# Intersecting Branes Flip SU(5) 

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#### Abstract

Within a toroidal orbifold framework, we exhibit intersecting brane-world constructions of flipped $S U(5) \times U(1)$ GUT models with various numbers of generations, other chiral matter representations and Higgs representations. We exhibit orientifold constructions with integer winding numbers that yield 8 or more conventional $S U(5)$ generations, and orbifold constructions with fractional winding numbers that yield flipped $S U(5) \times U(1)$ models with just 3 conventional generations. Some of these models have candidates for the $\mathbf{5}$ and $\overline{5}$ Higgs representations needed for electroweak symmetry breaking, but not for the $\mathbf{1 0}$ and $\overline{\mathbf{1 0}}$ representations needed for GUT symmetry breaking. We have also derived models with complete GUT and electroweak Higgs sectors, but these have undesirable extra chiral matter.


## 1 Introduction

In recent years, theoretical understanding of string has deepened enormously, but the route to a model capable of unifying all the particle interactions in a realistic way still remains a mystery. String theory certainly has sufficient degrees of freedom to accommodate all the known particles and their interactions, and recent theoretical advances have revealed additional ways in which this might occur. Historically, the first approach to string modelbuilding was to compactify string on a suitable manifold [1] or orbifold [2], and subsequently constructions using fermions on the world-sheet were made available [3]. These approaches all originated in the context of weakly-coupled string theory, and many more possibilities are now evident on non-perturbative string theory, also known as M theory. A new dimension appears in the strong-coupling limit, string theories that formerly appeared unrelated are now known to be connected by dualities, new gauge symmetries may appear at singularities in moduli space [4], and non-perturbative brane constructions can accommodate new types of matter $[5,6,7,8,9,10,11,12,13]$.

Different types of particle models have been sought using these various constructions. At first, it was thought that the four-dimensional gauge group would necessarily be some subgroup of $E_{6}$ [1], then it was thought that the rank of the gauge group might be as large as 22 [3], and now higher-rank possibilities are known [4]. The minimal option would be to embed just the Standard Model $S U(3) \times S U(2) \times U(1)$ gauge group, but almost every construction includes at least extra $U(1)$ factors. Numerous attempts have been made to embed conventional GUT groups such as $S U(5)$ or $S O(10)$ in string theory, but none of these has been completely satisfactory. In the bad old days of perturbative string theory, one of the issues was the origin of GUT symmetry breaking. In four-dimensional field theories, this required Higgs multiplets in adjoint or larger representations, which were not present in simple compactifications on manifolds or orbifolds, using for example Calabi-Yau spaces [1] or lowest-level world-sheet fermions [14] ${ }^{1}$.

This impasse led to the proposal [16] of flipped $S U(5) \times U(1)$ [17, 18] as a suitable framework for string GUTs, since its symmetry breaking requires only $\mathbf{1 0}$ and $\overline{\mathbf{1 0}}$ representations at the GUT scale, as well as $\mathbf{5}$ and $\overline{5}$ representations at the electroweak scale, and these were readily available in perturbative string constructions. Flipped $S U(5) \times U(1)$ has a number of attractive phenomenological features in its own right [16]. For example, it has a very elegant missing-partner mechanism for suppressing proton decay via dimension- 5 operators, and is probably the simplest GUT to survive experimental limits on proton decay [19]. These considerations motivated the derivation of a number of flipped $S U(5) \times U(1)$ models from constructions using fermions on the world-sheet [20].

Recently, models based on $S U(5)$ or $S O(10)$ GUT groups have been derived using more sophisticated constructions, notably using branes [8, 12, 13] (for an introduction to Dbranes, see [21]). Promising constructions involve Type-I strings on toroidal orbifolds with intersecting D9-branes, or $T$-dual formulations. The models known to us do not yet have

[^0]all the phenomenological features one might desire, but certainly merit being pursued as far as has been done for some flipped $S U(5) \times U(1)$ models. In parallel with this effort, the attractive phenomenological features of flipped $S U(5) \times U(1)$ models motivate us to understand more completely their possible moduli space, in particular by exploring how they may be derived from such brane constructions.

We explore in this paper the type of brane approach pioneered by $[8,12,6,5,7,9,10,11]$ and studied further in [22]-[34] (for alternative compactifications with D-branes, see [35][41]). Issues arising in this framework have included the breaking of supersymmetry, the stability of the vacuum, the number of generations and the appearance of Higgs representations suitable for both GUT and electroweak symmetry breaking. In particular, toroidal orientifold models with integer winding numbers have tended to have rather large numbers of chiral matter generations. The number of generations can be adjusted to three in models with fractional winding numbers [12, 10], although these do not provide any explanation why there are just three generations in Nature ${ }^{2}$. Moreover, the existing GUT models of this type do not contain Higgs multiplets suitable for electroweak symmetry breaking, whereas the adjoint Higgs representations needed for GUT symmetry breaking can be found. The majority of the models constructed so far are non-supersymmetric and thus they suffer from an intrinsic instability of the internal spacetime due to the presence of scalar tadpoles in the theory ${ }^{3}$. The constructions presented in $[12,10]$ provide some improvement in this respect, as they ensure the cancellation of both Ramond-Ramond (RR) and Neveu-Schwarz-NeveuSchwarz (NSNS) tadpoles in the theory.

We investigate in this paper whether intersecting-brane constructions can give rise to any flipped $S U(5) \times U(1)$ models. In the case of such constructions on a toroidal orientifold, we have managed to construct an $S U(5)$ GUT with eight generations, much less than in the previous GUT models with integer winding numbers. This model contains many singlet fields, but there is no phenomenological objection to their proliferation. It is shown, however, that the model does not support a flipped $S U(5)$ model but only a traditional version of it with an extra $U(1)$ symmetry. Turning to models with fractional winding numbers, we show that a flipped $S U(5) \times U(1)$ gauge group can arise very naturally in toroidal orbifold brane constructions, and we give examples with three generations. Moreover, many of these models also contain, by construction, $\mathbf{5}$ and $\overline{5}$ Higgs multiplets suitable for electroweak symmetry breaking and no extra chiral matter. Our attempts to include also the $\mathbf{1 0}$ and $\overline{\mathbf{1 0}}$ Higgs representations suitable for GUT symmetry breaking into the chiral spectrum have produced models with complete GUT and electroweak Higgs sectors, but they suffer from a proliferation of undesirable extra chiral matter fields. Since the GUT symmetry-breaking scale is close to the string/gravity scale, we find it quite plausible that some (higher-dimensional?) mechanism might be responsible for this first stage of symmetry breaking. Therefore the earlier models with neither GUT Higgs multiplets nor undesirable chiral matter fields may be a more promising basis for future development.

