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Instanton Moduli and Brane Creation

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ABSTRACT

We obtain new intersecting 5-brane, string and pp-wave solutions in the heterotic string on a torus and on a K3 manifold. In the former case the 5-brane is supported by Yang-Mills instantons, and in the latter case both the 5-brane and the string are supported by the instantons. The instanton moduli are parameterised by the sizes and locations of the instantons. We exhibit two kinds of phase transition in which, for suitable choices of the instanton moduli, a 5-brane and/or a string can be created. One kind of phase transition occurs when the size of an instanton vanishes, while the other occurs when a pair of Yang-Mills instantons coalesce. We also study the associated five-dimensional black holes and the implications of these phase transitions for the black-hole entropy. Specifically, we find that the entropy of the three-charge black holes is zero when the instantons are separated and of non-zero scale size, but becomes non-zero (which can be counted microscopically) after either of the phase transitions.

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1 Introduction

The BPS p -branes of supergravity theories describe non-perturbative states of the underlying string theory or M-theory. In general, the p -brane solitons are not exact solutions in supergravity, in the sense that delta-function singularities arise in the field equations, implying that external source terms are needed. These sources are in fact supplied by the associated fundamental p -brane actions [1]. There are a few examples of p -brane solitons in maximal supergravities where such source terms are absent, notably the M5-brane [2] and the D3-brane [3, 4].

In the heterotic string there is a different mechanism that can give rise to regular brane-like solutions with no singular source terms. Due to the the Bianchi identity

$$dF_{(3)} = \frac{1}{2}G_{(2)}^a \wedge G_{(2)}^a, \quad (1.1)$$

one can construct a solitonic 5-brane that is supported by a Yang-Mills instanton configuration living in the 4-dimensional space transverse to the 5-brane worldvolume [5]. This configuration, unlike its 5-brane counterpart in the maximal $D = 10$ supergravity, is a perfectly regular solution of the supergravity equations of motion and is not supported by any external source term.¹ The Bianchi identity (1.1) implies that the 5-brane charge is nothing but the total instanton number, providing a natural quantisation of the 5-brane charge that lies outside, but is consistent with, the usual Dirac quantisation condition.

In this paper, we obtain a new solution describing the intersection of a gauge 5-brane, a string and a pp-wave. In other words, we show that a string, with a wave propagating on its worldsheet, can lie on the worldvolume of the instanton-supported 5-brane. This configuration is of particular interest since it reduces to a three-charge black hole in $D = 5$. This means that we can study thermodynamic quantities such as the entropy.

The Yang-Mills instanton moduli are parameterised by the sizes of the instantons and their locations. The 5-brane charge, following from (1.1), is given by

$$Q_m = \frac{1}{8\pi^2} \int dF_{(3)} = \frac{1}{16\pi^2} \int G_{(2)}^a \wedge G_{(2)}^a = N, \quad (1.2)$$

where the integration is over the entire 4-volume of the transverse space and N is the instanton number. This charge is topological and is therefore independent of the Yang-Mills instanton moduli. This leads to the interesting question as what happens if the size

¹One might argue that calling such a regular solution a 5-brane is somewhat inappropriate, and that it were better thought of as an instanton solution which happens to have a Poincaré symmetry in a six-dimensional submanifold. We shall, however, follow the traditional terminology and refer to it as the gauge 5-brane [5].

of a instanton becomes zero, or if two instantons coalesce. We show that in each case there is a phase transition in which a fundamental 5-brane is created, while at the same time a gauge 5-brane is destroyed. Thus a gauge 5-brane turns into a fundamental 5-brane, while keeping the total magnetic charge conserved.

These two phase transitions have significant consequences for the associated 3-charge black holes that arise after a dimensional reduction to five dimensions. The horizon has a curvature singularity, and has zero area, when the instantons supporting the gauge 5-brane are of non-vanishing size, and are non-coincident. However, the horizon becomes regular (AdS \times sphere), with non-zero area, when either of the above phase transitions occurs. This phenomenon supports the idea that a fundamental 5-brane is created as a result of the phase transition, since the 3-charge black hole in $D = 5$ with non-zero horizon area can be interpreted, at the microscopic level, by counting the states in such an intersecting configuration [6].

We also study the phase transitions leading to brane creation in the context of the heterotic string compactified on the K3 manifold. In this case, both the string and 5-brane (in the ten-dimensional picture) can be supported by Yang-Mills instantons; this corresponds to gauge dyonic strings in $D = 6$ [7]. Thus either of the two kinds of phase transition discussed above will now lead to the creation not only of fundamental 5-branes, but also fundamental strings. We obtain a new intersection with an additional superposed pp-wave. This gives rise, upon a further reduction to $D = 5$, to 3-charge black holes with two instanton-supported charges and one point charge, whose entropies become non-vanishing under either of the two phase transitions.

Supergravity on an anti-de Sitter spacetime background is conjectured to be dual to an associated superconformal field theory on its boundary [8]. Thus the instanton phase transition can be viewed as a transition from a supergravity theory to the superconformal field theory.

2 Heterotic string on torus

The low-energy effective action of the heterotic string is $N = 1$ supergravity in $D = 10$, coupled to $E_8 \times E_8$ Yang-Mills matter fields. We shall focus on an $SU(2)$ subgroup of $E_8 \times E_8$. The Lagrangian for the bosonic sector is given by

$$e^{-1} \mathcal{L}_{10} = R * \mathbb{1} - \frac{1}{2} * d\phi \wedge d\phi - \frac{1}{2} e^{-\phi} * F_{(3)} \wedge F_{(3)} - \frac{1}{2} e^{-\frac{1}{2}\phi} * G_{(2)}^a \wedge G_{(2)}^a, \quad (2.1)$$

where the field $G_{(2)}^a$ is the Yang-Mills field strength given by

$$G_{(2)}^\alpha = dB_{(1)}^a + \frac{1}{2}\epsilon^{abc} B_{(1)}^b \wedge B_{(1)}^c , \quad (2.2)$$

and $F_{(3)}$ is the three-form field strength, given by

$$F_{(3)} = dA_{(2)} + \frac{1}{2}B_{(1)}^a \wedge dB_{(1)}^a + \frac{1}{6}\epsilon^{abc} B_{(1)}^a \wedge B_{(1)}^b \wedge B_{(1)}^c . \quad (2.3)$$

