# Scalar Absorption by Noncommutative D3-branes 

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#### Abstract

The classical cross section for low energy absorption of the RRscalar by a stack of noncommutative D3-branes in the large NS B-field limit is calculated. In the spirit of AdS/CFT correspondence, this cross section is related to two point function of a certain operator in noncommutative Yang-Mills theory. Compared at the same gauge coupling, the result agrees with that of obtained from ordinary D3-branes. This is consistent with the expectation that ordinary and noncommutative Yang-Mills theories are equivalent below the noncommutativity scale, but it is a nontrivial prediction above this scale.


[^0]Theories on noncommutative spaces arise naturally in string-M theory. In [1], the system of parallel D-branes has been reinterpreted as a quantum space [2]. DLCQ of M-theory with nonzero background three-form field along the null compactified direction has been argued to have matrix theory description of the gauge theory on noncommutative torus [3]. It has been shown in (4] that the D-brane world volume theories are noncommutative at a certain limit of the compactification moduli. Applying Dirac's constrained quantization method to open strings in the presence of NS two-form (B) field, the spacetime coordinates of open string end points have been shown not to commute [6] . Recently, by a direct string theoretic analysis, it has been shown in [7] that noncommutativity in the effective action is natural in the presence of constant background B-field.

A particularly interesting example of all these is the noncommutative Yang-Mills theory (NCYM) in four dimensions. The spectrum of IIB theory in the presence of D3-branes on constant B-field consists of open and closed string excitations coupled together. However, it is possible to take a low energy limit and scale some parameters such that the closed string modes decouple [7]. The resulting theory of open string modes turn out to be the NCYM. In [8] 9], a supergravity background has been proposed to be dual to this system. This background can be obtained by first constructing a solution which has nontrivial B-field dependence and then taking the decoupling limit of [7]. Closed string two point scattering amplitudes in the presence of B-field have been calculated and shown to be consistent with the ones encoded in the gravity solution [11]. In [10], the solution has been shown to have holographic features. Some properties of the NCYM have also been studied using its gravity dual [9] [12) (13].

In this paper we will consider the process of classical low energy absorption of the RR-scalar by noncommutative D3-branes in the large B-field limit. In the spirit of AdS/CFT correspondence, the cross section calculated from the gravity side is related to discontinuity of two point function of a certain operator in the dual NCYM [20]. This operator can be deduced from the coupling of the scalar to the D-brane world volume effective action [14]. The fluctuations turn out to be non-minimally coupled to background geometry. However, the coupling of RR-scalar to the effective world volume theory is relatively simple which may help one to identify the operator in NCYM. In calculating the absorption cross section, we will work with the D3-brane solution of (9] before taking the decoupling limit. As discussed in the context of ordi-
nary D3-branes in [21], the decoupling limit identifies the so called throat region with a certain limit of the world volume theory. On the other hand, a large B-field is also encountered in the decoupling limit considered in [7] [3] which means that the dual theory is "close" to its throat limit. For previous work on cross section calculations on black-brane backgrounds see, for instance, (15]- (22].

The type IIB supergravity action in the string frame can be written as

$$
\begin{array}{r}
S_{I I B}=\frac{1}{2 \kappa_{10}^{2}} \int d x^{10} \sqrt{-g}\left(e^{-2 \phi}\left[R+4(\partial \phi)^{2}-\frac{3}{4}\left(\partial B_{2}\right)^{2}\right]\right. \\
\left.-\frac{1}{2}(\partial C)^{2}-\frac{3}{4}\left(\partial C_{2}-C \partial B_{2}\right)^{2}-\frac{5}{6} F\left(C_{4}\right)^{2}-\frac{\epsilon_{10}}{48} C_{4} \partial C_{2} \partial B_{2}+\ldots\right) \tag{1}
\end{array}
$$

where $\partial B_{2}=\partial_{[\mu} B_{\nu \lambda]}$ etc., $F\left(C_{4}\right)=\partial C_{4}+3 / 4\left(B_{2} \partial C_{2}-C_{2} \partial B_{2}\right)$, and the self duality of $F\left(C_{4}\right)$ is imposed at the level of field equations. Specifically $B_{2}$ denotes the NS twoform field and $C$ is the RR-scalar. The solution corresponding to noncommutative D 3 -branes is given by [9],

