acting $SU(2)$ isometry of the 3-sphere. The constants α and β in (3) will be determined later.

It will prove convenient to work in a vielbein basis, which we take to be

$$\hat{e}^a = e^{\alpha\,\phi}\, e^a\,, \qquad \hat{e}^i = 2g^{-1}\, e^{\beta\,\phi}\, L^i_\alpha\, \nu^\alpha\,. \tag{5}$$

Here e^a is a vielbein basis for the n-dimensional metric ds^2, and L^i_α is a "square root" of $h_{\alpha\beta}$, and so

$$h_{\alpha\beta} = L^i_\alpha\, L^i_\beta\,, \qquad \det(L^i_\alpha) = 1\,. \tag{6}$$

Defining the Yang-Mills field strengths $F^\alpha = dA^\alpha + \tfrac{1}{2} g\, f^\alpha{}_{\beta\gamma}\, A^\beta \wedge A^\gamma$, we have:

$$\begin{aligned} D\,F^\alpha &\equiv dF^\alpha + g\, f^\alpha{}_{\beta\gamma}\, A^\beta \wedge F^\gamma = 0\,,\\ D\,\nu^\alpha &\equiv d\nu^\alpha + g\, f^\alpha{}_{\beta\gamma}\, A^\beta \wedge \nu^\gamma = -g\, F^\alpha - \tfrac{1}{2}\, f^\alpha{}_{\beta\gamma}\, \nu^\beta \wedge \nu^\gamma\,. \end{aligned} \tag{7}$$

It is also useful to define the Yang-Mills covariant exterior derivative acting on the scalars L^i_α:

$$D\, L^i_\alpha \equiv dL^i_\alpha - g\, f^\beta{}_{\gamma\alpha}\, A^\gamma\, L^i_\beta\,. \tag{8}$$

[4]In this paper we are using "supergravity conventions," in which the bosonic Lagrangian is written with normalisations of the form $\sqrt{-g}\,(\frac{1}{4}R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}(F^i_{(2)})^2 + \cdots)$. For the convenience of readers who prefer the customary $\sqrt{-g}\,(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}(F^i_{(2)})^2 + \cdots)$ convention, we include a "hidden" appendix which repeats sections 2 and 4 in this notation. It can be accessed by deleting the "\end{document}" in the Latex file at the end of the references.

Using these expressions, we find from (5) that

$$\begin{aligned} d\hat{e}^a &= -\alpha\, e^{-\alpha\phi}\, \partial_b\phi\ \hat{e}^a\wedge\hat{e}^b - \omega^a{}_b\wedge\hat{e}^b\,,\\ d\hat{e}^i &= e^{-\alpha\phi}\,(L^{-1})_j^\alpha\,(D_a L^i_\alpha)\,\hat{e}^a\wedge\hat{e}^j + \beta\, e^{-\alpha\phi}\,\partial_a\phi\,\hat{e}^a\wedge\hat{e}^i - e^{(\beta-2\alpha)\phi}\, F^i_{ab}\,\hat{e}^a\wedge\hat{e}^b\\ &\quad -\tfrac14 g\, e^{-\beta\phi}\, T^{i\ell}\,\epsilon_{jk\ell}\,\hat{e}^j\wedge\hat{e}^k\,, \end{aligned}\qquad (9)$$

where we have defined

$$F^i_{ab}\equiv L^i_\alpha\, F^\alpha\,,\qquad T^{ij}\equiv L^i_\alpha\, L^j_\alpha\,. \qquad (10)$$

(Note that T^{ij} is $SU(2)$ covariant, despite superficial appearances, since $\delta^{\alpha\beta}$ is an invariant tensor in $SO(3)\sim SU(2)$.) From (9), we calculate the torsion-free spin connection $\hat{\omega}^A{}_B$, defined by $d\hat{e}^A = -\hat{\omega}^A{}_B\wedge\hat{e}^B$ and $\hat{\omega}_{AB}=-\hat{\omega}_{BA}$, finding

$$\begin{aligned} \hat{\omega}_{ab} &= \omega_{ab} + \alpha\, e^{-\alpha\phi}\,(\partial_b\phi\,\eta_{ac}\,\hat{e}^c - \partial_a\phi\,\eta_{bc}\,\hat{e}^c) + e^{(\beta-2\alpha)\,\phi}\, F^i_{ab}\,\hat{e}^i\,,\\ \hat{\omega}_{ai} &= -e^{-\alpha\phi}\, P_{a\,ij}\,\hat{e}^j - \beta\, e^{-\alpha\phi}\,\partial_a\phi\,\hat{e}^i + e^{(\beta-2\alpha)\,\phi}\, F^i_{ab}\,\hat{e}^b\,,\\ \hat{\omega}_{ij} &= e^{-\alpha\phi}\, Q_{a\,ij}\,\hat{e}^a + \tfrac14 g\, e^{-\beta\phi}\,(T^{k\ell}\,\epsilon_{ij\ell} + T^{j\ell}\,\epsilon_{ik\ell} - T^{i\ell}\,\epsilon_{jk\ell})\,\hat{e}^k\,. \end{aligned}\qquad (11)$$

Here we have defined

$$P_{a\,ij}\equiv \tfrac12[(L^{-1})_i^\alpha\, D_a L^j_\alpha + (L^{-1})_j^\alpha\, D_a L^i_\alpha]\,,\qquad Q_{a\,ij}\equiv \tfrac12[(L^{-1})_i^\alpha\, D_a L^j_\alpha - (L^{-1})_j^\alpha\, D_a L^i_\alpha]\,. \quad (12)$$

The next step is to calculate the Ricci tensor $\hat{R}_{AB}=\hat{R}^C{}_{ACB}$, which can in principle be done by first calculating the curvature 2-forms $\hat{\Theta}^A{}_B = d\hat{\omega}^A{}_B + \hat{\omega}^A{}_C\wedge\hat{\omega}^C{}_B = \frac12 \hat{R}^A{}_{BCD}\,\hat{e}^C\wedge\hat{e}^D$. This is quite an involved calculation. In practice, a simpler way to find the Ricci tensor is to use an observation that was made in [3], which is that the dimensional reduction of the Einstein Hilbert Lagrangian $L=\hat{\rm e}\,\hat{R}$ is given, up to a total derivative, by[5]

$$L = \hat{\rm e}\,(\hat{\omega}_{A\,BC}\,\hat{\omega}^{C\,AB} + \hat{\omega}^A\,\hat{\omega}_A)\,, \qquad (13)$$

where

$$\hat{\omega}_{AB}\equiv \omega_{C\,AB}\,\hat{e}^C\,,\qquad \omega_A\equiv \eta^{BC}\,\hat{\omega}_{B\,CA}\,. \qquad (14)$$

It is convenient at this point to make the following choices for the constants α and β:

$$\alpha^2 = \frac{6}{(n-2)\,(n+1)}\,,\qquad \beta = -\tfrac13\alpha\,(n-2)\,. \qquad (15)$$

The second condition ensures that the lower-dimensional metric is also in the Einstein frame, and the first condition simply sets the scale for ϕ so that it has a canonically-normalised

[5]It is crucial, in order to apply this argument, that the reduction be a *consistent* one, meaning that the equations of motion derived from the dimensionally-reduced action coincide with those that follow from the dimensional reduction of the higher-dimensional equations of motion.

kinetic term. After making these choices, we find after a simple calculation from (11) and (13) that the higher-dimensional Einstein-Hilbert Lagrangian $L = \hat{R}\sqrt{-\hat{g}}$ reduces to give

$$\begin{aligned}\sqrt{-\hat{g}}\,\hat{R} &= \sqrt{-g}\Big[R - 2(\partial\phi)^2 - (P_{a\,ij})^2 - e^{-\frac{2}{3}\alpha\,(n+1)\,\phi}\,(F^i_{ab})^2 \\ &\quad -\tfrac{1}{4}g^2\,e^{\frac{2}{3}\alpha\,(n+1)\,\phi}\,(T^{ij}\,T^{ij} - \tfrac{1}{2}T^2)\Big] + \text{total derivative}\,,\end{aligned} \tag{16}$$

where $T \equiv T^{ii}$.