It satisfies the Bianchi identity

$$dF_{(3)} = \frac{1}{2}G_{(2)}^\alpha \wedge G_{(2)}^\alpha . \quad (2.4)$$

2.1 Instanton-supported intersections

The Lagrangian (2.1) admits a solution describing an intersection of a string, a 5-brane and a pp-wave, given by

$$\begin{aligned} ds_{10}^2 &= H_e^{-3/4} H_m^{-1/4} (-W^{-1} dt^2 + W (dz_1 + (W^{-1} - 1) dt)^2) \\ &\quad + H_e^{1/4} H_m^{-1/4} (dz_2^2 + \dots + dz_5^2) + H_e^{1/4} H_m^{3/4} dy^i dy^i , \\ \phi &= -\frac{1}{2} \log(H_e/H_m) , \\ F_{(3)} &= e^\phi *(dt \wedge d^5 z \wedge dH_m^{-1}) - dt \wedge dz_1 \wedge dH_e^{-1} , \end{aligned} \quad (2.5)$$

where the functions H_e , H_m and W , associated with the string, 5-brane and pp-wave respectively, depend only on the four coordinates y^i of the transverse space, and satisfy the equations

$$\square H_e = 0 , \quad \square W = 0 , \quad \square H_m = -\frac{1}{4} G_{ij}^a G_{ij}^a . \quad (2.6)$$

Note that here $\square \equiv \partial_i \partial_i$ is the Laplacian in the flat transverse metric $ds^2 = dy^i dy^i$, and the index contractions in $G_{ij}^a G_{ij}^a$ are performed simply using the metric δ_{ij} of the flat transverse space. The $SU(2)$ Yang-Mills fields $G_{(2)}^a$ satisfy the self-duality equations $*G_{ij}^a = G_{ij}^a$ in the four-dimensional flat transverse space, where $*$ denotes Hodge duality in this flat space. Single-charge and certain multi-charge $SU(2)$ instanton solutions are given in the Appendix.

For a single-center configuration, the solutions to equations (2.6) can be taken to be:

$$H_e = 1 + \frac{2Q_e}{r^2} , \quad H_m = 1 + \frac{2(r^2 + 2a^2)}{(r^2 + a^2)^2} , \quad W = 1 + \frac{2P}{r^2} , \quad (2.7)$$

where we have made use of (A.5) in order to solve the equation of motion for H_m . Note that the solution requires a fundamental string as its source term, but does not require any fundamental 5-brane, since the 5-brane is supported by the Yang-Mills instanton, which

provides one unit of 5-brane charge. This provides a discretisation of the 5-brane charge that lies outside the Dirac quantisation condition [5, 1]. This is possible due to the Bianchi identity (2.4). Of course, if there were also a fundamental 5-brane that could provide a delta-function source term, then we could have an additional term $2\tilde{Q}_m/r^2$ in H_m , giving

$$H_m = 1 + \frac{2\tilde{Q}_m}{r^2} + \frac{2(r^2 + 2a^2)}{(r^2 + a^2)^2} . \quad (2.8)$$

(We shall discuss the normalisation of the magnetic charge below. Note that a *unit* charge corresponds to a singularity of strength 2 in the harmonic function.)

For multi-centered configurations, we may take the solutions to (2.6) to be

$$H_e = 1 + \sum_{\alpha'} \frac{2Q_e^{\alpha'}}{|\vec{y} - \vec{y}'_{\alpha'}|^2} , \quad H_m = 1 + 2\psi , \quad W = 1 + \sum_{\alpha''} \frac{2P_{\alpha''}}{|\vec{y} - \vec{y}''_{\alpha''}|^2} , \quad (2.9)$$

where ψ , given in (A.10), is associated with the Yang-Mills instanton, discussed in detail in Appendix. The primes on the various indices and the locations of the singularities signify the fact that the number, and locations, of the singularities for H_e , W and f can all be different.

To determine the appropriate choice for the harmonic function h in (A.10), we must examine the behaviour of ψ in the vicinity of each singularity of the harmonic function f in (A.8). Noting that $\square \log f = f^{-1} \square f - f^{-2} (\partial_i f)^2$, we see that near the singularity at $\vec{y} = \vec{y}_\alpha$, we shall have $\psi = \frac{1}{4} \square \log f + h \sim -|\vec{y} - \vec{y}_\alpha|^{-2} + h$, since $f \sim \lambda_\alpha |\vec{y} - \vec{y}_\alpha|^{-2}$ near this singularity. We see from (2.9) that in the absence of any correction term from h , this would be of the form of a singular point source with magnetic charge (-1) . Therefore we may exploit the freedom of adding an harmonic function $h = |\vec{y} - \vec{y}_\alpha|^{-2}$ to ψ , in order to ensure that the only source for the magnetic charge of the 5-brane at $\vec{y} = \vec{y}_\alpha$ is from the Yang-Mills instanton. Carrying out this procedure for each singularity \vec{y}_α , we see that ψ in (2.9) should be chosen to be²

$$\psi = \frac{1}{4} \square \log \left(1 + \sum_{\alpha=1}^N \frac{\lambda_\alpha}{|\vec{y} - \vec{y}_\alpha|^2} \right) + \sum_{\alpha=1}^N \frac{1}{|\vec{y} - \vec{y}_\alpha|^2} . \quad (2.10)$$

²In [18], the delta-function singularities in $G_{ij}^a G_{ij}^a = -8 \square \psi$ that result from simply taking ψ to be given by $\frac{1}{4} \log f$ were eliminated by excising small spheres around the singularities in f . This could be done there because ψ itself had no direct physical significance (and indeed it was not explicitly introduced in [18]). In our case, however, ψ itself appears when we solve for H_m in (2.6) to obtain (2.7), and so we must ensure that ψ is free of singularities if we are to have a solution that has only non-singular instanton source-terms for the 5-brane charge. This procedure has the added advantage that $G_{ij}^a G_{ij}^a = -8 \square \psi$ is now an exactly correct expression, with no delta-function singularities, and so the excision of spheres performed in [18] is no longer necessary.