[^1]
## 2 Search for Flipped $S U(5) \times U(1)_{X}$ Brane Models on a Toroidal Orientifold

In this section, we focus on the four-dimensional models that follow by considering sets of D6-branes wrapping on a six-torus orientifold $[5,8]$. We assume that the internal sixdimensional space-time can be written as the direct product of three two-dimensional tori, $T^{6}=T^{2} \times T^{2} \times T^{2}$, which is made into an orientifold by the action of the world-sheet parity transformation $\Omega$. In the $T$-dual picture, the above construction is regarded as a model of D9-branes with non-vanishing magnetic fluxes and mixed Neuman-Dirichlet boundary conditions [6, 22]. However, we find the previous picture easier to conceptualize, as a construction of D6-branes wrapped around two-dimensional cycles and intersecting at angles. We denote by $i=1,2,3$ the two-dimensional tori that comprise the internal space-time, and by $\mu=a, b, c, \ldots$ the different stacks of D6-branes present in our models. The position of each brane is given by the sets of integer numbers $\left(n_{\mu}^{(i)}, m_{\mu}^{(i)}\right)$ that describe the number of times that each brane is wrapped around the $\left(X^{(i)}, Y^{(i)}\right)$ axes, respectively, of each torus.

A number of conditions on these wrapping numbers arise from the requirement that the Ramond-Ramond (RR) tadpoles in the model cancel, conditions that also imply the cancellation of all non-Abelian gauge anomalies. For the particular toroidal construction considered here, these tadpole cancellation conditions are [8]

$$
\begin{array}{ll}
\sum_{\mu} N_{\mu} n_{\mu}^{(1)} n_{\mu}^{(2)} n_{\mu}^{(3)}=16, & \sum_{\mu} N_{\mu} n_{\mu}^{(1)} m_{\mu}^{(2)} m_{\mu}^{(3)}=0, \\
\sum_{\mu} N_{\mu} m_{\mu}^{(1)} n_{\mu}^{(2)} m_{\mu}^{(3)}=0, & \sum_{\mu} N_{\mu} m_{\mu}^{(1)} m_{\mu}^{(2)} n_{\mu}^{(3)}=0 . \tag{2.2}
\end{array}
$$

The spectra of chiral matter given by such intersecting-brane constructions arise in a variety of ways. Strings stretching between a brane belonging to stack $(a)$ and a brane belonging to stack $(b)$, or its mirror image $(\Omega b)$ under the parity transformation, give rise to bifundamental representations, $\left(\bar{N}_{a}, N_{b}\right)$ and $\left(N_{a}, N_{b}\right)$, respectively, of chiral matter of the group $U\left(N_{a}\right) \times U\left(N_{b}\right)$, with multiplicities

$$
\begin{align*}
& \mathcal{M}\left(\bar{N}_{a}, N_{b}\right)=\left(n_{a}^{(1)} m_{b}^{(1)}-m_{a}^{(1)} n_{b}^{(1)}\right)\left(n_{a}^{(2)} m_{b}^{(2)}-m_{a}^{(2)} n_{b}^{(2)}\right)\left(n_{a}^{(3)} m_{b}^{(3)}-m_{a}^{(3)} n_{b}^{(3)}\right),  \tag{2.3}\\
& \mathcal{M}\left(N_{a}, N_{b}\right)=\left(n_{a}^{(1)} m_{b}^{(1)}+m_{a}^{(1)} n_{b}^{(1)}\right)\left(n_{a}^{(2)} m_{b}^{(2)}+m_{a}^{(2)} n_{b}^{(2)}\right)\left(n_{a}^{(3)} m_{b}^{(3)}+m_{a}^{(3)} n_{b}^{(3)}\right), \tag{2.4}
\end{align*}
$$

respectively. Strings stretching between a brane in stack $(a)$ and its mirror image $(\Omega a)$ yield chiral matter in the antisymmetric and symmetric representations of the group $U\left(N_{a}\right)$, with multiplicities

$$
\begin{gather*}
\mathcal{M}\left(A_{a}\right)=8 m_{a}^{(1)} m_{a}^{(2)} m_{a}^{(3)}  \tag{2.5}\\
\mathcal{M}\left(A_{a}+S_{a}\right)=4 m_{a}^{(1)} m_{a}^{(2)} m_{a}^{(3)}\left(n_{a}^{(1)} n_{a}^{(2)} n_{a}^{(3)}-1\right), \tag{2.6}
\end{gather*}
$$

respectively. Finally, the chiral matter yielded by strings starting and ending on the same brane of stack (a) corresponds to the spectrum of a $d=4, \mathcal{N}=4$ super Yang-Mills theory of the group $U\left(N_{a}\right)$.

The latter part of the spectrum is obviously supersymmetric, whilst the open string spectra described previously, although 'supersymmetric' in number [6], i.e., with equal numbers of fermionic and bosonic degrees of freedom, does not have a supersymmetric mass spectrum. The fermions are massless, whilst the scalars acquire masses that are proportional to the length of the string, and which depend on the details of the construction of the internal space-time [9]. The spectrum of scalars is in principle tachyonic, although models with 'local' supersymmetry [11] may be constructed. The tachyonic spectrum of the theory may actually be useful, as it provides a potential source for the Higgs fields.

The fermionic spectrum of an $S U(5)$ GUT model fits into three copies of $(\mathbf{1 0}, \mathbf{1})$ and $(\overline{5}, \mathbf{1})$ representations, additionally with $(\mathbf{1}, \mathbf{1})$ representations if singlet neutrinos are to be accommodated. In the minimal flipped $S U(5) \times U(1)_{X}$ model [17, 18], the particle spectrum also includes a pair of $\mathbf{1 0}$ and $\overline{\mathbf{1 0}}$ Higgs multiplets, that break the GUT gauge group down to the Standard Model group, and a pair of light Higgs bosons in 5 and $\overline{5}$ multiplets, for electroweak symmetry breaking. Moreover, the fermionic multiplets should have specific charges under the extra $U(1)_{X}$ gauge factor, since a linear combination of the $U(1)_{X}$ and the $U(1)$ gauge factor contained in $S U(5)$ gives rise to the hypercharge factor of the Standard Model gauge group. In the framework of the intersecting-brane models on a six-torus orientifold, we look in this paper for flipped $S U(5)$ GUT models with the minimal possible particle content.

We saw easily that such a model cannot arise in the minimal case with two stacks of branes. We considered the case with $N_{a}=5$ and $N_{b}=1$, and we concentrated first on the fermionic spectrum. We found that the demand for the minimum number of families predicted by the model, namely eight, could not be met with non-fractional wrapping numbers $\left(n_{\mu}^{(i)}, m_{\mu}^{(i)}\right)$. The situation did not ameliorate even when we tried to modify the spectrum so as to include also Higgs multiplets in the matter spectrum, at least in the form of the 'nearly supersymmetric' fermionic partners of the massive Higgs multiplets [6]. All attempts in this direction resulted in models with many extra chiral matter multiplets, but with no $\overline{5}$ representations or singlets.

