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Proton Decay and Flavor Violating Thresholds in SO(10) Models

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Discovery of neutrino mass has put the spotlight on supersymmetric SO(10) as a natural candidate for grand unification of forces and matter. However, the suppression of proton decay is a major problem in any supersymmetric grand unified models. In this paper we show how to alleviate this problem by simple threshold effect which raises the colored Higgsino masses and the grand unification scale to $\gtrsim 10^{17}$ GeV. There exist only four types of fields arising from different SO(10) representations which can generate this kind of threshold effects. Some of these fields also generate a sizable flavor violation in the quark sector compared to the lepton sector. The b-τ unification can work in these types of models even for intermediate values of $\tan\beta$.

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Supersymmetry (SUSY) is an attractive and widely discussed candidate for physics beyond the standard model (SM) at the TeV scale. In addition to solving the gauge hierarchy problem, it has the appealing feature of leading to unification of three SM gauge couplings at a high scale of around 2×10^{16} GeV if there is no new physics between the TeV and grand unified theory (GUT) scale. The coupling unification would suggest that the SM gauge groups are grand unified into one simple group. An additional boost to the argument for grand unification comes from the understanding of small neutrino masses via the seesaw mechanism, which suggests the breaking of B-L symmetry around the GUT scale. The simplest GUT that embodies all these features including the seesaw mechanism is SUSY SO(10) group, where all quarks and leptons are unified in one **16**-dimensional spinor representation.

The downside of quark-lepton unification is that it predicts an unstable proton. Searches for proton decay have so far have not been successful and put a lower limit on its lifetime of $\gtrsim 10^{33}$ years. Clearly, the observation of nucleon decay would provide the most direct test of the GUT models. However, the present lower bound on the nucleon lifetime imposes severe constraints on the SUSY GUT models [1, 2], if SUSY particles are less than 2-3 TeV. The main sources of these constraints are the dimension five operators induced by colored Higgsino fields which accompany the SM Higgs in the GUT theory [3]. The Higgsino couplings are then related to the fermion masses and are less free for adjustment. One way to escape these constraints is to add additional symmetries above and beyond SO(10), which eliminates such dimension five operators altogether. In this study, we assume that the dimension five operators do really exist and point out a new way to suppress their effects.

As is well known, in the presence of the Higgsino induced dimension five operators, the nucleon lifetime depends on SUSY particles' masses, colored Higgsino mass (chosen to be of order of the GUT scale), and Yukawa texture for fermions. In a recent paper, we pointed out that one way to suppress these contributions is to use suitable Yukawa texture [4] and presented an SO(10) example that uses only renormalizable couplings for fermion masses. This suppresses proton decay even for the color triplet Higgsino mass at 2×10^{16} GeV. Such a structure provides hierarchical mass for quarks and leptons, small U_{e3} (one of the neutrino mixings) and large atmospheric and solar neutrino mixings. Here we consider an alternative approach where we do not constrain the Yukawa texture; instead we introduce new thresholds to suppress nucleon decay while at the same time maintaining gauge coupling unification.

This approach is nontrivial due to constraints of gauge unification. For example, in the minimal SU(5) model, coupling unification imposes a very stringent upper bound on the colored Higgsino mass making it impossible to satisfy current experimental bounds of the nucleon lifetime [1]. On the other hand, the rich multiplet structure of SO(10) allows new sub-GUT scale thresholds even in its minimal version. Of course it is apriori not clear whether coupling unification will necessarily impose constraints on Higgsino mass when one includes new thresholds. We explore this issue here.

An interesting point about new thresholds is that they may be accessible at low energies via new flavor effects. The point is that in SUSY models, the flavor degeneracy of squarks and sleptons are often assumed at high scale to avoid flavor changing neutral currents (FCNCs). However loop corrections in the presence of new thresholds can induce flavor violation and can provide a way to test indirectly for GUT scale particle spectrum. In the MSSM, the induced FCNCs in the quark sector from the evolution of renormalization group equations (RGE) are small due to the small quark mixings whereas in the lepton sector, sizable effects can arise from the presence of right-handed neutrino thresholds in the seesaw framework [5]. New thresholds can make their presence felt both in the quark and the lepton sector by altering these

FCNC footprints in interesting ways.