From (16), we can easily obtain the lower-dimensional equations of motion for the metric, the Yang-Mills potentials, and the scalar fields. These equations in fact match up with the $\hat{R}_{ab}$, $\hat{R}_{ai}$ and $\hat{R}_{ij}$ vielbein components of the higher-dimensional Ricci tensor. The only subtlety is that there are overall scalings to be determined, involving certain specific powers of e^{ϕ}, and that the components $\hat{R}_{ab}$ are actually formed from a linear combination of the lower-dimensional Einstein equation and η_{ab} times a multiple of the trace of the scalar field equations. These scalings and combinations are easily determined by considering special cases. By this means, we therefore arrive at the expressions for the components of the higher-dimensional Ricci tensor with much less labour than by the direct approach via the curvature 2-forms. We find

$$\begin{aligned}\hat{R}_{ab} &= e^{-2\alpha\phi}\,[R_{ab} - 2\partial_a\phi\,\partial_b\phi - P_{a\,ij}\,P_{b\,ij} - \alpha\,\Box\phi\,\eta_{ab} - 2e^{-\frac{2}{3}\alpha\,(n+1)\,\phi}\,F^i_{ac}\,F^i_{bd}\,\eta^{cd}]\,, \\ \hat{R}_{ai} &= -e^{\frac{1}{3}\alpha\,(n-5)\,\phi}\,[\mathcal{D}^b\,(e^{-\frac{2}{3}\alpha\,(n+1)\,\phi}\,F^i_{ab}) + e^{-\frac{2}{3}\alpha\,(n+1)\,\phi}\,F^j_{ab}\,P^b{}_{ij} - \tfrac{1}{2}g\,\epsilon_{ijk}\,T^{k\ell}\,P_{a\,j\ell}]\,, \\ \hat{R}_{ij} &= -\tfrac{1}{2}e^{-2\alpha\phi}\,[\mathcal{D}_a\,P^a{}_{ij} - \tfrac{2}{3}\alpha\,(n-2)\,\Box\phi\,\delta_{ij} - 2e^{-\frac{2}{3}\alpha\,(n+1)\,\phi}\,F^i_{ab}\,F^j_{cd}\,\eta^{ac}\,\eta^{bd} \\ &\quad -g^2\,e^{\frac{2}{3}\alpha\,(n+1)\,\phi}\,(T^{ik}\,T^{jk} - \tfrac{1}{2}T\,T^{ij}) + \tfrac{1}{2}g^2\,e^{\frac{2}{3}\alpha\,(n+1)\,\phi}\,(T^{k\ell}\,T^{k\ell} - \tfrac{1}{2}T^2)\,\delta_{ij}]\,,\end{aligned} \tag{17}$$

where we have now defined a derivative $\mathcal{D}_a$ that is covariant not only with respect to general coordinate and Yang-Mills transformations, but also it involves the composite connection $Q_{a\,ij}$:

$$\mathcal{D}_a\,L^i_\alpha \equiv D_a\,L^i_\alpha + Q_{a\,ij}\,L^j_\beta\,, \tag{18}$$

(and similarly when acting on $P_{a\,ij}$ and F^i). Note that using $\mathcal{D}_a$, we have

$$(L^{-1})^\alpha_i\,\mathcal{D}_a\,L^j_\alpha = P_{a\,ij}\,. \tag{19}$$

2.2 Reduction of the 3-form

The self-dual 3-form in six dimensions can be written as $\hat{H}_{(3)} = \hat{G}_{(3)} + \hat{*}\hat{G}_{(3)}$. We find that the appropriate reduction ansatz is given by taking

$$\hat{G}_{(3)} = \frac{8m}{g^3}\,\Omega_{(3)} + \frac{2}{g^2}\epsilon_{\alpha\beta\gamma}\,B^\alpha\wedge\nu^\beta\wedge\nu^\gamma\,, \tag{20}$$

where m is a constant and $\Omega \equiv \nu^1 \wedge \nu^2 \wedge \nu^3$. Dualising (20) in the metric (3), we find that

$$\hat{*}\hat{G}_{(3)} = m\, e^{4\alpha\,\phi}\,\epsilon_{(3)} - \frac{2}{g}\, e^{\frac{4}{3}\alpha\,\phi} h_{\alpha\beta}\, {*B^\alpha} \wedge \nu^\beta\,, \tag{21}$$

where $h_{\alpha\beta} \equiv L^i_\alpha\, L^i_\beta$. (Here we have used equation (15), which for $n=3$ implies $\beta = -\frac{1}{3}\alpha$ and $\alpha^2 = \frac{3}{8}$.) Noting that $d\Omega_{(3)} = -\frac{1}{2}g\,\epsilon_{\alpha\beta\gamma}\, F^\alpha \wedge \nu^\beta \wedge \nu^\gamma$, we find that the Bianchi identity $d\hat{H}_{(3)} = 0$ implies the equation

$$(DB^\alpha)\, L^i_\alpha - 2m\, F^i + \tfrac{1}{2} g\, e^{\frac{4}{3}\alpha\,\phi}\, T^{ij}\, *B_j = 0\,. \tag{22}$$

We find that the vielbein components of the self-dual field strength $\hat{H}_{(3)}$ are given by

$$\begin{aligned} &\hat{H}_{abc} = m\, e^{\alpha\phi}\,\epsilon_{abc}\,, \qquad && \hat{H}_{abi} = -e^{-\frac{1}{3}\alpha\phi}\,\epsilon_{ab}{}^c\, B^i_c\,, \\ &\hat{H}_{ijk} = m\, e^{\alpha\phi}\,\epsilon_{ijk}\,, \qquad && \hat{H}_{aij} = e^{-\frac{1}{3}\alpha\phi}\,\epsilon_{ijk}\, B^k_a\,, \end{aligned} \tag{23}$$

where we have defined $B^i \equiv L^i_\alpha\, B^\alpha$.

