

# Penguin Contribution to the Phase of $B_s$ - $\bar{B}_s$ Mixing and $B_s \rightarrow \mu\mu$ in Grand Unified Theories

Bhaskar Dutta and Yukihiro Mimura

*Department of Physics, Texas A&M University, College Station, TX 77843-4242, USA*

## Abstract

We investigate the possibility of a large  $B_s$ - $\bar{B}_s$  mixing phase in the context of grand unified theory (GUT) models, e.g., SO(10) and SU(5). In these models, we find that a large phase of  $B_s$  mixing is correlated with  $\text{Br}(b \rightarrow s\gamma)$ ,  $\text{Br}(\tau \rightarrow \mu\gamma)$  and  $\text{Br}(B_s \rightarrow \mu\mu)$  for large  $\tan\beta$ . In the case of the SO(10) model, the large phase of  $B_s$  mixing is correlated with  $\text{Br}(b \rightarrow s\gamma)$  and  $\text{Br}(B_s \rightarrow \mu\mu)$  and we find that a large  $B_s$  mixing corresponds to an enhanced  $\text{Br}(B_s \rightarrow \mu\mu)$  about to be probed by the Tevatron. In the case of the SU(5) model, the large phase is correlated with  $\text{Br}(\tau \rightarrow \mu\gamma)$  and  $\text{Br}(B_s \rightarrow \mu\mu)$ . In this case, the  $\text{Br}(\tau \rightarrow \mu\gamma)$  constraint requires a smaller pseudo-scalar Higgs mass which in turn generates a large  $\text{Br}(B_s \rightarrow \mu\mu)$  almost at the edge of present experimental constraint. If the present observation of large phase of  $B_s$  mixing persists in the upcoming data, using all these branching ratios, we will be able to distinguish these models.

# 1 Introduction

Recently, CDF and DØ collaborations have announced the analysis of the flavor-tagged  $B_s \rightarrow J/\psi\phi$  decay. The decay width difference and the mixing induced CP violating phase,  $\phi_s$ , were extracted from their analysis [1]. In the Standard Model (SM), the CP violating phase is predicted to be small,  $\phi_s = 2\beta_s \equiv 2 \arg(-V_{ts}V_{tb}^*/V_{cs}V_{cb}^*) \simeq 0.04$ . However, the measurements of the phase are large:

$$\phi_s(\text{CDF}) \in [0.28, 1.29] \text{ (68\% C.L.)}, \quad (1)$$

$$\phi_s(\text{DØ}) = 0.57_{-0.24}^{+0.30}(\text{stat})_{-0.07}^{+0.02}(\text{syst}). \quad (2)$$

The UTfit group made a combined data analysis including the semileptonic asymmetry in the  $B_s$  decay, and find that the CP violating phase deviates more than  $2.5\sigma$  from the SM prediction [2]. If this large phase still persists in the upcoming results from Fermilab, it implies the existence of new physics (NP) beyond SM and that the NP model requires a flavor violation in  $b$ - $s$  transition as well as a phase in the transition.

The nature of flavor changing neutral currents (FCNCs) and the CP violating phase is very important to test the existence of new physics beyond the standard model. Supersymmetry (SUSY) is the most attractive candidate to build NP models. The gauge hierarchy problem can be solved and a natural aspect of the theory can be developed from the weak scale to the ultra high energy scale. In fact, the gauge coupling constants in the standard model can unify at a high scale using the renormalization group equations (RGEs) involving the particle contents of the minimal SUSY standard model (MSSM), which indicates the existence of grand unified theories (GUTs). The well motivated SUSY GUTs have always been subjects of intense experimental and theoretical investigations. Identifying a GUT model will be a major focus of the upcoming experiments.

In SUSY models, the SUSY breaking mass terms for squarks and sleptons must be introduced, and they have sources of FCNCs and CP violation beyond the Kobayashi-Maskawa theory. In general, they generate too large FCNCs, and thus the flavor universality is often assumed in squark and slepton mass matrices to avoid the large FCNCs in the meson mixings and the lepton flavor violations (LFV) [3]. The flavor universality is expected to be realized by the Planck scale physics. However, even if the flavor universality is realized at a scale such as the GUT scale or the Planck scale, the non-universality in the SUSY breaking sfermion masses is generated from the evolution of RGEs, and they can generate a small flavor violating transitions, which can be observed in the ongoing experiments.

In the MSSM with right-handed neutrinos, the induced FCNCs from RGE effects are not large in the quark sector, while sizable effects can be generated in the lepton sector due to the

large neutrino mixing angles [4]. In GUTs, the loop effects due to the large neutrino mixings can also induce sizable effects in the quark sector since GUT scale particles can propagate in the loops [5]. As a result, the patterns of the induced FCNCs highly depend on the unification scenario, and the contents of the heavy particles. Therefore, it is important to investigate the FCNC effects to obtain a footprint of the GUT models. If the quark-lepton unification is manifested in GUT models, the flavor violation in  $b$ - $s$  transition can be responsible for the large atmospheric neutrino mixing [6], and thus, the amount of the flavor violation in  $b$ - $s$  transition (the second and the third generation mixing), which is related to the  $B_s$ - $\bar{B}_s$  mixing and its phase, has to be related to the  $\tau \rightarrow \mu\gamma$  decay [7, 8, 9, 10] for a given particle spectrum. The branching ratio of the  $\tau \rightarrow \mu\gamma$  is being measured at the  $B$ -factory, and thus, the future results of LFV and the ongoing measurement of the phase of  $B_s$ - $\bar{B}_s$  mixing will provide an important information to probe the GUT scale physics.

In Ref.[9], we have studied the correlation between  $\text{Br}(\tau \rightarrow \mu\gamma)$  and  $\phi_s$ , the phase in  $B_s$ - $\bar{B}_s$  mixing, comparing SU(5) and SO(10) GUT models, and investigated the constraints in these models from the observations in order to decipher GUT models. The flavor violation originating from the loop correction via the heavy particles can be characterized by the CKM (Cabibbo-Kobayashi-Maskawa) quark mixing matrix and the MNSP (Maki-Nakagawa-Sakata-Pontecorvo) neutrino mixing matrix, as well as the size of the Yukawa couplings. Since the CKM mixings are small, it is expected that the neutrino mixings dominate the source of FCNCs at low energy. It is important to know whether the large neutrino mixings originate from the Dirac neutrino Yukawa coupling or the Majorana-type Yukawa coupling. When the large neutrino mixings originate from the Dirac neutrino Yukawa couplings in a GUT model, the (squared) right-handed down-type squark mass matrix,  $M_{\bar{D}c}^2$ , as well as the left-handed lepton doublet mass matrix,  $M_L^2$ , can have flavor non-universality. When the large mixings originate from the Majorana Yukawa couplings, the left-handed squark mass matrix,  $M_Q^2$ , can also have flavor non-universality in addition to the other sfermions.