$$
\begin{align*}
& d s^{2}=f^{-1 / 2}\left[-d t^{2}+d x_{1}^{2}+h\left(d x_{3}^{2}+d x_{4}^{2}\right)\right]+f^{1 / 2}\left[d r^{2}+r^{2} d \Omega_{5}^{2}\right],  \tag{2}\\
& B_{23}=\frac{\sin \theta}{\cos \theta} f^{-1} h, \quad e^{2 \phi}=g_{s}^{2} h, \quad \partial\left(C_{2}\right)_{01 r}=\frac{1}{g_{s}} \sin \theta \partial_{r} f^{-1},  \tag{3}\\
& F\left(C_{4}\right)_{0123 r}=\frac{1}{g_{s}} \cos \theta h \partial f^{-1}, \tag{4}
\end{align*}
$$

where $g_{s}$ is the asymptotic value of the string coupling constant, and the functions $f$ and $h$ are given by

$$
\begin{equation*}
f=1+\frac{R^{4}}{r^{4}}, \quad h=\frac{f}{\sin ^{2} \theta+\cos ^{2} \theta f} . \tag{5}
\end{equation*}
$$

The solution is asymptotically flat and there is a horizon at $r=0$. The near horizon geometry is $A d S_{5} \times S_{5}$. The mass per unit volume of (2) in string frame can be calculated to be

$$
\begin{equation*}
M=\frac{2 \pi^{3} R^{4}}{\kappa_{10}^{2} g_{s}^{2}} \tag{6}
\end{equation*}
$$

which is remarkably independent of $\theta$. Note that the asymptotic value of B-field is $\tan \theta$.

[^1]From (1), the fluctuations of the RR-scalar on this background obeys

$$
\begin{equation*}
\nabla^{2} C=\frac{3}{2}\left(\partial B_{2}\right)^{2} C \tag{7}
\end{equation*}
$$

and thus couple to the background geometry non-minimally. We note that the contraction $\left(\partial B_{2}\right)\left(\partial C_{2}\right)$ is zero. Respecting the translational invariance, we will assume that $C$ does not depend on the spatial coordinates of D3-branes. Separating the time dependence and considering a spherically symmetric fluctuation (s-wave), which is supposed to give the dominant contribution to cross section, we write

$$
\begin{equation*}
C=e^{-i w t} \phi(r) . \tag{8}
\end{equation*}
$$

Following from (7), $\phi$ obeys

$$
\begin{equation*}
\left(h r^{5}\right)^{-1} \frac{d}{d r}\left(h r^{5} \frac{d}{d r} \phi\right)+\omega^{2} f \phi-\frac{16 \sin ^{2} \theta \cos ^{2} \theta R^{8}}{r^{10}} f^{-3} h^{2} \phi=0 . \tag{9}
\end{equation*}
$$

Since this equation does not appear to be analytically solvable, following previous work, we try to find an approximate solution by matching three different regions dictated by the structure of the functions $f$ and $h$. Low energy scattering is characterized by $\omega \sqrt{\alpha^{\prime}} \ll 1$ and the $\alpha^{\prime}$ corrections to background is suppressed when $\sqrt{\alpha^{\prime}} / R \ll 1$. Consistent with these two restrictions, we will consider the double scaling limit of [18] and assume $\omega R \ll 1$. Large B-field corresponds to $\cos \theta \ll 1$ and we will further analyze the case where $\cos \theta \sim \omega R$.

