Improving the Efficiency of an Ideal Heat Engine:
The Quantum Afterburner

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By using a laser and maser in tandem, it is possible to obtain laser action in the hot exhaust gases involved in heat engine operation. Such a "quantum afterburner" involves the internal quantum states of working gas atoms or molecules as well as the techniques of cavity quantum electrodynamics and is therefore in the domain of quantum thermodynamics. As an example, it is shown that Otto cycle engine performance can be improved beyond that of the "ideal" Otto heat engine.

The laws of thermodynamics [1] are very useful in telling us how things work and what things will never work. For example, The ideal heat engine is a paradigm of modern science and technology. We read in the textbooks that:

"it might be supposed that the ideal cycle analysis is too unrealistic to be useful. In fact, this is not so. Real gas cycles are reasonably close to, although always less efficient than, the ideal cycles."

But as technology develops, it behooves us to reexamine thermodynamic dogma. The purpose of the present paper is to reconsider the operating limits of ideal heat engines in light of recent developments in quantum optics such as cavity QED [2], the micromaser [3], and quantum coherence effects such as lasing without inversion (LWI) [4] and cooling via coherent control [5]. In particular, we shall show that by extracting coherent laser radiation from the "exhaust" gas of a heat engine [6], e.g., the Otto cycle idealization of the automobile engine. We here show that it is indeed possible to improve on the efficiency of an ideal Otto cycle engine, operating between two fixed temperature reservoirs, by adding a quantum afterburner which extracts coherent energy from the hot exhaust gases of the heat engine.

In such a quantum Otto engine (QOE) the laser energy is supplied by a thermal reservoir in accord with the first law of thermodynamics, and entropy balance is maintained as required by the second law [1].

In what follows we present a physical picture for and thermodynamic analysis of the QOE. In the conclusion we make contact with related previous work. In the next section we present the quantum engine concept physically as an extension of the conventional Otto cycle engine as in Fig.(1). Then we analyze the QOE, calculating the efficiency and entropy flow, etc. The proposed scheme is simple enough to permit reasonably complete analysis; but, hopefully, realistic enough to be convincing. In the conclusion, we examine our results in the context of previous research on the subject.

In order to present the physics behind the QOE, consider Fig.(1) in which the working fluid passes through the cycle 1234561. As mentioned earlier, we extend the classical Otto engine to include a laser arrangement which can extract coherent laser energy from the internal atomic degrees of freedom. As depicted in Figs.(1a-e), the QOE operates in a closed cycle in the following steps:

a.$(1 \rightarrow 2)$ The hot gas expands isentropically doing useful ("good") work $W_g = C_v(T_1 - T_2)$ where $T_2 = T_1 \mathcal{R}^{-1}$, C_v is the heat capacity, $\mathcal{R} = (V_1/V_2)^{(\gamma-1)}$, and γ is the ratio of heat capacities at constant pressure to constant volume.

b.$(2 \rightarrow 3)$ Heat $Q_{out} = C_v(T_2 - T_3)$ is extracted at constant volume by a heat exchanger at temperature T_3.

c.$(3 \rightarrow 4)$ Maser-laser cavities are added and energy is extracted from the hot internal atomic degrees of freedom by cycling the gas from left to right to left through the laser-maser system held at temperature T_3, with an entropy decrease $\Delta S_{int} \cong Nk \ln 3$, as discussed later.

d.$(4 \rightarrow 5)$ The gas is then compressed isentropically to volume $V_4 = V_1$, requiring waste work $W_w = C_v(T_4 - T_3) = (\mathcal{R} - 1)T_3$.

e.$(5 \rightarrow 6)$ The gas is again put in contact with the heat exchanger (at temperature T_1) and the external-transnational degrees of freedom are heated isochorically to T_1 by heat energy Q_{ext}.

f. $(6 \rightarrow 1)$ Maser-laser cavities are again added and internal states are heated by an amount q_{in} extracted from the hot cavities at temperature T_1, completing the cycle.