In this Letter, we will show that there exists a simple threshold effect that makes the colored Higgsino mass and the symmetry breaking scale larger in SO(10) models, and thus naturally suppressing proton decay. In such a SO(10) breaking vacuum, it is possible that a sizable flavor violation is generated in the quark sector rather than the lepton sector. We also emphasize that investigating flavor violation in both quark and lepton sectors is important to select possible scenarios of GUT models and symmetry breaking vacua.

At first, we will briefly examine the SO(10) GUT model buildings. In order to break SO(10) symmetry down to the SM gauge symmetry, we employ **210** (four-antisymmetric tensor) and $\mathbf{126}(\Delta)+\overline{\mathbf{126}}(\bar{\Delta})$ representations. In this choice, the SO(10) breaking vacua down to SM can be dictated in the minimal number of parameters [6]. The Higgs representations above can be replaced to $\mathbf{45}+\mathbf{54}$ and $\mathbf{16}+\overline{\mathbf{16}}$, respectively. The Higgs spectrum and the breaking patterns of SO(10) can be found in the Ref.[7]. One can also employ only a pair of vector-spinor representations ($\mathbf{144}+\overline{\mathbf{144}}$) to break SO(10) down to SM at a single scale [8]. All fermions are unified in **16** representation ψ_i in each generation ($i=1,2,3$). The Higgs fields which couple to fermions in renormalizable Yukawa terms are **10** (H), $\overline{\mathbf{126}}$ ($\bar{\Delta}$) and **120** (D):

$$W_Y=\frac{1}{2}h_{ij}\psi_i\psi_j H+\frac{1}{2}f_{ij}\psi_i\psi_j\bar{\Delta}+\frac{1}{2}h'_{ij}\psi_i\psi_j D. \qquad (1)$$

The SO(10) invariance implies that the coupling matrices h and f are symmetric and h' is antisymmetric.

The Yukawa couplings for quarks and leptons can be written as

$$Y_u=\bar{h}+r_2\bar{f}+r_3\bar{h}', \qquad Y_d=r_1(\bar{h}+\bar{f}+\bar{h}'), \qquad (2)$$
$$Y_\nu=\bar{h}-3r_2\bar{f}+c_\nu\bar{h}', \qquad Y_e=r_1(\bar{h}-3\bar{f}+c_e\bar{h}'), \qquad (3)$$

where u,d,e,ν denote up-type quark, down-type quark, charged-lepton, Dirac neutrino Yukawa couplings, respectively. Notations such as $r_{1,2,3}$, which are the functions of Higgs mixings, and $\bar{h}$, which is an original coupling h multiplied by a Higgs mixing, are given in the Ref.[4]. The parameter r_1 provides a freedom of $\tan\beta$ (ratio of vacuum expectation values (VEVs) for up and down Higgs doublets in MSSM). When there is an exchange symmetry between Δ and $\bar{\Delta}$, r_1 turns out to be 1 and then $\tan\beta$ is ~ 50. If there is no such exchange symmetry, $\tan\beta$ is a free parameter in the model (t-b-τ Yukawa unification is not satisfied), while b-τ Yukawa unification is still realized approximately if $\bar{f}$ is small, which is natural to obtain Georgi-Jarskog relation, $3m_s/m_b\simeq m_\mu/m_\tau$ at GUT scale.

There has been efforts to fit all fermion masses and mixings using only h and f couplings (without **120** Higgs) [9]. However, such a minimal situation is disfavored [10]. In the minimal choice, one needs fine-tuning to fit electron mass since electron mass also becomes three times larger than down quark mass naively. In that case, the first and second generation component of the coupling matrices are large, and the proton decay suppression becomes really unnatural [4]. Even for the numerical fits of masses and mixings, the fits are excluded more than 3 sigma level if the minimal choice of Higgs content is considered [10]. This is because that (1) r_2 is determined by the SO(10) breaking vacua in the minimal choice. (2) SO(10) breaking pattern is limited to obtain proper neutrino mass scale. (3) If the restricted SO(10) breaking vacuum is chosen, the gauge coupling unification does not occur. However, such a disaster can be avoided if we introduce additional Higgs fields as done in Refs. [4, 11]. In these new scenarios, the constraint on r_2 from the fermion fit is relaxed. Therefore, in this Letter, we do not pay much attention to the detail fitting of the fermion masses and mixings, including neutrino mass scale since the fit is not restricted to the choice of a SO(10) breaking vacuum.