## Region I: $r \gg R$

In this region $f \sim h \sim 1$. Defining $\rho=\omega r$ and $\phi=\rho^{-5 / 2} \psi$, (9) simplifies as

$$
\begin{equation*}
\left(\frac{d^{2}}{d \rho^{2}}-\frac{15}{4 \rho^{2}}+1-\frac{16 \sin ^{2} \theta \cos ^{2} \theta(\omega R)^{8}}{\rho^{10}}\right) \psi=0 . \tag{10}
\end{equation*}
$$

Since $\rho \gg \omega R$, the last term in this equation is negligible compared to the second one. Ignoring this term, (10) can be solved in terms of Bessel and Neumann functions which in turn gives

$$
\begin{equation*}
\phi=a_{1} \quad \rho^{-2} J_{2}(\rho)+a_{2} \quad \rho^{-2} N_{2}(\rho), \tag{11}
\end{equation*}
$$

where $a_{1}$ and $a_{2}$ are constants.
Region II: $R \gg r \gg R \sqrt{\cos \theta}$
In this region $f$ and $h$ can be approximated as $f \sim h \sim R^{4} / r^{4}$. Using this form and defining $\phi=\rho^{-1 / 2} \chi$, (5) becomes

$$
\begin{equation*}
\left(\frac{d^{2}}{d \rho^{2}}+\frac{1}{4 \rho^{2}}+\frac{(\omega R)^{4}}{\rho^{4}}-\frac{16 \sin ^{2} \theta \cos ^{2} \theta(\omega R)^{4}}{\rho^{6}}\right) \chi=0 . \tag{12}
\end{equation*}
$$

In region II, $\omega R \gg \rho \gg \omega R \sqrt{\cos \theta}$ and in that interval; the third term can be ignored compared to the fourth one (with the assumption $\cos \theta \sim \omega R$ ), and the fourth term is always very small with respect to the second one. Therefore, $\chi$ approximately obeys

$$
\begin{equation*}
\frac{d^{2}}{d \rho^{2}} \chi+\frac{1}{4 \rho^{2}} \chi=0 . \tag{13}
\end{equation*}
$$

Two solutions of this equation are $\rho^{1 / 2}$ and $\rho^{1 / 2} \ln \rho$, and thus

$$
\begin{equation*}
\phi=b_{1}+b_{2} \ln \rho, \tag{14}
\end{equation*}
$$

where $b_{1}$ and $b_{2}$ are constants.
Region III: $R \sqrt{\cos \theta} \gg r$
In this region $f \sim R^{4} / r^{4}$ and $h \sim 1 / \cos ^{2} \theta$. Defining $z=\omega R \sqrt{\cos \theta} / \rho$ and $\phi=z^{3 / 2} Z$, equation (9) can be approximated to,

$$
\begin{equation*}
\left(\frac{d^{2}}{d z^{2}}-\frac{15}{4 z^{2}}+\frac{(\omega R)^{2}}{\cos \theta}-16 \frac{\sin ^{2} \theta}{z^{6}}\right) Z=0 \tag{15}
\end{equation*}
$$

In region III, $z \gg 1$ and thus the last term can be ignored compared to the second one. Dropping this term, two solutions of $Z$ can be found to be $z^{1 / 2} J_{2}(z \omega R / \sqrt{\cos \theta})$ and $z^{1 / 2} N_{2}(z \omega R / \sqrt{\cos \theta})$. This gives

$$
\begin{equation*}
\phi=c_{1} \rho^{-2} J_{2}\left(\frac{(\omega R)^{2}}{\rho}\right)+c_{2} \rho^{-2} N_{2}\left(\frac{(\omega R)^{2}}{\rho}\right), \tag{16}
\end{equation*}
$$

where $c_{1}$ and $c_{2}$ are constants.

## Matching the solutions:

In matching the solutions in different regions, we will use the small argument expansion of the Bessel and Neumann functions

$$
\begin{align*}
J_{2}(x) & \sim \frac{x^{2}}{8} \\
N_{2}(x) & \sim \frac{-4}{\pi x^{2}}\left(1+\frac{x^{2}}{4}\right)+\frac{1}{4 \pi} x^{2}(\ln x+c) \tag{17}
\end{align*}
$$

where $c$ is a constant. Close to the horizon, we want only an ingoing wave which implies

$$
\begin{equation*}
c_{1}=-i c_{2} . \tag{18}
\end{equation*}
$$

The overall normalization of $\phi$ can be fixed by imposing $c_{1}=i(\omega R)^{4}$. To be able to match the solution (16) to region II, we consider its behavior when $\rho \sim \omega R \sqrt{\cos \theta}$.