In the minimal-type of SU(5) GUT, the large neutrino mixing originates from the Dirac neutrino coupling if there is no fine-tuning in the seesaw neutrino matrix. On the other hand, in the minimal-type of SO(10) GUT, the large neutrino mixing can originate from the Majorana-type coupling. In general, since SU(5) is a subgroup of SO(10), one can construct a model where the neutrino mixing originate from the Majorana-type coupling in non-minimal-type of SU(5) GUT. Also, if we allow the fine-tuning in the Yukawa coupling matrices, the Dirac neutrino Yukawa coupling can be the source of the large mixing even in the SO(10) model. Actually, there is a little ambiguity to determine the minimal SU(5) or SO(10) GUT model, since minimal versions of the GUT models have problems with phenomenology. (That is why

we call them minimal-type.) Here, we call the typical boundary condition as minimal-type of SU(5) GUT condition when the off-diagonal elements of  $M_{\tilde{D}^c}^2$  and  $M_{\tilde{L}}^2$  are correlated due to the Dirac neutrino coupling in GUT models. The other boundary condition where the  $M_{\tilde{Q}}^2$  is also correlated to  $M_{\tilde{D}^c, \tilde{U}^c}^2$  and  $M_{\tilde{L}}^2$  due to the Majorana coupling in SO(10) model is called as minimal-type of SO(10) GUT boundary condition. The large phase of  $B_s$ - $\bar{B}_s$  mixing, as well as the other flavor violating processes, can tell us which type of boundary condition is preferable.

We analyzed the case of lower  $\tan\beta$  (which is a ratio of the vacuum expectation values of up- and down-type Higgs fields) in the Ref.[9]. In such a case, the box diagram contribution will dominate the SUSY contribution of  $B_s$ - $\bar{B}_s$  mixing amplitude, and we found that the SO(10) boundary condition is more important to obtain the large phase of  $B_s$ - $\bar{B}_s$  mixing. When  $\tan\beta$  is large, the so called double penguin contribution [11, 12] can dominate the SUSY contribution rather than the box contribution unless the pseudo Higgs field is heavy. In such cases, the  $B_s \rightarrow \mu\mu$  decay [13, 12] will be enhanced close to its experimental bound [14]. In other words, if the large phase of  $B_s$ - $\bar{B}_s$  mixing originates from the double penguin contribution, the  $B_s \rightarrow \mu\mu$  decay will be observed very soon, and it is worth to examine the constraints if a large phase is really generated from the double penguin contribution. In this paper, we will investigate the double penguin contribution of the  $B_s$ - $\bar{B}_s$  mixing, as well as the other flavor violating processes including  $B_s \rightarrow \mu\mu$ ,  $b \rightarrow s\gamma$ , and  $\tau \rightarrow \mu\gamma$  in the context of SO(10) and SU(5) models.

The paper is organized as follows: In section 2, we will describe the FCNC sources in SUSY GUT models. The two typical boundary conditions in both SU(5) and SO(10) model are considered. In section 3, we will describe the SUSY contributions of  $B_s$ - $\bar{B}_s$  mixing amplitudes, including the box diagram and the double penguin contribution. The constraint from  $B_s \rightarrow \mu\mu$ ,  $b \rightarrow s\gamma$ ,  $\tau \rightarrow \mu\gamma$  in the models are also noted. In section 4, we will show our numerical work on the both kinds of the GUT models. Section 5 devotes the conclusion and remarks.

## 2 FCNC sources in SUSY GUTs

In SUSY theories, the SUSY breaking terms can be the sources of flavor violations. In general, it is easy to include sources of flavor violation by hand since the SUSY breaking masses with flavor indices are parameters in the MSSM. However, if these parameters are completely general, too much FCNCs are induced [3]. Therefore, as a minimal assumption of the SUSY breaking, the universality of scalar masses is often considered, which means that all the SUSY breaking (squared) scalar masses are universal to be  $m_0^2$ , and the scalar trilinear couplings are proportional to Yukawa couplings (the coefficient is universal to be  $A_0$ ) at a unification scale. Even if the universality is assumed, the non-universality in scalar masses is generated from the evolution of the theory from the GUT scale down to the weak scale via RGEs. In the MSSM

with right-handed neutrino ( $N^c$ ), the induced FCNCs from RGE effects are not large in the quark sector, while sizable effects can be generated in the lepton sector due to the large neutrino mixings [4]. The sources of FCNCs in the model are the Dirac neutrino couplings.

In GUT models, the left-handed lepton doublet ( $L$ ) and the right-handed down-type squarks ( $D^c$ ) are unified in  $\bar{\mathbf{5}}$ , and the Dirac neutrino couplings can be written as  $Y_\nu \bar{\mathbf{5}} N^c H_{\mathbf{5}}$ . As a result, non-universality in the SUSY breaking mass matrix for  $D^c$  is also generated from the colored-Higgs and right-handed neutrino loop diagram, and the flavor violation in the quark sector can be generated from the Dirac neutrino couplings [5, 6].

The light neutrino mass matrix is written as

$$\mathcal{M}_\nu^{\text{light}} = f \langle \Delta_L \rangle - Y_\nu M_R^{-1} Y_\nu^T \langle H_u^0 \rangle^2, \quad (3)$$

where  $\Delta_L$  is an  $SU(2)_L$  triplet, and  $f$  is a Majorana coupling  $\frac{1}{2} LL \Delta_L$ . The second term is called type I seesaw term [15]. If the type I seesaw term dominates the light neutrino mass, the Dirac neutrino coupling will have large mixings to explain the large neutrino mixings in the basis where the charge-lepton Yukawa coupling  $Y_e$  is diagonal. On the other hand, when the first term (triplet term) dominates it (type II seesaw [16]), the Majorana coupling must have the large mixings. Distinguishing these two cases is very important in order to understand the source of FCNCs in the GUT models.

Let us first describe the non-universality from the Dirac neutrino couplings. We will work in a basis where the charged-lepton Yukawa matrix,  $Y_e$ , and the right-handed neutrino Majorana mass matrix,  $M_R$ , are diagonal,

$$M_R = \text{diag}(M_1, M_2, M_3). \quad (4)$$

The neutrino Dirac Yukawa coupling matrix is written as

$$Y_\nu = U_L Y_\nu^{\text{diag}} U_R^T, \quad (5)$$

where  $U_{L,R}$  are diagonalizing unitary matrices. We note that  $U_L$  corresponds to the (conjugate of) MNSP neutrino mixing matrix,  $U_{\text{MNSP}}$ , in type I seesaw, up to a diagonal phase matrix if  $U_R$  is exactly same as  $\mathbf{1}$  (identity matrix), which we will assume for simplicity. Through RGEs, the off-diagonal elements of the SUSY breaking mass matrix for the left-handed lepton doublet gets the following correction

$$\delta M_{Lij}^2 \simeq -\frac{1}{8\pi^2} (3m_0^2 + A_0^2) \sum_k (Y_\nu)_{ik} (Y_\nu^*)_{jk} \ln \frac{M_*}{M_k}, \quad (6)$$

where  $M_*$  is a cutoff scale and the SUSY breaking parameters are universal. Neglecting the threshold of the GUT and the Majorana mass scales, we can write down the boundary conditions

as

$$M_{\mathbf{5}}^2 = M_{D^c}^2 = M_L^2 = m_0^2 \left( \mathbf{1} - \kappa U_L \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} U_L^\dagger \right), \quad (7)$$

where  $\kappa \simeq (Y_\nu^{\text{diag}})_{33}^2 (3 + A_0^2/m_0^2)/8\pi^2 \ln M_*/M_{\text{GUT}}$ , and  $k_2 \simeq \sqrt{\Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2} M_2/M_3$ . We parameterize the unitary matrix  $U_L$  as