As a useful simplifying assumption, we consider the external and internal degrees of freedom to be decoupled. Only when the atoms are passing through the maser-laser system do they change their internal state. That is, the atomic states are chosen to be very long lived when not in the cavities. But they are strongly coupled to the radiation field in the maser and laser cavities due to the increased density of states of the radiation inside the cavity. Thus, when an atom in the $|b\rangle$ state is passed into the maser cavity it quickly comes into equilibrium with the thermal radiation in the cavity. For example in step $3 \rightarrow 4$, after passing through the cold maser, $|b\rangle$ state population is determined by the Boltzmann factor governed by temperature T_3. And for small enough T_3 the $|b\rangle$ state is effectively depopulated thus providing a population inversion between states $|a\rangle$ and $|b\rangle$ since the

population in state $|a\rangle$ is still determined by T_1. This is the basis for lasing off the thermal energy of the exhaust gases.

Thus the maser serves as the incoherent ("heat") energy removal mechanism, q_{in}, which enables the coherent (useful) energy, w_l, to be emitted by the laser. To understand the work w_l vs heat q_m aspect of the problem we need only compare the photon statistics for the incoherent thermal field in the maser cavity with the coherent laser field. The maser field density matrix is given by [7]:

$$r_{nn}^m = \bar{n}_m^n/(\bar{n}_m + 1)^{n+1} \qquad (1a)$$

where $\bar{n}_m = 1/[\exp(\hbar\nu_m/kT_2)-1]$, $\hbar\nu_m$ is the energy per quantum of the maser field.

The density matrix describing the laser field proceeds from an initial thermal state which is largest for small $\bar{n}_l$, to the sharply peaked coherent distribution given by

$$r_{n,n}^{(l)} = r_{0,0}\prod_{l=1}^{n}\left[\frac{A(l+1)}{1+(A/B)(l+1)} - C(l+1) + \frac{\bar{n}_l}{\bar{n}_l+1}\right] \quad (1b)$$

where A(C) is the linear hain (loss) and B is the nonlinear saturation parameter, $\bar{n}_l$ is the average number of thermal photons in the laser cavity at temperature T_2 with no atoms present.

Having established the fact that only the laser radiation contributes useful work, we write the efficiency of the QOE as:

$$\eta_{qo} = \frac{W_g - W_w + w_l}{Q_{in} + q_{in}}$$

and since $q_{in} = w_l + q_m$ we find:

$$\eta_{qo} = \eta_0 + \frac{w_l(1 - \eta_0) - \eta_o q_m}{Q_{in} + w_l + q_m}. \qquad (2)$$

where the ideal classical Otto engine efficiency is defined as $\eta_0 = (W_g - W_w)/Q_{in}$. Taking W_g, W_w, and Q_{in} as given in the discussion of QOE operation we have the alternative expressions $\eta_0 = 1 - 1/\mathcal{R} = 1 - T_2/T_1$.

In order to determine whether η_{q0}, Eq.(2), is an improvement over η_0 we now turn to the calculation of w_l and q_m. A rigorous calculation requires a quantum theory of the laser/maser type analysis and this will be given elsewhere. However it is sufficient for the present purposes to apply microscopic energy balance calculations to obtain good expressions for the important quantities.

