

CTP-TAMU-01/95

hep-ph/9501247

January 1995

Planck Scale Physics and the Testability of SU(5) Supergravity GUT

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Abstract

GUT scale threshold corrections in minimal SU(5) supergravity grand unification are discussed. It is shown that predictions may be made despite uncertainties associated with the high energy scale. A bound relating the strong coupling constant to the mass scales associated with proton decay and supersymmetry is derived, and a sensitive probe of the underlying theory is outlined. In particular, low energy measurements can in principle determine the presence of Planck scale ($1/M_{\text{Pl}}$) terms.

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Over the past two decades, much attention has been given to the possibility of unifying the three gauge groups of the Standard Model into one group. These GUT models are theoretically more appealing than the Standard Model for various reasons [1]. The 1990 precision LEP data strongly indicated that supersymmetry is needed to achieve grand unification [2], and spurred on many new analyses of different aspects of SUSY GUTs.

With the increasing precision of data, further predictions will be affected by supersymmetry threshold effects and the details of the model at high energy. In this note we examine the effects of the non-degeneracy of the super-heavy GUT spectrum, and the possible existence of non-renormalizable operators from Planck scale physics. A number of treatments of these issues have been given recently for the SU(5) model [3–6]. It was found that many of the predictions for low energy observables were blurred by the high scale effects. For example, it was argued that the SUSY scale cannot be determined by a more accurate measurement of α_3 [3], and the rate of proton decay cannot be predicted from low energy data if additional Planck scale terms are present [5].

Nevertheless, we will show here that there are still predictions to be made in this model:

a. The effect of Planck scale non-renormalizable terms becomes smaller as the value of α_3 is varied to lower values, so the lower limit on α_3 is not lost. This is especially interesting since there is currently a disparity in the values of α_3 between the measurements made at weak scale energies, and those made at lower energies [7]. Resolving this disparity and refining the measured value of α_3 will provide an important test of the model.

b. Since Planck scale physics smears the correlation between α_3 and the mass scale which governs proton decay, proton decay will be a sensitive probe of Planck scale physics. In particular it will be seen that by purely low-energy measurements one can determine experimentally the degree to which the dominant Planck scale term is present. Thus models of this type allow for the first time a test for the existence of Planck scale physics and whether Planck scale physics impinges on low-energy (electroweak scale) physics.

With respect to the second item, progress has been made recently in deriving models similar to the ones considered here from string theory [8,9], so that the gravitational smearing

may be calculable in principle. Proton decay would then be a sensitive test of string theory.

Our model of GUT physics is defined by superpotential [10,11]

$$\begin{aligned}
W = & \lambda_1[\frac{1}{3} \text{tr}(\Sigma^3) + \frac{1}{2}M \text{tr}(\Sigma^2)] \\
& + \lambda_2 \bar{H}_X (\Sigma_Y^X + 3M' \delta_Y^X) H^Y \\
& + \varepsilon_{UVWXY} H^U M^{VW} f_1 M^{XY} + \bar{H}_X M^{XY} f_2 \bar{M}_Y,
\end{aligned} \tag{1}$$

where Σ , H , and \bar{H} form a **24**, **5**, and $\bar{\mathbf{5}}$ of $SU(5)$ respectively, \bar{M} and M are $\bar{\mathbf{5}}$ and **10** matter superfields, f_1 and f_2 are Yukawa coupling constant matrices in the generation space, and the mass parameters M and M' are set equal to account for a light Higgs doublet. (This is a well known fine tuning problem with this model. We consider elsewhere alternate models such as those in Ref. [12] which avoid this fine tuning.) The gauge group is broken down to $SU(3) \times SU(2) \times U(1)$ when Σ grows a VEV: $\langle \Sigma \rangle = M \text{diag}(2, 2, 2, -3, -3)$. The resulting super-heavy spectrum includes a heavy color Higgs chiral multiplet $(\mathbf{3}, \mathbf{1}, \frac{2}{3})$ of mass $M_H = 5\lambda_2 M$, a vector multiplet $(\mathbf{3}, \mathbf{2}, \frac{5}{3})$ of mass $M_V = 5\sqrt{2}gM$, chiral multiplets $(\mathbf{8}, \mathbf{1}, 0)$ and $(\mathbf{1}, \mathbf{3}, 0)$ of mass $M_\Sigma = \frac{5}{2}\lambda_1 M$, and a Standard Model gauge singlet chiral multiplet $(\mathbf{1}, \mathbf{1}, 0)$ of mass $\frac{1}{2}\lambda_1 M$, where the numbers in parentheses are the $SU(3)$ and $SU(2)$ representations and hypercharge quantum numbers.