We therefore concentrate on the search for viable configurations of three stacks of branes with $N_{a}=5, N_{b}=1$ and $N_{c}=1$. The resulting gauge group is $S U(5) \times U(1)^{3}$. We focus again on the fermionic part of the spectrum, and we look for values of the wrapping numbers $\left(n_{\mu}^{(i)}, m_{\mu}^{(i)}\right)$ that would avoid any unnecessary proliferation of fermionic matter. For that purpose, we impose the constraints

$$
\begin{equation*}
m_{a}^{(1)} m_{a}^{(2)} m_{a}^{(3)}=1, \quad n_{a}^{(1)} n_{a}^{(2)} n_{a}^{(3)}=1 \tag{2.7}
\end{equation*}
$$

which lead to the minimal number of $(\mathbf{1 0}, \mathbf{1})$ representations, and none with symmetric $S U(5)$ indices.

The complete set of wrapping numbers that satisfies the aforementioned constraints, as well as the tadpole cancellation conditions, is given below:

$$
n_{\mu}^{(i)}=\left(\begin{array}{ccc}
1 & 1 & 1  \tag{2.8}\\
1 & 1 & 2 \\
1 & 3 & 4
\end{array}\right), \quad m_{\mu}^{(i)}=\left(\begin{array}{ccc}
1 & 1 & 1 \\
1 & 1 & -2 \\
1 & -5 & 0
\end{array}\right)
$$

This set of wrapping numbers leads to 8 copies of the antisymmetric representation $(\mathbf{1 0}, \mathbf{1})$, and the same number of bifundamental representations $(\overline{\mathbf{5}}, \mathbf{1})$. The full spectrum of fermionic matter is presented in Table I.

Table I

| multiplicity | representation | $\mathbf{U}(\mathbf{1})_{\mathbf{a}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{b}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{c}}$ | $\mathbf{U}(\mathbf{1})_{\text {free }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 8 | $(\mathbf{1 0}, \mathbf{1})$ | 2 | 0 | 0 | 2 |
| 8 | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | -1 | 0 | -2 |
| 40 | $(\mathbf{1}, \mathbf{1})$ | 0 | -2 | 0 | -2 |

In [8], the attempt to construct an $S U(5)$ GUT model in the framework of the same brane construction, led to a 24 -generation model with abundant extra chiral matter. The model presented above minimizes the number of fermionic representations, and makes a considerable reduction in the number of generations to 8 . The only extra chiral matter representations present are singlets, whose proliferation is not in disagreement with particle physics phenomenology. Neutrino masses suggest that at least three such states exist and mix with the light neutrino species, but do not exclude the possible existence of more than three such states.

The double vertical line in Table I separates the anomalous $U(1)$ gauge factors from the non-anomalous ones. All fermionic chiral matter is neutral under $U(1)_{c}$, so this gauge factor is automatically anomaly-free. Using the two remaining $U(1)$ factors, we may construct an anomaly-free combination in the following way

$$
\begin{equation*}
U(1)_{\text {free }}=U(1)_{a}+U(1)_{b}, \tag{2.9}
\end{equation*}
$$

whilst the orthogonal combination $U(1)_{a}-U(1)_{b}$ is anomalous. The charges of all the representations under the anomaly-free Abelian gauge factor are displayed in the last column of Table I. As can easily be seen, these charges do not correspond to the ones that the fermionic representations should have under the $U(1)_{X}$ gauge factor of the flipped $S U(5)$ GUT model, so we conclude that such a model cannot arise in the framework of this analysis. Moreover, we need to check whether the anomaly-free gauge factors remain massless after the generalised Green-Schwarz mechanism, that gives masses to the anomalous $U(1)$ factors, is implemented in the theory. According to [11], there are four RR fields, $B_{2}^{0}$ and $B_{2}^{I}$, with $I=1,2,3$, whose couplings to the $U(1)_{\mu}$ gauge factors are given by

$$
\begin{equation*}
c_{\mu}^{(0)}=N_{\mu} m_{\mu}^{(1)} m_{\mu}^{(2)} m_{\mu}^{(3)}, \quad c_{\mu}^{(I)}=N_{\mu} n_{\mu}^{(J)} n_{\mu}^{(K)} m_{\mu}^{(I)}, \quad I \neq J \neq K \neq I \tag{2.10}
\end{equation*}
$$

respectively. By using the wrapping numbers displayed in (2.8), we find that

$$
\begin{equation*}
\sum_{A} c_{c}^{(A)}=0, \quad \sum_{A} c_{\text {free }}^{(A)} \equiv \sum_{A}\left(c_{a}^{(A)}+c_{b}^{(A)}\right)=16 \tag{2.11}
\end{equation*}
$$

respectively, for the $U(1)_{c}$ and $U(1)_{\text {free }}$ anomaly-free gauge factors (in the above, $A$ sums over all RR scalar fields). Therefore, only the first Abelian gauge factor remains massless, whereas the latter acquires a mass.

We considered a number of alternative sets of wrapping numbers $\left(n_{\mu}^{(i)}, m_{\mu}^{(i)}\right)$, consistent with the tadpole cancellation conditions, in attempts to include the Higgs multiplets in the matter spectrum. As in the two-brane case, any attempt to generate the minimal number of $(\mathbf{1 0}, \mathbf{1})$ and $(\overline{\mathbf{1 0}}, \mathbf{1})$ representations led to the absence of any $(\overline{\mathbf{5}}, \mathbf{1})$ representations and singlets in the model. Abandoning the demand for the minimal number of families of chiral matter led to a very rapid proliferation of multiplets which still, however, failed to have the correct charges under the desired $U(1)_{X}$ gauge factor. We do not pursue further here the quest for a viable flipped $S U(5)$ model, as our analysis suggests that intersecting-brane models on a six-torus orientifold are unsuited for the construction of such GUT models. Even if a more persistent analysis of the possible combinations of wrapping numbers could lead to such a model, the result would still be marred by the large number of generations that these models generically predict, and orbifold constructions offer better prospects, as we now discuss.