After the atom makes one pass through the maser-laser system, the internal density matrix is given by:

$$\rho_{one}(3) = \frac{1}{2}(p_a^1 + p_b^3)(\Lambda_a + \Lambda_b) + (p_c^1 + p_b^1 - p_b^3)\Lambda_c \quad (3)$$

where $\Lambda_a = |\alpha\rangle\langle\alpha|, \alpha = a, b,$ and c, and in the notation of Fig.(1), p_α^i is the Boltzmann factor given by $p_\alpha^i = Z_i^{-1}\exp(-\beta_i\epsilon_\alpha)$ where $\beta_i = 1/kT_i$; T_i is the reservoir temperature T_1 or T_2 and $Z_i = \sum\exp-(\beta_i\epsilon_\alpha)$. But

an atom will bounce many times back and forth through the maser/laser cavities in moving the gas adiabatically from right to left. After many bounces the atom settles down into the mixed state:

$$\rho_{many}(3) = p_b^3\Lambda_a + p_b^3\Lambda_b + (1 - 2p_b^3)\Lambda_c \qquad (4)$$

We calculate q_m by noting that $(p_a^1 - p_b^3)N$ atoms go from $a \to b \to c$ and $(p_b^1 - p_b^3)N$ atoms go from $b \to c$, and in both cases add energy $\epsilon_b - \epsilon_c$ to the maser field. Thus on making the $b \to c$ transition, the total incoherent energy added to the maser field by all N atoms is:

$$q_m = (\epsilon_b - \epsilon_c)N[p_a^1 - p_b^3 + p_b^1 - p_b^3] \qquad (5)$$

Likewise w_l is obtained by noting that the number of atoms going from a to b with the coherent emission of laser radiation is $N(p_a^1 - p_b^3)$. Energy $\epsilon_a - \epsilon_b$ is given up by each atom, and the total coherent energy (i.e. useful work) given to the laser field is:

$$W_l = (\epsilon_a - \epsilon_b)N(p_a^1 - p_b^3) \qquad (6)$$

We now use Eqs.(5,6) for q_m and w_l in a form which allows us to determine the sign of the efficiency enhancement factor in Eq.(2). That is, we wish to establish the conditions for which:

$$(1 - \eta_0)w_l > \eta_0 q_m \qquad (7a)$$

We use Eqs.(5,6) and introduce the notation $\epsilon_{\alpha\beta} = \epsilon_\alpha - \epsilon_\beta$ to write Eq.(7a) as:

$$\left(\frac{1}{\eta_0} - 1\right)\left(\frac{\epsilon_{ac}}{\epsilon_{bc}} - 1\right) > 1 + \frac{p_b^1 - p_b^3}{p_a^1 - p_b^3} \qquad (7b)$$

As an example, we may take $\eta_0 = 1/4$, $\epsilon_{ac}/\epsilon_{bc} = 11$ so that the LHS of Eq.(7b) is 30; furthermore noting that for high enough T_1 that $p_\alpha^1 \simeq 1/3$ and so long as $p_b^3 < 1/3$ the RHS of Eq.(7b) equals 2. Hence the RHS (maser) factor is an order of magnitude less than the LHS (laser) factor in (7a,b), which indeed shows that $\eta_{q0} > \eta_0$ as desired. Finally we note that the von Neumann entropy, $S = -kNTr\rho\ln\rho$, added in the heating of the internal states to temperature T_1:

$$S_{int}(6 \to 1) = -kN\sum\left[p_\alpha^1\ln p_\alpha^1 - 2p_b^3\ln p_b^3 - p_c\ln p_c\right] \qquad (8)$$

where $p_c = 1 - 2p_a^3$, is equal and opposite to that removed in the $4 \to 4$ maser-laser energy-entropy extraction process. Hence when T_1 is high enough and T_3 is low enough that $p_c \cong 1$, $p_a = p_b = 0$ then (8) takes the simple form $S(6 \to 1) \cong kN\ln 3$ as noted earlier. We now turn to the relation of the present results to that of previous work.

The landmark paper by Ramsey [9] on negative temperatures in thermodynamics and statistical mechanics is

a pillar of quantum thermodynamics and is directly relevant to the present work. In his paper he shows that his work is contrary to the Kelvin-Planck [10] statement of the second law, which has now been revised to the Kelvin-Planck- Ramsey statement. Furthermore in ref [9] it is noted that: systems at negative temperatures have various novel properties of which one of the most intriguing is that a heat engine operating in a closed cycle can be constructed that will produce no other effect than the extraction of heat from a negative temperature reservoir with the performance of the equivalent amount of work. But it is also noted that at both positive and negative temperatures, cyclic heat engines which produce work have efficiencies less than unity, i.e. they absorb more heat than they produce work.