In the following we assume $0.1 \leq \lambda_{1,2} \leq 2.0$, i.e. $10^{-3} \lesssim \alpha_{\lambda_{1,2}} \lesssim 1/3$. The upper bound is imposed so that the model stays within the perturbative domain, while the lower bound excludes any anomalously small couplings.

In addition to the renormalizable interactions, one may add the dominant non-renormalizable operator from Planck scale physics [13],

$$\mathcal{L}_0 = \frac{c}{2M_{\text{Pl}}} \text{tr}(\text{FF}\Sigma), \tag{2}$$

where $M_{\text{Pl}} = 1/\sqrt{\kappa} = 1/\sqrt{8\pi G}$. In the first part of this paper we impose, for naturalness, $|c| < 1$. The main effect of this term is to modify the unification condition when Σ grows a VEV. Note for now that $\langle \Sigma \rangle$ will be $O(M_{\text{GUT}})$ so this term enters with a coefficient $M_{\text{GUT}}/M_{\text{Pl}} \approx (1/10 - 1/100)$ and we would naively expect its effects to be small [14].

We will concentrate mainly on gauge coupling unification. The running of the gauge couplings with respect to the energy scale, μ , is governed by the two-loop renormalization group equations [15],

$$\frac{d}{dt} \frac{1}{\alpha_i(t)} = -b_i - \frac{1}{4\pi} \sum_j b_{ij} \alpha_j(t) + \frac{1}{16\pi^2} \sum_f b_{if} h_f^2(t) \quad (3)$$

where $\alpha_i \equiv g_i^2/4\pi$, $t \equiv (\ln \hat{\mu})/2\pi$, $\hat{\mu} = \mu/(\text{arbitrary mass parameter})$, and h_f are the Yukawa couplings. In MSSM, one-loop coefficients are $b_i = (33/5, 1, -3)_i$. The two-loop coefficients are also well known and can be found elsewhere, e.g. in [16].

We first discuss the effect of GUT scale thresholds without the Planck term. The GUT degrees of freedom are included in the running at their respective thresholds by

$$\alpha_i^{-1}(\mu) = \alpha_{i0}^{-1}(\mu) - \sum_a \Delta b_{ia}^{th} \frac{1}{2\pi} \ln\left(\frac{\mu}{M_a}\right), \quad (4)$$

where $\alpha_{i0}(\mu)$'s are calculated numerically to two-loop accuracy from their low energy values via the RGE's using the MSSM beta functions. The low energy values of $\alpha_{i0}(\mu)$'s include SUSY threshold which will be discussed later. In Eq. (4), the index a sums over the GUT degrees of freedom with masses less than μ , $\Delta b_{i\Sigma}^{th} = (0, 2, 3)_i$, $\Delta b_{iH}^{th} = (2/5, 0, 1)_i$, and $\Delta b_{iV}^{th} = (-10, -6, -4)_i$. The largest of the M_a is called M_U , as this is where the coupling constants actually meet. Thus the unification condition is $\alpha_i(M_U) = \alpha_5(M_U)$.

