JHEP

Published for SISSA by Springer

Received: August 4, 2016
Revised: January 19, 2017
Accepted: April 6, 2017
Published: April 18, 2017

Bootstrapping mixed correlators in the five dimensional critical $O(N)$ models

Zhijin Li and Ning Su

George P. and Cynthia W. Mitchell Institute for Fundamental Physics and Astronomy, Texas A&M University, College Station, TX 77843, U.S.A.

E-mail: lizhijin@physics.tamu.edu, suning1985@gmail.com

Abstract: We use the conformal bootstrap approach to explore $5D$ CFTs with $O(N)$ global symmetry, which contain N scalars ϕ_i transforming as $O(N)$ vector. Specifically, we study multiple four-point correlators of the leading $O(N)$ vector ϕ_i and the $O(N)$ singlet σ. The crossing symmetry of the four-point functions and the unitarity condition provide nontrivial constraints on the scaling dimensions $(\Delta_\phi, \Delta_\sigma)$ of ϕ_i and σ. With reasonable assumptions on the gaps between scaling dimensions of ϕ_i (σ) and the next $O(N)$ vector ϕ_i' (singlet σ') scalar, we are able to isolate the scaling dimensions $(\Delta_\phi, \Delta_\sigma)$ in small islands. In particular, for large $N = 500$, the isolated region is highly consistent with the result obtained from large N expansion. We also study the interacting $O(N)$ CFTs for $1 \leq N \leq 100$. Isolated regions on $(\Delta_\phi, \Delta_\sigma)$ plane are obtained using conformal bootstrap program with lower order of derivatives Λ; however, they disappear after increasing Λ. For $N = 100$, no solution can be found with $\Lambda = 25$ under the assumptions on the scaling dimensions of next $O(N)$ vector $\Delta_{\phi_i'} \geq 5.0$ (singlet $\Delta_{\sigma'} \geq 3.3$). These islands are expected to be corresponding to interacting but nonunitary $O(N)$ CFTs. Our results suggest a lower bound on the critical value $N_c > 100$, below which the interacting $O(N)$ CFTs turn into nonunitary.

Keywords: Conformal Field Theory, Field Theories in Higher Dimensions, Global Symmetries

ArXiv ePrint: 1607.07077

doi:10.1007/JHEP04(2017)098

JHEP04(2017)098

Contents

JHEP04(2017)098

1 Introduction

The conformal bootstrap [1–4] provides a non-perturbative approach to solve conformal field theories (CFTs) using general consistency conditions of CFT. It has led to great successes in 2D, such as the seminal work [4] on solving 2D rational CFTs. In recent years the conformal bootstrap has been revived since the breakthrough discovery in [5], which shows that the crossing symmetry and the unitary conditions can provide strong constraints on the operator scaling dimensions without an explicit form of Lagrangian. The crossing symmetry of four-point correlator leads to an infinite set of constraints on the CFT data. These constraints are difficult to be solved analytically, instead, they are truncated to a finite set and reformulated as a convex optimization problem so that they can be solved numerically. Here the convexity of conformal block functions [6, 7] plays a crucial role. Since then the conformal bootstrap has been significantly developed and it becomes a remarkably powerful technique to obtain CFT data, including operator scaling dimensions and operator product expansion (OPE) coefficients in $D > 2$ dimensions [8–49]. Review of previous developments on conformal bootstrap is provided in [50].

From conformal bootstrap with single correlator $\langle\phi\phi\phi\phi\rangle$, one can obtain bounds on the conformal dimension or OPE coefficient of objective operator. The bounds may exhibit singular behaviors, such as kinks which are believed to be related to unitary CFTs. One can expect to obtain more information on CFTs through bootstrapping mixed correlators like $\langle\phi\phi\phi^2\phi^2\rangle$. Conformal bootstrap with mixed operators has been fulfilled in [28, 39]

for $3D$ Ising model and critical $O(N)$ vector models and the results are quite impressive — the allowed scaling dimensions are isolated in small islands. The accuracy can be improved further by refining the numerical techniques [37, 49]. Studies on the $3D$ $O(N)$ vector models are strongly motivated by their special importance in physics. For small $N \leq 3$ they describe second-order phase transitions occurring in real physical systems [51]. Besides, its $O(N)$-singlet sector is proposed to be dual to higher spin quantum gravity in AdS_4 with Dirichlet boundary conditions [52]. In the UV side, the $3D$ $O(N)$ vector model contains N free scalars ϕ_i, $i = 1, \cdots, N$ perturbed by quartic coupling $(\phi_i\phi_i)^2$. The RG flows to an IR fixed point which is strongly coupled. For the critical $O(N)$ vector models with large N or in $D = 4-\epsilon, \epsilon \ll 1$ dimensions, one can obtain reliable results using large N expansion or the well-known Wilson-Fisher ϵ expansion. Actually these analytical results have been used as consistency checks of conformal bootstrap in [18, 19]. Nevertheless, for the $3D$ ($\epsilon = 1$) critical $O(N)$ vector models with small N which are more physically attractive, these perturbative methods turn into less effective. In contrast, conformal bootstrap remains useful and has provided the most accurate results up to date [49].

Following the success of conformal bootstrap in critical $3D$ $O(N)$ vector models, one may expect to generalize the results to critical $O(N)$ vector models in higher dimensions. These models, if exist, are expected to provide examples on $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$ correspondence in higher dimensions. In 4D there is no critical $O(N)$ CFT, while in $D > 4$, the interaction term $(\phi_i\phi_i)^2$ is irrelevant in the free $O(N)$ theory so the UV free $O(N)$ theory perturbed by the quartic interaction does not lead to an interacting fixed point in the IR, instead, the theory admits a Gaussian fixed point in the IR which flows to an interacting UV fixed point under $(\phi_i\phi_i)^2$ perturbation [53, 54]. In $D = 4 + \epsilon$ such UV fixed point theory is weakly coupled for sufficient small ϵ and it requires a negative quartic coupling coefficient, which may introduce the problem of instability even though the scaling dimensions of the operators are above the unitary bound. A UV-completed formulation of the $O(N)$ model in $D > 4$ dimensions has been proposed in [55, 56]

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi_i)^2 + \frac{1}{2}(\partial_i\sigma)^2 + \frac{1}{2}g\sigma\phi_i^2 + \frac{1}{6}\lambda\sigma^3, \tag{1.1}$$

in which the ϕ_i constructs fundamental representation of $O(N)$ and the $O(N)$ singlet σ performs as composite field ϕ_i^2 in the UV side. The theory contains cubic interaction terms which are relevant in space with dimension $D < 6$. Using the combination of ϵ and large N expansion it has been shown that this theory admits an interacting IR fixed point [55, 56], which is unitary for $N > N_c$, while below the critical value $N < N_c$ the coupling turns into complex and the IR fixed point theory is nonunitary. At one-loop level the critical value N_c is about $N_c \approx 1038$. For $5D$ ($\epsilon = 1$) critical $O(N)$ theories, the small ϵ condition for ϵ expansion approach breaks down so the results obtained from ϵ expansion should be treated carefully. Actually the critical value decreases to $N_c \approx 64$ at three-loop level. In [57] the author has obtained a critical value $N_c \approx 400$ at four-loop level based on resummation methods. A non-perturbative method is desirable to determine the critical value N_c in $5D$. The $5D$ critical $O(N)$ models have been studied using the nonperturbative functional renormalization group equations [58–63]. In these works the $5D$ interacting $O(N)$ fixed

JHEP04(2017)098

points have been obtained while the effective potential is metastalbe. Specifically the analysis in [63] agrees with the results from the $D = 6 - \epsilon$ perturbative approach when $\epsilon \ll 1$ and predicts the $5D$ critical value $N_c = 1$.

The conformal bootstrap approach has been employed to study $5D$ critical $O(N)$ models in [25, 35, 36] following the proposal of the cubic model [55, 56]. In [25] the $5D$ critical $O(N)$ models have been assumed to saturate the minimum of the $O(N)$ current central charge c_J for large N and the existence of $5D$ critical $O(N)$ models are indicated from these minimums obtained from conformal bootstrap. The authors focused on bootstrapping the OPE coefficients rather than the scaling dimensions of conformal primary operators. In $3D$ conformal bootstrap the interacting $O(N)$ CFTs have been found to lie at the kinks of the bounds for the scaling dimension Δ_σ of the $O(N)$ singlet σ, which appears as lowest dimension operator in the S channel of the correlator $\langle\phi_i\phi_j\phi_k\phi_l\rangle$. However, in $5D$ cubic model the lowest dimension $O(N)$ singlet operator σ performs as ϕ_i^2, $\Delta_\sigma = 2\Delta_\phi = 3$ at the UV Gaussian fixed point which reduces to $\Delta_\sigma = 2 + O(1/N)$ near the IR fixed point. The IR fixed point is below the upper bound of scaling dimensions Δ_σ so there is no clue on the fixed point theory in the bound of scaling dimensions. This problem has been overcome in [35, 36] by imposing a gap on the scaling dimensions of σ and the second lowest $O(N)$ singlet conformal primary scalar. With a reasonable assumption on the gap, the allowed region of the scaling dimensions $(\Delta_\phi, \Delta_\sigma)$ can be carved out and forms two sharp kinks. The UV Gaussian fixed point lies at the higher kink while the lower kink agrees with the large N expansion predictions on IR interacting fixed point theories. Furthermore, the kink disappears for small $N \approx 15$ which may indicate a small critical value N_c [36].

However, one should be careful to consider the kinks in conformal dimension bound or the minimum of central charges as unitary CFTs. From perturbative methods it is known that in $D = 6-\epsilon$, $\epsilon \ll 1$ the IR fixed point of cubic $O(N)$ models is endowed with complex critical couplings for $N \leq 1000$.[1] Nevertheless, in [36] a sharp kink is still generated from conformal bootstrap for $D = 5.95$, $N = 600$ which is much lower than the threshold value and should be nonunitary. The reason seems to be that the precision adopted in [36] is not high enough to detect the small violation of unitary. A more powerful bootstrap approach is needed to study the $5D$ $O(N)$ models, especially on its critical value N_c.