## 3 Flipped $S U(5) \times U(1)_{X}$ Brane Models on a Toroidal Orbifold

Intersecting-brane models on tori, such as the one presented in the previous section, are known to have an additional weak point, apart from the large number of generations of chiral matter. A dynamical instability of the moduli space associated with the non-vanishing NSNS tadpoles is shared by all non-supersymmetric intersecting-brane models [12]. One solution to this problem, presented by the same authors [12], is the construction of nonsupersymmetric intersecting-brane models with a fixed moduli space. This can be accomplished by imposing a discrete symmetry $Z_{N}$ on the toroidal internal space-time, turning it into an orbifold. The problem of the large number of families has been tackled by introducing a discrete NSNS two-form field [10], which translates in the $T$-dual picture into a tilting of the two-dimensional tori. The RR tadpole cancellation conditions should also be modified, as well as the spectrum of the chiral matter predicted by the model. A simpler language was used for this purpose, through the introduction of effective wrapping numbers $\left(Y_{\mu}, Z_{\mu}\right)$ which could also be fractional, in terms of which the set of RR tadpole conditions reduced to the following, single requirement

$$
\begin{equation*}
\sum_{\mu} N_{\mu} Z_{\mu}=2 . \tag{3.1}
\end{equation*}
$$

Turning to the spectrum of chiral matter that arises in these models, it was shown that the net number of chiral bifundamental representations, that are yielded by strings stretching between a brane belonging to stack $(a)$ and a brane belonging to stack ( $b$ ), or its mirror
image $(\Omega b)$, would now be given by

$$
\begin{align*}
& \mathcal{M}\left(\bar{N}_{a}, N_{b}\right)=Z_{a} Y_{b}-Y_{a} Z_{b},  \tag{3.2}\\
& \mathcal{M}\left(N_{a}, N_{b}\right)=Z_{a} Y_{b}+Y_{a} Z_{b}, \tag{3.3}
\end{align*}
$$

respectively. Strings stretching between a brane in stack (a) and its mirror image ( $\Omega a$ ) give rise to chiral matter in the antisymmetric and symmetric representations of the group $U\left(N_{a}\right)$ as before, with multiplicities

$$
\begin{gather*}
\mathcal{M}\left(A_{a}\right)=Y_{a}  \tag{3.4}\\
\mathcal{M}\left(A_{a}+S_{a}\right)=Y_{a}\left(Z_{a}-\frac{1}{2}\right), \tag{3.5}
\end{gather*}
$$

respectively. The above part of the spectrum, corresponding to the open-string sector, breaks supersymmetry, as in the case of intersecting-brane models on a toroidal orientifold [8], with the scalar fields acquiring a non-vanishing mass. The closed-string sector still preserves supersymmetry, and leads again to the spectrum of a $d=4, \mathcal{N}=4$ super Yang-Mills theory of the $U\left(N_{a}\right)$ gauge group.

In what follows, we look for viable three-generation flipped $S U(5)$ GUT models. We first concentrate on the fermionic chiral representations that follow from two- and three-stack models, enquiring whether they have the correct charges under the $U(1)_{X}$ gauge factor. Subsequently, we study modifications of the spectrum of chiral matter, seeking to include the desired Higgs multiplets.

### 3.1 Two Stacks of Branes

We start again with the minimal case of two stacks of branes with $N_{a}=5$ and $N_{b}=1$. The final objective is to obtain three generations of the desired representations, that is $(\mathbf{1 0}, \mathbf{1})$, $(\overline{\mathbf{5}}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{1})$. However, we first make a general analysis for $n$ families, before specifying $n=3$. The demands for $n$ copies of the $(\mathbf{1 0}, \mathbf{1})$ representation and for the absence of any extra antisymmetric or symmetric representations lead, using (3.4)-(3.5), to $Y_{a}=n$ and $Z_{a}=1 / 2$, respectively. The choice of the value of the wrapping number $Z_{a}$ automatically determines the value of the second one, through the tadpole cancellation condition (3.1), to be $Z_{b}=-1 / 2$.

Then, from (3.2), (3.3) and (3.5), we find that the number of bifundamental and symmetric representations are given, respectively, by

$$
\begin{gather*}
\mathcal{M}(\overline{\mathbf{5}}, \mathbf{1})=\frac{1}{2}\left(Y_{b}+n\right), \quad \mathcal{M}(\mathbf{5}, \mathbf{1})=\frac{1}{2}\left(Y_{b}-n\right)  \tag{3.6}\\
\mathcal{M}(\mathbf{1}, \mathbf{1})=-Y_{b} \tag{3.7}
\end{gather*}
$$

The above part of the spectrum is characterized by the symmetry $Y_{b} \leftrightarrow-Y_{b}$ under which the spectrum remains essentially invariant. We choose the value $Y_{b}=+n$, and comment
briefly later on the differences appearing for the alternative choice $Y_{b}=-n$. The spectrum which follows in this case is displayed in Table II.

Table II

| multiplicity | representation | $\mathbf{U}(\mathbf{1})_{\mathbf{a}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{b}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{x}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $n$ | $(\mathbf{1 0}, \mathbf{1})$ | +2 | 0 | 1 |
| $n$ | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | 1 | -3 |
| $n$ | $(\mathbf{1}, \mathbf{1})$ | 0 | -2 | 5 |

Both of the two $U(1)$ gauge factors are anomalous. However, there is a single combination that turns out to be anomaly-free ${ }^{4}$, namely

$$
\begin{equation*}
U(1)_{X}=\frac{1}{2}\left[U(1)_{a}-5 U(1)_{b}\right] . \tag{3.8}
\end{equation*}
$$

The charges of the derived fermionic chiral spectrum under the $U(1)_{X}$ factor are shown in the last column of Table II and are exactly those that these representations should have in a flipped $S U(5) \times U(1)_{X}$ model. Note that these charges under the $U(1)_{X}$ gauge factor are reproduced for every number $n$ of families. We may, therefore, conclude that the particular two-stack intersecting-brane models studied here favour the construction of a flipped $S U(5)$ GUT model, without favouring a particular number of generations for the chiral matter. We are free to choose the phenomenologically relevant case $n=3$, but every other value of $n$ appears to be equally acceptable, from the theoretical point of view.

As mentioned above, the spectrum remains invariant under the change of the sign of the wrapping number $Y_{b}$. Indeed, if we choose $Y_{b}=-n$, we end up again with $n$ families of $(\overline{\mathbf{5}}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{1})$, the only differences being the opposite signs of the charges of these representations under the $U(1)_{b}$ gauge factor, which in turn leads to the opposite sign in front of the second term in the definition of $U(1)_{X}(3.8)$. We now check that both options survive the demand that $U(1)_{X}$ should remain massless despite potential couplings with RR scalar fields. In the present orbifold models [12], the imposed $Z_{3}$ symmetry projects out three of the four RR scalar fields, and the remaining coupling with $B_{2}^{0}$ is given by $c_{\mu}=N_{\mu} Y_{\mu}$. For both choices $Y_{b}=+n$ and $Y_{b}=-n$, we find

$$
\begin{equation*}
c_{X}^{( \pm)} \equiv \frac{1}{2}\left(c_{a} \mp 5 c_{b}\right)=0, \tag{3.9}
\end{equation*}
$$

so in neither case does the corresponding $U(1)_{X}$ acquire a mass.
Another comment is in order at this point. For $Y_{b}=+n$ and $n=3$, we find the same number of generations and types of representations for the fermionic matter as in the $S U(5)$ GUT model presented in [12]. Therefore, the single, anomaly-free $U(1)$ gauge