$$U_L = \begin{pmatrix} e^{i(\alpha_1-\delta)} & & \\ & e^{i\alpha_2} & \\ & & 1 \end{pmatrix} \begin{pmatrix} c_{12}^e c_{13}^e & s_{12}^e c_{13}^e & s_{13}^e e^{i\delta} \\ -s_{12}^e c_{23}^e - c_{12}^e s_{23}^e s_{13}^e e^{-i\delta} & c_{12}^e c_{23}^e - s_{12}^e s_{13}^e s_{23}^e e^{-i\delta} & c_{13}^e s_{23}^e \\ s_{12}^e s_{23}^e - c_{12}^e c_{23}^e s_{13}^e e^{-i\delta} & -c_{12}^e s_{23}^e - s_{12}^e s_{13}^e c_{23}^e e^{-i\delta} & c_{13}^e c_{23}^e \end{pmatrix}, \quad (8)$$

where  $s_{ij}^e$  and  $c_{ij}^e$  are sin and cos of mixing angles  $\theta_{ij}$ . In the limit  $k_{1,2} \rightarrow 0$ ,  $\alpha_1$  and  $\alpha_2$  are the phases of the 13 and the 23 element of  $M_{\mathbf{5}}^2$ . Since we are assuming that  $U_R = \mathbf{1}$ ,  $\theta_{12}$  and  $\theta_{23}$  correspond to solar and atmospheric neutrino mixings, respectively, which are large. Even if we do not assume  $U_R = \mathbf{1}$ , the angle  $\theta_{23}$  is expected to be large unless there exists a fine-tuned relation among  $Y_\nu^{\text{diag}}$  and  $M_N$ . Assuming that the Dirac neutrino Yukawa coupling is hierarchical ( $k_1, k_2 \ll 1$ ), we obtain the 23 element of  $M_{\mathbf{5}}^2$  as  $-1/2 m_0^2 \kappa \sin 2\theta_{23} e^{i\alpha_2}$ . Therefore, the magnitude of the FCNC between 2nd and 3rd generations is controlled by  $\kappa \sin 2\theta_{23}$ . The phase  $\alpha_2$  will be the origin of a phase of SUSY contribution of  $B_s$ - $\bar{B}_s$  mixing amplitude. The SUSY breaking mass for  $\mathbf{10}$  multiplet ( $Q, U^c, E^c$ ) is also corrected by the (colored-)Higgsino loop, but it arises from CKM mixings and the effect is small. So, the boundary condition at the GUT scale for  $\mathbf{10}$  multiplet is

$$M_{\mathbf{10}}^2 = M_Q^2 = M_{U^c}^2 = M_{E^c}^2 \simeq m_0^2 \mathbf{1}. \quad (9)$$

The boundary conditions, Eqs.(7,9), are the typical boundary conditions in the case of minimal kind of SU(5) GUT with type I seesaw [9, 17].

The Yukawa coupling matrices for up- and down-type quarks and charged-leptons are given as

$$Y_u = V_L V_{\text{CKM}}^T Y_u^{\text{diag}} P_u V_{uR}^T, \quad (10)$$

$$Y_d = V_L Y_d^{\text{diag}} P_d V_{dR}^T, \quad (11)$$

$$Y_e = Y_e^{\text{diag}} P_e, \quad (12)$$

where  $Y_{u,d,e}^{\text{diag}}$  are real (positive) diagonal matrices and  $P_{u,d,e}$  are diagonal phase matrices. In the minimal SU(5) GUT, in which only  $H_{\mathbf{5}}$  and  $H_{\bar{\mathbf{5}}}$  couple to matter fields, we have  $V_{uR} = V_{\text{CKM}}^T$ ,  $V_L = V_R = \mathbf{1}$ , and  $Y_d^{\text{diag}} = Y_e^{\text{diag}}$ . Because it will give us a wrong prediction to the quark and charged-lepton masses, we need at least a slight modification from the minimal assumption. Even if there is a slight modification of the Yukawa coupling, we assume that the unitary matrix

$V_{dR}$  does not have large mixings. If  $V_{dR}$  has a large mixing, the FCNC sources in  $M_{D^c}^2$  may be cancelled in the basis where the down-type quark mass matrix is diagonal.

Next, let us consider the case of type II seesaw in the framework of SO(10) GUT models [18, 19]. All matter fields are unified in the spinor representation **16** in the SO(10) models. Since the right-handed neutrino is also unified to other matter fields, the neutrino Dirac Yukawa coupling does not have large mixings (*i.e.*  $U_L \simeq \mathbf{1}$ ) if there is no large cancellation in the Yukawa couplings. In this case, as we have mentioned, the proper neutrino masses with large mixings can be generated from the Majorana couplings  $\frac{1}{2}fLL\Delta_L$ . Due to the unification under SO(10), the left-handed Majorana coupling,  $f$ , is tied to all other matter fields, and therefore, the off-diagonal terms in the sparticle masses are induced by loop effect which are proportional to  $ff^\dagger$ . Neglecting the GUT scale threshold, we can write the boundary condition in SO(10) as

$$M_{\mathbf{16}}^2 = m_0^2 \left( \mathbf{1} - \kappa U \begin{pmatrix} k_1 & & \\ & k_2 & \\ & & 1 \end{pmatrix} U^\dagger \right), \quad (13)$$

where  $\kappa \simeq 15/4 (f_{33}^{\text{diag}})^2 (3 + A_0^2/m_0^2)/8\pi^2 \ln M_*/M_{\text{GUT}}$ , and  $k_2 \simeq \Delta m_{\text{sol}}^2/\Delta m_{\text{atm}}^2$  in this case. Note that the parameters  $\kappa, k_{1,2}$  are of course different from those given in Eq.(7) using the set-up for type I seesaw, but we use the same notation to simplify the description. The unitary matrix  $U$  is the (conjugate of) MNSP neutrino mixing matrix up to a diagonal phase matrix, which is parameterized in the same way as Eq.(8). The Yukawa couplings are also given as Eq.(10,11,12). If we do not employ **120** Higgs fields, the Yukawa matrices are symmetric, and thus,  $V_{uR} = V_L V_{\text{CKM}}^T$ ,  $V_{dR} = V_L$ . The unitary matrix  $V_L$  is expected to be close to  $\mathbf{1}$  if there is no huge fine-tuning in the fermion mass fits.

Note that the sources of phases are not only in the unitary matrix  $U$  but also in the phase matrix  $P_d$  in the Yukawa coupling. Actually, in the basis where down-type quark mass matrix is a real (positive) diagonal matrix, the phases of the 23 elements in  $M_Q^2$  and  $M_{D^c}^2$  are independent, and these are two independent phase parameters which act as FCNC and CP violating sources for  $b$  to  $s$  transition.

If the SO(10) symmetry is manifested above the GUT symmetry breaking threshold, the off-diagonal elements of SUSY breaking sfermion mass matrices are unified at the GUT scale. However, depending on a Higgs spectrum, the symmetry breaking of the SO(10) symmetry may not happen at a single scale. Actually, the Higgs spectrum from **126** Higgs can be split depending on a vacuum of the SO(10) symmetry breaking. At that time, the magnitude of the off-diagonal elements depends on the sfermion species:

$$M_F^2 = m_0^2 [\mathbf{1} - \kappa_F U \text{diag}(k_1, k_2, 1) U^\dagger], \quad (14)$$

where  $F = Q, U^c, D^c, L, E^c$ . The quantity  $\kappa_F$  denotes the amount of the off-diagonal elements and it depends on the sfermion species. For example, only uncolored GUT particles are light compared to the others as a result of the SO(10) breaking,  $\kappa_L$  and  $\kappa_{E^c}$  are larger than the others, and the lepton flavor violation will be enhanced rather than the quark flavor violation.