Assuming $\cos \theta \sim \omega R$, the arguments of the Bessel and Neumann functions in (16) are small in this range and thus we can use the expansion (17). We see that there is no term in this expansion to be matched by $\ln \rho$ of region II, therefore, we should set $b_{2}=0$. The dominant contribution of the rest of the terms when $\rho \sim \omega R \sqrt{\cos \theta}$ is the constant $4 / \pi$, which fixes $b_{1}$ as

$$
\begin{equation*}
b_{1}=\frac{4}{\pi} . \tag{19}
\end{equation*}
$$

To match (11) to region II, we will consider its behavior when $\rho \sim \omega R$. The arguments of Bessel and Neumann functions in (11) are also small in this range. The leading contributions of their expansions are $a_{1} / 8$ and $\left(-4 a_{2}\right) /\left(\pi \rho^{4}\right)$, respectively. To be able to match these to (14), one should set

$$
\begin{equation*}
a_{2}=0, \quad a_{1}=\frac{32}{\pi} . \tag{20}
\end{equation*}
$$

Combining these, we obtain the following functions in three regions

$$
\begin{align*}
\phi_{I} & =\frac{32}{\pi} \rho^{-2} J_{2}(\rho) \\
\phi_{I I} & =\frac{4}{\pi} \\
\phi_{I I I} & =i(\omega R)^{4} \rho^{-2}\left[J_{2}\left(\frac{(\omega R)^{2}}{\rho}\right)+i N_{2}\left(\frac{(\omega R)^{2}}{\rho}\right)\right] \tag{21}
\end{align*}
$$

which smoothly overlaps and give an approximate solution to (9) To calculate the cross section one should compare the incoming flux at the horizon with the incoming flux at the infinity. At this stage, we recognize that the same functions appear in [18] in the solutions of the massless wave equation on ordinary D3-brane background. The cross section corresponding to the solution (21) can be read from [18] to be

$$
\begin{equation*}
\sigma_{a b s}=\frac{\pi^{4}}{8} \omega^{3} R^{8} . \tag{22}
\end{equation*}
$$

We now try to rewrite $\sigma_{a b s}$ in terms of the gauge coupling constant $\tilde{g}_{Y M}$ of NCYM.

The solution (2) can be shown to preserve $1 / 2$ supersymmetries of the theory. This can easily be seen by nothing that (2) is related to ordinary D3-brane solution by a chain of T-duality transformations (namely, first a T-duality along $x_{3}$, then a rotation by an angle $\theta$ along the $x_{2}-x_{3}$ plane, and then another T-duality along

[^2]$x_{3}$ ) and T-duality respects supersymmetry when the Killing spinor is independent of the direction of the duality [23]. From a world-sheet point of view, one can also see that the parallel D3-branes on constant, invertible, B-field backgrounds also preserve $1 / 2$ supersymmetries since boundary conditions identify the left moving supercurrents with the right moving ones. This is consistent with the identification of this configuration with the gravity solution. Due to this BPS property, the noncommutative D3-brane tension, when calculated from an effective action point of view, should be equal to the mass per unit volume (6). We will now carry out this effective field theory calculation to fix the value of $R$.

The Dirac-Born-Infeld action corresponding to a noncommutative D3-brane can be written as

$$
\begin{equation*}
S_{D B I}=T_{3} \int d^{4} \sigma \sqrt{-\operatorname{det}(\hat{g}+\hat{B})}, \tag{23}
\end{equation*}
$$

where $\sigma^{i}$ are coordinates on the D 3 brane, $T_{3}$ is the ordinary D 3 -brane tension when $B=0, \hat{g}$ and $\hat{B}$ are pull-backs of flat Minkowski metric $\eta_{\mu \nu}$ and constant B-field $B_{\mu \nu}$, respectively,

$$
\begin{align*}
\hat{g}_{i j} & =\partial_{i} X^{\mu} \partial_{j} X^{\nu} \eta_{\mu \nu}  \tag{24}\\
\hat{B}_{i j} & =\partial_{i} X^{\mu} \partial_{j} X^{\nu} B_{\mu \nu} \tag{25}
\end{align*}
$$

From $S_{D B I}$, one can calculate the conjugate momentum densities to coordinate fields $X^{\mu}$ as

$$
\begin{equation*}
P_{\mu}=\frac{\delta S_{D B I}}{\delta \partial_{\tau} X^{\mu}} \tag{26}
\end{equation*}
$$

where $\tau$ is the world volume time coordinate.