The present study, on the other hand, does not involve negative temperature reservoirs. But it is possible to envision a negative temperature as being associated with the $a \rightarrow b$ transition once inversion is produced by the maser interaction, and the present work has much in common with that of Ramsey.

The work of Ramsey led to the introduction of the quantum heat engine concept by Scovil and Schultz-Dubois. In their paper [11] entitled, *Three level masers as heat engines* they conclude that the limiting efficiency of their three level maser engine model is that of the Carnot cycle. And that their work may be regarded as another formulation of the second law of thermodynamics. In the present paper the atomic states are not to be viewed as the engine. We focus on the different problem of improving the efficiency of an ideal heat engine which has a laser-maser system integrated into an Otto cycle engine as in Fig. (1).

The present results are an extension of the work by the author and colleagues listed in ref [6] . In particular, the paper presented at the Dec. 1999 Japanese-American Conference on Coherent Control entitled *Using External Coherent Control Fields to Produce Laser Cooling Without Spontaneous Emission and Sharpen Thermodynamic Dogma* [12] , gave specific examples and direct calculation, based primarily on breaking emission symmetry as in lasing without inversion. Thus demonstrating that cooling of internal states by external coherent control fields is possible. There we also showed that such coherent schemes allow us to reach absolute zero in a finite number of steps, in contrast to usual third law of thermodynamics dogma.

It is interesting to compare this work with the paper [13] of Kosloff, Geva, and Gordon entitled *Quantum Refrigerators in the Quest of Absolute Zero* in which they have independently arrived at similar conclusions using a similar model. They have established a bound for the maximum cooling rate in the low temperature limit where quantum behavior dominates.

In conclusion: we have shown that it is possible, in principal, to improve on the efficiency of an ideal Otto cycle engine by extracting laser energy from exhaust gases; this is summarized in Fig.(2). However, as will be presented elsewhere, when a similar lasing-off-exhaust-atoms scheme is analyzed for a Carnot cycle engine, efficiency is not improved. The present results are in complete accord with the second law.

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[12] Workshop on Coherent Control, R. Gordon Ed., World Scientific, (2001); See also ref. [5].

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Figure Captions

Fig. (1) illustrates the steps, 1234561, in cyclic operation of the quantum Otto engine. The atomic internal populations are depicted for the three level atom (levels a, b, c) at each stage of operation. A detailed description of operating steps $a \to f$ is given in the text.

Fig. (2) is a temperature (T) entropy (S) plot for the quantum Otto cycle engine. Note that the entropy is the sum of entropy for the external (kinetic) and internal (quantum) degrees of freedom.