In general, the total magnetic charge Q_m is given by

$$Q_m = \frac{1}{8\pi^2} \int_{S^3} F_{(3)} . \quad (2.11)$$

This can receive contributions both from non-singular instanton-supported sources and from any singular sources corresponding to the possible additional presence of point charges. The instanton contributions can be calculated from the Bianchi identity (2.4), since the integral in (2.11) can be viewed as being over the sphere at infinity in the four-dimensional transverse space V_4 , and hence we can write

$$\begin{aligned} Q_m &= \frac{1}{8\pi^2} \int_{S^3} F_{(3)} = \frac{1}{8\pi^2} \int_{V_4} dF_3 \\ &= \frac{1}{16\pi^2} \int_{V_4} G_{(2)}^a \wedge G_{(2)}^2 \equiv N , \end{aligned} \quad (2.12)$$

where N is the instanton number defined by the integral in the second line. If there are, in addition, point-charge singular contributions, then these can be calculated from the expression for $F_{(3)}$ in (2.5). The first term gives the magnetic contribution $F_{(3)} = -\frac{1}{6}\partial_i H_m \epsilon_{ijkl} dy^j \wedge dy^k \wedge dy^l$, and hence a contribution to $dF_{(3)}$ of $dF_{(3)} = -\square H_m d^4y$. This implies that there will be a contribution to the magnetic charge

$$Q_m = \frac{1}{8\pi^2} \int_{S^3} F_{(3)} = -\frac{1}{8\pi^2} \int_{V_4} \square H_m . \quad (2.13)$$

Thus a term in H_m of the form $2k |\vec{y} - \vec{y}_\alpha|^{-2}$ will contribute a magnetic charge

$$Q_m = -\frac{1}{4\pi^2} \int_{S^3} \partial_i \left(\frac{k}{|\vec{y} - \vec{y}_\alpha|^2} \right) d\Sigma_i = k . \quad (2.14)$$

Putting this all together, we see that a solution with N instantons with scales λ_α centered at the points \vec{y}_α , and N' point magnetic charges $\tilde{Q}_m^{\tilde{\alpha}}$, centered at the points $\vec{y}_{\tilde{\alpha}}$, will be described in terms of a function H_m in (2.9) with

$$\psi = \left[\frac{1}{4}\square \log \left(1 + \sum_{\alpha=1}^N \frac{\lambda_\alpha}{|\vec{y} - \vec{y}_\alpha|^2} \right) + \sum_{\alpha=1}^N \frac{1}{|\vec{y} - \vec{y}_\alpha|^2} \right] + \sum_{\tilde{\alpha}=1}^{N'} \frac{\tilde{Q}_m^{\tilde{\alpha}}}{|\vec{y} - \vec{y}_{\tilde{\alpha}}|^2} . \quad (2.15)$$

The term enclosed in square brackets is the non-singular contribution from the instantons, and the final term is the singular contribution of the point charges. The total magnetic 5-brane charge will be

$$Q_m = N + \sum_{\tilde{\alpha}=1}^{N'} \tilde{Q}_m^{\tilde{\alpha}} . \quad (2.16)$$

2.2 Brane creation

We have seen that the moduli space of the instantons in the solutions we are discussing is parameterised by the sizes of the instantons λ_α and their positions \vec{y}_α . Two types of phase transitions can arise when one adjusts these modulus parameters. The first type is associated with the sizes of the instantons. If the scale-size of an instanton located at $\vec{y} = \vec{y}_\alpha$ is taken to zero, there is a point singularity left at \vec{y}_α . To see this explicitly, we note that in the vicinity of the instanton location \vec{y}_α , the function H_m defined by (2.9) and (2.10) becomes

$$H_m = 1 + \frac{2(r^2 + 2a^2)}{(r^2 + a^2)^2} \xrightarrow{a \rightarrow 0} 1 + \frac{2}{r^2}, \quad (2.17)$$

where $\vec{r} = \vec{y} - \vec{y}_\alpha$, and $a = \sqrt{\lambda_\alpha}$ is the scale-size of the instanton. In other words, the function H_m becomes a harmonic function, associated with a point singularity in the transverse space, when the instanton size vanishes. This point charge, unlike the case of the non-singular instanton, has a delta-function singularity, implying the need for a source term outside the $N = 1, D = 10$ supergravity. This external source is in fact provided by introducing a fundamental 5-brane action. Thus we see that a fundamental 5-brane is created when the instanton size is taken to zero. In this phase transition, the total magnetic charge measured by $\int F_{(3)}$ is conserved.

Another kind of phase transition occurs if two of the instanton centers are allowed to become coincident. Suppose, for example, that we take $\vec{y}_\alpha = \vec{y}_\beta$ for two specific instanton centers \vec{y}_α and \vec{y}_β . In the function $f = 1 + \sum_\alpha \lambda_\alpha |\vec{y} - \vec{y}_\alpha|^{-2}$, the effect is merely to coalesce a two-instanton configuration with instantons of size $a^2 = \lambda_\alpha$ and $a'^2 = \lambda_\beta$ into a *one*-instanton configuration of size $a''^2 = \lambda_\alpha + \lambda_\beta$. However, the harmonic function h will now have a term $2|\vec{y} - \vec{y}_\alpha|^{-2}$, whose strength is twice the value that is needed for cancelling out the singularity in ψ at $\vec{y} = \vec{y}_\alpha$. Thus, there is one unit of point charge (in the transverse space) left over. The upshot of this is that when two instanton centers are brought into coincidence, a configuration that previously described a non-singular gauge 5-brane with instanton number 2 undergoes a phase transition to a configuration describing two superposed 5-branes supported by one non-singular instanton charge and one singular point-magnetic-charge, which is nothing but the fundamental 5-brane charge. In particular, as must be, the net magnetic charge is conserved.