The tension $\tilde{T}_{3}$ of a noncommutative D 3 -brane can be defined as the energy density corresponding to a flat, non-exited brane. This can be described in a physical gauge by $X^{i}=\sigma^{i}, X^{\alpha}=$ const. For such a brane, (26) gives

$$
\begin{equation*}
P_{\mu}=T_{3} \sqrt{-\operatorname{det}\left(\eta_{i j}+B_{i j}\right)} \delta_{\mu}^{0}, \tag{27}
\end{equation*}
$$

which corresponds to the momentum density of a massive object with zero velocity. The energy per unit volume of such an object can be read from $P_{0}$ component of the momentum. This gives the tension for a noncommutative D3-brane as

$$
\begin{equation*}
\tilde{T}_{3}=T_{3} \sqrt{-\operatorname{det}\left(\eta_{i j}+B_{i j}\right)} . \tag{28}
\end{equation*}
$$

Note that the ordinary D3-brane tension and the 10-dimensional gravitational coupling constant are

$$
\begin{equation*}
T_{3}=\frac{1}{(2 \pi)^{3} \alpha^{\prime 2} g_{s}}, \quad 2 \kappa_{10}^{2}=(2 \pi)^{7} \alpha^{\prime 4} \tag{29}
\end{equation*}
$$

In the solution (2), B-field is a rank 2 matrix. Using (28) and (29) we obtain,

$$
\begin{equation*}
\tilde{T}_{3}=\frac{1}{(2 \pi)^{3} \alpha^{\prime 2} g_{s} \cos \theta} \tag{30}
\end{equation*}
$$

The total energy of $N$-coincident D3-branes is given by $N \tilde{T}_{3}$ which should be equal to (6). This gives the parameter $R$ as $[$

$$
\begin{equation*}
R^{4}=\frac{4 \pi \alpha^{\prime 2} g_{s} N}{\cos \theta} \tag{31}
\end{equation*}
$$

On the other hand the gauge coupling constant $\tilde{g}_{Y M}$ of NCYM, can be read from [7] to be

$$
\begin{align*}
\tilde{g}_{Y M}^{2} & =2 \pi g_{s} \sqrt{-\operatorname{det}\left(\eta_{i j}+B_{i j}\right)}, \\
& =\frac{2 \pi g_{s}}{\cos \theta} . \tag{32}
\end{align*}
$$

Note that for $\theta=0$, this gives the well known relation between the ordinary YangMills and string coupling constants. Combining (32) with (31), the cross section (22) can be rewritten in terms of $\tilde{g}_{Y M}$ as

$$
\begin{equation*}
\sigma_{a b s}=\frac{\pi^{4}}{2} \omega^{3} \alpha^{\prime 4} N^{2} \tilde{g}_{Y M}^{4} . \tag{33}
\end{equation*}
$$

Remarkably, all $\theta$ dependence is hidden in $\tilde{g}_{Y M}$. The classical cross section for low energy absorption of RR-scalar by ordinary D3-branes (which is identical to massless scalar absorption) has been calculated in [18] and the result is (33) in which $\tilde{g}_{Y M}$ is replaced with the gauge coupling $g_{Y M}$ of ordinary Yang-Mills theory. Comparing NCYM with the ordinary Yang-Mills at the same coupling

$$
\begin{equation*}
\tilde{g}_{Y M}=g_{Y M}, \tag{34}
\end{equation*}
$$

the result of 18] exactly agrees with (33).