[^2]factor is bound to be given by the same linear combination of $U(1)_{a}$ and $U(1)_{b}$, modulo an arbitrary coefficient. Indeed, by multiplying the charges of the fermionic representations under the $U(1)_{\text {free }}$ gauge factor presented in Table $2^{5}$ of Ref. [12] by a factor $5 / 2$, we recover the charges under the $U(1)_{X}$ gauge factor displayed on the last column of our Table II. If one interprets this anomaly-free $U(1)$ as an extra Abelian symmetry with no physical content, then a Higgs singlet needs to be found to break the unnecessary symmetry, which was the approach followed in [12]. However, the charges of the fermionic representations under the anomaly-free $U(1)$ call for a flipped $S U(5) \times U(1)_{X}$ GUT model instead of the traditional $S U(5)$ one. In this approach, which we follow here, this gauge factor does not need to be broken as it contributes to the building of the flipped version of the $S U(5)$ model.

### 3.2 Three Stacks of Branes

We now turn to the case with three stacks of branes, with $N_{a}=5, N_{b}=1$ and $N_{c}=1$. It is of interest to investigate whether the construction of flipped $\mathrm{SU}(5)$ models is generically favoured in the case of a toroidal orbifold, independently of the number of stacks of branes considered. The desired spectrum of fermionic representations remains the same as before: we need 3 generations, each one containing $(\mathbf{1 0}, \mathbf{1}),(\overline{\mathbf{5}}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{1})$ multiplets. For this purpose, and starting from (3.4) and (3.5), we assume that

$$
\begin{equation*}
\left(Y_{a}, Z_{a}\right)=\left(3, \frac{1}{2}\right) \tag{3.10}
\end{equation*}
$$

The tadpole cancellation condition (3.1) leads in this case to the constraint

$$
\begin{equation*}
Z_{b}+Z_{c}=-\frac{1}{2} \tag{3.11}
\end{equation*}
$$

which leaves an infinite number of possibilities for the values of the two wrapping numbers $Z_{\mu}$. In what follows, we present in detail two sample models that lead to an optimal spectrum of chiral matter, among the many examples we studied.

### 3.2.1 Model I: $Z_{b}=-1 / 3$ and $Z_{c}=-1 / 6$

We discuss first the spectrum of bifundamental representations. From (3.2) and (3.3), after substituting the values of the wrapping numbers already determined, we find that the number of copies of the $\overline{\mathbf{5}}$ and $\mathbf{5}$ multiplets predicted by the model are given, respectively, by the expressions

$$
\begin{equation*}
\mathcal{M}\left(\bar{N}_{a}, N_{b}\right)=\mathcal{M}(\overline{\mathbf{5}}, \mathbf{1})=\frac{Y_{b}}{2}+1, \quad \mathcal{M}\left(\bar{N}_{a}, N_{c}\right)=\mathcal{M}(\overline{\mathbf{5}}, \mathbf{1})=\frac{Y_{c}}{2}+\frac{1}{2} \tag{3.12}
\end{equation*}
$$

[^3]\[

$$
\begin{equation*}
\mathcal{M}\left(N_{a}, N_{b}\right)=\mathcal{M}(\mathbf{5}, \mathbf{1})=\frac{Y_{b}}{2}-1, \quad \mathcal{M}\left(N_{a}, N_{c}\right)=\mathcal{M}(\mathbf{5}, \mathbf{1})=\frac{Y_{c}}{2}-\frac{1}{2} . \tag{3.13}
\end{equation*}
$$

\]

On the other hand, the spectrum of singlets $(\mathbf{1}, \mathbf{1})$ can be derived from the following expressions

$$
\begin{array}{cl}
\mathcal{M}\left(S_{b}\right)=-\frac{5}{6} Y_{b}, & \mathcal{M}\left(S_{c}\right)=-\frac{2}{3} Y_{c}, \\
\mathcal{M}\left(\bar{N}_{b}, N_{c}\right)=-\frac{Y_{c}}{3}+\frac{Y_{b}}{6}, & \mathcal{M}\left(N_{b}, N_{c}\right)=-\frac{Y_{c}}{3}-\frac{Y_{b}}{6} . \tag{3.15}
\end{array}
$$

We need to make the correct choices for the wrapping numbers $Y_{b}$ and $Y_{c}$ that lead, first, to integer numbers for the above multiplicities and, secondly, to a total number of copies of the $(\overline{5}, \mathbf{1})$ representation that close to 3 . For non-vanishing $Y_{b}$, it turns out to be extremely difficult to perform successfully both tasks, so we choose $Y_{b}=0$ and $Y_{c}=+3$. In that case, the spectrum that we obtain is displayed in Table III.

The spectrum derived above indeed includes three generations with the desired representations for an $S U(5)$ GUT model. The double horizontal line separates those representations from the extra chiral matter obtained, whose importance is discussed in subsection 3.3. From among the three $U(1)$ gauge symmetries present in the model, the following linear combination turns out to be anomaly-free:

$$
\begin{equation*}
U(1)_{X}=\frac{1}{2} U(1)_{a}-\frac{5}{2}\left[U(1)_{b}+U(1)_{c}\right] . \tag{3.16}
\end{equation*}
$$

Table III

| multiplicity | representation | $\mathbf{U}(\mathbf{1})_{\mathbf{a}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{b}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{c}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{X}}$ | $\mathbf{U}(\mathbf{1})_{\text {free }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $(\mathbf{1 0}, \mathbf{1})$ | 2 | 0 | 0 | 1 | 0 |
| 2 | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | 0 | 1 | -3 | 0 |
| 1 | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | 1 | 0 | -3 | 1 |
| 2 | $(\mathbf{1}, \mathbf{1})$ | 0 | 0 | -2 | 5 | 0 |
| 1 | $(\mathbf{1}, \mathbf{1})$ | 0 | -1 | -1 | 5 | -1 |
| 1 | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | -1 | 0 | 2 | -1 |
| 1 | $(\mathbf{5}, \mathbf{1})$ | 1 | 0 | 1 | -2 | 0 |
| 1 | $(\mathbf{1}, \mathbf{1})$ | 0 | 1 | -1 | 0 | 1 |

The corresponding charges of all representations under this gauge factor are also displayed in the above Table, and they are the correct ones for a flipped $S U(5) \times U(1)_{X}$ GUT model. Moreover, one of the three gauge factors also turns out to be anomaly-free

$$
\begin{equation*}
U(1)_{\text {free }}=U(1)_{b}, \tag{3.17}
\end{equation*}
$$

while the anomalous $U(1)$ factor can be chosen to be

$$
\begin{equation*}
U(1)_{a n}=\frac{1}{3} \sum_{\mu} N_{\mu} Y_{\mu} U(1)_{\mu}=5 U(1)_{a}+U(1)_{c} \tag{3.18}
\end{equation*}
$$

and is the only one that decouples from the system by acquiring a mass.
A symmetry under the change of the signs of the wrapping numbers $Y_{b}$ and $Y_{c}$, similar to that encountered in the two-stack case, is also present here. The spectrum obtained in this case is identical to the one presented above, apart from the sign of the charges under the corresponding $U(1)$ gauge factors. Reversing the sign in front of $U(1)_{b}$ and $U(1)_{c}$ in the definition of $U(1)_{X}$ (3.16) restores the correct charges under the flipped $U(1)$ factor.