2.3 Three-dimensional bosonic equations of motion and Lagrangian

From (23), together with our expressions (17) for the Ricci tensor, we find that the six-dimensional bosonic equations (1) imply the following three-dimensional bosonic equations of motion. First, from the $\hat{R}_{ij}$ components of the Einstein equation, we obtain the scalar equations

$$\begin{aligned} \alpha\,\Box\phi &= 6m^2\, e^{4\alpha\phi} + \tfrac{1}{4} g^2\, e^{\frac{8}{3}\alpha\phi}\, (T^{ij}\, T^{ij} - \tfrac{1}{2}T^2) + 2e^{\frac{4}{3}\alpha\phi}\, (B^i_a)^2 - e^{-\frac{8}{3}\alpha\phi}\, (F^i_{ab})^2\,, \\ \mathcal{D}_a P^a{}_{ij} &= g^2\, e^{\frac{8}{3}\alpha\phi}\, [T^{ik}\, T_{jk} - \tfrac{1}{2} T\, T^{ij} - \tfrac{1}{3}(T^{k\ell}\, T^{k\ell} - \tfrac{1}{2}T^2)\,\delta_{ij}] \\ &\quad +8e^{\frac{4}{3}\alpha\phi}\, [B^i_a\, B^j_b\, \eta^{ab} - \tfrac{1}{3}(B^k_a)^2\,\delta_{ij}] + 2e^{-\frac{8}{3}\alpha\phi}\, [F^i_{ab}\, F^j_{cd}\, \eta^{ac}\, \eta^{bd} - \tfrac{1}{3}(F^k_{ab})^2\,\delta_{ij}]\,. \end{aligned} \tag{24}$$

From the $\hat{R}_{ai}$ components of the Einstein equation, we obtain the Yang-Mills equation

$$\mathcal{D}^b(e^{-\frac{8}{3}\alpha\phi}\, F^i_{ab}) = -e^{-\frac{8}{3}\alpha\phi}\, F^j_{ab}\, P^b{}_{ij} + \tfrac{1}{4} g^2\, \epsilon_{ijk}\, T^{k\ell}\, P_{a\,j\ell} - 4m\, e^{\frac{4}{3}\alpha\phi}\, B^i_a + 2\epsilon_{ijk}\,\epsilon_a{}^{bc}\, B^j_b\, B^k_c\,. \tag{25}$$

From the $\hat{R}_{ab}$ components of the Einstein equation we obtain, after using the ϕ equation above to replace a $\Box\phi$ term,

$$\begin{aligned} R_{ab} &= 2\partial_a\phi\,\partial_b\phi + P_{a\,ij}\, P_{b\,ij} + 4e^{\frac{4}{3}\alpha\phi}\, B^i_a\, B^i_b + 2e^{-\frac{8}{3}\alpha\phi}\, (F^i_{ac}\, F^i_{bd}\,\eta^{cd} - \tfrac{1}{2}(F^i_{cd})^2\,\eta_{ab}) \\ &\quad +4m^2\, e^{4\alpha\phi}\,\eta_{ab} + \tfrac{1}{4} g^2\, e^{\frac{8}{3}\alpha\phi}\, (T^{ij}\, T^{ij} - \tfrac{1}{2}T^2)\,\eta_{ab}\,. \end{aligned} \tag{26}$$

Finally, there is the equation (22) that came from $d\hat{H}_{(3)} = 0$.

We find that these equations of motions can be derived from the 3-dimensional Lagrangian

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{4}R\,{*\mathbb{1}} - \tfrac{1}{2}{*d\phi}\wedge d\phi - \tfrac{1}{4}{*P_{ij}}\wedge P_{ij} - e^{\frac{4}{3}\alpha\phi}\,{*B^i}\wedge B^i - \tfrac{1}{2}e^{-\frac{8}{3}\alpha\phi}\,{*F^i}\wedge F^i\\ &\quad -m^2\,e^{4\alpha\phi}\,{*\mathbb{1}} - \tfrac{1}{16}g^2\,e^{\frac{8}{3}\alpha\phi}\,(T^{ij}\,T^{ij} - \tfrac{1}{2}T^2)\,{*\mathbb{1}} + \mathcal{L}_{CS}\,, \end{aligned} \tag{27}$$

where the Chern-Simons contribution $\mathcal{L}_{CS}$ is given by

$$\mathcal{L}_{CS} = -\frac{2}{g}\,DB^\alpha\wedge B^\alpha + \frac{8m}{g}\,F^\alpha\wedge B^\alpha - \frac{8m^2}{g}\,\omega_{(3)}\,, \tag{28}$$

where

$$\omega_{(3)} \equiv A^\alpha\wedge dA^\alpha + \tfrac{1}{3}\epsilon_{\alpha\beta\gamma}\,A^\alpha\wedge A^\beta\wedge A^\gamma \tag{29}$$

is the usual Chern-Simons 3-form for the $SU(2)$ Yang-Mills fields, satisfying $d\omega_{(3)} = F^\alpha\wedge F^\alpha$. Note that the Lagrangian is invariant under $m\longrightarrow -m$, together with $B^\alpha\longrightarrow -B^\alpha$. On the other hand, it is invariant under $g\longrightarrow -g$ and $A^\alpha\longrightarrow -A^\alpha$ only if one also performs a parity or time-reversal transformation.

Although one cannot directly take the $g\longrightarrow 0$ limit in the Lagrangian, it can clearly be done at the level of the equations of motion. This is analogous to the case of seven-dimensional gauged supergravity, where the limit was discussed in detail in [29]. The $g\longrightarrow 0$ limit corresponds to a flattening of the reduction 3-sphere, which in the truncation to the massless sector can be replaced by a torus. The theory also admits a different limit, in which one instead sends m to zero. As can be seen from (20), this corresponds to setting the 3-form flux on S^3 to zero. The maximally-symmetric vacuum solution would then be six-dimensional Minkowski spacetime $d\hat{s}_6^2 = dx^\mu\,dx_\mu + dr^2 + r^2\,d\Omega_3^2$, instead of $\mathrm{AdS}_3\times S^3$.

It is interesting to note that in the Scherk-Schwarz reduction of ten-dimensional $N=1$ supergravity on S^3, one can consistently truncate out the scalar fields that are parameterised by L^i_α, whilst retaining the $SU(2)$ Yang-Mills fields. (This was proved in [30] for Scherk-Schwarz reductions of the low-energy effective action of the bosonic string in any dimension, reduced on any group manifold. The truncation of the scalars is possible provided that the vectors coming from the reduction of the 2-form potential are equated to the Yang-Mills potentials.) By contrast, in our present case we cannot consistently truncate the scalars L^i_α without also truncating the $SU(2)$ Yang-Mills fields, as can be seen from the scalar equations for $\mathcal{D}_a\,P^a_{ij}$ in (24). The key difference is that in the present case the 3-form field strength in six dimensions is subject to a self-duality condition.

2.4 Superpotential for the three-dimensional theory

The scalar potential in (27) can be expressed in terms of a superpotential W. To do so, it is useful first to absorb the dilaton ϕ into L^i_α, by defining

$$\widetilde{L}^i_\alpha \equiv e^{\frac{2}{3}\alpha\,\phi}\, L^i_\alpha\,. \tag{30}$$

The scalar sector of the Lagrangian (27) can then be written as

$$\mathcal{L}_{\rm scal} = -\tfrac{1}{4}{*\widetilde{P}_{ij}}\wedge \widetilde{P}_{ij} - V\,{*\rlap{1}\hskip 1.6pt 1}\,, \tag{31}$$

where the scalar potential is given by

$$V \equiv m^2\,\det \widetilde{T}^{ij} + \tfrac{1}{16} g^2\,(\widetilde{T}^{ij}\,\widetilde{T}^{ij} - \tfrac{1}{2}\widetilde{T}^2)\,, \tag{32}$$

and $\widetilde{T}^{ij} \equiv \widetilde{L}^i_\alpha\,\widetilde{L}^j_\alpha$.