The dimension five operators ($LLLL$ and $RRRR$ operators) induced by colored Higgsino are given as

$$-W_5=\frac{1}{2}C_L^{ijkl}q_kq_lq_i\ell_j+C_R^{ijkl}e_k^cu_l^cu_i^cd_j^c. \qquad (4)$$

In the minimal SU(5) GUT, C_L and C_R can be written by fermion Yukawa couplings, Y_u, Y_d (or Y_e) and there is no freedom to cancel. In a SO(10) model, however, C_L and C_R are written by combinations of h, f, and h' multiplied by colored Higgs mixings. Therefore, there is a freedom to cancel (even in the minimal model) [12]. However, such cancelation is quite unnatural if we make the general fitting of fermion masses and mixings since we have to introduce cancelation for each nucleon decay mode. Actually, the coefficients of f coupling in the C_L and C_R are opposite due to D-parity of the SO(10) symmetry, and it is hard to suppress both C_L and C_R and only small $\tan\beta\sim 2$ remains available for the solutions to satisfy current bounds. We then need to introduce suitable structures for h, f and h' couplings to suppress the operators naturally [4].

As stated, our proposal is to suppress the proton decay by increasing the mass of color-triplet Higgsinos since the dimension five operators are generated by integrating out these particles. We now study the condition under which this is possible consistent with unification.

The colored Higgsino mass is bounded due to the gauge coupling unification conditions. The unification of the three gauge couplings provides two independent relations on the particle mass spectrum below the symmetry breaking scale [13]. The lightest colored Higgsino mass M_{H_C} and X, Y super heavy gauge boson mass M_X are restricted by the following relations at 1-loop level:

$$-2\alpha_3^{-1}(m_Z)+3\alpha_2^{-1}(m_Z)-\alpha_1^{-1}(m_Z) \qquad (5)$$

TABLE I: List of the fields whose N_A and N_B are both positive. The definitions of N_A and N_B are given in the text.

	N_A	N_B	SO(10)	SU(5)
$(\mathbf{8},\mathbf{2},1/2)+c.c.$	$\frac{24}{5}$	24	$\mathbf{126}+\overline{\mathbf{126}}$, $\mathbf{120}$	$\mathbf{45},\mathbf{50}$
$(\mathbf{6},\mathbf{1},1/3)+c.c.$	$\frac{54}{5}$	6	$\mathbf{126}+\overline{\mathbf{126}}$, $\mathbf{120}$	$\mathbf{45}$
$(\mathbf{6},\mathbf{2},-1/6)+c.c.$	$\frac{12}{5}$	36	$\mathbf{210}$	$\mathbf{40}$
$(\mathbf{8},\mathbf{1},0)$	6	6	$\mathbf{210}$, $\mathbf{45}$, $\mathbf{54}$	$\mathbf{24},\mathbf{75}$

$$= \frac{1}{2\pi}\left(\frac{12}{5}\ln\frac{M_{H_C}}{m_Z}+\sum_I N_A^I\ln\frac{M_I}{\Lambda}-2\ln\frac{m_{\rm SUSY}}{m_Z}\right),$$

$$-2\alpha_3^{-1}(m_Z)-3\alpha_2^{-1}(m_Z)+5\alpha_1^{-1}(m_Z) \quad (6)$$

$$= \frac{1}{2\pi}\left(12\ln\frac{M_X^2\Lambda}{m_Z^3}+\sum_I N_B^I\ln\frac{M_I}{\Lambda}+8\ln\frac{m_{\rm SUSY}}{m_Z}\right),$$