In this work, we will study the conformal bootstrap with multiple correlators of conformal primaries ϕ_i and σ: $\langle\phi_i\phi_j\phi_k\phi_l\rangle$, $\langle\phi_i\phi_j\sigma\sigma\rangle$, $\langle\sigma\sigma\sigma\sigma\rangle$. Since there are more operators involved in the bootstrap program, it is expected that the results will provide more rigid restrictions on the scaling dimensions of $(\Delta_\phi, \Delta_\sigma)$. Actually we find that the scaling dimensions $(\Delta_\phi, \Delta_\sigma)$ obtained from bootstrapping multiple correlators of $5D$ $O(500)$ model is isolated in a rather small island, which is nicely compatible with the perturbative results. We also study the critical value N_c in $5D$. In preliminary numerical calculations we find small islands on the allowed scaling dimensions $(\Delta_\phi, \Delta_\sigma)$ for all $N \geq 1$. However, these islands disappear after improving the bootstrapping precisions. Taking $N = 100$ for example, it shows an apparent kink in the bound from bootstrapping single correlator

[1] CFTs in fractional dimensions are known to be nonunitary even with real couplings [64, 65]. However, the unitarity is violated by operators with high scaling dimensions so they are more difficult to be tested through conformal bootstrap approach.

JHEP04(2017)098

$\langle\phi_i\phi_j\phi_k\phi_l\rangle$. Using multiple correlator conformal bootstrap with small Λ, we obtain an island on $(\Delta_\phi, \Delta_\sigma)$ plane close to the kink from single correlator conformal bootstrap, while it vanishes after increasing Λ even though we relax the conformal dimension gap to $\Delta_{\phi_i'} \geq 5.0$ and $\Delta_{\sigma'} \geq 3.3$.[2] Therefore our results suggest a rather large critical value $N_c > 100$ unless the perturbative methods have drastically overestimated the scaling dimensions.

This paper is organized as follows. In section 2 we briefly review the cubic model of $O(N)$ vector model in $4 < D < 6$ and the perturbative results on scaling dimensions of lowest primary scalars. The scaling dimensions $(\Delta_\phi, \Delta_\sigma)$ obtained from large N and ϵ expansions provide consistency checks for the results from conformal bootstrap. In section 3 we introduce the numerical conformal bootstrap equations for $5D$ $O(N)$ vector models and their numerical implementation. Our results are presented in section 4. We show that through bootstrapping the multiple correlators the scaling dimensions $(\Delta_\phi, \Delta_\sigma)$ are isolated in a small island for large $N = 500$, while disappear with larger Λ for $N \leq 100$. Conclusions are made in section 5.

2 Perturbative results for $5D$ critical $O(N)$ models

The critical $O(N)$ vector model with quartic interaction in arbitrary dimensions $D = 4 - \epsilon$ has been analyzed using the large N expansions [66–72, 74–77]. In $2 < D < 4$ ($\epsilon > 0$), the quartic interaction is relevant and the RG flows from UV Gaussian fixed point to interacting IR fixed point perturbed by this coupling. The quartic interaction is irrelevant in $4 < D < 6$ ($\epsilon < 0$) so the long-range physics is described by free field theory. The quartic coupling generates RG flow from the IR Gaussian fixed point to an interacting UV fixed point. The perturbative result for small ϵ shows the interaction coupling is negative at interacting UV fixed point which may lead to the stability problem. However, the scaling dimensions of scalar operators obtained from the large N expansion are still above unitary bound and the unitary conditions remain unbroken for sufficient large N. One may expect the interacting UV fixed point from quartic model describes a universality class with $O(N)$ global symmetry in $4 < D < 6$ whose stable or metastable formulation may be realized in different model.

In $D = 5$ spacetime, the conformal dimensions of ϕ_i and σ have been evaluated at three-loop level

$$\Delta_\phi = \frac{3}{2} + \frac{0.216152}{N} - \frac{4.342}{N^2} - \frac{121.673}{N^3} + \cdots \tag{2.1}$$

$$\Delta_\sigma = 2 + \frac{10.3753}{N} + \frac{206.542}{N^2} + \cdots \tag{2.2}$$

$$\Delta_{\sigma^2} = 4 - \frac{13.8337}{N} - \frac{1819.66}{N^2} + \cdots \tag{2.3}$$

According to above $1/N$ expansion, the conformal dimension of ϕ_i is above the unitary bound ($\Delta_\phi > 3/2$ for scalar fields) given $N > 35$. The critical value $N_c = 35$ can be

[2] As a comparison, $\Delta_{\phi_i'} \approx 5.39$ up to the order $1/N$ and $\Delta_{\sigma'} \approx 3.68$ up to the order $1/N^2$ from perturbative methods.

JHEP04(2017)098

significantly modified by by higher order corrections. Actually the $5D$ $1/N$ expansions converge much slower than those in $3D$ [55].

Alternatively, the $5D$ quartic theory can also be studied using ϵ expansion [78]. Conformal dimensions of ϕ_i and ϕ^2 (σ) have been calculated up to five-loop [79]:

$$\Delta_\phi = 1 - \frac{\epsilon}{2} + \frac{N+2}{4(N+8)^2}\epsilon^2(1 + a_1\epsilon + a_2\epsilon^2 + a_3\epsilon^3) + \cdots , \tag{2.4}$$

where

$$\begin{aligned}
a_1 &= \frac{-N^2+56N+272}{4(N+8)^2},\\
a_2 &= -\frac{1}{16(N+8)^4}(5N^4+230N^3-1124N^2-17920N\\
&\quad -46144+384\zeta(3)(N+8)(5N+22)),\\
a_3 &= -\frac{1}{64(N+8)^6}\big(13N^6+946N^5+27620N^4+121472N^3-262528N^2-2912768N\\
&\quad -5655552-16\zeta(3)(N+8)(N^5+10N^4+1220N^3\\
&\quad -1136N^2-68672N-171264)+1152\zeta(4)(N+8)^3(5N+22)\\
&\quad -5120\zeta(5)(N+8)^2(2N^2+55N+186)\big),
\end{aligned}$$

and

$$\Delta_\sigma = 2 - \epsilon + \frac{N+2}{N+8}\epsilon\left(1 + c_1\epsilon + c_2\epsilon^2 + c_3\epsilon^3 + c_4\epsilon^4\right) + \cdots , \tag{2.5}$$

where

$$\begin{aligned}
c_1 &= \frac{13N+44}{2(N+8)^2},\\
c_2 &= -\frac{1}{8(N+8)^4}(3N^3-452N^2-2672N-5312+96\zeta(3)(N+8)(5N+22)),\\
c_3 &= -\frac{1}{32(N+8)^6}\big(3N^5+398N^4-12900N^3-81552N^2-219968N-357120\\
&\quad +16\zeta(3)(N+8)(3N^4-194N^3+148N^2+9472N+19488)\\
&\quad +288\zeta(4)(N+8)^3(5N+22)-1280\zeta(5)(N+8)^2(2N^2+55N+186)\big),\\
c_4 &= -\frac{1}{128(N+8)^8}\big(3N^7-1198N^6-27484N^5-1055344N^4-5242112N^3-5256704N^2\\
&\quad +6999040N-626688-16\zeta(3)(N+8)(19004N^4+102400N^3\\
&\quad +13N^6-310N^5-381536N^2-2792576N-4240640)\\
&\quad -1024\zeta(3)^2(N+8)^2(2N^4+18N^3+981N^2+6994N+11688)\\
&\quad +48\zeta(4)(N+8)^3(148N^2+3N^4-194N^3+9472N+19488)\\
&\quad +256\zeta(5)(N+8)^2(155N^4+3026N^3+989N^2-66018N-130608)\\
&\quad -6400\zeta(6)(2N^2+55N+186)(N+8)^4\\
&\quad +56448\zeta(7)(14N^2+189N+526)(N+8)^3\big).
\end{aligned}$$

Besides, for the next $O(N)$ vector operator $\phi_i' \equiv \phi^4\phi_i$, its scaling dimension has been provided in [73] at the first order

$$\Delta_{\phi_i'} = 5 - \frac{\epsilon}{2} + \frac{12}{N+8}\epsilon + \cdots . \tag{2.6}$$

JHEP04(2017)098

Taking $\epsilon = -1$ the results can be interpolated to $5D$. For large N the higher order coefficients c_is are of order $1/N$. In this case the ϵ expansion performs worse asymptotically in $5D$ comparing with the large N expansion. While for small N it is not clear at this stage which approach can provide better estimation. These perturbative results will be useful to estimate the conformal dimension gap which can be applied in the conformal bootstrap to improve the numerical efficiency.

Both the large N expansion and the ϵ expansion contain negative terms at higher loop level. For small Ns these negative contributions may play dominating roles in the perturbative expansion and result in negative anomalous dimension. Specifically the five-loop result (2.4) shows the conformal dimension $\Delta_\phi < 3/2$ for $N \leq 14$ [55]. In [36] the conformal bootstrap with single correlator has been applied to generate bound on Δ_σ. Interestingly the bounds are featured with kinks which are expected to relate to certain unitary fixed point theories while the kinks disappear near $N \approx 15$, close to the critical value estimated from ϵ expansion. However, as in the large N expansion, the ϵ expansion in $5D$ is not converged up to fifth order and the contributions from higher loops are likely to modify the threshold value N_c significantly.

The cubic $O(N)$ model (1.1) provides an approach to realize stable interacting $O(N)$ fixed point in $5D$ [55, 56]. The authors show that at one of the IR fixed point the cubic $O(N)$ model shares the same relevant critical exponents with the quartic $O(N)$ model so the two models are expected to describe the same universality class.[3] Like the quartic $O(N)$ model, the cubic $O(N)$ model also requires a critical value N_c from unitarity constraint. In the cubic model, the unitarity is violated in the way that the coupling coefficients acquire imaginary part when $N < N_c$. In [55, 56] the critical value N_c is evaluated up to order ϵ^2 in arbitrary dimension $D = 6 - \epsilon$. Four-loop results which include corrections on N_c at order ϵ^3 have been calculated in [57]

$$N_c = 1038.26605 - 609.83980\epsilon - 364.17333\epsilon^2 + 452.71060\epsilon^3 + O(\epsilon^4). \tag{2.7}$$

As usual, above perturbative result is not sufficient to make a solid estimation on $5D$ ($\epsilon = 1$) N_c due to its asymptotic performance. It is tempting to evaluate the critical value N_c using non-perturbative method. Besides the above interacting IR fixed point, the cubic model also admits extra fixed points with different critical value N_c'; however, they are not corresponding to the classical interacting quartic fixed point and will not be studied in this work.

