

## BPS Geometries and AdS Bubbles

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### Abstract

Recently,  $\frac{1}{2}$ -BPS AdS bubble solutions have been obtained by Lin, Lunin and Maldacena, which correspond to Fermi droplets in phase space in the dual CFT picture. They can be thought of as generalisations of  $\frac{1}{2}$ -BPS AdS black hole solutions in five or seven dimensional gauged supergravity. In this paper, we extend these solutions by invoking additional gauge fields and scalar fields in the supergravity Lagrangians, thereby obtaining AdS bubble generalisations of the previously-known multi-charge AdS black solutions of gauged supergravity. We also obtain analogous AdS bubble solutions in four-dimensional gauged supergravity. Our solutions generically preserve supersymmetry fractions  $\frac{1}{4}$ ,  $\frac{1}{8}$  and  $\frac{1}{8}$  in seven, five and four dimensions respectively. They can be lifted to M-theory or type IIB string theory, using previously known formulae for the consistent Pauli sphere reductions that yield the gauged supergravities. We also find similar solutions in six-dimensional gauged supergravity, and discuss their lift to the massive type IIA theory.

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# 1 Introduction

It was recently shown in [1] that there exist smooth BPS solutions in the ten-dimensional type IIB string and in M-theory, which preserve one half of the supersymmetry, and which admit an interpretation in the dual field theory, via the AdS/CFT correspondence, as droplets in a phase space occupied by fermions. In [1], the  $\frac{1}{2}$ -BPS M-theory solutions were obtained by first constructing solutions in seven-dimensional gauged supergravity, and then lifting them to eleven dimensions by making use of the consistent  $S^4$  reduction that was constructed in [2, 3]. The seven-dimensional solutions can be thought of as certain generalisations of  $\frac{1}{2}$ -BPS black-hole solutions of seven-dimensional gauged supergravity, which are contained within a construction of seven-dimensional AdS black holes in [4]. The generalisation involves turning on an additional scalar field that is set to zero in the original black-hole solution. Interestingly, the resulting “AdS bubble” solutions obtained in [1] are in general everywhere smooth, unlike the  $\frac{1}{2}$ -BPS extremal AdS black hole which suffers from a naked singularity that lies strictly outside the horizon.

The  $\frac{1}{2}$ -BPS solutions of type IIB supergravity that were obtained in [1] were found by a direct ten-dimensional construction.<sup>1</sup> They can also be viewed from a five-dimensional point of view, by making use of results obtained in [6] on the consistent reduction of type IIB supergravity on  $S^5$ . The type IIB solutions in [1] then acquire an interpretation as generalisations of  $\frac{1}{2}$ -BPS AdS black holes in five-dimensional supergravity, which were contained within a construction of five-dimensional AdS black holes in [7].

Since the AdS bubble solutions obtained in [1] are generalisations of  $\frac{1}{2}$ -supersymmetric AdS black holes in  $D = 7$  and  $D = 5$ , which carry just a single electric charge, a further extension naturally suggests itself, in which one turns on additional electric charges, thereby giving solutions which preserve smaller supersymmetry fractions. For the AdS black hole solutions themselves, one can turn on two independent charges in  $D = 7$ , giving solutions which preserve  $\frac{1}{4}$  supersymmetry in the BPS limit. Likewise, in five dimensions one can obtain AdS black hole solutions with three independent charges, which have BPS limits preserving  $\frac{1}{8}$  of the supersymmetry when all three charges are non-zero.

In this paper, we construct “AdS bubble” solutions in  $D = 7$ ,  $D = 5$  and  $D = 4$  gauged supergravities, which respectively generalise the 2-charge, 3-charge and 4-charge BPS black hole solutions of these theories. Making use of results in [2, 3] for  $D = 7$ , in [6] for  $D = 5$ , and in [8, 9, 10], we can straightforwardly lift all of the resulting solutions to

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<sup>1</sup>Analogous solutions in  $D = 6$  were found recently in [5].

ten or eleven dimensions. We therefore find AdS bubble solutions that generically preserve  $\frac{1}{4}$  of the supersymmetry in seven dimensions,  $\frac{1}{8}$  of the supersymmetry in five dimensions, and  $\frac{1}{8}$  of the supersymmetry in four dimensions. We also obtain AdS bubble solutions in six-dimensional gauged supergravity, and discuss their lift to the massive type IIA theory.

## 2 $\frac{1}{4}$ -BPS Geometries in Seven Dimensions

The relevant part of the  $\mathcal{N} = 4$  gauged supergravity theory that suffices for describing the supersymmetric solutions with  $\frac{1}{2}$  and  $\frac{1}{4}$  supersymmetry is derivable from the Lagrangian<sup>2</sup>

$$\mathcal{L} = R * \mathbf{1} - \frac{1}{4} T_{ij}^{-1} * D T_{jk} \wedge T_{k\ell}^{-1} D T_{\ell i} - \frac{1}{4} T_{ik}^{-1} T_{j\ell}^{-1} * F_{(2)}^{ij} \wedge F_{(2)}^{k\ell} - V * \mathbf{1}, \quad (1)$$

where the 14 scalars are described by the unimodular symmetric matrix  $T_{ij}$ , the covariant derivative is defined by

$$D T_{ij} = d T_{ij} + g A_{(1)}^{ik} T_{kj} + g A_{(1)}^{jk} T_{ik}, \quad (2)$$

and the  $SO(5)$  gauge fields are given by  $F_{(2)}^{ij} = d A_{(1)}^{ij} + g A_{(1)}^{ik} \wedge A_{(1)}^{kj}$ . The scalar potential  $V$  is given by