where $N_A^I = 2T_3(\phi_I)-3T_2(\phi_I)+T_1(\phi_I)$ and $N_B^I = 2T_3(\phi_I)+3T_2(\phi_I)-5T_1(\phi_I)$ for SM decomposed fields ϕ_I (GUT particles except for the lightest colored Higgs and X, Y gauge bosons) with mass M_I. Λ is the GUT scale. The Dynkin indices $T_{3,2,1}$ are given for $SU(3)_c\times SU(2)_L\times U(1)_Y$ with canonical $U(1)_Y$ normalization by factor 3/5. We assume a single scale threshold $m_{\rm SUSY}$ for SUSY particle, just for simplicity to describe. The GUT scale Λ is cancelled if we take into account the full multiplets in the equations. For instance, in the minimal SU(5) model, additional Higgs fields are $(\mathbf{8},\mathbf{1},0)$ and $(\mathbf{1},\mathbf{3},0)$, and $M_{(\mathbf{8},\mathbf{1},0)} = M_{(\mathbf{1},\mathbf{3},0)}$ is satisfied, and then $\sum_I N_A^I\ln M_I/\Lambda = 0$ and $\sum_I N_B^I\ln M_I/\Lambda = 12\ln M_{(\mathbf{8},\mathbf{1},0)}/\Lambda$. As a result, M_{H_C} and $M_X^2 M_{(\mathbf{8},\mathbf{1},0)}$ are constrained from the measurements of the gauge couplings, and the bound of colored Higgsino mass is $M_{H_C}\le 3.6\times10^{15}$ GeV [1, 13].

The lightest colored Higgs mass scale are always comparable to (or smaller than) the heavy gauge boson masses since it depends on the SO(10) breaking VEVs. Therefore, to increase the M_{H_C} bound, we need negative contribution for both $\sum_I N_A^I\ln M_I/\Lambda$ and $\sum_I N_B^I\ln M_I/\Lambda$. To realize such a situation, we need a light field whose N_A and N_B are both positive. If this field splits from the other SM decomposed fields and become light for a given vacuum, M_{H_C} can be larger. We list such candidate SM decomposed fields in the SO(10) multiplets in TABLE I. For reader's convenience, we also give the SU(5) representation which includes the candidates in terms of SM decomposed fields. All four candidates are also included in $\mathbf{144}+\overline{\mathbf{144}}$. We comment that larger dimensional representations such as three- and four-symmetric tensors also include candidate representations whose N_A and N_B are larger, but there is no motivation to employ such fields.

In the well motivated SO(10) representations, the candidates for gauge unification at a higher scale are only four decomposed fields as shown in the list. Among them, the $SU(3)_c$ adjoint field $(\mathbf{8},\mathbf{1},0)$ are already well known to increase M_{H_C} in the context of SU(5) GUT models [14]. However, the other three candidates have not been discussed in the literature. It is obvious that larger N_B fields can increase SO(10) breaking scale by a smaller hierarchy between the breaking scale and the mass of the fields. Therefore, $(\mathbf{8},\mathbf{2},1/2)$ and $(\mathbf{6},\mathbf{2},-1/6)$ are more suitable candidates compared to others. In fact, if the particle contents are MSSM particles plus $(\mathbf{8},\mathbf{2},1/2)+c.c$, the 1-loop beta coefficients of $SU(3)_c$ and $SU(2)_L$ are same, $b_3=b_2=9$, $b_1=57/5$. If we adopt $(\mathbf{6},\mathbf{2},-1/6)+c.c.$, all three beta coefficients are same, $b_3=b_2=b_1=7$ above the threshold. Therefore, it is possible that the scale where the three gauge couplings meets (approximately) is the mass scale of neither $(\mathbf{8},\mathbf{2},1/2)$ nor $(\mathbf{6},\mathbf{2},-1/6)$, which are lighter compared to the SO(10) breaking scale. Actually, we have checked that there is a SO(10) breaking vacuum where these candidate fields are light and no other decomposed field is light by using a full expression of the Higgs spectrum [7]. We plot the gauge coupling running with $(\mathbf{8},\mathbf{2},1/2)$ threshold in FIG.1. We use 2-loop RGEs for our numerical calculation and find the gauge coupling unification at the string scale. Above the threshold of SO(10) breaking scale (possibly at the Planck scale) the gauge couplings will blow up rapidly due to the large representations such as $\mathbf{126}+\overline{\mathbf{126}}$, however at that point the theory is presumably described by string theory.