On the other hand, if the Planck term is included, then when the VEV of Σ is inserted into the dominant Planck scale operator of Eq. (2), the kinetic terms for the gauge bosons will receive a contribution. Thus the unification condition will be modified by replacing $\alpha_5(M_U)^{-1}$ by

$$\alpha_5^{-1}(M_U) \left(1 - c \frac{M}{M_{\text{Pl}}}, 1 - 3c \frac{M}{M_{\text{Pl}}}, 1 + 2c \frac{M}{M_{\text{Pl}}}\right), \quad (5)$$

where M is the mass parameter entering in $\langle \Sigma \rangle$.

At low energies we must consider the decoupling of supersymmetric degrees of freedom. This can be described at the one-loop level by three SUSY threshold parameters M_i , one for each coupling constant [6]. The meaning of these parameters is as follows: if above

M_i we assume the threshold particles to be massless, and below M_i we assume them to be completely integrated out, and we assume the couplings meet smoothly at M_i , then at scales far from M_i our running coupling constants will match the exact ones [17]. Such an M_i can always be found for each i so long as the one-loop beta function above the threshold is different from that below [18]. Thus the effect of SUSY thresholds is given by [19]

$$\alpha_i^{-1}(M_Z) = \alpha_{i0}^{-1}(M_Z) + \Delta b_i^{th} \frac{1}{2\pi} \ln\left(\frac{M_Z}{M_i}\right). \quad (6)$$

Here, $\alpha_i(M_Z)$ are the couplings at M_Z , while $\alpha_{i0}(M_Z)$ are the couplings one would obtain at M_Z if one ran with the full SUSY beta function down to M_Z . $\Delta b_i^{th} = (5/2, 25/6, 4)_i$ gives the contribution to the β function from the additional SUSY degrees of freedom. This is sufficient for a two-loop analysis as well, so long as the SUSY thresholds are not too far from M_Z . We treat all Standard Model degrees of freedom except the top as degenerate with or lighter than M_Z . We take the top mass to be 174 GeV in accordance to the latest experimental indication [20]. Uncertainties in its value do not affect our results significantly.

There are two subtleties involved in relating the M_i 's to the sparticle spectrum. First, the coupling constants do not actually jump in slope when the scale reaches the mass of a particle, rather they change gradually. If our measurements are performed in the region of changing slope, “match and run” will be inaccurate. Second, if the threshold region is close to the electroweak symmetry breaking scale, there are always mass splittings among the particles in the gauge multiplets. This potentially large effect has never been fully treated, and is essential for incorporating detailed SUSY spectra in unification analyses. Regardless of these subtleties, any supersymmetric spectrum can be accommodated by Eq. (6) if the M_i 's are allowed to vary below M_Z as well as above [6].

Combining Eqs. (3), (4), (6), and the unification condition as modified by (5), we arrive at the equation which we use for our calculations:

$$\begin{aligned} & \alpha_5^{-1}(M_U) \left(1 - c \frac{M}{M_{\text{Pl}}}, 1 - 3c \frac{M}{M_{\text{Pl}}}, 1 + 2c \frac{M}{M_{\text{Pl}}} \right)_i \\ & = \alpha_i^{-1}(M_Z) - b_i \frac{1}{2\pi} \ln\left(\frac{M_U}{M_Z}\right) \end{aligned}$$

$$\begin{aligned}
& - \sum_j b_{ij} \frac{1}{4\pi} \int_{t_Z}^{t_U} \alpha_{j0}(t) dt + \sum_f b_{if} \frac{1}{16\pi^2} \int_{t_Z}^{t_U} h_f^2(t) dt \\
& - \Delta b_i^{th} \frac{1}{2\pi} \ln\left(\frac{M_Z}{M_i}\right) - \sum_a \Delta b_{ia}^{th} \frac{1}{2\pi} \ln\left(\frac{M_U}{M_a}\right).
\end{aligned} \tag{7}$$

In order to provide a bound relating α_3 to M_H and M_i we fix c and $\sin^2(\theta_W)$ to take their maximum values [21] and λ_1 to take its minimum value, while λ_2 varies. At each point we iterate numerically in order to find the solution of Eq. (7) and thus α_3 and M_H . Some of the results are displayed in Fig. 1, where we have chosen the degenerate case of $M_1 = M_2 = M_3 = M_{\text{SUSY}}$, and we have plotted a bound each for $M_{\text{SUSY}} = 10$ GeV, 100 GeV, and 1000 GeV.