3 Conformal bootstrap with multiple correlators

Conformal bootstrap with multiple correlators has been developed in [28, 39] which aimed to solve the $3D$ Ising model and $O(N)$ vector model. This approach has obtained the most accurate solutions on $3D$ Ising model and $O(N)$ vector model up to date [49]. Here we

[3] The renormalization group approach suggests the cubic model admits an extra RG relevant direction with positive critical exponent at the IR fixed point [63]. In this sense the universality class of the quartic $O(N)$ model is a subset of that of cubic $O(N)$ model.

JHEP04(2017)098

briefly introduce the conformal bootstrap program for $5D$ $O(N)$ vector model analogous to that for $3D$ $O(N)$ vector model [39]. More details on this program are provided in [37].

3.1 Bootstrap equations from crossing symmetry

Conformal partial wave function is the crucial ingredient for conformal bootstrap. In even dimensions $D = 2, 4, 6$, the conformal partial wave functions have been solved analytically [6, 7]. In odd dimensions there is no analytical expression for conformal partial wave function; however, it can be calculated recursively with arbitrary precision [18, 28, 80].[4] The general four-point function of scalar operators can be expanded in terms of conformal partial waves

$$\langle\sigma_1\sigma_2\sigma_3\sigma_4\rangle = \frac{1}{x_{12}^{\Delta_1+\Delta_2}x_{34}^{\Delta_3+\Delta_4}}\left(\frac{x_{24}}{x_{14}}\right)^{\Delta_{12}}\left(\frac{x_{14}}{x_{13}}\right)^{\Delta_{34}}\sum_{\mathcal{O}}\lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}}g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(u,v), \tag{3.1}$$

where σ_is are scalar operators with conformal dimension Δ_i ($\Delta_{ij} = \Delta_i - \Delta_j$) and $\mathcal{O}$ is the conformal primary operator appears in the OPE expansion of $\sigma_1\sigma_2 \sim \lambda_{12\mathcal{O}}\mathcal{O}$ (and also $\sigma_3\sigma_4 \sim \lambda_{34\mathcal{O}}\mathcal{O}$), whose conformal dimension and spin are $(\Delta,\ \ell)$. The conformal invariant cross ratios $u,\ v$ are of the standard form $u = \frac{x_{12}^2x_{34}^2}{x_{13}^2x_{24}^2}$ and $v = \frac{x_{14}^2x_{23}^2}{x_{13}^2x_{24}^2}$, $x_{ij} = |x_i - x_j|$.

The four-point function can be evaluated equivalently in different channels, as suggested by crossing symmetry, and it leads to the following equations

$$\sum_{\mathcal{O}}\left(\lambda_{12\mathcal{O}}\lambda_{34\mathcal{O}}F_{\mp,\Delta,\ell}^{12,34}(u,v) \pm \lambda_{32\mathcal{O}}\lambda_{14\mathcal{O}}F_{\mp,\Delta,\ell}^{32,14}(u,v)\right) = 0, \tag{3.2}$$

in which

$$F_{\mp,\Delta,\ell}^{12,34}(u,v) = v^{\frac{\Delta_2+\Delta_3}{2}}g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(u,v) \mp u^{\frac{\Delta_2+\Delta_3}{2}}g_{\Delta,\ell}^{\Delta_{12},\Delta_{34}}(v,u). \tag{3.3}$$

To study the $5D$ $O(N)$ vector model, we apply the crossing relations for correlators $\langle\phi_i\phi_j\phi_k\phi_l\rangle$, $\langle\sigma\sigma\sigma\sigma\rangle$ and $\langle\phi_i\phi_j\sigma\sigma\rangle$. The $O(N)$ indices in the correlators are decomposed into three irreducible structures: the $O(N)$ invariant, traceless symmetric and antisymmetric tensors. The conformal primaries appearing in the OPE of $O(N)$ vector representations ϕ_i can be classified into three irreducible representations:

$$\phi_i \times \phi_j \sim \sum_S \lambda_{\phi\phi\mathcal{O}_S}\mathcal{O}\delta_{ij} + \sum_T \lambda_{\phi\phi\mathcal{O}_T}\mathcal{O}_{(ij)} + \sum_A \lambda_{\phi\phi\mathcal{O}_A}\mathcal{O}_{[ij]}, \tag{3.4}$$

in which $S,\ T$ and A denote $O(N)$ singlet, traceless symmetric tensor and anti-symmetric tensor representations. Consequently, the four-point correlator $\langle\phi_i\phi_j\phi_k\phi_l\rangle$ and its crossing symmetric partners are separated into three channels: S, T, A. For the mixed four-point correlator $\langle\phi_i\sigma\phi_j\sigma\rangle$, one needs to consider the OPE $\phi_i\sigma \sim \sum_V \lambda_{\phi_i\sigma\mathcal{O}_i}\mathcal{O}_i$ which introduces the vector representations (denoted by V) as propagating operators in the mixed four-point correlator and its crossing symmetric partner.

[4] Details on calculating conformal block function in arbitrary dimensions are provided in [33] as part of an open-source numerical conformal bootstrap program JuliBootS. In this work we will use the JuliBoots code to calculate the conformal block functions of scalar operators in $5D$.

JHEP04(2017)098

The crossing relations for bootstrapping $5D$ $O(N)$ critical theories are essentially the same as those for $3D$ $O(N)$ vector model [39]. These equations can be written in a compact form [39] which are presented below for later reference

$$
\begin{aligned}
0 = & \sum_{\mathcal{O}_S} \begin{pmatrix} \lambda_{\phi\phi\mathcal{O}_S} & \lambda_{\sigma\sigma\mathcal{O}_S} \end{pmatrix} \vec{V}_{S,\Delta,\ell} \begin{pmatrix} \lambda_{\phi\phi\mathcal{O}_S} \\ \lambda_{\sigma\sigma\mathcal{O}_S} \end{pmatrix} \\
& + \sum_{\mathcal{O}_T} \lambda^2_{\phi\phi\mathcal{O}_T} \vec{V}_{T,\Delta,\ell} + \sum_{\mathcal{O}_A} \lambda^2_{\phi\phi\mathcal{O}_A} \vec{V}_{A,\Delta,\ell} + \sum_{\mathcal{O}_V} \lambda^2_{\phi\sigma\mathcal{O}_V} \vec{V}_{V,\Delta,\ell}.
\end{aligned} \tag{3.5}
$$

Explicit forms of the 7-component vectors $\vec{V}_S, \vec{V}_T, \vec{V}_A, \vec{V}_V$ are provided in the appendix.

3.2 Bounds from crossing relations

The equations from crossing symmetry (3.5) provide nontrivial constraints on the CFT data. The numerical approach to study these equations was first proposed in [5] and the following developments show this method is extremely powerful. The logic of numerical conformal bootstrap is firstly to make assumptions on the CFT spectra. If the assumptions are physical they are required to satisfy the crossing relations (3.5) and the unitary condition. Numerical conformal bootstrap provides a systematical way to check the consistency between the assumptions and general constraints on CFTs. Bounds on the CFT data, including conformal dimensions of primary operators and OPE coefficients can be obtained by falsifying possible assumptions on the CFT spectra.

Specifically for any hypothetical spectra $(\Delta_\phi, \Delta_\sigma)$ above the unitary bounds, they should be consistent with the crossing relations (3.5). However, if there are linear functionals $\vec{\alpha} = (\alpha_1, \alpha_2, \cdots, \alpha_7)$ satisfying

$$
\begin{aligned}
& (1\ 1)\ \vec{\alpha} \cdot \vec{V}_{S,0,0} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1, \\
& \vec{\alpha} \cdot \vec{V}_{S,\Delta,\ell} \succeq 0, \qquad \Delta \geq \Delta^*_{S,0} \ \text{ for the } O(N) \text{ singlet scalars except } \sigma, \\
& \vec{\alpha} \cdot \vec{V}_{T,\Delta,\ell} \geq 0, \\
& \vec{\alpha} \cdot \vec{V}_{A,\Delta,\ell} \geq 0, \\
& \vec{\alpha} \cdot \vec{V}_{V,\Delta,\ell} \geq 0, \qquad \Delta \geq \Delta^*_{V,0} \ \text{ for the } O(N) \text{ vector scalars except } \phi_i, \\
& \vec{\alpha} \cdot \left(\vec{V}_{S,\Delta_\sigma,0} + \vec{V}_{V,\Delta_\phi,0} \otimes \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right) \succeq 0,
\end{aligned} \tag{3.6}
$$

then the crossing relations (3.5) can never be satisfied and initial assumption on the spectra $(\Delta_\phi, \Delta_\sigma)$ have to be abandoned as unphysical. In the bootstrap conditions (3.6), we have required the $O(N)$ singlet scalars (except σ) have conformal dimensions above a lower bound $\Delta^*_{S,0}$, and similarly a lower bound for $\Delta^*_{V,0}$ for $O(N)$ vector scalars in addition to ϕ_i. Besides, we have implicitly assumed that all the extra operators accord with the unitary bound. In the last equation of (3.6), it is the summation of contributions in S (from σ)

JHEP04(2017)098

and V channels (from ϕ) that is required to be positive-semidefinite due to the equality of OPE coefficients $\lambda_{\phi\phi\sigma} = \lambda_{\phi\sigma\phi}$.