As an example, consider a 2-instanton solution, where the two instanton centers are initially located at \vec{y}_1 and \vec{y}_2 . The function ψ will be given by

$$\psi = \frac{1}{4} \square \log \left(1 + \frac{\lambda_1}{|\vec{y} - \vec{y}_1|^2} + \frac{\lambda_2}{|\vec{y} - \vec{y}_2|^2} \right) + \frac{1}{|\vec{y} - \vec{y}_1|^2} + \frac{1}{|\vec{y} - \vec{y}_2|^2}. \quad (2.18)$$

After allowing the instanton centers to coalesce, say at $\vec{y} = \vec{y}_1$, the function ψ becomes

$$\psi = \left[\frac{1}{4} \square \log \left(1 + \frac{\lambda_1 + \lambda_2}{|\vec{y} - \vec{y}_1|^2} \right) + \frac{1}{|\vec{y} - \vec{y}_1|^2} \right] + \frac{1}{|\vec{y} - \vec{y}_1|^2}, \quad (2.19)$$

where the term enclosed in square brackets is the non-singular contribution to the function H_m in (2.7) coming from the remaining instanton-supported charge, while the final term describes the singular contribution to H_m coming from a unit point-charge located at \vec{y}_1 .

In both of the phase transitions that lead to the creation of fundamental 5-branes, the fundamental 5-brane charge that is generated is quantised and is equal to the decrease in the Yang-Mills instanton number. The 5-branes that are created all contribute positively to the total mass.

One could have asked the question purely in the framework of four-dimensional Yang-Mills theory as to what happens when two instanton centers in a multi-instanton solution are allowed to coalesce. However, unlike the situation that we have been discussing here, the question in four-dimensional Yang-Mills theory is an entirely non-dynamical one, in the sense that there is no external “time” coordinate and, thus, no possibility of a “slow motion” confluence of the instanton centers. Thus four-dimensional Yang-Mills theory does not really in itself demand that one give a precise interpretation to the question of what happens if two instanton centers coalesce. In our case, however, where the instantons reside in a four-dimensional space transverse to the 5-branes, it does make sense to envisage a slow-motion approximation in which the locations of the instanton centers vary as a function of time. Thus it is important in this context that one should be able to give a sensible interpretation, of the kind that we have supplied, to the question of what happens when two instanton centers coalesce.

2.3 $D = 5$ black hole and its entropy

The intersection solution (2.5) is invariant under translational symmetry of the coordinates $\{z_1, z_2, \dots, z_5\}$. It follows that we can dimensionally reduce the solution on the five-torus T^5 associated with these coordinates, giving rise to a three-charge black hole in $D = 5$. This torus reduction can also be consistently performed on the Lagrangian (2.1). The relevant part of the five-dimensional Lagrangian of the associated three-charge black hole is given by

$$\begin{aligned} e^{-1} \mathcal{L}_5 = & R * \mathbb{1} - \frac{1}{2} * d\vec{\phi} \wedge d\vec{\phi} - \frac{1}{2} e^{\vec{a} \cdot \vec{\phi}} * F_{(3)} \wedge F_{(3)} - \frac{1}{2} e^{\vec{d} \cdot \vec{\phi}} * F_{(2)} \wedge F_{(2)} \\ & - \frac{1}{2} e^{\vec{b} \cdot \vec{\phi}} * \mathcal{F}_{(2)} \wedge \mathcal{F}_{(2)} - \frac{1}{2} e^{\vec{c} \cdot \vec{\phi}} * G_{(2)}^a \wedge G_{(2)}^a, \end{aligned} \quad (2.20)$$

where $\mathcal{F}_{(2)} = d\mathcal{A}_{(1)}$ is the Kaluza-Klein two-form field strength. The dilaton vectors \vec{a} , \vec{b} , \vec{c} and \vec{d} in (2.20) satisfy the following product rules

$$\begin{aligned} \vec{a} \cdot \vec{a} = \vec{b} \cdot \vec{b} = \vec{d} \cdot \vec{d} = 4\vec{c} \cdot \vec{c} = \frac{8}{3}, \quad \vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{d} = \vec{a} \cdot \vec{c} = \frac{4}{3}, \\ \vec{b} \cdot \vec{c} = \vec{d} \cdot \vec{c} = \frac{2}{3}, \quad \vec{b} \cdot \vec{d} = -\frac{4}{3}. \end{aligned} \quad (2.21)$$

We can realise these by the two-component vectors

$$\vec{a} = (-\sqrt{2}, \sqrt{\frac{2}{3}}), \quad \vec{b} = (0, \sqrt{\frac{8}{3}}), \quad \vec{c} = (-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{6}}), \quad \vec{d} = (-\sqrt{2}, -\sqrt{\frac{2}{3}}). \quad (2.22)$$

The three-charge five-dimensional black hole, which is the dimensional reduction of (2.5) and a solution to (2.20), is given by

$$\begin{aligned} ds_5^2 &= -(H_e H_m W)^{-2/3} dt^2 + (H_e H_m W)^{1/3} (dr^2 + r^2 d\Omega_3^2), \\ \vec{\phi} &= -\frac{1}{2}\vec{a} \log H_m + \frac{1}{2}\vec{b} \log W + \frac{1}{2}\vec{d} \log H_e, \\ \mathcal{F}_{(2)} &= dt \wedge dW^{-1}, \quad F_{(2)} = dt \wedge dH_e^{-1}, \quad F_{(3)} = e^{-\vec{a} \cdot \vec{\phi}} *(dt \wedge dH_m^{-1}). \end{aligned} \quad (2.23)$$