The relevant bound on α_3 , the lower horizontal line in Fig. 1, can be parametrized by [22]

$$\begin{aligned}
\alpha_{3,min} &= 0.040 + 0.0139 t_H \\
&\quad - 0.00579 t_{\text{SUSY}} - 0.00454 t_{\text{SUSY}}^2,
\end{aligned} \tag{8}$$

where $t_\alpha \equiv (1/2\pi) \ln(M_\alpha/M_Z)$ for mass M_α . For the case that the M_i 's are non-degenerate, we may still use Eq. (8) to a close approximation, when we replace t_{SUSY} using the following formula:

$$\begin{aligned}
t_{\text{SUSY}} &= 0.305 t_1 + 7.738 t_2 - 7.043 t_3 \\
&\quad + 2.38 t_1^2 + 20.91 t_2^2 + 6.90 t_3^2 \\
&\quad - 20.21 t_1 t_2 + 12.54 t_1 t_3 - 22.53 t_2 t_3.
\end{aligned} \tag{9}$$

We find it useful to think of M_{SUSY} as defined in this formula as an effective mass scale to account for the SUSY thresholds in Eq. (8). Thus, in Fig. 1, although we have only plotted the results for the degenerate SUSY thresholds case, we may think of the bottom curves (which give the α_3 bound) as valid for all M_i with corresponding M_{SUSY} as given by Eq. (9) [23].

Current bounds on the $p \rightarrow \bar{\nu} + K^+$ decay mode [24] imply $M_H > 1.2 \times 10^{16}$ GeV [25]. Using $M_{\text{SUSY}} = 100$ GeV, (a characteristic value consistent with proton decay data), one

finds $\alpha_3 > 0.112$. Thus resolving the α_3 measurements is a crucial test of the model.

We also plot in Fig. 1 the allowed region (shaded) where $c = 0$, and all GUT thresholds are taken to be degenerate (i.e. $M_H = M_V = M_\Sigma = M_U$), for the case of degenerate SUSY thresholds, and M_i between 10 GeV to 1000 GeV. We see that the lower bound on M_H implies now $\alpha_3 > 0.1145$, and there is a strong correlation between $\alpha_3(M_Z)$ and M_H .

We turn now to the Planck scale parameter c . Due to a cancellation, fixing $\sin^2(\theta_W)$, $\alpha_3(M_Z)$, and the M_i 's uniquely determines M_H when $c = 0$ [26]. Assuming in the future we have a complete picture of the low energy physics, including the SUSY spectrum and the proton decay rate in both major modes, we can compare this value for $c = 0$ ($M_H(0)$) with the correct M_H as determined from proton decay, and therefore determine c . The result can be summarized in the formula

$$c = \frac{\alpha_5 M_{\text{Pl}}}{10\pi M} \ln \frac{M_H}{M_H(0)}. \quad (10)$$

M could be determined by the $p \rightarrow e^+ + \pi^0$ decay mode, since this decay is governed by $M_V = 5\sqrt{8\pi\alpha_5} M$. One may obtain α_5 from Eq. (7). An approximate value is $\alpha_5 = 1/23$. Note that a rough determination of c is possible even without the pion decay mode, e.g. by taking $\lambda_2 = 1$ so that $M = M_H/5$. In this approximation, the front factor in Eq. (10) is ≈ 0.3 , and while the ratio $M_H/M_H(0)$ can vary substantially for reasonable range of low energy physics data, owing to the logarithmic dependence, c determined this way is in fact $O(1)$ and not expected to be anomalously large.