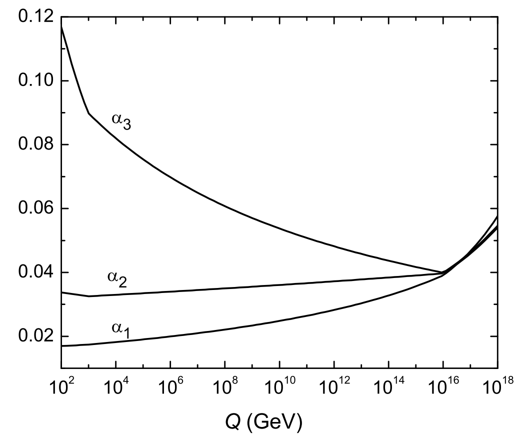

FIG. 1: Gauge coupling evolutions for MSSM with $(\mathbf{8},\mathbf{2},1/2)$ threshold. We choose $M_{(\mathbf{8},\mathbf{2},1/2)} = 10^{16}$ GeV and $m_{\rm SUSY}=1$ TeV. Although the gauge symmetry does not recover at 10^{16} GeV, the gauge couplings run unitedly above the threshold.

It is important that the suppression of the dimension five operator can be easily done by such a simple assumption. This mechanism holds for any GUT model, independent of the detail of the model buildings. For example, in the SU(5) model, the $\mathbf{45}$ representation which is used to fit fermion masses provides one of the candidate fields.

In models of the type discussed here, if the mass scale of the candidate fields are 10^{16} GeV, the colored Higgs fields can be heavier than 10^{17} GeV and the current nucleon decay bounds can be satisfied. If $\tan\beta$ is large enough $\gtrsim 20$, the proton decay via dimension five operator (such as $p \to K\bar{\nu}$) can be observable in the megaton class detector. However, the proton decay via the dimension six operator (such as $p \to \pi e$) may not be observed since it is suppressed by M_X^4, while the dimension five nucleon decay is suppressed by $M_{H_C}^2$.

We comment that in the minimal SU(5) model, a heavy gluino lowers the M_{H_C} and is therefore not admissible, although heavy gluinos (heavier than other gauginos) are generic to minimal supergravity models. However, in this SO(10) model, a heavy gluino is favored if we adopt light $(\mathbf{8},\mathbf{2},1/2)$. Since $b_3 = b_2 < b_1$ (asymptotic non-freeness of U(1) is stronger than the others) is satisfied above the threshold, a heavier gluino threshold at low energy makes $SU(3)_c$ and $SU(2)_L$ gauge couplings meet before $U(1)_Y$ coupling finally meets them at a common unification.

It is important that if $(\mathbf{8},\mathbf{2},1/2)$ and/or $(\mathbf{6},\mathbf{1},1/3)$ is much lighter than the SO(10) breaking scale, a sizable flavor violation can be generated since those fields originate from $\overline{\mathbf{126}}$ or $\mathbf{120}$ which couple to fermions. The couplings can be written as $qu^c\phi_{(\mathbf{8},\mathbf{2},1/2)} + qd^c\phi_{(\mathbf{8},\mathbf{2},-1/2)} + qq\phi_{(\bar{\mathbf{6}},\mathbf{1},-1/3)} + u^cd^c\phi_{(\mathbf{6},\mathbf{1},1/3)}$. Note that the $qq\phi$ coupling matrix is generation antisymmetric and will give rise to specific flavor violation pattern. In general the flavor violating effects will depend on the origin of the light fields. For example, if $(\mathbf{8},\mathbf{2},1/2)$ field is light, it can generate off-diagonal elements for both left- and right-handed squark mass matrices. On the other hand, if the light $(\bar{\mathbf{6}},\mathbf{1},-1/3)$ field comes form $\overline{\mathbf{126}}$, it can generate off-diagonal elements only for right-handed squarks. In both cases we have found SO(10) breaking vacua where only each of these above fields is light.