It is interesting that the flavor violation pattern in the lepton sector and the quark sector can depend on the SO(10) symmetry breaking vacua. Actually, in order to forbid a rapid proton decay, the quark flavor violation should be larger than the lepton flavor violation among the symmetry breaking vacua [20]. Namely, it is expected that  $\kappa_Q, \kappa_{U^c}$ , and  $\kappa_{D^c}$  are much larger than  $\kappa_L$  and  $\kappa_{E^c}$ . For example, if only the Higgs fields  $(\mathbf{8}, \mathbf{2}, \pm 1/2)$  are light compared to the breaking scale (which is the most suitable case), one obtains  $\kappa_Q = \kappa_{U^c} = \kappa_{D^c}$ , and only quark flavor violation is generated, while lepton flavor violation is not. On the other hand, when the flavor violation is generated from the minimal-type of SU(5) vacua with type I seesaw, the quantities  $\kappa$ 's have relations as  $\kappa_L \sim \kappa_{D^c}$ , and  $\kappa_Q, \kappa_{U^c}, \kappa_{E^c} \sim 0$ , effectively. Actually, when we take into account the threshold effect, it is expected that  $\kappa_L$  is always larger than  $\kappa_{D^c}$  since the right-handed Majorana mass scale is less than the scale of colored Higgs mass. Therefore, the existence of  $b$ - $s$  transition indicated by the experimental results in Fermilab predicts the sizable lepton flavor violation in the minimal-type of SU(5) model. Thus, if the results of large  $B_s$ - $\bar{B}_s$  phase is really an evidence of NP, the GUT models are restricted severely [8, 9, 10].

Therefore, investigating the quark and lepton flavor violation is very important to decipher the GUT symmetry breaking, when the  $B_s$ - $\bar{B}_s$  phase is large [9].

### 3 $B_s$ - $\bar{B}_s$ mixing and the other flavor violating processes

Let us briefly see the phase of  $B_s$ - $\bar{B}_s$  mixing. We use the model-independent parameterization of the NP contribution:

$$C_{B_s} e^{2i\phi_{B_s}} = M_{12}^{\text{full}} / M_{12}^{\text{SM}}, \quad (15)$$

where ‘full’ means the SM plus NP contribution,  $M_{12}^{\text{full}} = M_{12}^{\text{SM}} + M_{12}^{\text{NP}}$ . The NP contribution can be parameterized by two real parameters  $C_{B_s}$  and  $\phi_{B_s}$ . The time dependent CP asymmetry ( $S = \sin \phi_s$ ) in  $B_s \rightarrow J/\psi\phi$  is dictated by the argument of  $M_{12}^{\text{full}}$ :  $\phi_s = -\arg M_{12}^{\text{full}}$ , and thus  $\phi_s = 2(\beta_s - \phi_{B_s})$ . It is important to note that the large SUSY contribution is still allowed even though the mass difference of  $B_s$ - $\bar{B}_s$  [21] is fairly consistent with the SM prediction. This is because the mass difference,  $\Delta M_{B_s}$ , can be just twice the absolute value of  $M_{12}^{\text{full}}$ . The consistency of the mass difference between the SM prediction and the experimental measurement just means  $C_{B_s} \sim 1$ , and a large  $\phi_{B_s}$  is still allowed. For example, when  $C_{B_s} \simeq 1$ , the phase  $\phi_{B_s}$  is related as  $2 \sin \phi_{B_s} \simeq A_s^{\text{NP}} / A_s^{\text{SM}}$ , where  $A_s^{\text{NP,SM}} = |M_{12}^{\text{NP,SM}}|$ . In the model-independent global analysis



by the UTfit group, the fit result is

$$A_s^{\text{NP}}/A_s^{\text{SM}} \in [0.24, 1.38] \cup [1.50, 2.47] \quad (16)$$

at 95% probability [2]. The argument of  $M_{12}^{\text{NP}}$  being free in GUT models is due to the phase in off-diagonal elements in SUSY breaking mass matrix (in the basis where  $Y_d$  is a real diagonal matrix), and one can choose an appropriate value for the new phase in the NP contribution. Therefore, the experimental data constrains  $A_s^{\text{NP}}/A_s^{\text{SM}}$ , and therefore,  $\kappa \sin 2\theta_{23}$  is constrained for a given SUSY particle spectrum.

### 3.1 Box contribution

In the MSSM with flavor universality, the chargino box diagram dominates the SUSY contribution for  $M_{12}(B_s)$ . In a general parameter space for the soft SUSY breaking terms, the gluino box diagram can dominate the SUSY contribution. The gluino contribution can be written naively in the mass insertion form [7]

$$\frac{M_{12}^{\tilde{g}}}{M_{12}^{\text{SM}}} \simeq a [(\delta_{LL}^d)_{32}^2 + (\delta_{RR}^d)_{32}^2] - b (\delta_{LL}^d)_{32} (\delta_{RR}^d)_{32}, \quad (17)$$

where  $a$  and  $b$  depend on squark and gluino masses, and  $\delta_{LL,RR}^d = (M_d^2)_{LL,RR}/\tilde{m}^2$  ( $\tilde{m}$  is an averaged squark mass). The mass matrix  $M_d^2$  is a down-type squark mass matrix  $(\tilde{Q}, \tilde{U}^{c\dagger})M_d^2(\tilde{Q}^\dagger, \tilde{U}^c)^T$  in the basis where down-type quark mass matrix is real (positive) diagonal. When squark and gluino masses are less than 1 TeV,  $a \sim O(1)$  and  $b \sim O(100)$ . We also have contributions from  $\delta_{LR}^d$ , but we neglect it since it is suppressed by  $(m_b/m_{\text{SUSY}})^2$ .

Due to the fact that  $b \gg a$ , the gluino contribution is enhanced if both left- and right-handed squark mass matrices have off-diagonal elements. Therefore, it is expected that the SUSY contribution to the  $B_s$ - $\bar{B}_s$  mixing amplitude is large for the SO(10) model with type II seesaw [9].

### 3.2 Higgs penguin contribution and $B_s \rightarrow \mu\mu$

The box diagram does not depend on  $\tan\beta$  (ratio of the vacuum expectation values of two Higgs fields) explicitly. However, the flavor changing Higgs interaction (through so-called Higgs penguin diagram) directly depend on the  $\tan\beta$ , and the Higgs penguin contribution can become more important than the box diagram when  $\tan\beta$  is large [11, 12].