Introducing coordinates ϕ^I on the $GL(3,\mathbb{R})/SO(3)$ scalar coset manifold, a vielbein on the coset can be written as

$$V^{ij}_I = (\widetilde{L}^{-1})_i^\alpha\,\bar{D}_I \widetilde{L}^j_\alpha\,, \tag{33}$$

where

$$\bar{D}_I \widetilde{L}^i_\alpha \equiv \frac{\partial \widetilde{L}^i_\alpha}{\partial\phi^I} + \bar{Q}^{ij}_I\,\widetilde{L}^j_\alpha\,,\qquad \bar{Q}^{ij}_I \equiv \tfrac{1}{2}\Big[(\widetilde{L}^{-1})_i^\alpha\,\frac{\partial \widetilde{L}^j_\alpha}{\partial\phi^I} - (\widetilde{L}^{-1})_j^\alpha\,\frac{\partial \widetilde{L}^i_\alpha}{\partial\phi^I}\Big]\,. \tag{34}$$

In terms of the coset metric $G_{IJ} \equiv V^{ij}_I\, V^{ij}_J$, the scalar Lagrangian (31) can be written as $\mathcal{L}_{\rm scal} = -\frac{1}{4} G_{IJ}\,{*d\phi^I}\wedge d\phi^J - V\,{*\rlap{1}\hskip 1.6pt 1}$. The potential V can be expressed in terms of a superpotential W as

$$V = G^{IJ}\,\frac{\partial W}{\partial \phi^I}\,\frac{\partial W}{\partial\phi^J} - W^2\,, \tag{35}$$

where we find that W is given by

$$\begin{aligned} W &= -\sqrt{2} m\,\sqrt{\det \widetilde{T}^{ij}} + \tfrac{1}{4\sqrt{2}}\, g\,\widetilde{T}\,,\\ &= -\sqrt{2} m\, e^{2\alpha\,\phi} + \tfrac{1}{4\sqrt{2}}\, g\, e^{\frac{4}{3}\alpha\,\phi}\, T\,. \end{aligned} \tag{36}$$

If m has the same sign as g, (36) has an extremum at $T^{ij} = \delta^{ij}$, corresponding to the pure AdS$_3$ supersymmetric vacuum solution. More generally, there are solutions that break half the supersymmetry, which can be lifted back to six dimensions where they correspond to the standard self-dual string. If, on the other hand, m has the opposite sign to g, then AdS$_3$ is not included among the solutions of the associated first-order equations coming from W. There will instead be a half-supersymmetric domain-wall solution, which can be lifted back into six dimensions where it acquires an interpretation as a disjoint "interior" branch of a negative-mass self-dual string. (This issue was discussed extensively in [31].)

2.5 A consistent $U(1)$ truncation

It is straightforward to see that we can perform a consistent truncation of the three-dimensional theory obtained in the previous section, in which we set to zero two of the three Yang-Mills potentials A^α and two of the B^α 1-forms (for $\alpha = 1$ and 2), and at the same time we truncate the 5 scalars in the unimodular matrix L^i_α to a single diagonal scalar:

$$L^i_\alpha = \begin{pmatrix} e^{\gamma\varphi} & 0 & 0 \\ 0 & e^{\gamma\varphi} & 0 \\ 0 & 0 & e^{-2\gamma\varphi} \end{pmatrix}, \tag{37}$$

where $\gamma \equiv 1/\sqrt{3}$. After establishing that the truncation is consistent (by looking at the previous field equations), we can then simply impose the truncation in the Lagrangian (27). This gives

$$\begin{aligned} \mathcal{L} \;=\; & \tfrac14 R\,{*\mathbb{1}} - \tfrac12 {*d\phi}\wedge d\phi - \tfrac12 {*d\varphi}\wedge d\varphi - e^{\frac43\alpha\phi - 4\gamma\varphi}\, {*B}\wedge B - \tfrac12\, e^{-\frac83\alpha\phi-4\gamma\varphi}\, {*F}\wedge F \\ & - m^2\, e^{4\alpha\phi}\, {*\mathbb{1}} - \tfrac1{32} g^2\, e^{\frac83\alpha\phi}\,(e^{-8\gamma\varphi} - 4e^{-2\gamma\varphi})\,{*\mathbb{1}} + \mathcal{L}_{CS}\,, \end{aligned} \tag{38}$$

where the Chern-Simons contribution $\mathcal{L}_{CS}$ is given by

$$\mathcal{L}_{CS} = -\frac{2}{g}\, dB\wedge B + \frac{8m}{g}\, F\wedge B - \frac{8m^2}{g}\, dA\wedge A\,. \tag{39}$$

Note that the Lagrangian is invariant under $m \longrightarrow -m$, together with $B^\alpha \longrightarrow -B^\alpha$. On the other hand, it is invariant under $g \longrightarrow -g$ and $A^\alpha \longrightarrow -A^\alpha$ only if one also performs a parity or time-reversal transformation.

The three-dimensional equations of motion following from this truncated Lagrangian are:

$$\begin{aligned} &\alpha\Box\phi = 6m^2\, e^{4\alpha\phi} + \tfrac18 g^2\, e^{\frac83\alpha\phi}\,(e^{-8\gamma\varphi} - 4e^{-2\gamma\varphi}) - e^{-\frac83\alpha\phi-4\gamma\varphi}\, F^2 + \tfrac12 g^2\, e^{\frac43\alpha\phi-4\gamma\varphi}\, B^2\,, \\ &\gamma\Box\varphi = -\tfrac1{12} g^2\, e^{\frac83\alpha\phi}\,(e^{-8\gamma\varphi} - e^{-2\gamma\varphi}) - \tfrac13 e^{-\frac83\alpha\phi-4\gamma\varphi}\, F^2 - \tfrac13 g^2\, e^{\frac43\alpha\phi-4\gamma\varphi}\, B^2\,, \\ &d(e^{-\frac83\alpha\phi-4\gamma\varphi}\, {*F}) = -4m\, e^{\frac43\alpha\phi-4\gamma\varphi}\, {*B}\,, \\ &e^{\frac43\alpha\phi-4\gamma\varphi}\, {*B} = \frac{4m}{g}\, F - \frac{2}{g}\, dB\,, \\ &R_{ab} = \tfrac12\partial_a\phi\,\partial_b\phi + 6\partial_a\varphi\,\partial_b\varphi + 4e^{\frac43\alpha\phi-4\gamma\varphi}\, B_a\, B_b + 2e^{-\frac83\alpha\phi-4\gamma\varphi}\,(F^2_{ab} - \tfrac12 F^2\,\eta_{ab}) \\ &\qquad + 4m^2\, e^{4\alpha\phi}\,\eta_{ab} + \tfrac18 g^2\, e^{\frac83\alpha\phi}\,(e^{-8\gamma\varphi} - 4e^{-2\gamma\varphi})\,\eta_{ab}\,. \end{aligned} \tag{40}$$

The corresponding truncation in the $SU(2)$ reduction ansatz is given by

$$\begin{aligned} d\hat s_6^2 &= e^{2\alpha\phi}\, ds_3^2 + \frac{4}{g^2}\, e^{-\frac23\alpha\phi}\,[e^{2\gamma\varphi}\,(\sigma_1^2 + \sigma_2^2) + e^{-4\gamma\varphi}\,(\sigma_3 - g\,A)^2]\,, \\ \hat G_{(3)} &= \frac{8m}{g^3}\,\sigma_1\wedge\sigma_2\wedge(\sigma_3 - g\,A) + \frac{4}{g^2}\, B\wedge\sigma_1\wedge\sigma_2\,. \end{aligned} \tag{41}$$