### 3.2.2 Model II: $Z_{b}=-1 / 2$ and $Z_{c}=0$

We start again with the spectrum of bifundamental representations. Their multiplicities, for the chosen values of the wrapping numbers, are given by the expressions

$$
\begin{array}{ll}
\mathcal{M}\left(\bar{N}_{a}, N_{b}\right)=\mathcal{M}(\overline{\mathbf{5}}, \mathbf{1})=\frac{1}{2}\left(Y_{b}+3\right), & \mathcal{M}\left(\bar{N}_{a}, N_{c}\right)=\mathcal{M}(\overline{\mathbf{5}}, \mathbf{1})=\frac{Y_{c}}{2} \\
\mathcal{M}\left(N_{a}, N_{b}\right)=\mathcal{M}(\mathbf{5}, \mathbf{1})=\frac{1}{2}\left(Y_{b}-3\right), & \mathcal{M}\left(N_{a}, N_{c}\right)=\mathcal{M}(\mathbf{5}, \mathbf{1})=\frac{Y_{c}}{2} \tag{3.20}
\end{array}
$$

We also need to compute the spectrum of singlets $(\mathbf{1}, \mathbf{1})$. They come again from both symmetric and bifundamental representations, and have the multiplicities:

$$
\begin{array}{cl}
\mathcal{M}\left(S_{b}\right)=-Y_{b}, & \mathcal{M}\left(S_{c}\right)=-\frac{Y_{c}}{2}, \\
\mathcal{M}\left(\bar{N}_{b}, N_{c}\right)=-\frac{Y_{c}}{2}, & \mathcal{M}\left(N_{b}, N_{c}\right)=-\frac{Y_{c}}{2} . \tag{3.22}
\end{array}
$$

The values of the wrapping numbers $Y_{b}$ and $Y_{c}$ that lead to integer multiplicities, close to three, for both bifundamentals and singlets are $Y_{b}=1$ and $Y_{c}=2$. The spectrum of chiral matter obtained in this case ${ }^{6}$ is shown in Table IV.

Table IV

| multiplicity | representation | $\mathbf{U}(\mathbf{1})_{\mathbf{a}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{b}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{c}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{X}}$ | $\mathbf{U}(\mathbf{1})_{\text {free }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $(\mathbf{1 0}, \mathbf{1})$ | 2 | 0 | 0 | 1 | 0 |
| 2 | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | 1 | 0 | -3 | -1 |
| 1 | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | 0 | 1 | -3 | $1 / 2$ |
| 1 | $(\mathbf{1}, \mathbf{1})$ | 0 | -2 | 0 | 5 | 2 |
| 1 | $(\mathbf{1}, \mathbf{1})$ | 0 | 0 | -2 | 5 | -1 |
| 1 | $(\mathbf{1}, \mathbf{1})$ | 0 | -1 | -1 | 5 | $1 / 2$ |
| 1 | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | -1 | 0 | 2 | 1 |
| 1 | $(\mathbf{5}, \mathbf{1})$ | 1 | 0 | 1 | -2 | $1 / 2$ |
| 1 | $(\mathbf{1}, \mathbf{1})$ | 0 | 1 | -1 | 0 | $-3 / 2$ |

[^4]Focusing first on the $U(1)$ gauge factors of the model, we easily see that each one of them is anomalous. However, there are two anomaly-free combinations. The first one is

$$
\begin{equation*}
U(1)_{X}=\frac{1}{2} U(1)_{a}-\frac{5}{2}\left[U(1)_{b}+U(1)_{c}\right] \tag{3.23}
\end{equation*}
$$

and corresponds to the gauge factor necessary for the construction of the flipped $S U(5)$ model. The second one is

$$
\begin{equation*}
U(1)_{\text {free }}=\frac{1}{2} U(1)_{c}-U(1)_{b} \tag{3.24}
\end{equation*}
$$

and the charges of all chiral matter under this gauge factor are displayed in the last column of Table IV. Finally, the remaining, anomalous $U(1)$ factor, that acquires a mass via its coupling with the RR field, can be chosen to be

$$
\begin{equation*}
U(1)_{a n}=\sum_{\mu} N_{\mu} Y_{\mu} U(1)_{\mu}=15 U(1)_{a}+U(1)_{b}+2 U(1)_{c} \tag{3.25}
\end{equation*}
$$

As is clear from the entries in the Table, we have again obtained three generations of fermionic chiral matter, together with the same extra chiral spectrum as in the previous case. Both models have an extra pair of 5 and $\overline{5}$ multiplets in the spectrum, together with a singlet that is charged under the $U(1)_{\text {free }}$ gauge factor but neutral under the $U(1)_{X}$ factor. The pair of non-singlet multiplets may be identified with the 'supersymmetric' partners of the Higgs multiplets for the electroweak symmetry breaking, while the extra singlet serves to break the $U(1)_{\text {free }}$ gauge factor. Therefore, these two models successfully generate a three-generation fermionic spectrum with a flipped $S U(5) \times U(1)_{X}$ GUT gauge symmetry, together with a complete electroweak Higgs sector and no extra chiral matter.

Let us finally note that by choosing alternative values of the two wrapping numbers $Z_{b}$ and $Z_{c}$ that satisfied the constraint (3.11), a number of additional models were also constructed that successfully led to a three-generation fermionic chiral spectrum with the correct charges for a flipped $S U(5)$ GUT model. However, these models were accompanied by a moderate, but unnecessary, proliferation of 5 and $\overline{5}$ multiplets, and therefore we do not present them here.

### 3.3 Search for a Viable Higgs Spectrum

The successful derivation of the desired fermionic representations with the correct charges under the $U(1)_{X}$ gauge factor is only part of the attempt to construct a flipped $S U(5) \times$ $U(1)_{X}$ GUT model. The models derived in the previous subsection already included the $(\mathbf{5}, \mathbf{1})$ and $(\overline{\mathbf{5}}, \mathbf{1})$ Higgs multiplets needed for electroweak symmetry breaking. However, one would also like to augment the matter spectrum so as to include the $(\mathbf{1 0}, \mathbf{1})$ and $(\overline{\mathbf{1 0}}, \mathbf{1})$ Higgs multiplets needed for the breaking of the GUT group. This modification should take place in such a way that the number of generations, as well as the charges under the extra $U(1)_{X}$, are preserved and, if possible, the appearance of any extra chiral matter is avoided.