The bootstrap conditions in (3.6) are not the only way to break the crossing relations (3.5). In particular, to bootstrap certain OPE coefficient of operator (Δ_0, ℓ_0) in channel X, one may set $\vec{\alpha} \cdot \vec{V}_{X,\Delta_0,\ell_0} = 1$ instead of choosing the unit operator as in (3.6). The bootstrap conditions are further refined in [49]. The lower bounds $\Delta^*_{S,0}$ and $\Delta^*_{V,0}$ introduced in (3.6) are necessary to isolate the conformal dimensions $(\Delta_\phi, \Delta_\sigma)$ in a small island. A higher but remaining physical lower bound can improve the numerical efficiency to carve out the allowed parameter space. For sufficient large N, these lower bounds can be justified from perturbative expansions. The $O(N)$ singlet scalar next to ϕ^2 is ϕ^4 in the quartic model, and its conformal dimension can be evaluated through the large N expansion (2.3). In the cubic theory (1.1) this is given by a mixing of σ^2 and ϕ^2. One of the linear combination of σ^2 and ϕ^2 is actually the descendent of σ, while another orthogonal mixing constructs a primary $O(N)$ singlet that shares the same conformal dimension as obtained from quartic theory [55, 56]. The candidate of next $O(N)$ vector scalar ϕ'_i is $\phi^2\phi_i$ (or $\sigma\phi_i$ in the cubic theory). However, as argued in [39] for the $3D$ theories, in $D = 6 - \epsilon$, $\epsilon \ll 1$ dimension the quartic theory generates the following equation of motion for ϕ_i:

$$\partial^2\phi_i \propto \phi^2\phi_i, \tag{3.7}$$

which suggests that the operator $\phi^2\phi_i$ is a descendent of ϕ_i rather than a conformal primary scalar. One can get the same conclusion in cubic theory (1.1) with replacement $\phi^2 \to \sigma$. The next candidate is $\phi^4\phi_i$ (in $D = 6 - \epsilon$, $\epsilon \ll 1$ dimension operators with derivatives, like $\phi^2\partial^2\phi_i, (\partial_\mu\phi)^2\phi_i$ have different bare conformal dimensions given $\epsilon \neq 0$ so they do not mix with $\phi^4\phi_i$). At the interacting fixed point, the conformal dimension of $\phi^4\phi_i$ has been studied in [73]. At tree level the conformal dimension of ϕ^2 near the interacting fixed point is 2, so the conformal dimension of $\phi^4\phi_i$ is 5.5 with $1/N$ corrections, as shown in (2.6). In the cubic theory the potential second $O(N)$ vector scalar is a mixing of $\sigma^2\phi_i$ and $\phi^2\phi_i$, which has not been explicitly studied yet. One can expect that one of the mixing is actually a descendent of ϕ_i while another primary mixing has the same conformal dimension as $\phi^4\phi_i$ in quartic theory, like the quadratic and cubic $O(N)$ singlet operators [55, 56]. The lower bound of the $\phi^4\phi_i$ conformal dimension would be rather subtle for small N. Fortunately we will show that a unitary interacting fixed point disappears even for $N = 100$ (corresponding to $\Delta_{\phi'_i} = 5.39$ at order $1/N$), indicating a large critical value N_c.

3.3 Numerical implementation of conformal bootstrap

Equations from crossing symmetry (3.5) provide an infinite set of constraints (3.6) on the CFT data. For the numerical implementation the constraints need to be truncated to a large but finite set. In (3.6) the constraints are parameterized by (Δ, ℓ). The spins ℓ construct an infinite tower of spectra while in conformal bootstrap only these spectra with small ℓ will be considered. Contributions from operators with large spin are exponentially suppressed. The linear functionals $\vec{\alpha}$ can be expanded as

$$\alpha_i = \sum_{m+n\leq\Lambda} a_{imn}\partial_z^m\partial_{\bar{z}}^n, \tag{3.8}$$

JHEP04(2017)098

where $(z, \bar{z})$ are defined in terms of (u, v) through: $u = z\bar{z}$, $v = (1-z)(1-\bar{z})$. Moreover, for the linear functional α_i, the number of derivatives is also truncated up to Λ. Taking higher order of derivatives in (3.8), we have more chances to find the linear function satisfying (3.6). As a result, the conformal bootstrap program can exclude larger regions in parameter space. In practice the parameter Λ is restricted by computation power. The setups of parameter Λ and spins used in this work are as follows

$$\begin{aligned} S_{\Lambda=19} &= \{0, 1, \cdots, 30\} \cup \{49, 50\}, \\ S_{\Lambda=21} &= \{0, 1, \cdots, 30\} \cup \{47, 48, 49, 50, 51, 52\}, \\ S_{\Lambda=23} &= \{0, 1, \cdots, 30\} \cup \{47, 48, 49, 50, 51, 52, 53, 54\}, \\ S_{\Lambda=25} &= \{0, 1, \cdots, 30\} \cup \{47, 48, 49, 50, 51, 52, 53, 54, 55, 56\}. \end{aligned} \tag{3.9}$$

The problem to find the linear functions $\vec{\alpha}$ under truncated constraints can be solved with SDPB program [37].

4 Results

4.1 Bootstrapping $5D$ $O(500)$ vector model

The $5D$ $O(500)$ vector model has been studied in [25, 35, 36] using conformal bootstrap with single correlator $\langle \phi_i \phi_j \phi_k \phi_l \rangle$. At the fixed point the conformal dimensions $(\Delta_\phi, \Delta_\sigma)$ of the lowest $O(N)$ vector ϕ_i and $O(N)$ singlet σ can be evaluated from the large N expansion in (2.1), (2.2) or the ϵ expansion in (2.4), (2.5). Taking $N = 500$, we get $(\Delta_\phi, \Delta_\sigma) = (1.500414, 2.02158)$ from 3-loop large N expansion and $(\Delta_\phi, \Delta_\sigma) = (1.500400, 2.02156)$ from 5-loop ϵ expansion. These predictions will be compared with the results obtained from conformal bootstrap.

In figure 1 we present the bounds on $(\Delta_\phi, \Delta_\sigma)$ obtained through bootstrapping the single correlator $\langle \phi_i \phi_j \phi_k \phi_l \rangle$ (light blue region) and the multiple correlators (dark blue island). To bootstrap the single correlator we have assumed that the next $O(N)$ singlet scalar has dimension above the gap $\Delta^*_{S,0} = 3.965$, which can be justified from the large N expansion result (2.3): $\Delta_{\sigma^2} \approx 3.972$. This gap is also employed in [36]. The upper part of light blue region is similar to the bound provided in [36]. Besides, there is an extra kink in the lower region and the whole region actually forms a sharp tip like presented in [35], although a much larger gap was used in that work. Results of perturbative methods are also shown in figure 1. Prediction from the large N expansion (denoted by the black cross) lies in the allowed region while prediction from the ϵ expansion (denoted by the black dot) is outside of the bound so is excluded. According to the conformal bootstrap results, the large N expansion does provide a better estimation on the conformal dimensions for large $N = 500$. Difference between the two perturbative approaches appears at the order $10^{-5} \approx O(1/N^2)$, as discussed before.

Remarkably, the allowed region of $(\Delta_\phi, \Delta_\sigma)$ obtained from the multiple correlator bootstrap is enclosed in a small island, which is colored in dark blue in figure 1. Besides the dimension gap $\Delta^*_{S,0} = 3.965$ in S-channel, we have employed another dimension gap $\Delta^*_{V,0} = 5$ in V-channel that the next primary $O(N)$ vector scalar has dimension $\Delta \geq$

JHEP04(2017)098

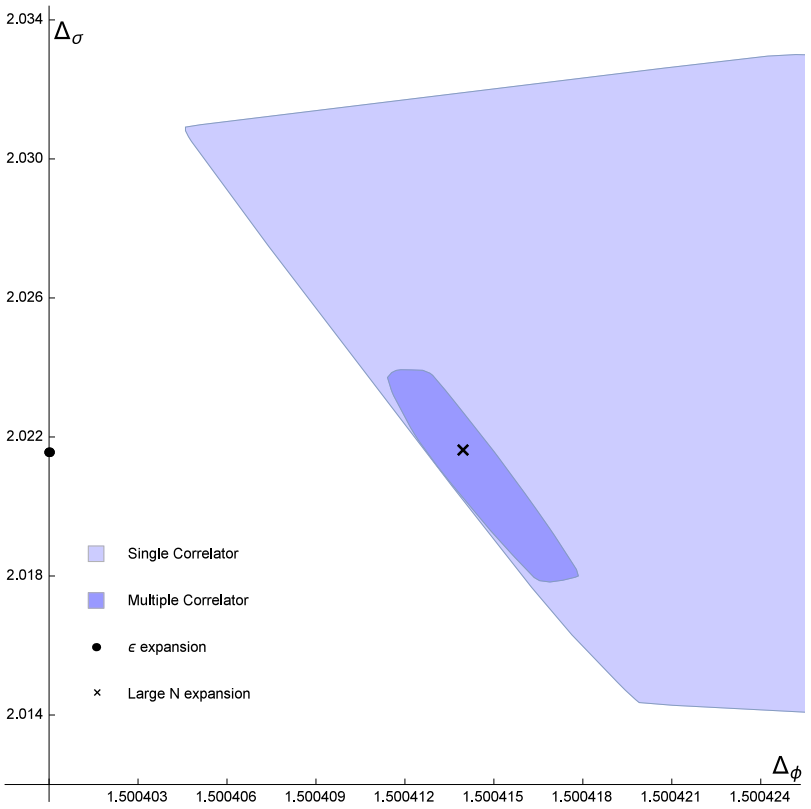

Figure 1. Bounds on the conformal dimensions $(\Delta_\phi, \Delta_\sigma)$ in the interacting $5D$ $O(500)$ CFT. The colored regions represent the conformal dimensions allowed by conformal bootstrap. Specifically the light blue region is obtained from single correlator bootstrap, while the dark blue island is isolated through bootstrapping the multiple correlators. We used the derivative at order $\Lambda = 19$ and spins $S_{\Lambda=19}$ in the numerical calculations. Besides, we assumed a gap $\Delta^*_{S,0} = 3.965$ in the S-channel. An extra gap $\Delta^*_{V,0} = 5$ has been used in the V-channel for bootstrapping multiple correlators. The black dot and cross relate to the predictions from ϵ expansion and large N expansion, respectively.

5. The dark blue island lies in the center of the tip, and the black cross denoting the large N prediction is rather close to the center of this island. Such a high coincidence is extraordinary in view of only crossing symmetry and unitary condition are applied to carve out the island. On the other hand, the conformal bootstrap result also shows that the large N expansion is reliable at third order.[5]

However, it should be careful to make statement based on results from conformal bootstrap with lower order of derivatives. Actually in preliminary study we have obtained isolated islands even for $N = 1$ with $\Lambda \sim 15$; however, they disappear after increasing Λ. As to the model with $N = 500$, we have checked the performance of the island with larger Λ. The results are provided in figure 2. The allowed regions shrink notably from $\Lambda = 21$ to $\Lambda = 25$. Interestingly, the fixed point predicted by large N expansion remains located in the center of the small island even though the allowed region has contracted significantly.

[5]Strictly speaking, such consistency check is not completely self-contained since we have already used the large N expansion result in setting the dimension gaps.