The Higgs penguin contribution originates from the finite correction of the down-type quark mass. The effective Yukawa coupling is given as

$$\mathcal{L}^{\text{eff}} = Y_d Q D^c H_d + \epsilon Q D^c H_u^*. \quad (18)$$

The second term is a non-holomorphic term, which can arise from the finite correction due to the SUSY breaking. The effective down-type quark mass matrix is  $M_d = Y_d v_d + \epsilon v_u$ . In the basis where the effective mass matrix is flavor diagonal, flavor changing Higgs interaction can be written as

$$\epsilon Q D^c H_u^* - \epsilon \frac{v_u}{v_d} Q D^c H_d. \quad (19)$$

Therefore, the flavor changing Higgs penguin coupling is proportional to the finite mass correction of the down-type quark mass matrix. The finite coupling  $\epsilon$  is naively proportional to  $\tan \beta$ , and thus, the dominant flavor changing Higgs interaction (second term) is proportional to  $\tan^2 \beta$ . Since the  $B_s$ - $\bar{B}_s$  mixing can be generated from a double penguin diagram, the mixing amplitude is proportional to  $\tan^4 \beta$ .

The effective flavor changing Higgs couplings are written as

$$X_{RL}^{Sij} (\bar{d}_i P_R d_j) S^0 + X_{LR}^{Sij} (\bar{d}_i P_L d_j) S^0, \quad (20)$$

where  $S^0$  represents for the neutral Higgs fields,  $S = [H, h, A]$ , where  $H$  and  $h$  stand for heavier and lighter CP even neutral Higgs fields, and  $A$  is a CP odd neutral Higgs field (pseudo Higgs field). The couplings are

$$X_{RL}^{Sij} = \epsilon_{ij} \frac{1}{\sqrt{2} \cos \beta} [\sin(\alpha - \beta), \cos(\alpha - \beta), -i], \quad (21)$$

$$X_{LR}^{Sij} = \epsilon_{ji} \frac{1}{\sqrt{2} \cos \beta} [\sin(\alpha - \beta), \cos(\alpha - \beta), i], \quad (22)$$

where  $\alpha$  is a mixing angle for  $h$  and  $H$ . The  $B_s$ - $\bar{B}_s$  mixing can be generated from the double left-handed penguin (which can be generated even in the universal SUSY breaking). However, it is proportional to the factor

$$\frac{\sin^2(\alpha - \beta)}{m_H^2} + \frac{\cos^2(\alpha - \beta)}{m_h^2} - \frac{1}{m_A^2}, \quad (23)$$

and the factor is almost zero since  $\cos(\alpha - \beta) \simeq 0$  and  $m_A \simeq m_H$  when  $m_A > M_Z$  and  $\tan \beta \gg 1$ . In the same reason, the double right-handed penguin contribution is negligible. On the other hand, the double penguin diagram including both left- and right-handed Higgs penguin which is proportional to the factor

$$\frac{\sin^2(\alpha - \beta)}{m_H^2} + \frac{\cos^2(\alpha - \beta)}{m_h^2} + \frac{1}{m_A^2}, \quad (24)$$

and the double penguin contribution is naively proportional to  $X_{RL}^{23} X_{LR}^{23} / m_A^2$ . In the flavor universal SUSY breaking, the right-handed penguin coupling  $X_{RL}^{23}$  is tiny, and the double penguin contribution can not be sizable even for a large  $\tan \beta$ . However, when the right-handed

mixing is generated in the SUSY GUT models, the double penguin diagram can be sizable for large  $\tan\beta$ . We note that if there is a FCNC source in the right-handed squark mass matrix, we do not need the off-diagonal elements in the left-handed squark mass matrix in order to generate the sizable double penguin contribution. Therefore, even in the minimal-type of SU(5) model, the double penguin contribution can be sizable when  $\tan\beta$  is large. When the off-diagonal elements of left-handed squark mass matrix are generated, the left-handed flavor changing contribution to different processes can be modified.

In the case where  $\tan\beta$  is about 10, the box contribution is dominant, and the contribution can be enhanced in the case of SO(10) model with type II seesaw, and a sizable contribution is not expected in the case of the minimal-type SU(5) GUT with type I seesaw. However, when  $\tan\beta$  is around 30 or more, the double penguin contribution can be sizable, even in the SU(5) GUT model. The difference between the SU(5) and the SO(10) models will not be so significant if only a large  $B_s$ - $\bar{B}_s$  mixing phase is observed. In order to distinguish these models, we, however, need to probe other flavor changing effects, such as  $B_s \rightarrow \mu\mu$ ,  $b \rightarrow s\gamma$  and  $\tau \rightarrow \mu\gamma$ .

The  $B_s \rightarrow \mu\mu$  decay can be generated by a single Higgs penguin diagram [13, 12]. The decay amplitude is proportional to the muon Yukawa coupling, and thus the amplitude is proportional to  $\tan^3\beta$ . Therefore, the branching ratio is proportional to  $\tan^6\beta$ . Since it can be generated by a single penguin, this decay occurs even in the universal SUSY breaking model like the mSUGRA (minimal supergravity) [22]. The current bound of the branching ratio is  $\text{Br}(B_s \rightarrow \mu\mu) = 4.7 \times 10^{-8}$  [23]. When  $\tan\beta$  is large, this bound gives an important constraint to the parameter space [8, 14]. In other words, one would expect that the  $B_s \rightarrow \mu\mu$  decay will be observed very soon.

### 3.3 $b \rightarrow s\gamma$ constraint

Another important constraint for the  $bs$  flavor violation is given by  $b \rightarrow s\gamma$  decay [24]:

$$\text{Br}(b \rightarrow s\gamma) = (3.55 \pm 0.26) \times 10^{-4}. \quad (25)$$

The  $b \rightarrow s\gamma$  decay can be generated even in the standard model, and the NNLO determination of the branching ratio is  $(3.15 \pm 0.23) \times 10^{-4}$  [24], which constrains the parameter space of the MSSM. We will choose a parameter region to make the branching ratio to be between 2.2 to 4.2 ( $\times 10^{-4}$ ) in 1-loop. In the MSSM, the chargino, gluino, and charged-Higgs contribution will be important to the amplitude. In the mSUGRA model, the chargino contribution will be dominant, and low gaugino masses are excluded especially for large  $\tan\beta$ , because the amplitude is proportional to  $\tan\beta$ . When the charged Higgs field is light, it will generate a positive contribution to the amplitude. Since the chargino contribution will be negative when

the Higgsino mass  $\mu$  is positive, the positive  $\mu$  has a wider parameter region in the MSSM. When there is no FCNC source in SUSY breaking, the gluino contribution is tiny. However, if there is a FCNC source in the right-handed down-type squark mass matrix, the right-handed operator (often called  $C_{7R}, C_{8R}$ ) due to the gluino loop can become large when  $\mu$  is large. When the left-handed squark mass matrix has off-diagonal elements, the chargino contribution for the left-handed operator (often called  $C_{7L}, C_{8L}$ ) can be modified from the mSUGRA case. It is hard to describe the allowed region in a general parameter space with flavor violation, but here, we give a simple correlation between the left-handed operator for  $b \rightarrow s\gamma$  and the left-handed Higgs penguin contribution. As we have described, the Higgs penguin contribution comes from the finite mass correction. The  $b \rightarrow s\gamma$  diagram is one-loop diagram with a photon emission. As a result, the signs of the contribution from the left-handed flavor violation in the squark masses are related for the  $b \rightarrow s\gamma$  contribution and the left-handed Higgs penguin contribution. This gives a correlation between  $b \rightarrow s\gamma$  and  $B_s \rightarrow \mu\mu$ . If  $\mu$  is positive, the branching ratio of  $B_s \rightarrow \mu\mu$  is enhanced when the SUSY contribution arises from the flavor violating terms cancels the chargino contribution in the mSUGRA.