The dual of $\hat{G}_{(3)}$ is given by

$$\hat{*}\hat{G}_{(3)} = m\, e^{4\alpha\phi}\, \epsilon_{(3)} - \frac{2}{g}\, e^{\frac{4}{3}\alpha\phi - 4\gamma\varphi}\, {*B} \wedge (\sigma_3 - g\, A)\,. \tag{42}$$

It is also of interest to look for superpotentials from which the scalar potential $V = m^2\, e^{4\alpha\phi} + \frac{1}{32} g^2\, e^{\frac{8}{3}\alpha\phi}\, (e^{-8\gamma\varphi} - 4e^{-2\gamma\varphi})$ can be derived. In this case, V will be expressed as

$$V = \tfrac{1}{4}\Big(\frac{\partial W}{\partial\phi}\Big)^2 + \tfrac{1}{4}\Big(\frac{\partial W}{\partial\varphi}\Big)^2 - W^2\,. \tag{43}$$

We find that the following choices for W are possible:

$$W = -\sqrt{2} m\, e^{2\alpha\,\phi} + \tfrac{1}{4\sqrt{2}}\, g\, e^{\frac{4}{3}\alpha\,\phi}\, (e^{-4\gamma\,\varphi} + 2e^{2\gamma\,\varphi})\,, \tag{44}$$

$$W = -\sqrt{2} m\, e^{2\alpha\,\phi} + \tfrac{1}{4\sqrt{2}}\, g\, e^{\frac{4}{3}\alpha\,\phi}\, (\pm e^{-4\gamma\,\varphi} + 4e^{-\gamma\,\varphi})\,. \tag{45}$$

The choice in (44) correspond to the superpotential given in (36), specialised to the $U(1)$ truncation given in (37). The choices in (45) do not arise from the specialisation of (36). A particular solution, if the minus sign is chosen, is the same AdS_3 vacuum. But in general, the solutions of the first-order equations associated with (45) are disjoint from those associated with (44).

3 The fermionic supersymmetry transformations

The $D = 6$, $(1, 0)$ supergravity multiplet consists of the vielbein, 2-form potential with self-dual field strength and a gravitino which is symplectic Majorana-Weyl spinor in doublet representation of the R-symmetry group $Sp(1)$. As is well known, a manifestly covariant action containing these fields alone cannot be written down due to the the self-duality condition. However, the coupling of this multiplet to a tensor multiplet consisting of a two-form potential with anti-self dual field strength, a dilaton and anti-chiral symplectic-Majorana spinor does admit a Lagrangian formulation. Indeed, the complete Lagrangian, field equations and supersymmetry transformation rules for the coupled system have been constructed in [34]. Starting from these field equations and transformation laws, we can obtain the corresponding ones for the pure supergravity theory by setting the dilaton and the tensor-multiplet spinor to zero, and imposing the exact, supersymmetric self-duality condition

$$H^-_{ABC} + \tfrac{1}{8}\bar{\psi}_D \Gamma^{[D}\Gamma_{ABC}\Gamma^{E]}\psi_E = 0\ . \tag{46}$$

One can show that the supersymmetric variation vanishes modulo the gravitino field equation. To implement this self-duality condition, we need to substitute

$$H_{ABC} = H^{+}_{ABC} - \tfrac{1}{8}\bar{\psi}_D\Gamma^{[D}\Gamma_{ABC}\Gamma^{E]}\psi_E \ , \tag{47}$$

everywhere H_{ABC} occurs in the equations of motion and the transformation laws.

In the present paper, we shall just derive the fermionic supersymmetry transformation rules for the Scherk-Schwarz reduced theree-dimensional theory; these suffice for testing the supersymmetry of three-dimensional bosonic solutions. In a later paper, we shall present the entire fermionic three-dimensional Lagrangian and transformation rules.

The supersymmetry transformations obtained from [34] by the truncation of the tensor multiplet are given up to leading order in fermions by

$$\begin{aligned} \delta e_M{}^A &= \bar{\epsilon}\,\Gamma^A\psi_M \ , & (48)\\ \delta B_{MN} &= -\bar{\epsilon}\Gamma_{[M}\psi_{N]} \ , & (49)\\ \delta\psi_M &= \big(\nabla_M + \tfrac{1}{4}H^{+}_M{}^{AB}\Gamma_{AB}\big)\,\epsilon\,, & (50) \end{aligned}$$

where $\nabla_M = \partial_M + \frac{1}{4}\omega_M{}^{AB}\,\Gamma_{AB}$. The chiral truncation leading to the a transformation rules have also been obtained in [8].

To perform the reduction of the fermionic transformation rules, we begin by making an ansatz for the reduction of the gravitino field. In doing so, we shall make use of the original treatment of this problem in [3], and [35] where it has been studied further in the context of $SU(2)$ reduction of $D = 11$ supergravity. One technical aspect of the reduction is the diagonalisation of the lower dimensional gravitino and spinor kinetic terms. It is convenient to treat the diagonalisation problem after performing the $SU(2)$ reduction, at the level of determining the field re-definitions in the lower dimensional Lagrangian that will yield diagonal fermionic kinetic terms. Thus, we begin with the ansatz

$$\hat{\psi}_a(x,y) = e^{\delta\phi}\psi_a(x) \ , \qquad \hat{\psi}_i(x,y) = e^{\delta\phi}\chi_i(x) \ , \tag{51}$$

where δ is a constant. To ensure that the gravitino kinetic term is canonical, with no dilaton prefactor, we must set

$$\delta = -\tfrac{1}{2}\,[(n-1)\alpha + 3\beta] \tag{52}$$

when reducing from $n+3$ to n dimensions. In our case, with $n = 3$, we therefore have

$$\beta = -\tfrac{1}{3}\alpha \ , \qquad \delta = -\tfrac{1}{2}\alpha \ , \tag{53}$$

We also specify our conventions as follows:

$$\Gamma^a \ = \ \gamma^a \times 1 \times \sigma^1 \ , \qquad\qquad \Gamma^i = 1 \times \gamma^i \times \sigma^2 \tag{54}$$

$$C_{(6)} = (i\sigma^2)\times(i\sigma^2)\times\sigma^1 \ , \qquad \Gamma_7 = 1\times 1\times\sigma^3 \ , \tag{55}$$

$$\gamma^{abc} = \epsilon^{abc} \ , \quad \gamma^{ijk} = i\epsilon^{ijk} \ , \qquad \{\gamma_a,\gamma_b\} = 2\eta_{ab} \ , \quad \{\gamma_i,\gamma_j\} = 2\delta_{ij} \ . \tag{56}$$

We use the metrics $\eta_{AB} = (-+++++)$ and $\eta_{ab} = (-++)$. Furthermore, $\epsilon^{012} = 1 = \epsilon^{345}$. In our convention, $\epsilon_{\mu\nu\rho}$ and $\epsilon^{\mu\nu\rho}$ are constant, and thus, $e\epsilon_{\mu\nu\rho}$ and $e^{-1}\epsilon^{\mu\nu\rho}$ are tensors.