If both left- and right-handed squark mass matrices have sizable off-diagonal elements, the meson mixing via box diagram is enhanced and thus, it can have impact on the modification of the unitarity triangle, and D-$\bar{D}$ mixing [15]. If the flavor violation is generated from the symmetric couplings f, B_s-$\bar{B}_s$ mixing phase can be enhanced (when we generate large atmospheric neutrino mixing) rather than K-$\bar{K}$ and B_d-$\bar{B}_d$ mixings. If they are generated form the antisymmetric couplings h', modification of B_d-$\bar{B}_d$ phase and D-$\bar{D}$ mixing can be sizable rather than the B_s-$\bar{B}_s$ phase. This is because the induced flavor non-universality is proportional to $h'h'^\dagger$ and

$$h'h'^\dagger = \begin{pmatrix} |b|^2+|c|^2 & -a^*b & -a^*c \\ -ab^* & |c|^2+|a|^2 & -b^*c \\ -ac^* & -bc^* & |a|^2+|b|^2 \end{pmatrix} \qquad (7)$$

where $(h'_{23}, h'_{13}, h'_{12}) = (a, b, c)$. In a natural fit of the fermion mass, we have $a \gg b, c$. As a result, we can distinguish the origins of f and h' in the ongoing experiments. In both cases, the 1-2 elements can be canceled in the Y_d diagonal basis and then K-$\bar{K}$ mixing can be consistent with the SM prediction. However, we cannot cancel both K-$\bar{K}$ and D-$\bar{D}$ mixing amplitudes [15]. Therefore, this flavor violating threshold can affect the recently measured D-$\bar{D}$ mixing.

We emphasize that none of the four candidates in the TABLE I couples to leptons directly, thus the enhancement of lepton flavor violations (LFV) such as $\tau \to \mu\gamma$ is not favored in the context of the proton decay suppression. Actually, if a decomposed field from the heavy Higgs fields which couples to leptons is light, it decreases either M_{H_C} or M_X. We comment that in the SO(10) models with simple fermion mass and mixing fitting, not much LFV is generated from right-handed neutrino loops, since the Dirac neutrino Yukawa coupling does not have large mixings.

It is also worthwhile to emphasize that the LFV depends upon which particles are light, though it is not favored to increase M_{H_C} For example, to realize type II seesaw, $SU(2)_L$ triplet $(\mathbf{1},\mathbf{3},1)$ needs to light at the scale of intermediate scale $\lesssim 10^{14}$ GeV. Then, the f coupling can generate sizable effects to the LFV. Also, if the $SU(2)_R$ breaking scale is smaller than the SO(10) breaking scale, the $SU(2)_R$ would-be-Goldstone Higgsino $(\mathbf{1},\mathbf{1},-1)$ is light, and then off-diagonal elements of the right-handed slepton can be generated. If the contribution arising from the $\mathbf{120}$ coupling dominates, the Br$[\tau \to e\gamma]$ can be more enhanced compared to Br$[\tau \to \mu\gamma]$, which is an important prediction of this scenario. Further analysis of quark and lepton flavor violations depending on the SO(10) breaking vacua can be found elsewhere.

Finally, we comment on the b-τ unification which is one of the important predictions of GUT models. In a natural fit, $\bar{f}_{33}$ is small and thus b-τ unification is realized at the SO(10) breaking scale approximately (up to 5% $\sim m_\mu/m_\tau$). From the numerical study of RGE evolution of Yukawa couplings, one finds that the b-τ unification is satisfied only when $\tan\beta \sim 2$ or ~ 50. Now, $\bar{f}$ and $\bar{h}'$ are small but since these are multiplied by Higgs doublet mixings, the original couplings f_{33} and h'_{23} can be order 1 and, surely, the coupling to the $(\mathbf{8},\mathbf{2},1/2)$ is not multiplied by the Higgs doublet mixings. Then, due to the $(\mathbf{8},\mathbf{2},1/2)$ threshold, the RGE evolution of b-τ Yukawa coupling can be modified, and thus it is possible to realize the b-τ unification even for a moderate $\tan\beta$. In this situation, due to the large f and h couplings, the induced FCNCs can be also sizable.

In conclusion, we point out that the lightest colored Higgsino mass can be made heavy when the fields listed in the TABLE I are light compared to the SO(10) breaking scale and this pectrum is realizable in a particular SO(10) breaking vacuum. This will suppress the dimension five proton decay which however still may be observable in the next generation of detector. We point out that the constraints from nucleon decays may imply sizable quark FCNCs rather than leptonic FCNCs, which can be tested in future. This can probe the detailed nature of GUT theories.

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