### 3.4 $\tau \rightarrow \mu\gamma$ constraint

The current experimental bound of the branching ratio of  $\tau \rightarrow \mu\gamma$  decay is [25]

$$\text{Br}(\tau \rightarrow \mu\gamma) = 4.5 \times 10^{-8}. \quad (26)$$

When the lepton flavor violation is correlated to the flavor violation in the right-handed down-type squark as in the minimal-type of SU(5) model, the  $\tau \rightarrow \mu\gamma$  decay will give us the most important constraint to obtain the large  $B_s$ - $\bar{B}_s$  phase [9, 10]. The minimal-type of SU(5) GUT model is predictive due to the correlation between the amount of quark and lepton flavor violation as we have noted previously. Furthermore, the squark masses are raised much more compared to the slepton masses due to the gaugino loop contribution since the gluino is heavier than the Bino and the Wino at low energy, and thus the lepton flavor violation will be more sizable compared to the quark flavor violation.

In order to allow for a large phase in the  $B_s$ - $\bar{B}_s$  mixing in the minimal-type of SU(5) model, a large flavor universal scalar mass (often called  $m_0$ ) at the cutoff scale is preferable. The reasons are as follows. The gaugino loop effects are flavor invisible and they enhance the diagonal elements of the scalar mass matrices while keeping the off-diagonal elements unchanged. If the flavor universal scalar masses at the cutoff scale become larger, both  $\text{Br}(\tau \rightarrow \mu\gamma)$  and  $\phi_{B_s}$  are suppressed. However,  $\text{Br}(\tau \rightarrow \mu\gamma)$  is much more suppressed compared to  $\phi_{B_s}$  for a given  $\kappa \sin 2\theta_{23}$  because the low energy slepton masses are sensitive to  $m_0$  while the squark masses

are not so sensitive due to the gluino loop contribution to their masses. In this case, however, it is hard to satisfy the muon  $g - 2$  [26] and the stau-neutralino co-annihilation region for the dark matter [27].

When  $\tan\beta$  is large, the  $\tau \rightarrow \mu\gamma$  constraint is relaxed for a large  $B_s\text{-}\bar{B}_s$  phase, because the double-penguin contribution to the  $B_s\text{-}\bar{B}_s$  mixing is proportional to  $\tan^4\beta$  while the  $\tau \rightarrow \mu\gamma$  is proportional to  $\tan^2\beta$ . However, the  $B_s \rightarrow \mu\mu$  constraint becomes very severe in this case since it is proportional to  $\tan^6\beta$ .

As we have noted, in the SO(10) model, on the other hand, the suppression of lepton flavor violation is related to the selection of the symmetry breaking vacua, and in fact, it is preferable that the quark flavor violation is sizable but the lepton flavor violation is suppressed [20].

## 4 Numerical results

We plot the figures when the NP/SM ratio of the  $B_s\text{-}\bar{B}_s$  amplitude is 0.5,  $A_s^{\text{NP}}/A_s^{\text{SM}} = 0.5$ , and the absolute value of the full amplitude is same as SM amplitude,  $C_{B_s} = 1$ . Under these choices, one can obtain that  $|2\phi_{B_s}|$  is about 0.5 (rad). We choose the unified gaugino mass  $m_{1/2} = 500$  GeV, and the sfermion mass  $m_0 = 500$  GeV, and the universal trilinear scalar coupling,  $A_0 = 0$ , and  $\tan\beta = 40$ . We consider that the SUSY breaking Higgs squared masses  $m_{H_u}^2$  and  $m_{H_d}^2$  are not related to other scalar masses in order to make  $m_A$  and  $\mu$  free parameters, since these two parameters are important for Higgs penguin contribution and the SUSY contribution of  $b \rightarrow s\gamma$ . The absolute values of 23 off-diagonal elements of squark mass matrix is fixed to make the ratio  $A_s^{\text{NP}}/A_s^{\text{SM}} = 0.5$ .

In figure 1, we plot the figure in the case of the SO(10) boundary condition with type II seesaw. In this case, even if we fix the phase of  $B_s\text{-}\bar{B}_s$  amplitude, we still have one more phase degree of freedom. We show the cases where the 23 off-diagonal element of left-handed squark mass matrix is real in the basis where the down-type mass matrix is real diagonal. (Under this choice, the modification of the left-handed penguin contribution will be maximized.) The other phase in the off-diagonal element in the right-handed squark mass matrix is fixed when we choose  $C_{B_s} = 1$ . The two plots in figure 1 corresponds to the two signs of the off-diagonal element in the left-handed squark mass matrix. Since the off-diagonal elements can have continuous phase parameter, the two figures will morph into each other continuously by the phase degree of freedom. In the left plot, the  $\text{Br}(B_s \rightarrow \mu\mu)$  is enhanced, while it is suppressed in the right plot. As one can see that the  $\text{Br}(b \rightarrow s\gamma)$  excludes most regions of the right plot, while all this region is allowed in the left side plot, which is explained in the previous section. If  $\mu$  is large, the gluino contributions for  $C_{7R}, C_{8R}$  become large and the  $\text{Br}(b \rightarrow s\gamma)$  constraint is relaxed in the right plot. We can say that, in this parameter region, a large  $B_s\text{-}\bar{B}_s$  mixing phase

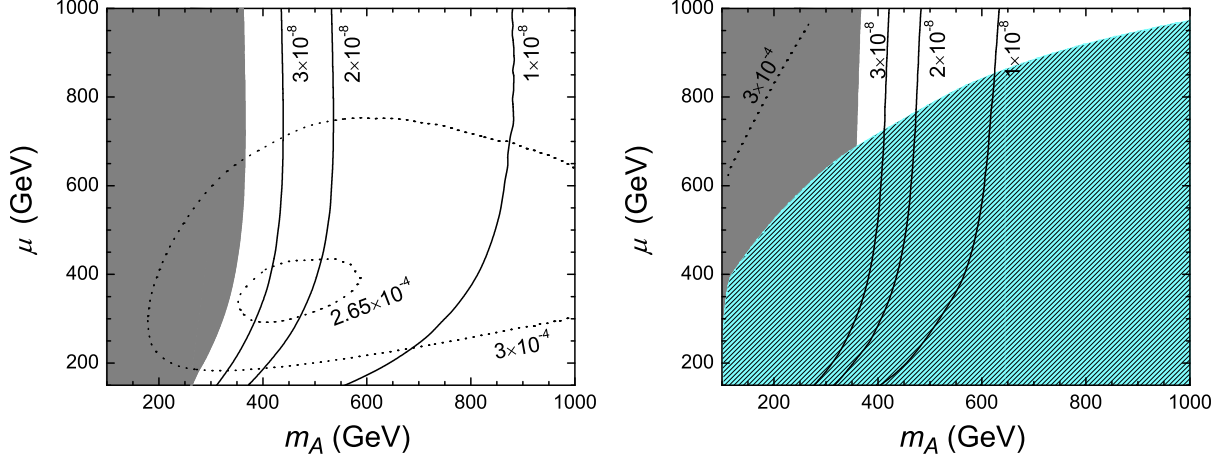


Figure 1: Plots for the SO(10) boundary condition when  $A_s^{\text{NP}}/A_s^{\text{SM}} = 0.5$  and  $C_{B_s} = 1$ . Solid lines show the contours of  $\text{Br}(B_s \rightarrow \mu\mu)$ . Dot lines show the contours of  $\text{Br}(b \rightarrow s\gamma)$ . Gray region is excluded by experimental bound of  $\text{Br}(B_s \rightarrow \mu\mu)$ . Blue shaded region is excluded by  $\text{Br}(b \rightarrow s\gamma)$ . Two figures are given for two signs of the 23 off-diagonal element of  $M_{\tilde{Q}}^2$ . Details are given in the text.

can be generated by the double penguin diagram, and the extra phase will be constrained by the  $\text{Br}(b \rightarrow s\gamma)$  constraint.