JHEP04(2017)098

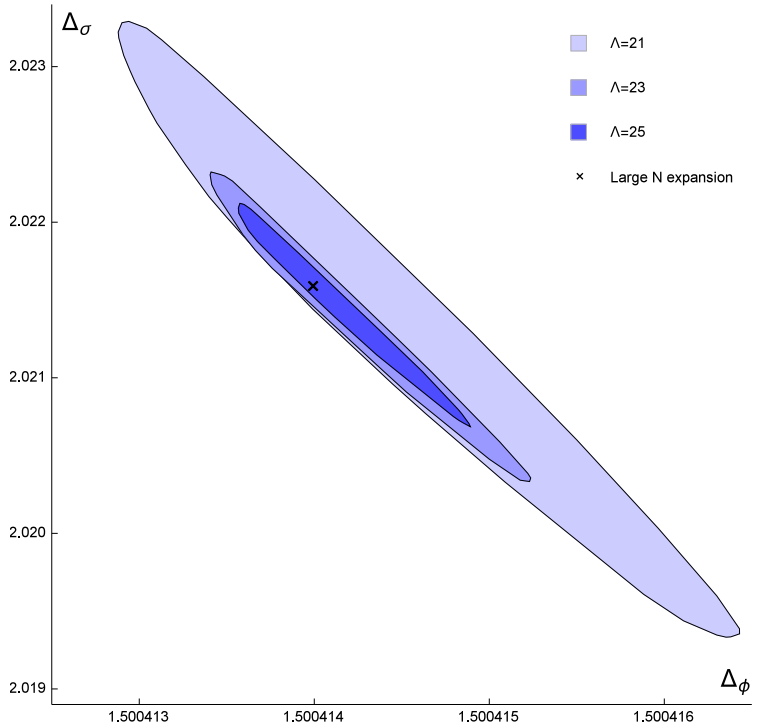

Figure 2. Isolated regions for the conformal dimensions $(\Delta_\phi, \Delta_\sigma)$ in $5D$ $O(500)$ vector model. The light, medium and dark blue regions are corresponding to the results from multiple correlator conformal bootstrap with $\Lambda = 21,\ 23,\ 25$, respectively. In the graph we have used the dimension gaps $\Delta^*_{S,0} = 3.965$ and $\Delta^*_{V,0} = 5$. The black cross denotes the prediction from large N expansion.

4.2 Bootstrapping $5D$ $O(N)$ ($N \leq 100$) vector models and the critical N_c

In $5D$ there is an interesting problem on the unitarity of the interacting $O(N)$ CFTs, that there is a threshold value N_c below which the CFTs become nonunitary [55, 56]. In contrast, the interacting $O(N)$ CFTs in $3D$ are unitary for any integer $N \geq 1$. Prior to our work, there are several evidences from conformal bootstrap which prefer to small N_c [25, 35, 36]. There are also some clues from perturbative results that the critical value $N_c < 100$. In this part we apply the conformal bootstrap with multiple correlators to study the $5D$ $O(N)$ vector model for small Ns. The multiple correlator conformal bootstrap involves in more $O(N)$ sectors and provides stronger constraints on the CFT data comparing with the conformal bootstrap with single correlator only.

We have searched the allowed regions on $(\Delta_\phi, \Delta_\sigma)$ plane for $N \leq 100$. The isolated islands can be obtained for small Ns with assumptions on the dimension gaps $(\Delta^*_{S,0}, \Delta^*_{V,0})$. However, these islands disappear after increasing the number of derivatives Λ. For $N \sim O(10)$ or smaller, the perturbative approaches cannot provide an approximate estimation on the conformal dimension Δ_{σ^2}. One may argue that the islands disappear due to the reason of the unphysical dimension gaps $(\Delta^*_{S,0},\ \Delta^*_{V,0})$ used in the bootstrap program instead of the nonunitarity of the CFTs. While for sufficient large Ns the perturbative predictions are expected to provide rough estimations on the fixed point. This

JHEP04(2017)098

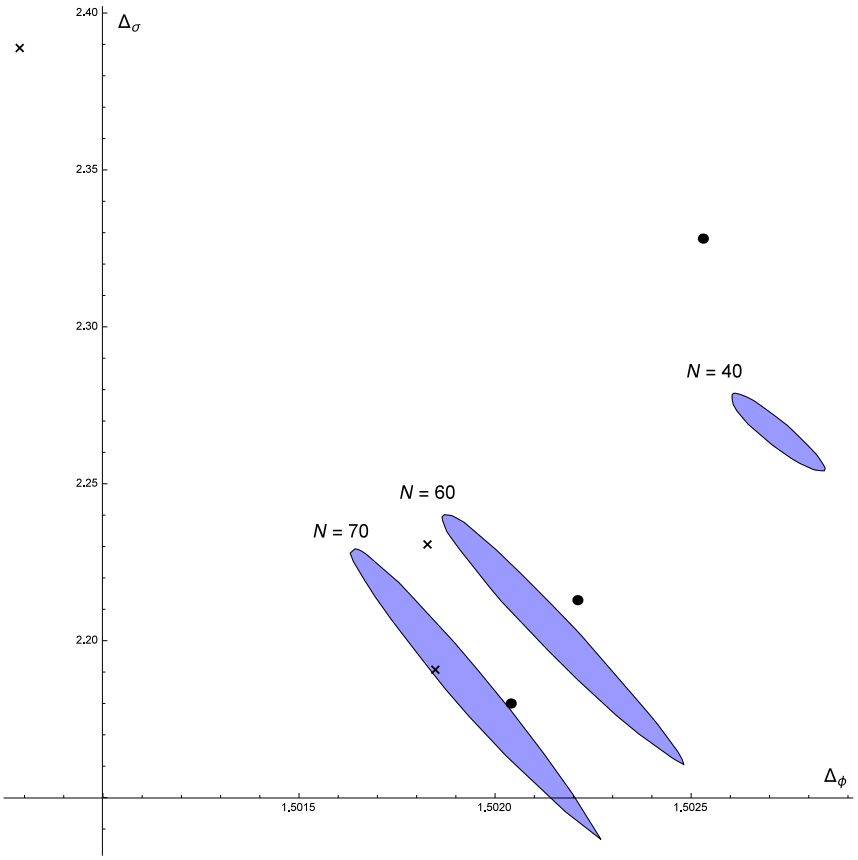

Figure 3. From top to bottom, the islands represent the allowed regions of $(\Delta_\phi, \Delta_\sigma)$ in the $5D$ $O(N)$ $N = 40, 60, 70$ vector models. The results are obtained from conformal bootstrap with $\Lambda = 19$ and spins $S_{\Lambda=19}$. The black dots and crosses denote predictions from ϵ expansion and large N expansions, respectively. The dimension gaps used in conformal bootstrap program are: $(\Delta^*_{S,0}, \Delta^*_{V,0}) = (3.4, 4.1)$ for $N = 40$, $(\Delta^*_{S,0}, \Delta^*_{V,0}) = (3.5, 4.3)$ for $N = 60, 70$. The perturbative methods, especially the large N expansion get abnormal and stay away from the region allowed by conformal bootstrap at $N = 40$.

can be seen from the fact that the isolated islands obtained from conformal bootstrap are close to the perturbative predictions before vanishing. In figure 3 we present the isolated regions for $N = 40, 60, 70$ from conformal bootstrap.[6] At derivative order $\Lambda = 19$, the conformal bootstrap program generates closed regions on the $(\Delta_\phi, \Delta_\sigma)$ plane, which disappear for larger $\Lambda \geq 23$. According to the results from conformal bootstrap, for $N = 60, 70$ the perturbative approaches can still provide approximate estimations on the conformal dimensions at the interacting fixed points, although the theories are likely to be nonunitary. While for $N = 40$, the perturbative approaches, especially the large N expansion cannot provide reliable estimations on the interacting fixed point. One may note that the

[6]The perturbative results on conformal dimension gaps are subtle for not so large Ns, and it is possible that the gaps used in figure 3 are unphysical, however, in this figure we are more interested in the comparison with the predictions from perturbative approach. We will adopt more "safe" scaling dimension gaps for $N = 100$ to study the unitarity problem.

JHEP04(2017)098

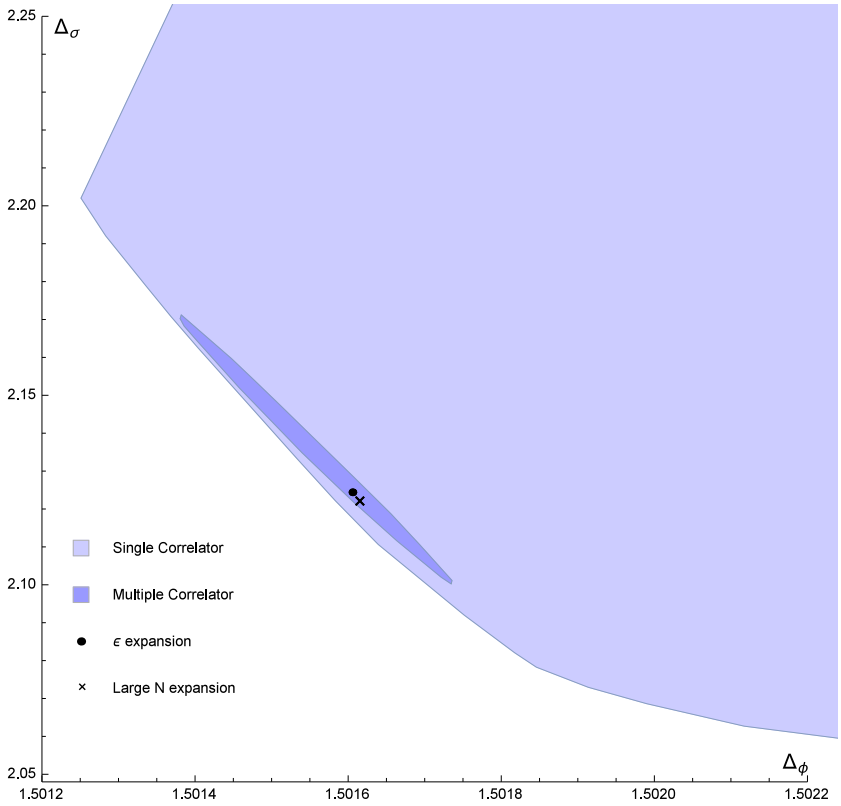

Figure 4. Bounds on the conformal dimensions $(\Delta_\phi, \Delta_\sigma)$ in $5D$ $O(100)$ vector model. The light blue region is obtained from single correlator bootstrap. The multiple correlators bootstrap leads to a small island colored in dark blue. In the bootstrap program we adopt the setup with $\Lambda = 19$ and the correspond spins provided in (3.9). We apply a dimension gap $\Delta^*_{S,0} = 3.6$ in the S-channel. Besides, an extra dimension gap $\Delta^*_{V,0} = 5$ has been used in the V-channel for bootstrapping multiple correlators. The black dot and cross relate to the predictions from ϵ expansion and large N expansion, respectively.

island corresponding to $N = 40$ shown in figure 3 is rather close to the kink from single correlator bootstrap presented in [36], where the kink was considered to indicate a unitary CFT. However, our studies based on multiple correlator bootstrap show that bootstrap results from single correlator or mixed correlators with low derivatives can be significantly modified in a more precise evaluation.