Although we shall not derive the complete fermionic sector of the three-dimensional theory in the present paper, it is nonetheless useful to examine the structure of the fermionic kinetic terms, in order to see how the diagonalisation of three-dimensional fermion fields should be performed. To do this we can write down the Lagrangian of the six-dimensional tensor multiplet coupled supergravity Lagrangian in which the dilaton field and the tensor multiplet spinor are set to zero, namely

$$e^{-1}\mathcal{L} = \tfrac{1}{4}R - \tfrac{1}{12}H_{ABC}H^{ABC} - \tfrac{1}{2}\bar{\psi}_A\Gamma^{ABC}\psi_{BC} - \tfrac{1}{24}\bar{\psi}^D\Gamma_{[D}\Gamma^{ABC}\Gamma_{E]}\psi^E H_{ABC}\,. \tag{57}$$

After variation, one then needs to impose the self-duality condition (46). Of course for our present purposes, only the gravitino kinetic term is relevant. Substituting (51) into the Lagrangian, we obtain the three-dimensional kinetic terms

$$e^{-1}\mathcal{L}_{\rm kin} = -\tfrac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu\rho}\mathcal{D}_\nu\psi_\rho - i\bar{\chi}_i\gamma^i\gamma^{\mu\nu}\mathcal{D}_\mu\psi_\nu + \tfrac{1}{2}\bar{\chi}_i\gamma^{ij}\gamma^\mu\mathcal{D}_\mu\chi_j\,. \tag{58}$$

where have defined

$$\psi_\mu = e_\mu^a\psi_a \ , \qquad \Gamma_\mu = e_\mu^a\Gamma_a\,. \tag{59}$$

As expected, the gravitino and spinor kinetic terms are mixed. We have verified that they can be simultaneously be diagonalised for any dimension n. In particular, the field redefinitions which do the job for $n=3$ are given by

$$\begin{aligned} \psi_\mu &= \psi'_\mu - \tfrac{i}{2}\gamma_\mu\gamma^k\chi'_k \ , \\ \chi_i &= -\tfrac{1}{2}\gamma^k\gamma_i\chi'_k \ . \end{aligned} \tag{60}$$

It follows that the inverse transformation is

$$\begin{aligned} \psi'_\mu &= \psi_\mu + i\gamma_\mu\gamma^k\chi_k \ , \\ \chi' i &= \gamma_{ij}\chi^j \ . \end{aligned} \tag{61}$$

The kinetic terms become diagonal in terms of the primed fields, and in particular the spinor kinetic term becomes $-\frac{1}{2}\bar{\chi}'^i\gamma^\mu\mathcal{D}_\mu\chi'_i$.

The supersymmetry transformations of the vielbein and gravitino in pure $(1,0)$ supergravity in six dimensions are

$$\hat{e}_A{}^M \delta \hat{e}_{MB} = \bar{\hat{\epsilon}}\Gamma_B \hat{\psi}_A\,, \tag{62}$$

$$\delta\hat{\psi}_A = \hat{\nabla}_A\,\hat{\epsilon} + \tfrac{1}{4}\hat{H}_A^{+\ CD}\Gamma_{CD}\hat{\epsilon}\,. \tag{63}$$

As for the potential $\hat{B}_{MN}$, it is more convenient to work with its field strength since its 3D field B_μ^α it gives rise to is related directly to the field strength $\hat{H}_{abi}$ and $\hat{H}_{aij}$ as in (23). From (49) we have

$$\delta\hat{H}^+_{ABC} = -\tfrac{3}{2}\hat{\nabla}_{[A}\Big(\bar{\hat{\epsilon}}\Gamma_B\hat{\psi}_{C]}\Big) - \tfrac{1}{4}\epsilon_{ABCDEF}\hat{\nabla}^D\Big(\bar{\hat{\epsilon}}\Gamma^E\hat{\psi}^F\Big)\,. \tag{64}$$

To perform the $SU(2)$ reduction of the above transformation laws, we begin by making an ansatz for the supersymmetry parameter $\hat{\epsilon}(x,y)$. Obtaining $\delta\psi_a(x) = \nabla_a\epsilon(x) + \cdots$ requires the ansatz

$$\hat{\epsilon}(x,y) = e^{\alpha\phi/2}\epsilon(x)\,, \tag{65}$$

where we have used (53). This allows us to carry out the reduction of the gravitino transformation rule (63); to leading order in fermions, we find that this gives

$$\begin{aligned}
\delta\psi_\mu &= \mathcal{D}_\mu\epsilon - \tfrac{1}{4}ie^{-4\alpha\phi/3}\left(F^i_{\mu\nu} - 2e^{2\alpha\phi}\gamma_\mu B^i_\nu\right)\gamma^\nu\gamma_i\epsilon \\
&\quad + \tfrac{1}{2}\alpha\gamma_\mu\gamma^\nu\partial_\nu\phi\epsilon - \tfrac{1}{2}me^{2\alpha\phi}\gamma_\mu\epsilon,
\end{aligned} \tag{66}$$

$$\begin{aligned}
\delta\chi_i &= -\tfrac{i}{2}\gamma^\mu\gamma^j\left(P_{\mu ij} - \tfrac{1}{3}\alpha\delta_{ij}\partial_\mu\phi\right)\epsilon + \tfrac{1}{4}e^{-4\alpha\phi/3}\left(F_{\mu\nu i} - ee^{2\alpha\phi}\gamma_i\gamma^k\epsilon_{\mu\nu\rho}B^\rho_k\right)\gamma^{\mu\nu}\epsilon \\
&\quad + \tfrac{i}{2}ge^{4\alpha\phi/3}\left(T_{ij} - \tfrac{1}{2}\delta_{ij}T\right)\gamma^j\epsilon + \tfrac{i}{2}me^{2\alpha\phi}\gamma_i\epsilon\ .
\end{aligned} \tag{67}$$

Note that an expected T-dependent term in the gravitino transformation rule will emerge after performing the redefinition (60).

After performing the redefinitions given in (59), we find that the supersymmetry transformation rules for the diagonalised fermionic fields can be expressed as

$$\begin{aligned}
\delta\psi'_\mu &= \mathcal{D}_\mu\,\epsilon + \sqrt{2}\,W\,\gamma_\mu\,\epsilon - i\,e^{\frac{2}{3}\alpha\,\phi}\,B^i_\nu\,\gamma_\mu\,\gamma^\nu\,\gamma_i\,\epsilon + \tfrac{i}{4}\,e^{-\frac{4}{3}\alpha\,\phi}\,(F^i_{\mu\nu}\,\gamma^\nu + F^i_{\rho\sigma}\,\gamma_\mu{}^{\rho\sigma})\,\gamma_i\,\epsilon\,, \\
\delta\chi'_i &= \tfrac{i}{2}\,V_I^{ij}\,\gamma_j\,(\gamma^\mu\,\partial_\mu\phi^I - \sqrt{2}G^{IJ}\,\frac{\partial W}{\partial\phi^J})\,\epsilon - e^{\frac{2}{3}\alpha\,\phi}\,\gamma^\mu\,B^k_\mu\,\gamma_i\,\gamma_k\,\epsilon + \tfrac{1}{4}e^{-\frac{4}{3}\alpha\,\phi}\,F^j_{\mu\nu}\,\gamma_{ij}\,\epsilon\,,
\end{aligned} \tag{68}$$

where W, G_{IJ} and ϕ^I are the superpotential, scalar sigma-model metric and target-space coordinates introduced in section 2.4.