For convenience, we have assumed here, as we did for the previous gauge 5-brane solution, that the asymptotic values of the dilatons vanish; $\vec{\phi}_0 = 0$. Here, for simplicity, we consider only the isotropic black hole, where all the charges are located at the origin. In this case, there is only a single instanton, contributing one unit of the charge associated with $F_{(3)}$. The metric in (2.23) has an horizon at $r = 0$. For any non-vanishing size a of the instanton, the metric (2.20) is singular at the horizon, which has vanishing area. It follows that the entropy is exactly zero. On other hand, when the instanton size is zero, the instanton is replaced by a point charge in the transverse space. In this case, the horizon becomes regular and has a non-zero area. Thus the entropy undergoes a phase transition as the scale-size of the instanton vanishes:

$$S = \begin{cases} 0 & : a > 0 \\ \frac{1}{4}A_{\text{horizon}} = \pi^2 \sqrt{2Q_e P} & : a = 0 \end{cases} \quad (2.24)$$

An analogous phenomenon occurs if $N + 1$ instantons coalesce. The entropy, which is initially zero, becomes non-vanishing and is given by

$$S = \pi^2 \sqrt{2NQ_e P}. \quad (2.25)$$

This non-vanishing of the area of the horizon in either of the two kinds of phase transition supports the earlier proposal that 5-branes are created, since the entropies of these black-hole configurations can be independently evaluated in terms of a microscopic counting of

string states propagating on the intersecting D1-D5 brane system [6]. One might envisage that although the black hole entropy, which is equal to one quarter of the area of the black hole event horizon, vanishes when the Yang-Mills instanton size is non-zero or the instantons are separated, it is possible that the total entropy, which is the sum of the black hole entropy and the entropy of the Yang-Mills excitations, may be conserved in the phase transition. It is worth mentioning that the dilaton behavior is quite different before and after the phase transition. Before the phase transition, the dilatonic scalars diverge on the horizon, with the consequence that the classical black-hole solution is not reliable for extracting information about physical quantities such as the entropy. After the phase transition, in which a fundamental 5-brane is created, the dilatons are stabilised on the horizon, and consequently the non-vanishing entropy can be evaluated by independent microscopic methods.

We have seen that the horizon has a curvature singularity when the instantons are of non-zero size and are separated but that, under either of the phase transitions, the horizon becomes regular once the point-source limit is reached. In $D = 5$, the near-horizon structure after the phase transition is $\text{AdS}_2 \times S^3$. From the ten-dimensional point of view, it is $\text{AdS}_3 \times S^3 \times T^4$. The AdS_3 is also known as the extremal BTZ black hole [9], which is a special case of the generalised Kaigorodov metric [10, 11]. Supergravity on this AdS_3 background is conjectured to be dual to a two-dimensional superconformal field theory on the boundary of the AdS_3 [8]. Thus the instanton phase transition can be viewed as a transition from supergravity theory to a two-dimensional conformal field theory.

Another physical quantity that undergoes a phase transition is the absorption rate for massless scalar waves. For the case when the instanton size a is non-zero, the near-horizon structure of the black hole is dominated by the electric string and wave charges and its low energy absorption cross-section is proportional to the frequency of the wave [12]. On the other hand, when the instanton scale size a goes to zero, the absorption cross-section approaches the area of the horizon in the low-frequency limit. To summarise, we have

$$\sigma \sim \begin{cases} 2\pi^2 (Q_e P) \omega & : a > 0 \\ A_{\text{horizon}} & : a = 0 \end{cases} \quad (2.26)$$

3 Heterotic string on K3

In the previous section, we obtained intersections of a string, a 5-brane and a pp-wave in the heterotic string, where the 5-brane carries magnetic charge supported by a Yang-Mills instanton or multi-instanton configuration. When dimensionally reduced on a torus to

$D = 6$, the intersection becomes that of a dyonic string with a pp-wave, where the magnetic charge of the string is supported by the Yang-Mills instanton, while the electric charge is associated with singular sources. Thus, in this case, the electric and the magnetic strings play very different rôles. In this section, we shall consider the heterotic string compactified on K3 rather than a 4-torus, in which case not only the magnetic strings, but also the electric strings, can be supported by Yang-Mills instantons.

3.1 $N = 1$ supergravity in $D = 6$

The heterotic string admits a compactification to $D = 6$ in which the internal four-dimensional manifold is taken to be K3. Various different six-dimensional theories can be obtained, with different Yang-Mills gauge groups, depending upon precisely how the $SU(2)$ -valued spin connection of the Ricci-flat Kähler K3 is embedded in the $E_8 \times E_8$ or $SO(32)$ gauge group of the ten-dimensional theory [13]. There will also be quantum corrections to the six-dimensional effective action, whose 1-loop structures can be determined by general arguments based on the necessary anomaly-freedom of the theory. The resulting six-dimensional theories are described by $N = 1$ supergravity, coupled to an $N = 1$ hypermultiplet and a Yang-Mills multiplet. The bosonic sectors comprise the metric $g_{\mu\nu}$, a dilaton ϕ , a 3-form field strength $F_{(3)}$, and the Yang-Mills fields $G_{(2)}^a$. The self-dual part of the 3-form field belongs to the gravity multiplet, while the anti-self-dual part and the dilaton belong to the hypermultiplet. The field equations, including the 1-loop terms, take the form [13]

$$\begin{aligned}
R_{\mu\nu} &= \frac{1}{2}\partial_\mu\phi\partial_\nu\phi + \frac{1}{4}e^{-2\alpha\phi}[F_{\mu\nu}^2 - \frac{1}{6}F_{(3)}^2 g_{\mu\nu}] \\
&\quad + \frac{1}{4}(v e^{-\alpha\phi} + \tilde{v} e^{\alpha\phi})[(G^a)_{\mu\nu}^2 - \frac{1}{8}(G_{(2)}^a)^2 g_{\mu\nu}], \\
d*d\phi &= \alpha e^{-2\alpha\phi} *F_{(3)} \wedge F_{(3)} + \frac{1}{2}\alpha(v e^{-\alpha\phi} - \tilde{v} e^{\alpha\phi}) *G_{(2)}^a \wedge G_{(2)}^a, \\
d(e^{-2\alpha\phi} *F_{(3)}) &= \frac{1}{2}v G_{(2)}^a \wedge G_{(2)}^a, \\
D[(v e^{-\alpha\phi} + \tilde{v} e^{\alpha\phi}) *G_{(2)}^a] &= v e^{-2\alpha\phi} *F_{(3)} \wedge G_{(2)}^a + \tilde{v} F_{(3)} \wedge G_{(2)}^a,
\end{aligned} \tag{3.1}$$