In fact there is no stable island from conformal bootstrap even at $N = 100$. The perturbative methods predict that the interacting $O(100)$ fixed point locates in the position with conformal dimensions $(\Delta_\phi, \Delta_\sigma) = (1.50161, 2.124)$ from large N expansion and $(\Delta_\phi, \Delta_\sigma) = (1.50162, 2.122)$ from ϵ expansion. In figure 4 we show the conformal bootstrap results of $O(100)$ vector model with $\Lambda = 19$. The single correlator conformal bootstrap generates a kinked bound similar to that of $O(500)$ vector model. The isolated region from multiple correlator conformal bootstrap lies in the middle of the tip. Here we have assumed a dimension gap $\Delta^*_{S,0} = 3.6$ in the S-channel, lower than the large N prediction $\Delta_{\sigma^2} \approx 3.68$. Besides, in the V-channel a dimension gap $\Delta^*_{V,0} = 5$ has been used, much lower than the

JHEP04(2017)098

one loop prediction $\Delta_{\phi_i'} \approx 5.39$. Predictions from large N and ϵ expansions are presented in the graph, both of which are nicely consistent with the conformal bootstrap bounds. In particular they locate in the isolated small island. All these features indicate a promising fixed point satisfying the crossing symmetry and unitarity constraints. However, the island disappears by taking higher order of derivatives $\Lambda = 23$! No isolated region can be found at $\Lambda = 25$ even we relax the S-channel gap to $\Delta^*_{S,0} = 3.3$. Unless the "true" island shrinks so drastically at certain order of Λ that it is hardly to be detected by scanning the parameter space, our bootstrap results disprove a unitary $5D$ $O(N)$ vector model even with $N = 100$!

Vanishing of the "allowed region" for $N \leq 100$ suggests that the theories with small N actually are not unitary. The violation of unitarity is rather small so that it cannot be uncovered by the bootstrap program with smaller Λ. This reminds us other examples on "pseudo" unitarity in conformal bootstrap. In [19] the $O(N)$ vector models in fractional dimensions $2 < D < 4$ have been studied using conformal bootstrap. In the work pronounced kinks are obtained in the bounds of conformal dimension of the lowest $O(N)$ singlet σ and are well consistent with the results obtained from extra approaches. However, careful studies in [64, 65] have shown that the CFTs in fractional dimensions are necessarily to be nonunitary, which are too subtle to be discovered in numerical conformal bootstrap. In the $5D$ $O(N)$ single correlator conformal bootstrap [36], sharp kinks are also generated in the fractional dimension $D = 5.95$ with $N = 600$, notably lower than the critical value $N_c \approx 1000$. We have studied this model through bootstrapping multiple correlators. There remains isolated allowed region even at $\Lambda = 21$, though it is quite small. The uncertainty on Δ_σ shown in the island is about 2×10^{-3}, while as shown in [36], the magnitude of imaginary part in Δ_σ is of the same order $\sim 1.5 \times 10^{-3}$ so it is expected that current conformal bootstrap program cannot capture the tiny unitarity violation unless the numerical accuracy can be improved significantly.

To summarize, the numerical conformal bootstrap provides a powerful approach to falsify assumptions on unitary CFTs. However, it is premature to validate the unitary CFTs using conformal bootstrap due to these "pseudo" unitary solutions. As to the $5D$ $O(500)$ model, although our results have provided strong evidence, they are still not sufficient to make a strict conclusion on its unitarity. On the other hand, it is surprising that the $5D$ $O(N)$ vector model is nonunitary even for $N = 100$. Consequenctly, the critical value $N_c > 100$, which is considerably larger than the value estimated before.

5 Conclusions

In this work, we have studied the interacting $5D$ CFTs with global $O(N)$ symmetry using the conformal bootstrap with multiple correlators. The multiple correlator conformal bootstrap has been developed in [28, 39] and obtained remarkable successes in $3D$ Ising and $O(N)$ vector models. The approach employs the correlators of the $O(N)$ vector scalar ϕ_i as well as the $O(N)$ singlet scalar σ. Since there are more operators involved in the crossing symmetry relations, the new method is expected to generate more strong constraints on the CFT data. Indeed the allowed regions on $(\Delta_\phi, \Delta_\sigma)$ plane is limited in a small island under reasonable assumptions on the dimension gaps.

JHEP04(2017)098

Specifically, in this study we have shown that by bootstrapping multiple correlators from the interacting $5D$ CFTs with $O(N)$ symmetry ($N = 500$), the allowed conformal dimensions $(\Delta_\phi, \Delta_\sigma)$ are strongly limited in a closed region, which is highly consistent with predictions from large N expansion. In order to uncover the isolated region we also applied assumptions on the dimension gaps both in the $O(N)$ singlet sector and the $O(N)$ vector sector. Our results suggest that the interacting fixed point of $O(N)$ vector model is unitary for sufficient large N and support the asymptotic free $5D$ $O(N)$ cubic model proposed in [55, 56]. Evidence of such fixed point has already been shown in the single correlator conformal bootstrap studied in [25, 35, 36]. The island obtained in this work is rather close to the kink in the bound of conformal dimension Δ_σ obtained from bootstrapping correlator of four ϕ_is [36]. We have studied the performance of the island under higher order of derivatives Λ. The island shrinks notably from $\Lambda = 19$ to $\Lambda = 25$, while the large N expansion predictions remain staying in the center of the allowed region. Such coincidence is surprising in considering of that only crossing symmetry and unitary conditions are employed to generate the allowed region. Besides we only input the $O(N)$ global symmetry for this model while even did not use its Lagrangian at all.

We are particularly interested in the critical value N_c of $5D$ $O(N)$ vector model below which the interacting fixed point theory loses unitarity. The problem on the critical value N_c can also be seen from the perturbative expansions of conformal dimension Δ_ϕ, that below the critical value the scalar ϕ_i acquires conformal dimension smaller than the unitary bound and breaks the unitary condition. However, in $5D$ the perturbative expansions converges much slower comparing with these of $3D$. In [55, 56] the critical value N_c has been evaluated based on large N expansion in $D = 6 - \epsilon$ spacetime. The critical value $N_c \simeq 1038$ at one-loop level; however, it oscillates drastically order by order. Conformal bootstrap provides a nonperturbative approach to study CFTs, and it has been applied to estimate N_c in [36]. The authors found that the pronounced kink in the bound of Δ_σ disappears near $N \sim 15$, which may suggest $N_c \sim 15$ in view of the observation that the singular behaviors, like kink in the dimension bound usually relate to unitary CFTs. In $3D$ such observation has helped to numerically solve the Ising model [23] and $O(N)$ vector model [18]. However, the unitarity condition becomes subtle for $5D$ CFTs and the unitarity violation may be too small to be detected by the bootstrap program with low order of derivatives. Therefore a kink does not necessarily guarantee unitarity, instead, it may relate to an interacting but nonunitary CFTs.

We have searched the allowed regions using multiple correlator conformal bootstrap for $1 \leq N \leq 100$. The isolated regions on the $(\Delta_\phi, \Delta_\sigma)$ plan can be obtained from conformal bootstrap program with lower order of derivatives. Moreover, the islands actually locate in the position close to the predictions from perturbative approaches given the Ns are not too small. However, the islands disappear after increasing the number of derivatives in bootstrap program. We believe these islands relate to interacting while nonunitary CFTs and the violation of unitarity can not be observed unless the program is equipped with sufficient high precision. In particular, our results suggest the critical value $N_c > 100$, much larger than the value estimated before. The bounds of N_c is expected to be improved further using conformal bootstrap. However, for larger N the unitarity violation in $O(N)$

JHEP04(2017)098

fixed point theory gets smaller and more difficult to be detected. It requires higher accuracy in the bootstrap program to determine the critical value N_c and we leave this problem for future work. On the other hand, for a sufficient large N_c, the large N expansion approach is validated. The critical value N_c can be effectively studied based on this perturbative approach as well. Due to the asymptotic behavior of perturbative expansions in $5D$, probably one needs to calculate several orders higher than in [55–57] to get a sufficient good estimation.

Acknowledgments

We are particularly grateful to David Simmons-Duffin for helpful correspondences on the numerical conformal bootstrap. We thank Simone Giombi for his comments on the spectra of $O(N)$ vector sector. We also thank Daliang Li, Tianjun Li, Daniel Robbins and Junchen Rong for valuable discussions. To calculate the conformal block functions, we used the code from an open-source program JuliBootS which is available at https://github.com/mfpaulos/JuliBoots. The work of N.S. was supported by NSF grants PHY-1214333 and PHY-1521099. The computations in this work were run on the Condor Computing Cluster supported by the Computer Support Group in Physics Department at Texas A&M University.