4 Charged AdS$_3$ black hole

In this section, we shall show how a charged black hole solution in the three-dimensional theory of section 2.5 can be obtained, by performing a dimensional reduction of a rotating self-dual string in the six-dimensional $(1,0)$ supergravity. A rotating dyonic string solution of six-dimensional $(1,1)$ supergravity was constructed in [32]. This involved two angular momentum parameters ℓ_1 and ℓ_2, associated with two commuting $U(1)$ factors in the $SO(4)$ rotation group acting on 3-spheres in the four-dimensional transverse space of the string. There were also two parameters δ_1 and δ_2 that characterised the electric and magnetic charges of the string. The configuration can be viewed as a solution in the $(1,0)$ supergravity if one sets $\delta_1 = \delta_2 = \delta$, since then the electric and magnetic charges become equal and consequently the 3-form field strength becomes self-dual and the dilaton decouples. If the two angular momentum parameters are also set equal, $\ell_1 = \ell_2 = \ell$, the rotating string solution then fits within our $SU(2)$ reduction ansatz, and in fact it fits within the $U(1)$ truncation of section 2.5. The metric of this self-dual rotating string solution can be read off from [32]:

$$\begin{aligned} d\hat{s}_6^2 &= -H^{-1}\,(1-\frac{2k}{r^2+\ell^2})\,dt^2 + H^{-1}\,dx^2 - \frac{H\,h^4\,\ell^2\,(r^2+\ell^2)}{(1+h^2\,\ell^2)}\,(c^2\,dt+s^2\,dx)^2 \\ &\quad + \frac{H\,r^2\,(r^2+\ell^2)}{(r^2+\ell^2)^2-2k\,r^2}\,dr^2 + \tfrac14 H\,(r^2+\ell^2)\,[\sigma_1^2+\sigma_2^2+(1+h^2\,\ell^2)\,(\sigma_3-\widetilde{A})^2]\,, \end{aligned} \tag{69}$$

where

$$H = 1+\frac{2k\,s^2}{r^2+\ell^2}\,,\qquad h^2 = \frac{2k}{H^2\,(r^2+\ell^2)^2}\,,\qquad \widetilde{A} = \frac{2h^2\,\ell}{1+h^2\,\ell^2}\,(c^2\,dt+s^2\,dx)\,, \tag{70}$$

and we have defined $c\equiv\cosh\delta$, $s\equiv\sinh\delta$.

We find that the self-dual 3-form is given by

$$\begin{aligned} \hat{H}_{(3)} &= -\frac{k\,s\,c}{\sqrt{2}}\Big\{\sigma_1\wedge\sigma_2\wedge(\sigma_3-\widetilde{A}) + [\widetilde{A} - \frac{2\ell}{H\,(r^2+\ell)^2}\,(dt+dx)]\wedge\sigma_1\wedge\sigma_2 \\ &\quad - \frac{4r\,\ell}{H^2\,(r^2+\ell^2)^2}\,(dt+dx)\wedge dr\wedge(\sigma_3-\widetilde{A}) + \frac{8r}{H^2\,(1+h^2\,\ell^2)(r^2+\ell^2)^2}\,dt\wedge dx\wedge dr\Big\}\,. \end{aligned} \tag{71}$$

Comparing the metric (69) and the field strength (71) with the reduction ansatz in section 2.5, we obtain a three-dimensional AdS charged black hole solution, given by

$$\begin{aligned} ds_3^2 &= \frac{g^6}{64}H^3\,(r^2+\ell^2)^3(1+h^2\,\ell^2)\Big\{-H^{-1}\,(1-\frac{2k}{r^2+\ell^2})\,dt^2 + H^{-1}\,dx^2 \\ &\quad - \frac{H\,h^4\,\ell^2\,(r^2+\ell^2)}{(1+h^2\,\ell^2)}\,(c^2\,dt+s^2\,dx)^2 + \frac{H\,r^2\,(r^2+\ell^2)}{(r^2+\ell^2)^2-2k\,r^2}\,dr^2\Big\}\,, \\ A &= \frac{2h^2\,\ell}{g\,(1+h^2\,\ell^2)}\,(c^2\,dt+s^2\,dx)\,, \end{aligned} \tag{72}$$

$$
\begin{aligned}
B &= \sqrt{2}\,\coth\delta\,[-g\,A + \frac{2\ell}{H\,(r^2+\ell^2)}\,(dt+dx)]\,,\\
e^{-2\alpha\phi} &= \frac{g^6}{64}\,H^3\,(r^2+\ell^2)^3(1+h^2\,\ell^2)\,, \qquad e^{-6\varphi} = 1+h^2\,\ell^2\,,
\end{aligned}
$$

where $m = -\sqrt{2}\,g\,\coth\delta$ and $g = \sqrt{2/(k\,s^2)}$. One can indeed directly verify that this configuration satisfies the three-dimensional equations of motion (40).

In the extremal limit, which is obtained by sending k to zero and δ to infinity, keeping the charge parameter $Q = k\,\sinh 2\delta$ fixed, the solution becomes

$$
\begin{aligned}
ds_3^2 &= \frac{W}{Q^3}\Big[\frac{r^2+\ell^2}{W}(-dt^2+dx^2) - \frac{Q^2\,\ell^2}{W^3}\,(dt+dx)^2 + \frac{W\,r^2\,dr^2}{(r^2+\ell^2)^2}\Big]\\
A &= \frac{\sqrt{Q^3}\,\ell\,(dt+dx)}{(r^2+\ell^2+Q)^2}\,, \quad B = \frac{2\ell\,(r^2+\ell^2+\frac{1}{2}Q)\,(dt+dx)}{(r^2+\ell^2+Q)^2}\,,\\
e^{-\frac{2}{3}\alpha\phi} &= Q^{-1}\,(r^2+\ell^2+Q)\,, \qquad \varphi = 0\,,
\end{aligned} \tag{73}
$$

where $W = r^2+\ell^2+Q$.

The charged AdS$_3$ black hole (72) also admits a decoupling limit, namely

$$
r = \epsilon\,\tilde{r}\,, \quad \ell = \epsilon\,\tilde{\ell}\,, \quad k = \epsilon^4\,\tilde{k}\,, \quad \sinh 2\delta = \frac{Q}{\tilde{k}\,\epsilon^2}\,, \tag{74}
$$

with ϵ taken to be small. In this limit, the solution becomes

$$
\begin{aligned}
ds_3^2 &= \epsilon^2\Big[-N^2\,dt^2 + \frac{1}{N^2}\,dr^2 + r^2\Big(\frac{dx}{\sqrt{Q}} - \frac{\ell^2}{\sqrt{Q}\,r^2}dt\Big)^2\Big]\,,\\
A &= \frac{\epsilon\,\ell}{\sqrt{Q}}\,(dt+dx)\,, \qquad B = 0\,, \qquad \phi = 0\,, \qquad \varphi = 0\,,\\
g &= \frac{2}{\epsilon\,\sqrt{Q}}\,, \qquad m = -\frac{2\sqrt{2}}{\epsilon\,\sqrt{Q}}\,.
\end{aligned} \tag{75}
$$

where

$$
N^2 = \frac{r^2}{Q} + \frac{2(\ell^2-k)}{Q} + \frac{\ell^4}{r^2\,Q}\,. \tag{76}
$$