Returning to (3.4)-(3.5), we note that the sector of open strings stretching between a brane in stack $(a)$ and its mirror image $(\Omega a)$, is the only possible source for the antisymmetric representation of the $S U(5)$ group and its conjugate. Therefore, in order to obtain an additional $(\mathbf{1 0}, \mathbf{1})$ together with a $(\overline{\mathbf{1 0}}, \mathbf{1})$ multiplet for the Higgs spectrum, we need to modify first the wrapping numbers that correspond to stack $(a)$. The values of those numbers that were found to serve best this purpose, while leading to a minimal spectrum of extra chiral matter, are the following

$$
\begin{equation*}
\left(Y_{a}, Z_{a}\right)=\left(6, \frac{1}{3}\right) . \tag{3.26}
\end{equation*}
$$

Then, (3.4)-(3.5) lead to $3+1(\mathbf{1 0}, \mathbf{1})$ and one $(\overline{\mathbf{1 0}}, \mathbf{1})$, as desired, together, however, with one copy of the symmetric representation $(\overline{\mathbf{1 5}}, \mathbf{1})$ of $S U(5)$ and two extra $(\overline{\mathbf{1 0}}, \mathbf{1})^{7}$.

In the two-stack case, a large number of models, including the one that corresponds to the optimum choice of wrapping numbers (3.26), were studied, but did not contain any $\overline{5}$ 's. Considering alternative values of the winding numbers and demanding the appearance of $3+1(\overline{5}, \mathbf{1})$ multiplets, we are led to a spectrum of chiral matter that contains three generations of fermionic multiplets and a GUT Higgs sector with the required pair of $(\mathbf{1 0}, \mathbf{1})$ and $(\overline{\mathbf{1 0}}, \mathbf{1})$. However, these are accompanied by a large number of extra chiral matter fields, and the flipped $S U(5) \times U(1)_{X}$ gauge symmetry is not there any more: the sole anomalyfree $U(1)$ gauge factor that can be built out of the anomalous $U(1)_{a}$ and $U(1)_{b}$ does not correspond to the flipped $U(1)_{X}$ gauge factor. The model fails to lead even to a traditional $S U(5)$ GUT model with an extra $U(1)$ symmetry, due to the absence of the $(\mathbf{5}, \mathbf{1})$ Higgs multiplet with the correct charges. Vice versa, any attempt to preserve the symmetry $S U(5) \times U(1)_{X}$ leaves incomplete both the electroweak and GUT Higgs sectors.

In the three-stack case, the same stack with $N_{a}=5$ branes leads to the group $S U(5)$ and its fermionic representations, and therefore we will try to modify the corresponding wrapping numbers as in $(3.26)$, in order to include an extra $(\mathbf{1 0}, \mathbf{1})$ and $(\overline{\mathbf{1 0}}, \mathbf{1})$ in the chiral spectrum. Then, the tadpole cancellation condition leads to

$$
\begin{equation*}
Z_{b}+Z_{c}=\frac{1}{3} \tag{3.27}
\end{equation*}
$$

The number of different combinations for the above wrapping numbers that respect this constraint is again infinite. We have studied various combinations leading to a number of models with different features in their fermionic and Higgs spectra. For a reason to be discussed shortly, we present here the one which corresponds to the following wrapping numbers:

$$
\begin{equation*}
\left(Y_{b}, Z_{b}\right)=\left(3,-\frac{1}{2}\right), \quad\left(Y_{c}, Z_{c}\right)=\left(3, \frac{5}{6}\right) . \tag{3.28}
\end{equation*}
$$

The spectrum of chiral matter that follows in this case is displayed in Table V.

[^5]Table V

| multiplicity | representation | $\mathbf{U}(\mathbf{1})_{\mathbf{a}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{b}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{c}}$ | $\mathbf{U}(\mathbf{1})_{\mathbf{x}}$ | $\mathbf{U}(\mathbf{1})_{\text {free }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | $(\mathbf{1 0}, \mathbf{1})$ | 2 | 0 | 0 | 1 | 0 |
| 3 | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | 1 | 0 | -3 | 1 |
| 3 | $(\mathbf{1}, \mathbf{1})$ | 0 | -2 | 0 | 5 | -2 |
| 1 | $(\mathbf{1 0}, \mathbf{1})$ | 2 | 0 | 0 | 1 | 0 |
| 1 | $(\overline{\mathbf{1}}, \mathbf{1})$ | -2 | 0 | 0 | -1 | 0 |
| 1 | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | -1 | 0 | 2 | -1 |
| 1 | $(\mathbf{5}, \mathbf{1})$ | 1 | 0 | 1 | -2 | -1 |
| 4 | $(\mathbf{1}, \mathbf{1})$ | 0 | 1 | -1 | 0 | 2 |
| 1 | $(\overline{\mathbf{1 5}}, \mathbf{1})$ | -2 | 0 | 0 | -1 | 0 |
| 2 | $(\mathbf{1 0}, \mathbf{1})$ | 2 | 0 | 0 | 1 | 0 |
| 1 | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | 1 | 0 | -3 | 1 |
| 1 | $(\overline{\mathbf{5}}, \mathbf{1})$ | -1 | -1 | 0 | 2 | -1 |
| 4 | $(\mathbf{5}, \mathbf{1})$ | 1 | 0 | -1 | 3 | 1 |
| 5 | $(\mathbf{5}, \mathbf{1})$ | 1 | 0 | 1 | -2 | -1 |
| 1 | $(\mathbf{1}, \mathbf{1})$ | 0 | 0 | 2 | -5 | -2 |
| 1 | $(\mathbf{1}, \mathbf{1})$ | 0 | 1 | 1 | -5 | 0 |

The following comments can be made concerning the derived spectrum:

- The first part of Table V contains the fermionic multiplets, that again come in 3 generations, as desired.
- We have managed to recover the correct charges of all the fermionic chiral spectrum under the $U(1)_{X}$ gauge factor, which is defined as:

$$
\begin{equation*}
U(1)_{X}=\frac{1}{2} U(1)_{a}-\frac{5}{2} U(1)_{b}-\frac{5}{2} U(1)_{c} . \tag{3.29}
\end{equation*}
$$

The above Abelian factor is indeed anomaly-free and massless, and leads to a $S U(5) \times$ $U(1)_{X}$ gauge symmetry for the flipped GUT model.