In figure 2, we plot the case of SU(5) boundary condition. We choose the  $\kappa$  values to be exactly same for  $\tilde{L}$  and  $\tilde{D}^c$  for simplicity. The parameters are same as before,  $m_0 = m_{1/2} = 500$  GeV,  $A_0 = 0$  and  $\tan\beta = 40$ . In this case, the phase is fixed (up to sign) when we choose  $A_s^{\text{NP}}/A_s^{\text{SM}} = 0.5$  and  $C_{B_s} = 1$ . One can see that, the  $\text{Br}(\tau \rightarrow \mu\gamma)$  constraint excludes most regions of the plot. In other words,  $\text{Br}(B_s \rightarrow \mu\mu)$  has to be large enough to be detected under this boundary condition for large  $\tan\beta$ . This is because of the following reasons. When  $m_A$  is large, the double penguin contribution is suppressed. Then,  $\kappa$  has to be large in order to obtain a large  $B_s$ - $\bar{B}_s$  mixing phase. However, a large  $\kappa$  is excluded by the  $\text{Br}(\tau \rightarrow \mu\gamma)$  constraint. As a result, a heavy pseudo Higgs field is excluded, and thus the  $\text{Br}(B_s \rightarrow \mu\mu)$  has to be large. As we have noted, the  $\text{Br}(\tau \rightarrow \mu\gamma)$  constraint is relaxed when  $m_0$  is large.

We note that using the universal scalar mass boundary condition for sfermion and Higgs fields ( $m_0 = m_{H_u} = m_{H_d}$ ), it is very hard to obtain the large  $B_s$ - $\bar{B}_s$  mixing phase due to  $\text{Br}(\tau \rightarrow \mu\gamma)$  constraint under the SU(5) boundary condition with type I seesaw, since  $m_A$  and  $\mu$  is not free in this case. Actually,  $m_A$  is not so low to enhance the double penguin diagram in this universal boundary condition as a consequence of the fact that the gaugino mass should be large enough to satisfy the  $b \rightarrow s\gamma$  constraint especially for large  $\tan\beta$ . However, when  $\tan\beta$  is about 50, the pseudo Higgs mass  $m_A$  becomes lower due to bottom Yukawa contribution in RGEs. Furthermore, as we have noted, the double-penguin contribution to the  $B_s$ - $\bar{B}_s$  mixing

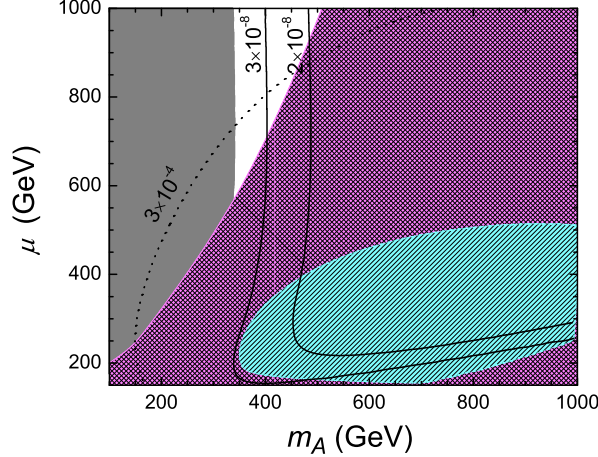


Figure 2: Plots for the SU(5) boundary condition when  $A_s^{\text{NP}}/A_s^{\text{SM}} = 0.5$  and  $C_{B_s} = 1$ . Solid lines show the contours of  $\text{Br}(B_s \rightarrow \mu\mu)$ . Dot lines show the contours of  $\text{Br}(b \rightarrow s\gamma)$ . Gray region is excluded by experimental bound of  $\text{Br}(B_s \rightarrow \mu\mu)$ . Blue shaded region is excluded by  $\text{Br}(b \rightarrow s\gamma)$ . Pink shaded region is excluded by  $\text{Br}(\tau \rightarrow \mu\gamma)$ . Details are given in the text.

is proportional to  $\tan^4 \beta$  while the  $\tau \rightarrow \mu\gamma$  is proportional to  $\tan^2 \beta$ . Therefore, in this case, a large  $B_s$ - $\bar{B}_s$  mixing phase can survive satisfying the  $\tau \rightarrow \mu\gamma$  constraint even in the universal scalar mass condition. In this case, the branching ratio of  $B_s \rightarrow \mu\mu$  has to be at the edge of the current bound.

## 5 Conclusion

We investigated the GUT models when the  $B_s$ - $\bar{B}_s$  mixing phase can become really large as indicated in the Fermilab experiments. We considered two cases: one is the minimal-type of SU(5) model with type I seesaw. The other is the minimal-type of SO(10) model with type II seesaw. The difference between the two boundary condition is whether there exists a sizable off-diagonal element in the left-handed squark mass matrix. It is important to note that the sources of FCNC will be restricted in the GUT models if the large phase of  $B_s$ - $\bar{B}_s$  mixing persists in the upcoming result in the Fermilab.

For small  $\tan \beta$ , the SUSY contribution of  $B_s$ - $\bar{B}_s$  mixing amplitude is dominated by the gluino box contribution, and the phase of  $B_s$ - $\bar{B}_s$  mixing will be more enhanced under the SO(10) boundary condition compared the SU(5) boundary condition [7, 9]. When  $\tan \beta$  is large, the double penguin contribution will dominate in both SU(5) and SO(10) boundary condition. Under the SO(10) boundary condition, the left-handed FCNC source will modify the left-handed Higgs penguin as well as the  $C_{7L}, C_{8L}$  operators for the  $\text{Br}(b \rightarrow s\gamma)$  decay, depending the phase of the 23 off-diagonal element. When the phase of  $B_s$ - $\bar{B}_s$  mixing is large,

the phase of the 23 off-diagonal element in the left-handed squark mass matrix is restricted especially when Higgsino mass  $\mu$  is small as shown in Fig. 1. When the phase is suitable to satisfy the  $\text{Br}(b \rightarrow s\gamma)$  bound, the left-handed penguin contribution is slightly enhanced and the  $\text{Br}(b \rightarrow s\gamma)$  is larger compared to the case with no left-handed FCNC source. Under the SU(5) boundary condition, the pseudo Higgs mass should be low enough to satisfy the  $\text{Br}(\tau \rightarrow \mu\gamma)$  constraint for a given parameter as shown in Fig. 2, and then the  $\text{Br}(B_s \rightarrow \mu\mu)$  has to be sizable, and it can be detected very soon.