where $\alpha = 1/\sqrt{2}$. Here, D denotes the Yang-Mills-covariant exterior derivative, defined by

$$DX^a = dX^a - \epsilon_{abc} X^b \wedge B_{(1)}^c, \tag{3.2}$$

where, as previously, we restrict attention to an $SU(2)$ subgroup of the Yang-Mills gauge group. The constants v and \tilde{v} are rational numbers characteristic of the embedding of the $SU(2)$ holonomy group of K3 in the original $E_8 \times E_8$ or $SO(32)$ Yang-Mills gauge group in

$D = 10$. The terms associated with \tilde{v} come from 1-loop corrections. The field strengths are given in terms of potentials as follows:

$$\begin{aligned} F_{(3)} &= dA_{(2)} + \frac{1}{2}v\omega, \\ G_{(2)}^a &= dB_{(1)}^a + \frac{1}{2}\epsilon_{abc} B_{(1)}^b \wedge B_{(1)}^c. \end{aligned} \quad (3.3)$$

Here, ω is given by

$$\omega = B_{(1)}^a \wedge dB_{(1)}^a + \frac{1}{3}\epsilon_{abc} B_{(1)}^a \wedge B_{(1)}^b \wedge B_{(1)}^c, \quad (3.4)$$

and by construction it satisfies $d\omega = G_{(2)}^a \wedge G_{(2)}^a$.

The field equations (3.1) cannot be obtained from any Lagrangian. However, there is a closely-related system of field equations which, in particular, admit the same set of solutions that we wish to consider, which *can* be derived from a Lagrangian. If we consider the Lagrangian

$$\mathcal{L}_6 = R * \mathbb{1} - \frac{1}{2} * d\phi \wedge d\phi - \frac{1}{2} e^{-2\alpha\phi} * F_{(3)} \wedge F_{(3)} - \frac{1}{2} (v e^{-\alpha\phi} + \tilde{v} e^{\alpha\phi}) * G_{(2)}^a \wedge G_{(2)}^a + \frac{1}{2} \tilde{v} G_{(2)}^a \wedge G_{(2)}^a \wedge A_{(2)}, \quad (3.5)$$

it is easily seen that it correctly produces all except one of the equations of motion given in (3.1). The exception is the Yang-Mills equation, which turns out to be

$$D[(v e^{-\alpha\phi} + \tilde{v} e^{\alpha\phi}) * G_{(2)}^a] = v e^{-2\alpha\phi} * F_{(3)} \wedge G_{(2)}^a + \tilde{v} dA_{(2)} \wedge G_{(2)}^a - \frac{1}{4} v \tilde{v} G_{(2)}^a \wedge G_{(2)}^a \wedge B_{(1)}^a, \quad (3.6)$$

rather than the corresponding equation in (3.1).

The discrepancy between the Yang-Mills equations in (3.1) and (3.6) is a term of the form

$$\begin{aligned} &(F_{(3)} - dA_{(2)}) \wedge G_{(2)}^a + \frac{1}{4} v G_{(2)}^b \wedge G_{(2)}^b \wedge B_{(1)}^a \\ &= \frac{1}{2} v \left[(B_{(1)}^b \wedge dB_{(1)}^b + \frac{1}{3} \epsilon_{bcd} B_{(1)}^b \wedge B_{(1)}^c \wedge B_{(1)}^d) \wedge G_{(2)}^a + \frac{1}{2} G_{(2)}^b \wedge G_{(2)}^b \wedge B_{(1)}^a \right]. \end{aligned} \quad (3.7)$$

It is therefore evident, since this involves only the Yang-Mills fields, that if we consider instanton solutions where $B_{(1)}^a$ is non-vanishing only in the four-dimensional transverse space, then this 5-form will vanish. Thus for such configurations, the solutions of (3.1) and those following from (3.5) will coincide. Note that the theory admits two different types of global limit [14]. In one of the limits, the resulting flat-space theory admits a tensionless string as a solution [7]. The other is a further specialisation of the flat-space theory and had also been obtained in [15].

3.2 Gauge dyonic strings with pp-wave, and $D = 5$ black hole

The equations of motion (3.1) admit solutions describing the intersection of a dyonic string with a pp-wave, given by

$$\begin{aligned} ds_6^2 &= (H_e H_m)^{-1/2} (-W^{-1} dt^2 + W(dz + (W^{-1} - 1)dt)^2 + (H_e H_m)^{1/2} dy^i dy^i) , \\ \phi &= \frac{1}{\sqrt{2}} \log(H_m/H_e) , \\ F_{(3)} &= e^{-\sqrt{2}\phi} *(dt \wedge dz \wedge dH_m^{-1}) - dt \wedge dz \wedge dH_e^{-1} . \end{aligned} \quad (3.8)$$

Here, H_e , H_m and W satisfy

$$\square H_e = -\frac{1}{4}\tilde{v} G_{ij}^a G_{ij}^a , \quad \square H_m = -\frac{1}{4}v G_{ij}^a G_{ij}^a , \quad \square W = 0 . \quad (3.9)$$

Thus we have

$$H_e = 1 + 2\tilde{v}\psi , \quad H_m = 1 + 2v\psi , \quad W = 1 + \sum_{\alpha'} \frac{2P_{\alpha'}}{|\vec{y} - \vec{y}'_{\alpha'}|^2} , \quad (3.10)$$

where for multi-instantons, ψ is given by (2.10). (The solution with no pp-wave was obtained in [7].) Thus we see that when the size of an instanton vanishes, or when two instantons coalesce, there is a creation not only of a magnetic string, coming from the dimensional reduction of the 5-brane in $D = 10$, but also of an electric string.