Although we have not yet observed proton decay, nor been able to determine the SUSY spectrum, using our naturalness conditions that $0.1 \leq \lambda_{1,2} \leq 2.0$ and $10 \text{ GeV} \leq M_{1,2,3} \leq 1000 \text{ GeV}$, and requiring $M_H \geq 1.2 \times 10^{16} \text{ GeV}$ [25], we can already place bounds on c for different values of $\alpha_3(M_Z)$ and $\sin^2(\theta_W)$. We plot in Fig. 2 the allowed region for $\sin^2(\theta_W) = 0.2294$ and for $\sin^2(\theta_W) = 0.2327$ in the c - $\alpha_3(M_Z)$ plane for $M_1 = M_2 = M_3$. We find that in this analysis where M_i 's are taken to be degenerate, the Planck scale term must exist for some values of $\sin^2(\theta_W)$ and $\alpha_3(M_Z)$. We further note that ranging over the values of $\sin^2(\theta_W)$ between 0.2294 and 0.2327, the LEP range for $\alpha_3(M_Z)$, i.e. $0.117 \leq \alpha_3 \leq 0.129$

corresponds to $-6.0 \leq c \leq 3.4$; and the low energy data [7] range, $0.107 \leq \alpha_3 \leq 0.117$, corresponds to $-2.2 \leq c \leq 8.1$.

In contrast to the conclusions of Refs. [3–5], we have found that GUT models of the type considered here do have significant experimental consequences. Thus for models without Planck scale terms ($c = 0$), measurements made purely at the low energy (electroweak) scale can allow a prediction of the proton lifetime, and thus allow a direct test of the model. When c is left arbitrary, this is no longer possible. However, then low energy measurements will allow an experimental determination of c , and one has the remarkable possibility of seeing experimentally, for the first time, whether Planck scale physics exists. We note further, that the presence of GUT threshold effects and Planck scale terms do not qualitatively change the grand unification conclusions of Ref. [2].

This work was supported in part by the National Science Foundation under grant number PHY-9411543.

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- [14] Claims in the literature [4] that the GUT scale can approach the Planck scale have relied on one-loop RGE's. With two-loop analysis, we find in general that M_{GUT} stays well below M_{Pl} .
- [15] The Yukawa's enter in the RGE's at the two-loop level. Although its effect is small, we include the top Yukawa term in our calculations. We use $b_{it} = (2/3, 1/2, 17/30)_i$.
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- [17] The advantage of the $\overline{\text{DR}}$ renormalization scheme here is that the M_i 's for a single particle threshold will equal the particle's mass. Thus the "match and run" method sets the M_i 's to the mass of the particle.
- [18] At the GUT threshold, this condition is not met for β_3 . Consequently, we do not use such a parametrization to treat the GUT threshold.
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- [21] We take $\sin^2(\theta_W)$ to lie between 0.2294 and 0.2327 and $\alpha(M_Z)$ to be 1/127.9. All low energy data is considered renormalized at M_Z in the $\overline{\text{MS}}$ scheme, while the couplings run in the $\overline{\text{DR}}$ scheme. The conversion between schemes gives a negligible contribution to the results.

- [22] Conceptually, the relation in Eq. (8) is found from Eq. (7) by eliminating α_5 and λ_2 , and then solving for α_3 in terms of M_i and M_H while fixing c , λ_1 , and $\sin^2(\theta_W)$ at their extrema values of 1, 0.1, and 0.2327, respectively. We avoid treating $\sin^2(\theta_W)$ as a free parameter in Eq. (8) in order not to further complicate the dependences. We note, however, that as the value of $\sin^2(\theta_W)$ is varied to its lowest allowed value (0.2294), $\alpha_{3,min}$ may move upwards by as much as 0.007 for some choices of M_i and M_H .
- [23] Eq. (9) was gotten from a numerical fit. We have confirmed that the fit is good for variations of M_i by as much as a factor of 2 from M_{SUSY} . Preliminary study does indicate that a range for M_{SUSY} of 10 to 1000 GeV corresponds to a reasonable range of low energy sparticle spectrum in this model.
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- [26] This can be seen by dotting in the vector $(1, -3, 2)_i$ into Eq. (7) [5].