In this paper, we have concentrated on the importance of the 2nd and 3rd generation FCNC effects such as  $\text{Br}(\tau \rightarrow \mu\gamma)$  and  $\phi_{B_s}$  correlation in GUT models, since they can be correlated directly by 23 mixing. The constraints from  $\text{Br}(\mu \rightarrow e\gamma)$  decay,  $K-\bar{K}$  and  $B_d-\bar{B}_d$  mixings, may be also important, but these effects depend on the details of flavor structure which can have a freedom of cancellation. We refer to the Ref.[17] for an analysis of flavor violation including the first generation.

## Acknowledgments

This work is supported in part by the DOE grant DE-FG02-95ER40917.

## References

- [1] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **100**, 161802 (2008) [arXiv:0712.2397 [hep-ex]]; V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **101**, 241801 (2008) [arXiv:0802.2255 [hep-ex]]; D. Tonelli [CDF Collaboration], arXiv:0810.3229 [hep-ex].
- [2] M. Bona *et al.* [UTfit Collaboration], arXiv:0803.0659 [hep-ph].
- [3] F. Gabbiani and A. Masiero, Nucl. Phys. B **322**, 235 (1989); J. Hagelin, S. Kelley and T. Tanaka, Nucl. Phys. B **415**, 293 (1994); F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B **477**, 321 (1996) [hep-ph/9604387].
- [4] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986); J. Hisano, T. Moroi, K. Tobe, M. Yamaguchi and T. Yanagida, Phys. Lett. B **357**, 579 (1995) [hep-ph/9501407].
- [5] L. J. Hall, V. A. Kostelecky and S. Raby, Nucl. Phys. B **267**, 415 (1986); R. Barbieri and L. J. Hall, Phys. Lett. B **338**, 212 (1994) [hep-ph/9408406]; J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Lett. B **391**, 341 (1997) [hep-ph/9605296].



- [6] T. Moroi, Phys. Lett. B **493**, 366 (2000) [hep-ph/0007328].
- [7] B. Dutta and Y. Mimura, Phys. Rev. Lett. **97**, 241802 (2006) [hep-ph/0607147].
- [8] J. K. Parry, Nucl. Phys. B **760**, 38 (2007) [hep-ph/0510305]; Mod. Phys. Lett. A **21**, 2853 (2006) [hep-ph/0608192]; J. K. Parry and H. h. Zhang, Nucl. Phys. B **802**, 63 (2008) [arXiv:0710.5443 [hep-ph]].
- [9] B. Dutta and Y. Mimura, Phys. Rev. D **78**, 071702 (2008) [arXiv:0805.2988 [hep-ph]].
- [10] J. Hisano and Y. Shimizu, Phys. Lett. B **669**, 301 (2008) [arXiv:0805.3327 [hep-ph]]; J. h. Park and M. Yamaguchi, Phys. Lett. B **670**, 356 (2009) [arXiv:0809.2614 [hep-ph]].
- [11] C. Hamzaoui, M. Pospelov and M. Toharia, Phys. Rev. D **59**, 095005 (1999) [hep-ph/9807350]; M. Gorbahn, S. Jager, U. Nierste and S. Trine, arXiv:0901.2065 [hep-ph].
- [12] A. J. Buras, P. H. Chankowski, J. Rosiek and L. Slawianowska, Nucl. Phys. B **619**, 434 (2001) [hep-ph/0107048]; Phys. Lett. B **546**, 96 (2002) [hep-ph/0207241]; Nucl. Phys. B **659**, 3 (2003) [hep-ph/0210145].
- [13] S. R. Choudhury and N. Gaur, Phys. Lett. B **451**, 86 (1999) [hep-ph/9810307]; K. S. Babu and C. F. Kolda, Phys. Rev. Lett. **84**, 228 (2000) [hep-ph/9909476]; C. S. Huang, W. Liao, Q. S. Yan and S. H. Zhu, Phys. Rev. D **63**, 114021 (2001) [Erratum-ibid. D **64**, 059902 (2001)] [hep-ph/0006250]. P. H. Chankowski and L. Slawianowska, Phys. Rev. D **63**, 054012 (2001) [hep-ph/0008046]; C. Bobeth, T. Ewerth, F. Kruger and J. Urban, Phys. Rev. D **64**, 074014 (2001) [hep-ph/0104284]; Phys. Rev. D **66**, 074021 (2002) [hep-ph/0204225].
- [14] J. Foster, K. Okumura and L. Roszkowski, Phys. Lett. B **609**, 102 (2005) [hep-ph/0410323]; JHEP **0508**, 094 (2005) [hep-ph/0506146]; Phys. Lett. B **641**, 452 (2006) [hep-ph/0604121].
- [15] P. Minkowski, Phys. Lett. B **67**, 421 (1977); T. Yanagida, in proc. of KEK workshop, eds. O. Sawada and S. Sugamoto (Tsukuba, 1979); M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979); R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).
- [16] J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. D **23**, 165 (1981); G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B **181**, 287 (1981).

- [17] B. Dutta and Y. Mimura, Phys. Rev. D **75**, 015006 (2007) [hep-ph/0611268]; Phys. Rev. D **77**, 051701 (2008) [arXiv:0708.3080 [hep-ph]].
- [18] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993) [hep-ph/9209215].
- [19] B. Dutta, Y. Mimura and R. N. Mohapatra, Phys. Rev. D **69**, 115014 (2004) [hep-ph/0402113]; Phys. Lett. B **603**, 35 (2004) [hep-ph/0406262]; Phys. Rev. Lett. **94**, 091804 (2005) [hep-ph/0412105]; Phys. Rev. D **72**, 075009 (2005) [hep-ph/0507319].
- [20] B. Dutta, Y. Mimura and R. N. Mohapatra, Phys. Rev. Lett. **100**, 181801 (2008) [arXiv:0712.1206 [hep-ph]].
- [21] V. Abazov *et al.* [D0], Phys. Rev. Lett. **97**, 021802 (2006) [hep-ex/0603029]; A. Abulencia *et al.* [CDF], Phys. Rev. Lett. **97**, 062003 (2006) [hep-ex/0606027].
- [22] A. Dedes, H. K. Dreiner and U. Nierste, Phys. Rev. Lett. **87**, 251804 (2001) [hep-ph/0108037]; R. L. Arnowitt, B. Dutta, T. Kamon and M. Tanaka, Phys. Lett. B **538**, 121 (2002) [hep-ph/0203069]; J. K. Mizukoshi, X. Tata and Y. Wang, Phys. Rev. D **66**, 115003 (2002) [hep-ph/0208078]; J. R. Ellis, K. A. Olive and V. C. Spanos, Phys. Lett. B **624**, 47 (2005) [hep-ph/0504196].
- [23] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **100**, 101802 (2008) [arXiv:0712.1708 [hep-ex]].
- [24] E. Barberio *et al.* [Heavy Flavor Averaging Group], arXiv:0808.1297 [hep-ex].
- [25] K. Hayasaka *et al.* [Belle Collaboration], Phys. Lett. B **666**, 16 (2008) [arXiv:0705.0650 [hep-ex]]; B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **95**, 041802 (2005) [hep-ex/0502032].
- [26] Muon  $g - 2$  Collaboration, G. W. Bennett *et al.*, Phys. Rev. Lett. **92**, 161802 (2004) [hep-ex/0401008].
- [27] V. Khotilovich, R. Arnowitt, B. Dutta and T. Kamon, Phys. Lett. B **618**, 182 (2005) [hep-ph/0503165].