The dyonic string solution (3.8) can be dimensionally reduced on the z coordinate, giving rise to a $D = 5$ three-charge black hole. The form of the solution is the same as given in (2.23), except that now the functions H_e , H_m are given by (3.10) instead of (2.9). Equations of motion that describe these black holes can be derived from the five-dimensional Lagrangian

$$\begin{aligned} \mathcal{L}_5 &= R * \mathbf{1} - \frac{1}{2} * d\vec{\phi} \wedge d\vec{\phi} - \frac{1}{2} e^{\vec{a} \cdot \vec{\phi}} * F_{(3)} \wedge F_{(3)} - \frac{1}{2} e^{\vec{d} \cdot \vec{\phi}} * F_{(2)} \wedge F_{(2)} \\ &\quad - \frac{1}{2} e^{\vec{b} \cdot \vec{\phi}} * \mathcal{F}_{(2)} \wedge \mathcal{F}_{(2)} - \frac{1}{2} v e^{\vec{c} \cdot \vec{\phi}} * G_{(2)}^a \wedge G_{(2)}^a - \frac{1}{2} \tilde{v} e^{\vec{c} \cdot \vec{\phi}} * G_{(2)}^a \wedge G_{(2)}^a \\ &\quad + \frac{1}{2} \tilde{v} A_{(1)} \wedge G_{(2)}^a \wedge G_{(2)}^a , \end{aligned} \quad (3.11)$$

where the dilaton vectors \vec{a} , \vec{b} , \vec{c} and \vec{d} are given by (2.22), and $\vec{c} = (1/\sqrt{2}, 1/\sqrt{6})$. This is obtained by dimensional reduction of the $D = 6$ Lagrangian (3.5). Again, this produces equations of motion which do not coincide precisely with those of the dimensionally-reduced string (which cannot themselves be derived from a Lagrangian). However, the discrepancies between the string equations of motion and those following from (3.11) are terms which vanish for the configurations we are considering.

The discussion of the entropy of the 3-charge black hole is analogous to the previous case. When the instanton size is non-zero, the entropy vanishes; when the size becomes

zero, the area of the horizon becomes non-zero and hence the entropy is non-vanishing. The singular horizon becomes regular and, in terms of the ten-dimensional point of view, the near horizon structure is $\text{AdS}_3 \times S^3 \times \text{K3}$. Before the phase transition, the metric has an *almost* naked singularity, which can be reached in a logarithmically-divergent time by a null geodesic. A closely-related feature is that the absorption cross-section for scalar waves vanishes below a certain frequency threshold [12]. After the phase transition, the metric becomes regular and at low frequencies the absorption cross-section is approximately equal to non-vanishing area of the horizon. Prior to any phase transition, there is one significant difference between this solution and the one obtained from the heterotic string on torus, which we discussed previously. Here, owing to the fact that both the string and the 5-brane are supported by Yang-Mills instantons, it follows that the six-dimensional dilaton remains finite as r tends to zero. Thus the six-dimensional dyonic string with a pp-wave has a regular horizon both before and after the phase transition. This should be contrasted with the previous 5-brane example, where the associated intersection has a regular horizon only after the phase transition occurs.

4 Conclusions

In this paper, we have studied certain extremal p -brane configurations in which one or more of the charges are supplied by Yang-Mills instantons in a four-dimensional transverse space. Previously known examples were the gauge 5-brane in the ten-dimensional heterotic theory [5] and the gauge dyonic string in the theory in six dimensions obtained by compactifying the heterotic string on K3 [7]. If the gauge 5-brane is compactified on T^5 , or the gauge dyonic string is compactified on S^1 , one obtains in either case a five-dimensional black hole. The former gives a 1-charge magnetic black hole, while the latter gives a 2-charge dyonic black hole. In both cases, the charges are “smeared out” by the Yang-Mills instanton construction.

It is of interest to study configurations that correspond to 3-charge black holes in five dimensions, since then one has the possibility of having a non-zero entropy even for extremal configurations. For this reason, we constructed generalisations of the previously-known gauge solutions, namely a gauge 5-brane intersecting with a string and a pp-wave in $D = 10$ and a gauge dyonic string intersecting with a pp-wave in $D = 6$. These give rise to five-dimensional 3-charge black holes with one smeared charge or two smeared charges respectively, with the remainder being standard point-source charges.

We showed that, as long as the Yang-Mills instantons are non-degenerate, the entropies

of the 3-charge black holes vanish. Indeed, from this point of view, the smeared charges coming from the instantons contribute little to the horizon structures and the black holes are more like those with the correspondingly fewer number of “genuine” point charges. However, we also showed that, if certain singular limits of the instanton configurations are taken, the resulting black holes undergo phase transitions in which they acquire the non-vanishing entropy associated with the usual 3-charge black holes.

We exhibited two different kinds of degenerate limits for the Yang-Mills instanton configurations, each of which leads to such phase transitions. One of these is the situation where the scale-size of an instanton goes to zero, leading to the appearance of a single unit of point charge at the location of the associated instanton center. Another, perhaps more surprising, degenerate limit occurs if two previously-separated instantons come into coincidence. This leads to a configuration with a superposed instanton and a unit point charge at the coincidence point. In either of these cases, the emergence of the point charge in the transverse space in the singular limit gives rise to the phase transition. This singularity is nothing but the fundamental 5-brane or string charge.