As previously noted, (33) is related to discontinuity of the cut in the two point function of a certain operator in NCYM. The discontinuity in momentum space is

[^3]evaluated at $p^{2}=\omega^{2}$. Since $\omega \sqrt{\alpha^{\prime}} \ll 1$, (33) is valid below the string scale. On the other hand, since the noncommutativity (NC) scale is roughly given by $\sqrt{\alpha^{\prime} \cos \theta}$ (see, for instance, [7]) and $\cos \theta \ll 1, \mathrm{NC}$ scale is also below the string scale. Below the NC scale, ordinary Yang-Mills theory is a good approximation to NCYM, and the equivalence of cross sections is consistent with this fact. Between NC and string scales, (33) is a nontrivial prediction for NCYM. We note that, (33) is the first term in an expansion in the parameter ( $\omega R$ ), which in NCYM corresponds to perturbing field theory by higher and higher dimensional operators and a loop expansion.

The corresponding operators in dual theories can be deduced from the coupling of the scalars at hand to the effective world volume theories. For the ordinary Yang-Mills theory, leading order coupling of RR-scalar to the world volume is

$$
\begin{equation*}
\epsilon^{i j k l} C \operatorname{Tr} F_{i j} F_{k l .} . \tag{35}
\end{equation*}
$$

The classical cross section for the absorption of RR-scalar by ordinary D3-branes has been shown to agree with a tree level world volume calculation which involves the above coupling [19]. This indicates (with a non-renormalization theorem) that the leading term of the corresponding operator in the dual theory is $\epsilon^{i j k l} \operatorname{Tr} F_{i j} F_{k l}$.

Naturally, one may try to identify the coupling of RR-scalar to a noncommutative D3-brane world volume. As discussed in [7] , it is very convenient to write the effective action in terms of the noncommutative gauge fields and open string parameters. When expressed in these variables, the effective action in the presence of B-field can be deduced form the ordinary one. From (35), the coupling of RR-scalar to the noncommutative D3-brane world volume can be written as

$$
\begin{equation*}
\epsilon^{i j k l} C * \operatorname{Tr} \hat{F}_{i j} * \hat{F}_{k l}, \tag{36}
\end{equation*}
$$

where $\hat{F}$ is the noncommutative gauge field strength, $\epsilon$-tensor and raising of indices refer to open string metric and *-product is defined by

$$
\begin{equation*}
(f * g)(x)=\left.e^{\frac{1}{2} \theta^{i j} \frac{\partial}{\chi^{2}} \frac{\partial}{\eta^{j}}} f(x+\chi) g(x+\eta)\right|_{\chi=\eta=0} . \tag{37}
\end{equation*}
$$

The open string metric and noncommutativity parameter $\theta^{i j}$ is fixed in terms of the background Minkowski metric and B-field [7]. This indicates that the leading order term of the operator in the NCYM is $\epsilon^{i j k l} \operatorname{Tr} \hat{F}_{i j} * \hat{F}_{k l}$. However, this term is not gauge invariant unless it is integrated over noncommuting directions.

Although it brakes the translational invariance, it is interesting to consider the effects of adding non-zero momentum along the noncommuting directions $x_{2}$ and $x_{3}$. For this, one modifies ( $(8)$ as

$$
\begin{equation*}
C=e^{-i w t} e^{i \vec{k} . \vec{x}} \phi(r), \tag{38}
\end{equation*}
$$

where $\vec{k} \cdot \vec{x}=k_{2} x_{2}+k_{3} x_{3}$. Due to this modification, (9) picks up an extra term to the left hand side which is

$$
\begin{equation*}
-k^{2} f h^{-1} \phi, \tag{39}
\end{equation*}
$$

where $k^{2}=k_{2}^{2}+k_{3}^{2}$. It is easy to see that perturbative scattering requires $\omega>k$. In regions II and III, this term can be neglected compared to another term in (9), namely $\omega^{2} f \phi$, since $h \gg 1$ in these regions and $\omega>k$. On the other hand, in region $\mathrm{I}, \phi_{I}$ is modified as

$$
\begin{equation*}
\phi_{I}=\frac{32}{\pi}(s \rho)^{-2} J_{2}(s \rho), \tag{40}
\end{equation*}
$$

where the parameter $s$ is

$$
\begin{equation*}
s=\sqrt{1-\frac{k^{2}}{\omega^{2}}} \tag{41}
\end{equation*}
$$