A Bootstrap equations

In (3.5) the crossing symmetry relations have been summarized in a compact form, as in [39] for $3D$ $O(N)$ vector model. The seven bootstrap equations obtained from $O(N)$ singlet (S), traceless symmetric tensor (T), antisymmetric tensor (A) and vector (V) sectors of multiple correlators are summarized in a 7-component vector equation (3.5), in which the vectors $\vec{V}_S, \vec{V}_T, \vec{V}_A, \vec{V}_V$ are:

$$\vec{V}_{T,\Delta,\ell} = \begin{pmatrix} F^{\phi\phi,\phi\phi}_{-,\Delta,\ell} \\ \left(1-\frac{2}{N}\right) F^{\phi\phi,\phi\phi}_{-,\Delta,\ell} \\ -\left(1+\frac{2}{N}\right) F^{\phi\phi,\phi\phi}_{+,\Delta,\ell} \\ \mathbf{0}_{4\times 1} \end{pmatrix}, \quad \vec{V}_{A,\Delta,\ell} = \begin{pmatrix} -F^{\phi\phi,\phi\phi}_{-,\Delta,\ell} \\ F^{\phi\phi,\phi\phi}_{-,\Delta,\ell} \\ -F^{\phi\phi,\phi\phi}_{+,\Delta,\ell} \\ \mathbf{0}_{4\times 1} \end{pmatrix}, \quad \vec{V}_{V,\Delta,\ell} = \begin{pmatrix} \mathbf{0}_{4\times 1} \\ (-1)^\ell F^{\phi\sigma,\phi\sigma}_{-,\Delta,\ell} \\ F^{\sigma\phi,\phi\sigma}_{-,\Delta,\ell} \\ -F^{\sigma\phi,\phi\sigma}_{+,\Delta,\ell} \end{pmatrix},$$

$$\vec{V}_{S,\Delta,\ell} = \begin{pmatrix} \mathbf{0}_{2\times 2} \\ \begin{pmatrix} F^{\phi\phi,\phi\phi}_{-,\Delta,\ell}(u,v) & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} F^{\phi\phi,\phi\phi}_{+,\Delta,\ell}(u,v) & 0 \\ 0 & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & 0 \\ 0 & F^{\sigma\sigma,\sigma\sigma}_{-,\Delta,\ell}(u,v) \end{pmatrix} \\ \mathbf{0}_{2\times 2} \\ \begin{pmatrix} 0 & \frac{1}{2}F^{\phi\phi,\sigma\sigma}_{-,\Delta,\ell}(u,v) \\ \frac{1}{2}F^{\phi\phi,\sigma\sigma}_{-,\Delta,\ell}(u,v) & 0 \end{pmatrix} \\ \begin{pmatrix} 0 & \frac{1}{2}F^{\phi\phi,\sigma\sigma}_{+,\Delta,\ell}(u,v) \\ \frac{1}{2}F^{\phi\phi,\sigma\sigma}_{+,\Delta,\ell}(u,v) & 0 \end{pmatrix} \end{pmatrix}. \tag{A.1}$$

Here our convention differs from [39] by a factor $(-1)^\ell$.

JHEP04(2017)098

References

[1] S. Ferrara, A.F. Grillo and R. Gatto, *Tensor representations of conformal algebra and conformally covariant operator product expansion*, *Annals Phys.* **76** (1973) 161 [INSPIRE].

[2] A.M. Polyakov, *Nonhamiltonian approach to conformal quantum field theory*, *Zh. Eksp. Teor. Fiz.* **66** (1974) 23 [INSPIRE].

[3] G. Mack, *Duality in quantum field theory*, *Nucl. Phys.* **B 118** (1977) 445 [INSPIRE].

[4] A.A. Belavin, A.M. Polyakov and A.B. Zamolodchikov, *Infinite conformal symmetry in two-dimensional quantum field theory*, *Nucl. Phys.* **B 241** (1984) 333 [INSPIRE].

[5] R. Rattazzi, V.S. Rychkov, E. Tonni and A. Vichi, *Bounding scalar operator dimensions in 4D CFT*, *JHEP* **12** (2008) 031 [arXiv:0807.0004] [INSPIRE].

[6] F.A. Dolan and H. Osborn, *Conformal four point functions and the operator product expansion*, *Nucl. Phys.* **B 599** (2001) 459 [hep-th/0011040] [INSPIRE].

[7] F.A. Dolan and H. Osborn, *Conformal partial waves and the operator product expansion*, *Nucl. Phys.* **B 678** (2004) 491 [hep-th/0309180] [INSPIRE].

[8] V.S. Rychkov and A. Vichi, *Universal constraints on conformal operator dimensions*, *Phys. Rev.* **D 80** (2009) 045006 [arXiv:0905.2211] [INSPIRE].

[9] F. Caracciolo and V.S. Rychkov, *Rigorous limits on the interaction strength in quantum field theory*, *Phys. Rev.* **D 81** (2010) 085037 [arXiv:0912.2726] [INSPIRE].

[10] D. Poland and D. Simmons-Duffin, *Bounds on 4D conformal and superconformal field theories*, *JHEP* **05** (2011) 017 [arXiv:1009.2087] [INSPIRE].

[11] R. Rattazzi, S. Rychkov and A. Vichi, *Central charge bounds in 4D conformal field theory*, *Phys. Rev.* **D 83** (2011) 046011 [arXiv:1009.2725] [INSPIRE].

[12] R. Rattazzi, S. Rychkov and A. Vichi, *Bounds in 4D conformal field theories with global symmetry*, *J. Phys.* **A 44** (2011) 035402 [arXiv:1009.5985] [INSPIRE].

[13] A. Vichi, *Improved bounds for CFT's with global symmetries*, *JHEP* **01** (2012) 162 [arXiv:1106.4037] [INSPIRE].

[14] D. Poland, D. Simmons-Duffin and A. Vichi, *Carving out the space of 4D CFTs*, *JHEP* **05** (2012) 110 [arXiv:1109.5176] [INSPIRE].

[15] S. El-Showk, M.F. Paulos, D. Poland, S. Rychkov, D. Simmons-Duffin and A. Vichi, *Solving the 3D Ising model with the conformal bootstrap*, *Phys. Rev.* **D 86** (2012) 025022 [arXiv:1203.6064] [INSPIRE].

[16] P. Liendo, L. Rastelli and B.C. van Rees, *The bootstrap program for boundary CFT_d*, *JHEP* **07** (2013) 113 [arXiv:1210.4258] [INSPIRE].

[17] C. Beem, L. Rastelli and B.C. van Rees, *The $\mathcal{N} = 4$ superconformal bootstrap*, *Phys. Rev. Lett.* **111** (2013) 071601 [arXiv:1304.1803] [INSPIRE].

[18] F. Kos, D. Poland and D. Simmons-Duffin, *Bootstrapping the $O(N)$ vector models*, *JHEP* **06** (2014) 091 [arXiv:1307.6856] [INSPIRE].

JHEP04(2017)098

[19] S. El-Showk et al., *Conformal field theories in fractional dimensions*, *Phys. Rev. Lett.* **112** (2014) 141601 [arXiv:1309.5089] [INSPIRE].

[20] L.F. Alday and A. Bissi, *The superconformal bootstrap for structure constants*, *JHEP* **09** (2014) 144 [arXiv:1310.3757] [INSPIRE].

[21] D. Gaiotto, D. Mazac and M.F. Paulos, *Bootstrapping the 3D Ising twist defect*, *JHEP* **03** (2014) 100 [arXiv:1310.5078] [INSPIRE].

[22] M. Berkooz, R. Yacoby and A. Zait, *Bounds on $\mathcal{N} = 1$ superconformal theories with global symmetries*, *JHEP* **08** (2014) 008 [*Erratum ibid.* **01** (2015) 132] [arXiv:1402.6068] [INSPIRE].

[23] S. El-Showk et al., *Solving the 3d Ising model with the conformal bootstrap II. c-minimization and precise critical exponents*, *J. Stat. Phys.* **157** (2014) 869 [arXiv:1403.4545] [INSPIRE].

[24] Y. Nakayama and T. Ohtsuki, *Approaching the conformal window of $O(n) \times O(m)$ symmetric Landau-Ginzburg models using the conformal bootstrap*, *Phys. Rev.* **D 89** (2014) 126009 [arXiv:1404.0489] [INSPIRE].

[25] Y. Nakayama and T. Ohtsuki, *Five dimensional $O(N)$-symmetric CFTs from conformal bootstrap*, *Phys. Lett.* **B 734** (2014) 193 [arXiv:1404.5201] [INSPIRE].

[26] L.F. Alday and A. Bissi, *Generalized bootstrap equations for $\mathcal{N} = 4$ SCFT*, *JHEP* **02** (2015) 101 [arXiv:1404.5864] [INSPIRE].

[27] S.M. Chester, J. Lee, S.S. Pufu and R. Yacoby, *The $\mathcal{N} = 8$ superconformal bootstrap in three dimensions*, *JHEP* **09** (2014) 143 [arXiv:1406.4814] [INSPIRE].

[28] F. Kos, D. Poland and D. Simmons-Duffin, *Bootstrapping mixed correlators in the 3D Ising model*, *JHEP* **11** (2014) 109 [arXiv:1406.4858] [INSPIRE].

[29] F. Caracciolo, A. Castedo Echeverri, B. von Harling and M. Serone, *Bounds on OPE coefficients in 4D conformal field theories*, *JHEP* **10** (2014) 020 [arXiv:1406.7845] [INSPIRE].

[30] Y. Nakayama and T. Ohtsuki, *Bootstrapping phase transitions in QCD and frustrated spin systems*, *Phys. Rev.* **D 91** (2015) 021901 [arXiv:1407.6195] [INSPIRE].

[31] J. Golden and M.F. Paulos, *No unitary bootstrap for the fractal Ising model*, *JHEP* **03** (2015) 167 [arXiv:1411.7932] [INSPIRE].

[32] S.M. Chester, J. Lee, S.S. Pufu and R. Yacoby, *Exact correlators of BPS operators from the 3D superconformal bootstrap*, *JHEP* **03** (2015) 130 [arXiv:1412.0334] [INSPIRE].

[33] M.F. Paulos, *JuliBootS: a hands-on guide to the conformal bootstrap*, arXiv:1412.4127 [INSPIRE].

[34] C. Beem, M. Lemos, P. Liendo, L. Rastelli and B.C. van Rees, *The $\mathcal{N} = 2$ superconformal bootstrap*, *JHEP* **03** (2016) 183 [arXiv:1412.7541] [INSPIRE].

[35] J.-B. Bae and S.-J. Rey, *Conformal bootstrap approach to $O(N)$ fixed points in five dimensions*, arXiv:1412.6549 [INSPIRE].

[36] S.M. Chester, S.S. Pufu and R. Yacoby, *Bootstrapping $O(N)$ vector models in $4 < d < 6$*, *Phys. Rev.* **D 91** (2015) 086014 [arXiv:1412.7746] [INSPIRE].