Fig. (3) depicts the evolution of internal atomic populations for the case in which the atom first passes through the maser-laser system; and then (because everything is adiabatic and involves long times and many bounces) bounces back and forth through the cavities many times. As discussed in the text, after, a large number of bounces the atom settles down into a configuration wherein most of the population is in state c.
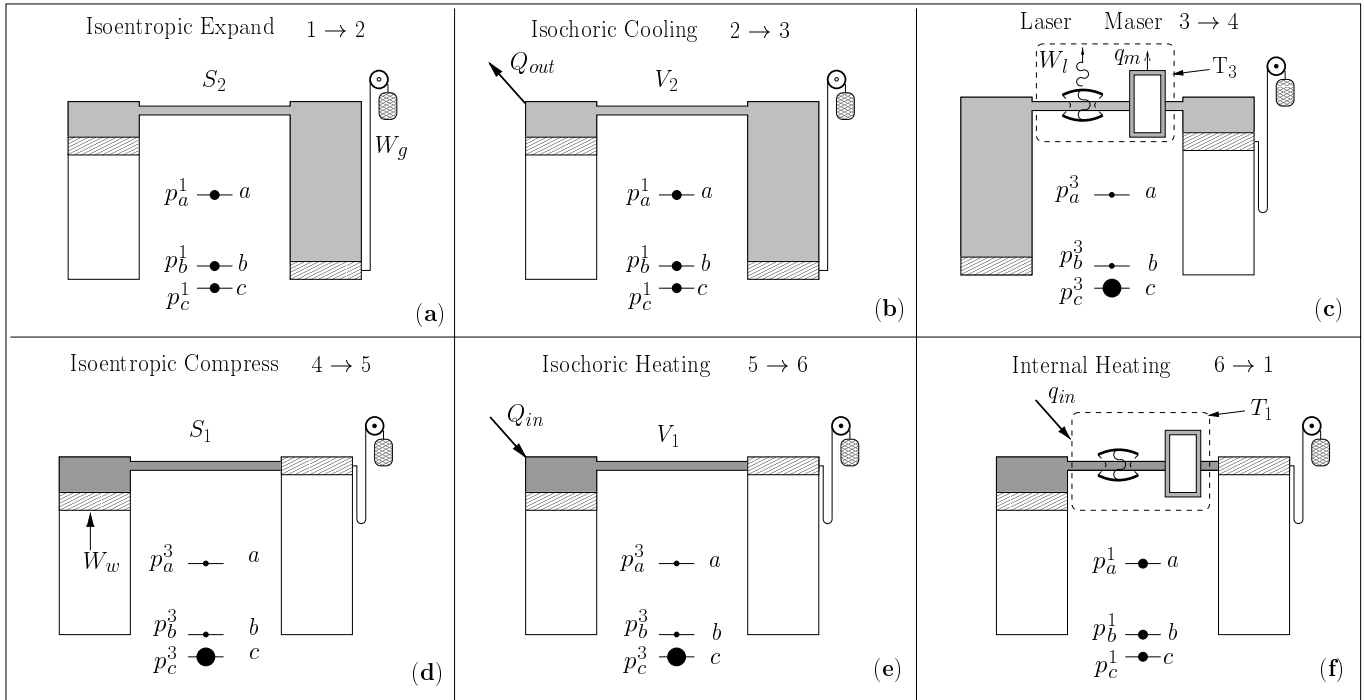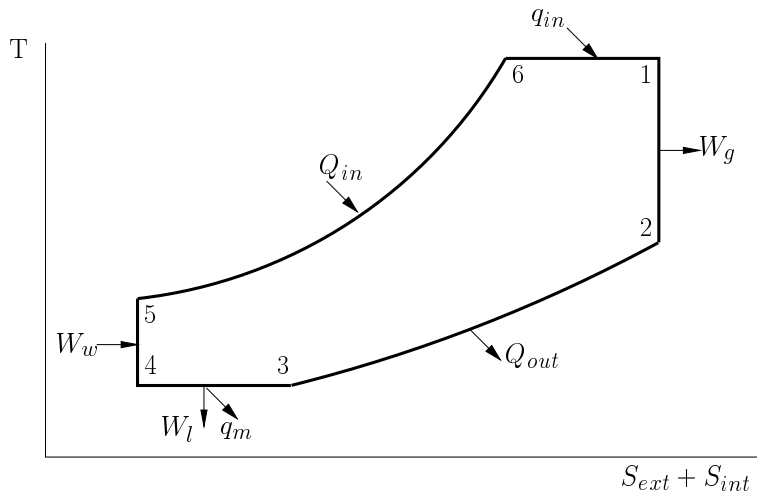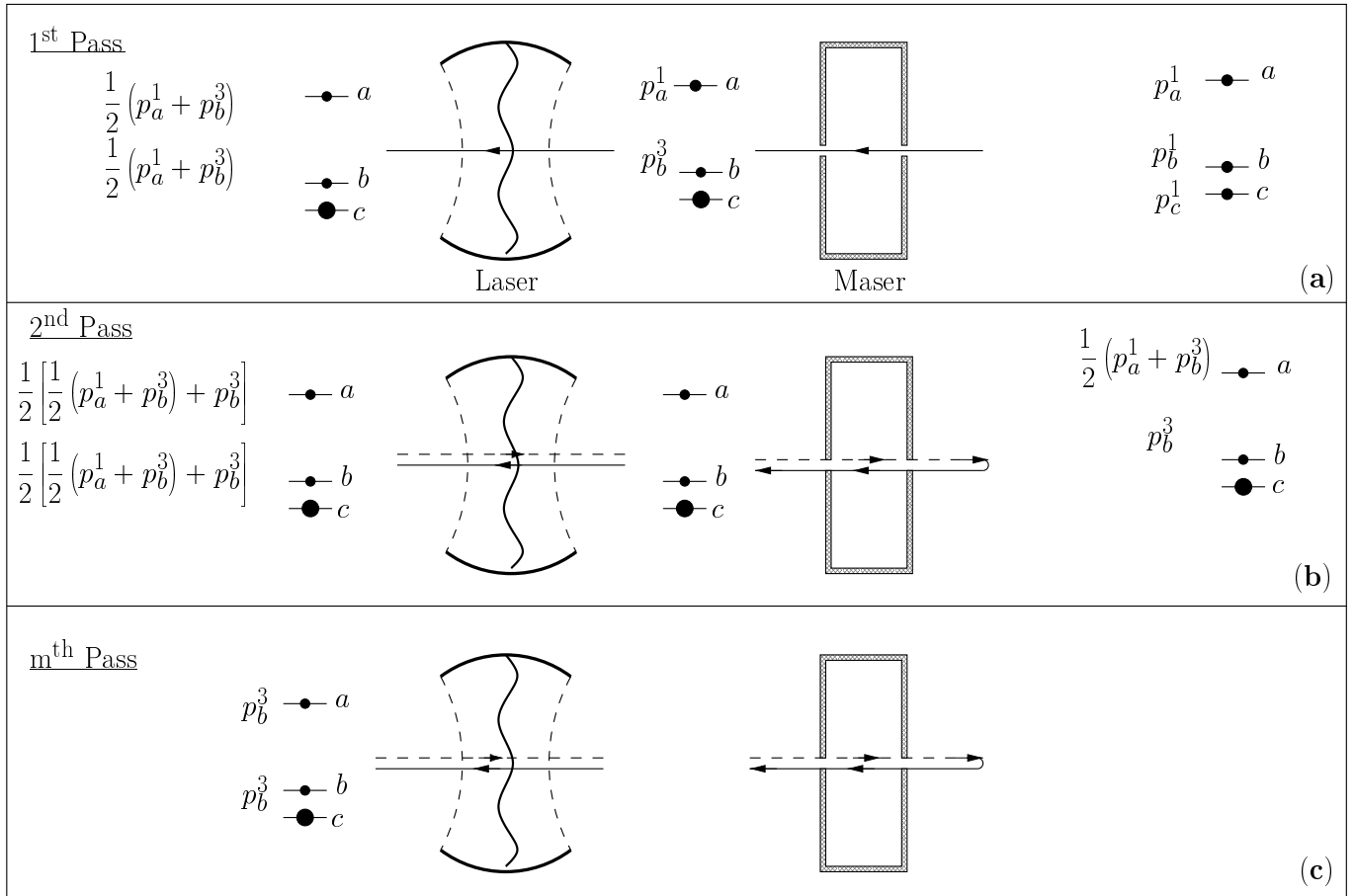

FIG. 1.

FIG. 2.

FIG. 3.