The flux at infinity calculated from the modified $\phi_{I}$ is changed by a factor of $1 / s^{4}$ and thus

$$
\begin{equation*}
\sigma_{a b s}(k)=s^{4} \sigma_{a b s}(0) \tag{42}
\end{equation*}
$$

For ordinary D3-branes, this factor can be calculated to be $s^{8}$ which seems to imply a disagreement for dual theories. However, we note that it is difficult to give a world volume interpretation to cross sections when $s \neq 0$. For instance, following [18], the factor of $s^{8}$ cannot be obtained by a world volume scattering calculation for ordinary D3-branes. Furthermore, due to the fact that the corresponding operator in dual Yang-Mills theory is also a scalar operator, the two point function of this operator can only depend on $p^{2}$ and the discontinuity in the complex plane can only depend on $\omega^{2}$ (or may be with a slight modification on $\left(\omega^{2}-k^{2}\right)$ ) which is not consistent with the factor $s^{8}$. We believe that the broken translational invariance ruins the connection between cross sections and the two point functions, and is responsible for these disagreements. Finally, it is also worth to mention that the wave equation for minimally coupled scalars can be solved exactly in Einstein frame and the cross sections agree to all orders when $k=0$ (24].

One may try to repeat same calculations for noncommutative M5-branes using the solution given in [9]. For ordinary M5-branes, the traceless metric perturbations
polarized along the 5 -branes have been shown to obey minimally coupled scalar equations (19]. One can easily show that this is also true for noncommutative M5-branes. Furthermore, the massless scalar equation on noncommutative M5-branes turns out to be the same with the one on the ordinary M5 branes. Therefore, the classical cross sections corresponding to absorption of traceless metric perturbations polarized along the 5 -branes are identical for both type of branes.

Note added for the hep-th version: After the submission of the present paper to the net, we received [24] and [25], which also consider scalar absorption by noncommutative D3-branes. In [24], the authors claim that to see effects of noncommutativity one should consider waves propagating along noncommuting directions on the brane. However, as discussed above, interpretation of this from a world volume theory point of view is not clear. In [25], the authors claim that in the $B \rightarrow \infty$, i.e. $\theta \rightarrow \pi / 2$, limit the noncommutativity effects are turned on, and in this limit the RR-scalar is nonpropagating. But, as shown in [7], even in the presence of constant and finite Bfield, noncommutativity in the effective field theory is inevitable. On the other hand, taking $B \rightarrow \infty$ limit is a delicate issue. For instance, to make contact with string theory, the parameter $R$ should be fixed as in (31) and thus diverges at $\theta=\pi / 2$. To avoid such infinities, one should scale other parameters in a suitable way and this gives the geometry found in [9]. Therefore, contrary what is claimed in [25], the NS 3 -form and the RR 5 -form field strengths are not zero at $\theta=\pi / 2$. Beside that, even for the solution in which NS 3-form and RR 5 -form field strengths are zero, one can still introduce fluctuations of these fields. In either case, it can be shown that RR -scalar is not necessarily nonpropagating when $\theta=\pi / 2$.

## Acknowledgements

I would like to thank S. Deg̃er for reading the manuscript.

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[^1]:    ${ }^{2}$ The conformal limit in the case of ordinary D3-branes.
    ${ }^{3}$ In taking the decoupling limit one keeps B fixed and let $\alpha^{\prime}$ go to zero which corresponds to a very large B-field for finite (but small) $\alpha^{\prime}$.

[^2]:    ${ }^{4}$ To ensure that $\phi$ can be differentiated twice, one has to let $b_{2}=O(\omega R)$ and $a_{2}=$ $O\left((\omega R)^{5}\right)$, instead of setting them to zero. To zeroth order in $\omega R$, this modification does not change the main result (22).

[^3]:    ${ }^{5}$ Exactly the same expression for $R$ is given in (9).