FIGURES

FIG. 1. Allowed regions in the parameter space projected onto the M_H - $\alpha_3(M_Z)$ plane. The curves are for $M_{\text{SUSY}} = 10$ GeV (solid), 100 GeV (dashed), and 1000 GeV (dot dashed). The shaded region corresponds to the case where $c = 0$, with all the GUT threshold mass scales taken to be degenerate, and M_{SUSY} allowed to vary between 10 GeV to 1000 GeV.

FIG. 2. Allowed regions in the parameter space projected onto the $\alpha_3(M_Z)$ - c plane. The curves are for $\sin^2(\theta_W) = 0.2294$ (solid) and $\sin^2(\theta_W) = 0.2327$ (dashed).

FIGURES

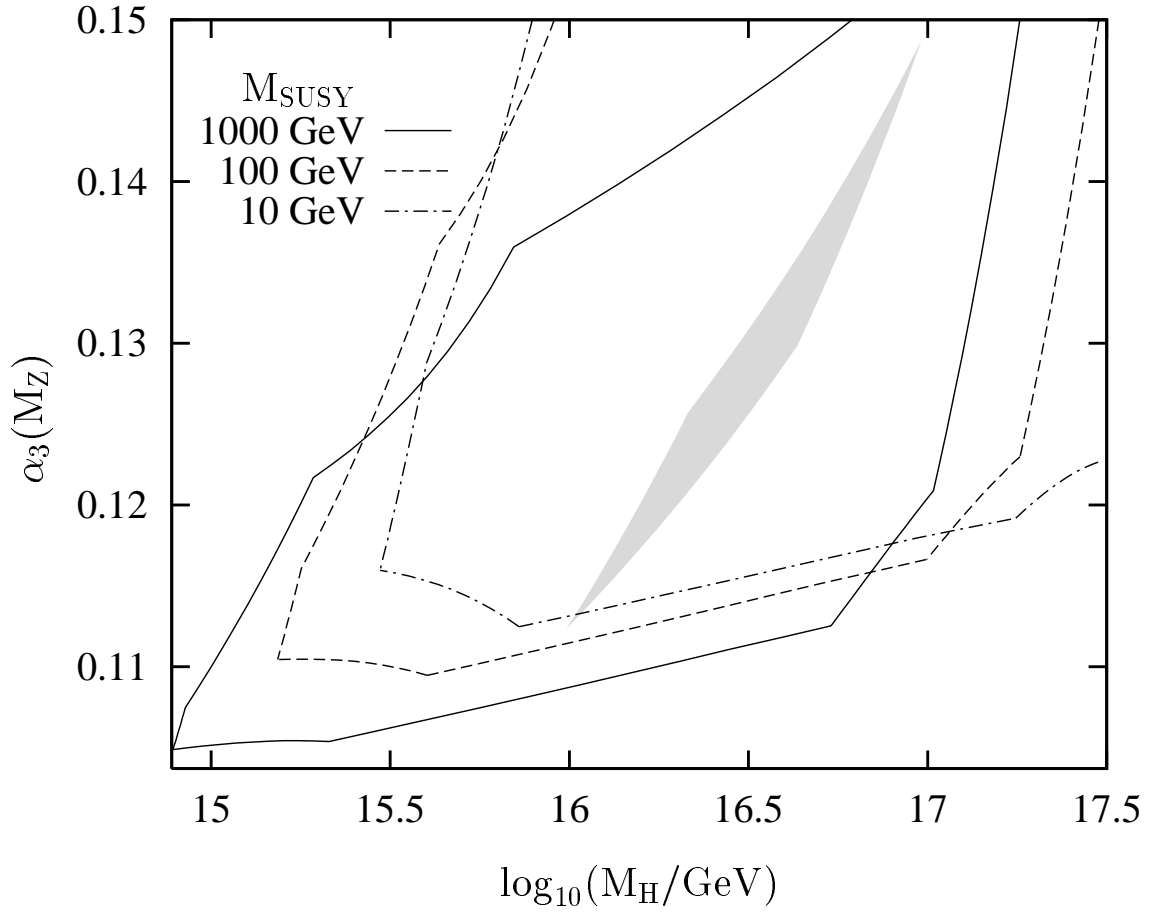


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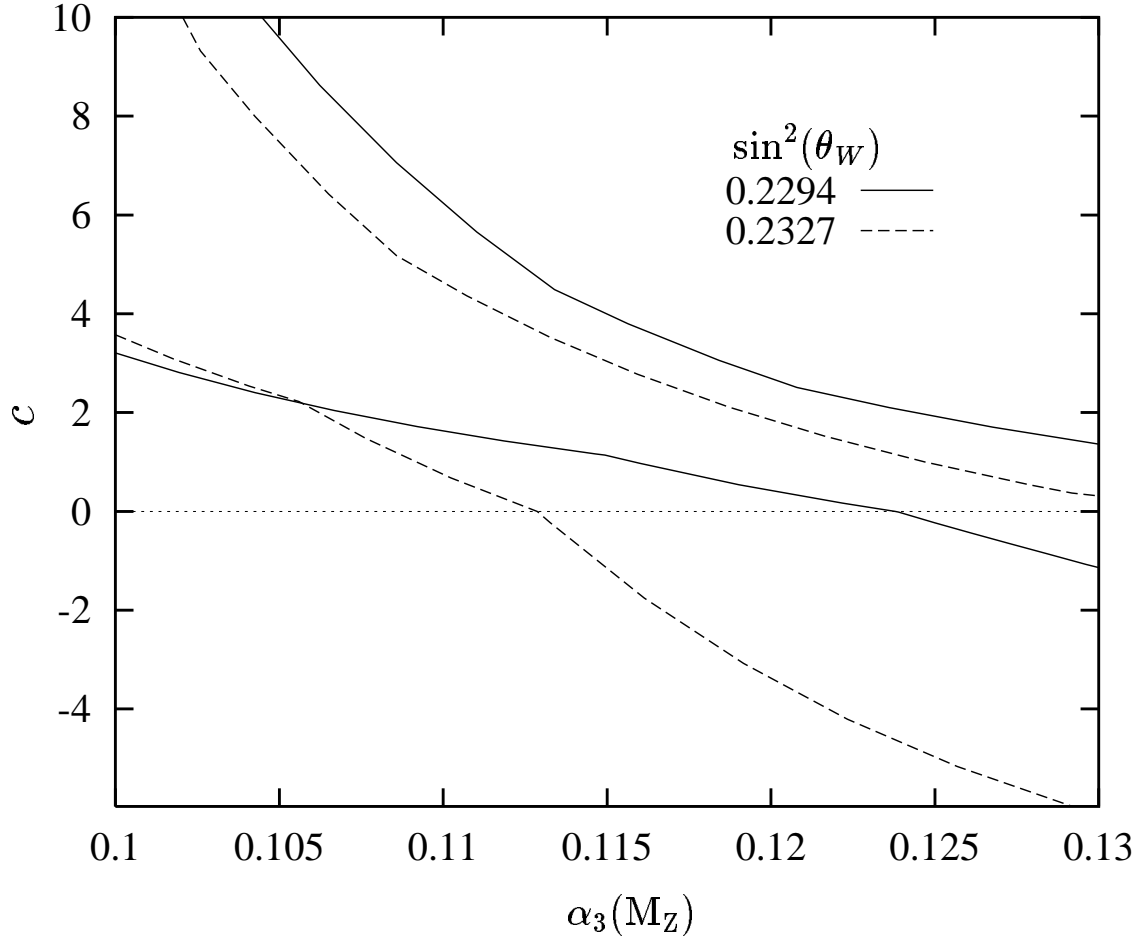


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