Appendix

A $SU(2)$ Yang-Mills instantons

The solutions that we consider in this paper all involve the use of an $SU(2)$ Yang-Mills instanton in the four-dimensional transverse space. The simplest such solution is the BPST single instanton, which is spherically symmetric. This is most elegantly described by writing the metric on the flat transverse space in terms of hyperspherical polar coordinates, as

$$ds^2 = dr^2 + \frac{1}{4}r^2 (\sigma_1^2 + \sigma_2^2 + \sigma_3^2) , \quad (\text{A.1})$$

where the σ_a are the three left-invariant 1-forms on the 3-sphere, satisfying the equation $d\sigma_a = -\frac{1}{2}\epsilon_{abc} \sigma_b \wedge \sigma_c$. The instanton is obtained by making the ansatz

$$B_{(1)}^a = h \sigma_a , \quad (\text{A.2})$$

where h is a function only of r . A simple symmetrical ansatz of this type is possible because we are considering a Yang-Mills instanton with $SU(2)$ gauge group, which coincides with the left-acting symmetry group of the 3-sphere. It is elementary to calculate the Yang-Mills field strengths $G_{(2)}^a$ for the ansatz (A.2) and then to show that the self-duality equations

are satisfied if $r h' = 2h(h - 1)$. The general solution of this equation is

$$h = \frac{a^2}{a^2 + r^2}, \quad (\text{A.3})$$

where a is an arbitrary constant which sets the scale-size of the instanton. The Yang-Mills field strength is therefore given by

$$G_{(2)}^a = -\frac{4a^2}{(a^2 + r^2)^2} (e^0 \wedge e^a + \frac{1}{2}\epsilon_{abc} e^b \wedge e^c), \quad (\text{A.4})$$

where $e^0 = dr$ and $e^a = \frac{1}{2}r \sigma_a$ is a vielbein basis for (A.1). Note that $G_{(2)}^a$ is manifestly self dual. One easily verifies from (A.4) that $G_{ij}^a G_{ij}^a = 192a^4 (a^2 + r^2)^{-4}$ and, hence, that

$$G_{ij}^a G_{ij}^a = -8\Box\psi, \quad \text{where} \quad \psi = \frac{r^2 + 2a^2}{(r^2 + a^2)^2}, \quad (\text{A.5})$$

where \Box here denotes the scalar Laplacian in the four-dimensional flat transverse-space metric (A.1). Note that the local solution for ψ is ambiguous up to the addition of a harmonic term k/r^2 , and we have resolved this ambiguity by choosing k so that ψ has no singularity at $r = 0$.

The general multi-instanton solutions are most completely described by the ADHM construction [16]. Sub-classes of solution are describable using more elementary methods [17, 18]. For this purpose, it is convenient to write the metric on the four-dimensional transverse space in Cartesian coordinates y^i , for $i = 0, 1, 2, 3$, as $ds^2 = dy^i dy^i$. Let us define the anti-self-dual 't Hooft tensors η_{ij}^a , which are antisymmetric and anti-self-dual in ij . Thus

$$\eta^a = \frac{1}{2}\eta_{ij}^a dy^i \wedge dy^j = -dy^0 \wedge dy^a + \frac{1}{2}\epsilon_{abc} dy^b \wedge dy^c. \quad (\text{A.6})$$

In other words, $\eta_{0b}^a = -\delta_b^a$, $\eta_{b0}^a = \delta_b^a$ and $\eta_{bc}^a = \epsilon_{abc}$. The ansatz for the Yang-Mills potentials is

$$B_{(1)}^a = -\eta_{ij}^a \partial_i \tilde{f} dy^j. \quad (\text{A.7})$$

After a little algebra, one finds that self-duality $*G_{(2)}^a = G_{(2)}^a$ implies the equation $\Box\tilde{f} + \partial_i \tilde{f} \partial_i \tilde{f} = 0$, which is solved by taking $\tilde{f} = \log f$, where f satisfies $\Box f = 0$. Thus we have instanton solutions with

$$f = \epsilon + \sum_{\alpha=1}^N \frac{\lambda_\alpha}{|\vec{y} - \vec{y}_\alpha|^2}, \quad (\text{A.8})$$

where ϵ is a constant that can be taken to be either 1 or 0, and λ_α and \vec{y}_α are constant strengths and positions for the singularities in f . When $\epsilon = 1$, they have rather direct interpretations as scale sizes and positions for N separated Yang-Mills instantons [17].

When $\epsilon = 0$, the interpretation is more subtle and (A.8) then actually describes an $(N - 1)$ -instanton solution, with scale sizes and locations that are rather complicated functions of the λ_α and \vec{y}_α parameters.

After further algebra, one can show that, for these multi-instanton solutions, we have

$$G_{ij}^a G_{ij}^a = -2\Box\Box \log f . \quad (\text{A.9})$$

This means that we may write $G_{ij}^a G_{ij}^a$ as

$$G_{ij}^a G_{ij}^a = -8\Box\psi , \quad \text{where} \quad \psi = \frac{1}{4}\Box \log f + h , \quad (\text{A.10})$$

f is the harmonic function given in (A.8) and h is an arbitrary harmonic function. As in the single-instanton example discussed above, we may exploit the freedom to add such an harmonic function in order to ensure that ψ itself is non-singular at the locations of the instantons. This is discussed in section 2. We consider, for convenience, the case where $\epsilon = 1$ in (A.8), since then the parameters λ_α and \vec{y}_α have clearer interpretations.

To see how the parameters may be interpreted, consider the special case $N = 1$ in (A.8), with $\epsilon = 1$. Without loss of generality, we may take $\vec{y}_\alpha = 0$ and $\lambda_\alpha = \lambda$, so that

$$f = 1 + \frac{\lambda}{r^2} , \quad (\text{A.11})$$

where $r = |\vec{y}|$. Evaluating ψ as given in (A.10), with h chosen to be $1/r^2$, we obtain $\psi = (2\lambda + r^2)(\lambda + r^2)^{-2}$. Comparing with (A.5), we see that the $N = 1$ solution has precisely the interpretation of a single Yang-Mills instanton of size $a = \sqrt{\lambda}$, located at $\vec{y} = 0$. The general N -instanton solution (A.8) with $\epsilon = 1$ describes instantons of size $\sqrt{\lambda_\alpha}$ centered on locations \vec{y}_α .

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