After performing the rescaling of three-dimensional fields and coupling constants

$$
g_{\mu\nu} = \epsilon^2\,\tilde{g}_{\mu\nu}\,, \quad A_\mu = \epsilon\,\widetilde{A}_\mu\,, \quad B_\mu = \epsilon\,\widetilde{B}_\mu\,, \quad g = \epsilon^{-1}\,\tilde{g}\,, \quad m = \epsilon^{-1}\,\tilde{m}\,, \tag{77}
$$

which leaves the equations of motion invariant, we obtain in the $\epsilon \longrightarrow 0$ limit the three-dimensional BTZ black hole metric [33], with the mass M and angular momentum J given by

$$
M = \frac{2(k-\ell^2)}{Q}\,, \qquad J = \frac{2\ell^2}{\sqrt{Q}} \tag{78}
$$

It is worth remarking that in our solution, the mass M can have either sign according to the relative values of k and ℓ. It becomes negative when the non-extremality parameter

k goes to zero. The BTZ black hole, which has a horizon, requires that M be positive, and $M \geq |J|/\sqrt{Q}$. In our parameterisation it is then necessary that $2\ell^2 \leq k \leq \infty$. It is rather surprising that the extremal BTZ black hole arises from the decoupling limit of the non-extremal rotating black self-dual string with the non-extremality parameter $k = 2\ell^2$.

If on the other hand, we let $\ell \to \mathrm{i}\,\ell$, the mass and angular momentum becomes

$$M = \frac{2(k+\ell^2)}{Q}\,, \qquad J = \frac{-2\ell^2}{\sqrt{Q}}\,. \tag{79}$$

The mass then satisfies the bound $M \geq |J|/\sqrt{Q}$ for all $k \geq 0$, with the extremal limit corresponding to $k = 0$.

5 Conclusion and discussion

In this paper we have carried out the Scherk-Schwarz reduction of pure $N = (1,0)$ chiral six-dimensional supergravity. Although the reduction ansatz is guaranteed to be consistent, there is a subtlety that the reduction can only be performed at the level of the equations of motion, because the six-dimensional chiral theory cannot be described in terms of a Lagrangian. The resulting three-dimensional theory, however, can be derived from a Lagrangian, and we have constructed this explicitly in the bosonic sector.

The three-dimensional theory that we have constructed comprises the metric, the Yang-Mills fields of $SU(2)$, three "massive" vectors in the adjoint of $SU(2)$, six scalar fields in the coset $GL(3,\mathbb{R})/SO(3)$, and their fermionic partners. The theory admits and AdS_3 vacuum solution. Although there exist many AdS_3 gauged supergravities, ours is the only known example that has a higher-dimensional string origin. One feature of the Scherk-Schwarz reduction that the breathing mode is part of the lower-dimensional massless supermultiplet. This contrasts with the situation in the exceptional cases where a consistent Kaluza-Klein supergravity reduction on a sphere such as S^4, S^5 or S^7 can be performed; in these cases the breathing mode would be massive, but must be excluded in the consistent reduction. In particular, the inclusion of the breathing mode in our Scherk-Schwarz reduction implies that we can reduce the full six-dimensional self-dual string solution, and not merely its $\mathrm{AdS}_3 \times S^3$ near-horizon limit, to give rise to a domain wall in $D = 3$. Indeed, we obtained such a solution, with rotation as well, by reducing the six-dimensional self-dual rotating string.

It is interesting to compare the spectrum of our model with that arising in the linearised analysis of the $\mathrm{AdS}_3 \times S^3$ compactification of the $N = (1,0)$ supergravity, obtained in [4]. The massless bosonic sector of that reduction comprises a dilaton, nine scalars and the gauge fields of $SO(4)$, all of which originate from the metric, together with an additional six vectors that come from the six-dimensional 2-form potential. This suggests the existence of a gauged supergravity theory with $SO(4)$ gauge symmetry and a scalar sector describing a $GL(4,R)/SO(4)$ coset, and admitting an AdS$_3$ vacuum. Note however that the total bosonic degrees of freedom in this case will be 16, in contrast to the 12 of our model obtained by $SU(2)$ reduction, and thus it differs also from the count of 12 obtained from toroidal compactification. The model with $SO(4)$ gauge fields is therefore not expected to arise from a consistent Kaluza-Klein reduction of the pure $N = (1,0)$ supergravity in $D = 6$. Moreover, it must differ from the $SO(4)$ gauged supergravity implied by the results of [16], which admits a domain wall, but not AdS$_3$, as a solution.

Of course, an AdS$_3$ supergravity with propagating $SO(4)$ gauge sector may exist in its own right, notwithstanding the fact that it is not expected to arise from a consistent Kaluza-Klein ansatz. This latter statement would mean that caution is necessary when utilizing such a theory in an AdS$_3$/CFT$_2$ correspondence, since the the massive Kaluza-Klein modes may have to be taken into account in this case. In fact, the situation is similar to that encountered in the $T^{1,1}$ compactification of the type IIB string, for which the linearised analysis yields a minimal gauged supergravity coupled to matter in $D = 6$. A supergravity theory with the same massless spectrum does exist in its own right, but it cannot arise from a consistent truncation of the massive Kaluza-Klein modes in the $T^{1,1}$ reduction of type IIB supergravity, since there can be no such consistent reduction ansatz [37].

A model of great interest in the AdS$_3/CFT_2$ context is the AdS$_3 \times S^3$ compactification of the $N = (2,0)$ supergravity in $D = 6$. In this case, the theory has 16 real supersymmetries, both in $D = 6$ and $D = 3$, and it was determined in [4] that the propagating massless Kaluza-Klein spectrum consists of 21 hyper-multiplets and a special 32+32 vector multiplet which consists of $SO(4)$ Yang-Mills fields, 6 additional vector fields and 26 scalar fields, which include a dilaton and other scalars in various representations of the $SO(4)_{\rm local} \times SO(4)_{\rm global}$ groups involved. The presence of this multiplet has not been emphasized in the literature so far, but it is clearly of great relevance to the ultimate construction of the AdS supergravity theory that describes the full massless spectrum. The model presented in [25] describes the coupling of arbitrary number of hyper-multiplets to an AdS$_3$ supergravity with 16 real supersymmetries, but it lacks the coupling of the propagating vector multiplet mentioned above, and as such the problem of constructing the desired supergravity theory still remains open. It is tempting to conjecture that the 26 scalars in the model parameterise an E_6/F_4 coset. This problem is currently under investigation [36].

There are other interesting aspects of gauged or AdS supergravities in $D = 3$, having to do, for example, with the understanding of the relation between the models constructed in [22] and [24, 25, 26]. Ultimately, progress in this area ought to lead to a better understanding of various interesting $\mathrm{AdS}_3/\mathrm{CFT}_2$ issues as well. Of course, the supersymmetric field theories in $D = 3$ may have a number of other applications too, including some aspects of brane dynamics. It is interesting that after years of progress in more complicated higher-dimensional supergravities, it is relatively recently that the interest in $D = 3$ supergravities has increased. It is becoming increasingly clear that this is a very rich area with a wealth of phenomena still waiting to be uncovered.

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