- We can build a second anomaly-free, massless $U(1)$ gauge factor in the following way

$$
\begin{equation*}
U(1)_{\text {free }}=U(1)_{b}-U(1)_{c} . \tag{3.30}
\end{equation*}
$$

In order to avoid having an $S U(5) \times U(1)_{X} \times U(1)$ gauge symmetry, we need to break this extra Abelian factor with a Higgs singlet that is charged under this $U(1)_{\text {free }}$ gauge factor, but neutral under the $U(1)_{X}$ factor. The model presented above has indeed singlets of this type.

- In the second part of the Table, we display, in addition to the Higgs singlets for the breaking of the $U(1)_{\text {free }}$ gauge factor, the derived GUT and electroweak Higgs sector. We see that the $(\mathbf{1 0}, \mathbf{1})$ and $(\overline{\mathbf{1 0}}, \mathbf{1})$ multiplets have been successfully included into the spectrum, which completes the GUT symmetry breaking sector. In addition, a complete electroweak Higgs sector with $(\mathbf{5}, \mathbf{1})$ and $(\overline{\mathbf{5}}, \mathbf{1})$ multiplets has also been generated.

The main conclusion that one can draw from the above analysis of the three-stack case is that, contrary to what happens in the two-stack case, the attempt to include the Higgs sector in the chiral spectrum does not lead to the breakdown of the $S U(5) \times U(1)_{X}$ gauge construction. In all of the models studied, the derived spectrum contained three generations of fermionic matter, with the correct charges under the flipped gauge group, as well as a complete GUT Higgs sector. Some of those models had an incomplete electroweak Higgs sector, and hence would require an alternative way of breaking the electroweak symmetry. Other models, an example of which is the one presented above, had a complete electroweak Higgs sector. However, both groups of models are characterized by a large number of extra chiral multiplets with undesirable quantum numbers.

In our opinion, a more natural scheme for symmetry breaking arises, together with a phenomenologically preferred spectrum, if one abandons the attempt to include the GUT Higgs sector into the spectrum. An alternative method for breaking the high-energy GUT group would need to be invoked, maybe higher-dimensional. We find this more plausible for the GUT sector than for the electroweak sector, retaining the more traditional Higgs mechanism for the low-energy electroweak symmetry breaking. If we adopt this line of thinking, the most successful models are those derived in Section 3.2. Both models presented there had a three-generation fermionic spectrum with the appropriate charges for a flipped $S U(5) \times U(1)_{X}$ model, a Higgs singlet for the breaking of the extra $U(1)_{\text {free }}$ gauge factor, and the pair of 5 and $\overline{5}$ needed for the electroweak symmetry breaking. However, an alternative way of breaking of the GUT model would need to be introduced in each model.

## 4 Conclusions

We have explored in this paper the possibility of constructing a flipped $S U(5) \times U(1)_{X}$ GUT model in the framework of intersecting-brane scenarios. After the construction of other GUT models in the literature, based on either $S U(5)$ or $S O(10)$ gauge groups, we felt that the attractive phenomenological features of this model motivated a study of the flipped version of $S U(5)$.

We considered, first, sets of $D 6$-branes wrapped on a six-dimensional $T^{6}$ toroidal orientifold. This brane construction is characterized by integer wrapping numbers and, in general, a large number of generations for chiral matter. In the case with three stacks of branes, we have managed to obtain an $S U(5)$ GUT model with just 8 families of fermionic matter, considerably smaller than the number of families predicted in previous brane constructions. This model has an $S U(5) \times U(1)$ gauge symmetry group, but the extra $U(1)$
factor did not correspond to the $U(1)_{X}$ factor in flipped $S U(5)$. Moreover, this gauge factor, although non-anomalous, had a non-vanishing coupling with the RR two-form fields and thus acquired a mass.

Whilst brane constructions on a toroidal orientifold seem not to favour the flipped version of $S U(5)$, intersecting-brane models on a toroidal orbifold give rise to a flipped $S U(5) \times U(1)_{X}$ GUT gauge group quite naturally. Thanks to the fractional nature of the wrapping numbers in this case, we were able to obtain models with 3 generations of chiral fermions with the correct charges for the $U(1)_{X}$ flipped gauge factor. A number of models were constructed, both in the case of two and three stacks of branes, manifesting a generic tendency of these intersecting-brane configurations to give rise to the flipped version of the $S U(5)$ GUT group. In the three-stack case, the derived spectrum contained also the Higgs multiplets required for electroweak symmetry breaking and Higgs singlet suitable for the breaking of the extra $U(1)$ Abelian factor present in the models. It is worth noting that, in all models that led successfully to a flipped $S U(5)$ GUT group, the $U(1)_{X}$ gauge factor always remained massless, despite the presence of a RR two-form field in the theory.

The final step in our study involved exploring modifications of the derived models with a view to including the Higgs multiplets needed for GUT symmetry breaking. In the case with two stacks of branes, all our attempts in this direction resulted in the breakdown of the flipped $S U(5)$ symmetry. In the three-stack case, the same procedure led to a 3 -generation flipped $S U(5)$ model with complete GUT and electroweak Higgs sectors and a Higgs singlet, suitable for the breaking of the extra $U(1)_{\text {free }}$ gauge factor. However, this model had a large amount of extra chiral matter. The previous three-stack models, though lacking a GUT Higgs sector, may be a more attractive starting-point for future work. They naturally accommodate a complete electroweak symmetry breaking sector, the Higgs singlet and no extra chiral matter, and it may be possible to find an alternative mechanism for breaking the high-energy GUT symmetry.

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[^0]:    ${ }^{1}$ For constructions using higher-level fermions, see [15].

[^1]:    ${ }^{2}$ An argument in this direction was given in [11]. For other works trying to explain the number of generations of fermionic matter, although in a different framework, see [42].
    ${ }^{3}$ Supersymmetric constructions have been build in Ref. [13], however, they contain exotic chiral matter.

[^2]:    ${ }^{4}$ The remaining anomalous $U(1)$ gauge boson becomes massive because of the coupling of the RR-forms to the gauge fields, and decouples from the system [12].

[^3]:    ${ }^{5}$ A typographical error in the third row of the last column in that Table has erroneously changed the correct charge of the singlets under the $U(1)_{\text {free }}$ gauge factor from 2 to -2 .

[^4]:    ${ }^{6}$ Note that the transformations $Y_{b} \leftrightarrow-Y_{b}$ and $Y_{c} \leftrightarrow-Y_{c}$ still leave the spectrum of chiral fermionic matter invariant.

[^5]:    ${ }^{7}$ The choice $\left(Y_{a}, Z_{a}\right)=\left(4, \frac{1}{4}\right)$, which leads to exactly $3+1(\mathbf{1 0}, \mathbf{1})$ and one $(\overline{\mathbf{1 0}}, \mathbf{1})$ together with only one $(\overline{\mathbf{1 5}}, \mathbf{1})$, is not allowed in the model by the construction of [12]. We thank R. Blumenhagen, B. Körs, D. Lüst and T. Ott for communicating this to us.