[37] D. Simmons-Duffin, *A semidefinite program solver for the conformal bootstrap*, *JHEP* **06** (2015) 174 [arXiv:1502.02033] [INSPIRE].

JHEP04(2017)098

[38] N. Bobev, S. El-Showk, D. Mazac and M.F. Paulos, *Bootstrapping the three-dimensional supersymmetric Ising model*, *Phys. Rev. Lett.* **115** (2015) 051601 [arXiv:1502.04124] [INSPIRE].

[39] F. Kos, D. Poland, D. Simmons-Duffin and A. Vichi, *Bootstrapping the $O(N)$ archipelago*, *JHEP* **11** (2015) 106 [arXiv:1504.07997] [INSPIRE].

[40] S.M. Chester, S. Giombi, L.V. Iliesiu, I.R. Klebanov, S.S. Pufu and R. Yacoby, *Accidental symmetries and the conformal bootstrap*, *JHEP* **01** (2016) 110 [arXiv:1507.04424] [INSPIRE].

[41] C. Beem, M. Lemos, L. Rastelli and B.C. van Rees, *The $(2,0)$ superconformal bootstrap*, *Phys. Rev.* **D 93** (2016) 025016 [arXiv:1507.05637] [INSPIRE].

[42] L. Iliesiu, F. Kos, D. Poland, S.S. Pufu, D. Simmons-Duffin and R. Yacoby, *Bootstrapping $3D$ fermions*, *JHEP* **03** (2016) 120 [arXiv:1508.00012] [INSPIRE].

[43] D. Poland and A. Stergiou, *Exploring the minimal $4D$ $\mathcal{N}=1$ SCFT*, *JHEP* **12** (2015) 121 [arXiv:1509.06368] [INSPIRE].

[44] M. Lemos and P. Liendo, *Bootstrapping $\mathcal{N}=2$ chiral correlators*, *JHEP* **01** (2016) 025 [arXiv:1510.03866] [INSPIRE].

[45] Y.-H. Lin, S.-H. Shao, D. Simmons-Duffin, Y. Wang and X. Yin, *$N=4$ superconformal bootstrap of the $K3$ CFT*, arXiv:1511.04065 [INSPIRE].

[46] S.M. Chester, L.V. Iliesiu, S.S. Pufu and R. Yacoby, *Bootstrapping $O(N)$ vector models with four supercharges in $3 \leq d \leq 4$*, *JHEP* **05** (2016) 103 [arXiv:1511.07552] [INSPIRE].

[47] S.M. Chester and S.S. Pufu, *Towards bootstrapping QED_3*, *JHEP* **08** (2016) 019 [arXiv:1601.03476] [INSPIRE].

[48] H. Iha, H. Makino and H. Suzuki, *Upper bound on the mass anomalous dimension in many-flavor gauge theories: a conformal bootstrap approach*, *PTEP* **2016** (2016) 053B03 [arXiv:1603.01995] [INSPIRE].

[49] F. Kos, D. Poland, D. Simmons-Duffin and A. Vichi, *Precision islands in the Ising and $O(N)$ models*, *JHEP* **08** (2016) 036 [arXiv:1603.04436] [INSPIRE].

[50] D. Simmons-Duffin, *TASI lectures on the conformal bootstrap*, arXiv:1602.07982 [INSPIRE].

[51] A. Pelissetto and E. Vicari, *Critical phenomena and renormalization group theory*, *Phys. Rept.* **368** (2002) 549 [cond-mat/0012164] [INSPIRE].

[52] I.R. Klebanov and A.M. Polyakov, *AdS dual of the critical $O(N)$ vector model*, *Phys. Lett.* **B 550** (2002) 213 [hep-th/0210114] [INSPIRE].

[53] G. Parisi, *The theory of nonrenormalizable interactions. 1. The large-N expansion*, *Nucl. Phys.* **B 100** (1975) 368 [INSPIRE].

[54] G. Parisi, *On nonrenormalizable interactions*, PRINT-77-0054 (1977).

[55] L. Fei, S. Giombi and I.R. Klebanov, *Critical $O(N)$ models in $6-\epsilon$ dimensions*, *Phys. Rev.* **D 90** (2014) 025018 [arXiv:1404.1094] [INSPIRE].

[56] L. Fei, S. Giombi, I.R. Klebanov and G. Tarnopolsky, *Three loop analysis of the critical $O(N)$ models in $6-\epsilon$ dimensions*, *Phys. Rev.* **D 91** (2015) 045011 [arXiv:1411.1099] [INSPIRE].

[57] J.A. Gracey, *Four loop renormalization of ϕ^3 theory in six dimensions*, *Phys. Rev.* **D 92** (2015) 025012 [arXiv:1506.03357] [INSPIRE].

JHEP04(2017)098

[58] O.J. Rosten, *Triviality from the exact renormalization group*, *JHEP* **07** (2009) 019 [arXiv:0808.0082] [INSPIRE].

[59] R. Percacci and G.P. Vacca, *Are there scaling solutions in the $O(N)$-models for large-N in $d > 4$?*, *Phys. Rev.* **D 90** (2014) 107702 [arXiv:1405.6622] [INSPIRE].

[60] P. Mati, *Vanishing β-function curves from the functional renormalization group*, *Phys. Rev.* **D 91** (2015) 125038 [arXiv:1501.00211] [INSPIRE].

[61] P. Mati, *Critical scaling in the large-N $O(N)$ model in higher dimensions and its possible connection to quantum gravity*, *Phys. Rev.* **D 94** (2016) 065025 [arXiv:1601.00450] [INSPIRE].

[62] K. Kamikado and T. Kanazawa, *Nonperturbative RG analysis of five-dimensional $O(N)$ models with cubic interactions*, arXiv:1604.04830 [INSPIRE].

[63] A. Eichhorn, L. Janssen and M.M. Scherer, *Critical $O(N)$ models above four dimensions: small-N solutions and stability*, *Phys. Rev.* **D 93** (2016) 125021 [arXiv:1604.03561] [INSPIRE].

[64] M. Hogervorst, S. Rychkov and B.C. van Rees, *Truncated conformal space approach in d dimensions: a cheap alternative to lattice field theory?*, *Phys. Rev.* **D 91** (2015) 025005 [arXiv:1409.1581] [INSPIRE].

[65] M. Hogervorst, S. Rychkov and B.C. van Rees, *Unitarity violation at the Wilson-Fisher fixed point in $4-\epsilon$ dimensions*, *Phys. Rev.* **D 93** (2016) 125025 [arXiv:1512.00013] [INSPIRE].

[66] A.N. Vasiliev, M.Y. Pismak and Yu.R. Khonkonen, *Simple method of calculating the critical indices in the $1/N$ expansion*, *Theor. Math. Phys.* **46** (1981) 104 [INSPIRE].

[67] A.N. Vasiliev, Yu.M. Pismak and Yu.R. Khonkonen, *$1/N$ expansion: calculation of the exponents η and ν in the order $1/N^2$ for arbitrary number of dimensions*, *Theor. Math. Phys.* **47** (1981) 465 [INSPIRE].

[68] A.N. Vasiliev, Yu.M. Pismak and Yu.R. Khonkonen, *$1/N$ expansion: calculation of the exponent eta in the order $1/N^3$ by the conformal bootstrap method*, *Theor. Math. Phys.* **50** (1982) 127 [INSPIRE].

[69] K. Lang and W. Rühl, *Field algebra for critical $O(N)$ vector nonlinear σ-models at $2 < d < 4$*, *Z. Phys.* **C 50** (1991) 285 [INSPIRE].

[70] K. Lang and W. Rühl, *The critical $O(N)$ σ-model at dimension $2 < d < 4$ and order $1/n^2$: operator product expansions and renormalization*, *Nucl. Phys.* **B 377** (1992) 371 [INSPIRE].

[71] K. Lang and W. Rühl, *The critical $O(N)$ σ-model at dimensions $2 < d < 4$: fusion coefficients and anomalous dimensions*, *Nucl. Phys.* **B 400** (1993) 597 [INSPIRE].

[72] K. Lang and W. Rühl, *The critical $O(N)$ σ-model at dimensions $2 < d < 4$: a list of quasiprimary fields*, *Nucl. Phys.* **B 402** (1993) 573 [INSPIRE].

[73] S. Kehrein, F. Wegner and Y. Pismak, *Conformal symmetry and the spectrum of anomalous dimensions in the N vector model in four epsilon dimensions*, *Nucl. Phys.* **B 402** (1993) 669 [INSPIRE].

[74] A. Petkou, *Conserved currents, consistency relations and operator product expansions in the conformally invariant $O(N)$ vector model*, *Annals Phys.* **249** (1996) 180 [hep-th/9410093] [INSPIRE].

JHEP04(2017)098

[75] A.C. Petkou, *C(T) and C(J) up to next-to-leading order in $1/N$ in the conformally invariant 0(N) vector model for $2 < d < 4$*, *Phys. Lett.* **B 359** (1995) 101 [hep-th/9506116] [INSPIRE].

[76] D.J. Broadhurst, J.A. Gracey and D. Kreimer, *Beyond the triangle and uniqueness relations: Nonzeta counterterms at large-N from positive knots*, *Z. Phys.* **C 75** (1997) 559 [hep-th/9607174] [INSPIRE].

[77] J.A. Gracey, *Crossover exponent in $O(N)$ ϕ^4 theory at $O(1/N^2)$*, *Phys. Rev.* **E 66** (2002) 027102 [cond-mat/0206098] [INSPIRE].

[78] K.G. Wilson and J.B. Kogut, *The renormalization group and the ϵ-expansion*, *Phys. Rept.* **12** (1974) 75 [INSPIRE].

[79] H. Kleinert, J. Neu, V. Schulte-Frohlinde, K.G. Chetyrkin and S.A. Larin, *Five loop renormalization group functions of $O(n)$ symmetric phi**4 theory and ϵ-expansions of critical exponents up to ϵ^5*, *Phys. Lett.* **B 272** (1991) 39 [*Erratum ibid.* **B 319** (1993) 545] [hep-th/9503230] [INSPIRE].

[80] J. Penedones, E. Trevisani and M. Yamazaki, *Recursion relations for conformal blocks*, *JHEP* **09** (2016) 070 [arXiv:1509.00428] [INSPIRE].

JHEP04(2017)098