$$V = \frac{1}{2} g^2 (2 T_{ij} T_{ij} - T_{ii}^2). \quad (3)$$

We consider the following restriction of the gauge and scalar fields, which arises as a consistent truncation of the complete set of equations of motion:

$$\begin{aligned} A_{(1)}^{12} &= A_{(1)}^1, & A_{(1)}^{34} &= A_{(1)}^2, \\ T_{ij} &= \text{diag}(X_1 e^{-\varphi_1}, X_1 e^{\varphi_1}, X_2 e^{-\varphi_2}, X_2 e^{\varphi_2}, (X_1 X_2)^{-2}), \end{aligned} \quad (4)$$

with all other gauge fields being zero. This corresponds to a truncation to gauge fields in the maximal torus  $U(1) \times U(1)$  in  $SO(5)$ . We are retaining four scalar fields in the truncation, namely  $(X_1, X_2, \varphi_1, \varphi_2)$ . In terms of a canonical parameterisation, we can write

$$X_1 = e^{-\frac{1}{\sqrt{2}} \phi_1 - \frac{1}{\sqrt{10}} \phi_2} \quad X_2 = e^{\frac{1}{\sqrt{2}} \phi_1 - \frac{1}{\sqrt{10}} \phi_2}, \quad (5)$$

whereupon the Lagrangian for this truncated system becomes

$$\begin{aligned} \mathcal{L}_7 &= R * \mathbf{1} - \frac{1}{2} \sum_{i=1}^2 * d \varphi_i \wedge d \varphi_i - \frac{1}{2} \sum_{\alpha=1}^2 * d \phi_\alpha \wedge d \phi_\alpha - \frac{1}{2} \sum_{i=1}^2 X_i^{-2} * F_{(2)}^i \wedge F_{(2)}^i \\ &\quad - 2g^2 \sum_{i=1}^2 \sinh^2 \varphi_i * A_{(1)}^i \wedge A_{(1)}^i - V * \mathbf{1}, \end{aligned} \quad (6)$$

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<sup>2</sup>This corresponds to a consistent truncation of the full  $SO(5)$  gauged supergravity, provided that one restricts the gauge fields to satisfy  $F_{(2)}^{[ij} \wedge F_{(2)}^{k\ell]} = 0$ , so that the 3-form fields are not excited. This restriction is indeed satisfied in the AdS bubble solutions we shall consider.

with the potential's being given by

$$\begin{aligned}
V = & g^2 (2X_1^2 \sinh^2 \varphi_1 + 2X_2^2 \sinh^2 \varphi_2 - 2X_1^{-1} X_2^{-2} \cosh \varphi_1 - 2X_2^{-1} X_1^{-2} \cosh \varphi_2 \\
& - 4X_1 X_2 \cosh \varphi_1 \cosh \varphi_2 + \frac{1}{2} X_1^{-4} X_2^{-4}).
\end{aligned}
\tag{7}$$

Guided by the form of the previously-known 2-charge black-hole solutions [4, 10], and the recent single-charge ‘‘AdS bubble’’ solutions obtained in [1], we are led to propose the following ansatz for general 2-charge seven-dimensional AdS bubble solutions:

$$\begin{aligned}
ds_7^2 &= -(H_1 H_2)^{-4/5} f dt^2 + (H_1 H_2)^{1/5} (f^{-1} dr^2 + r^2 d\Omega_5^2), \\
A_{(1)}^i &= -H_i^{-1} dt, \quad X_i = (H_1 H_2)^{2/5} H_i^{-1}, \quad \cosh \varphi_i = (RH_i)', \\
f &= 1 + \frac{1}{4} g^2 r^2 H_1 H_2,
\end{aligned}
\tag{8}$$

where  $R \equiv \frac{1}{4} r^4$  and a prime denotes a derivative with respect to  $R$ .

Substituting into the equations of motion that follow from (6), we find that they are indeed satisfied by the ansatz (8), provided that the functions  $H_i$  satisfy the equations

$$2R^{1/2} f (RH_i)'' = -g^2 [(RH_i)'^2 - 1] (H_1 H_2) H_i^{-1}.
\tag{9}$$

The 2-charge solutions that we have obtained generalise the single-charge  $\frac{1}{2}$ -BPS solutions of [1]. When both charges are non-zero, our solutions preserve  $\frac{1}{4}$  of the supersymmetry. If one or other of the gauge fields is set to zero (achieved by setting  $H_1 = 1$  or  $H_2 = 1$ ), the solutions reduce to those in [1].

### 3 $\frac{1}{8}$ -BPS Geometries in Five Dimensions

In a similar fashion, we can construct 3-charge AdS bubble solutions in five dimensions that correspond to generalisations of the  $\frac{1}{2}$ -BPS solutions of type IIB supergravity that were obtained in [1]. Although they were not described in gauged five-dimensional supergravity in [1], the type IIB solutions obtained there can be viewed as the lifting, via the consistent  $S^5$  Pauli reduction obtained in [6], of single-charge solutions in five-dimensional gauged supergravity. Our generalisation is obtained by considering a consistent truncation of five-dimensional maximal gauged supergravity in which gauge fields in the  $U(1)^3$  maximal torus of the  $SO(6)$  gauge group are retained, together with a total of five scalar fields. Prior to the truncation of the  $SO(6)$  gauge group, a consistently truncated subset of the bosonic fields, comprising the  $SO(6)$  gauge fields and an irreducible 20 of scalars, together with the metric, can be described by the Lagrangian (1), where now the  $i, j$  indices are in the

vector representation of  $SO(6)$ . The further consistent truncation that we are making then involves setting

$$A_{(1)}^{12} = A_{(1)}^1, \quad A_{(1)}^{34} = A_{(1)}^2, \quad A_{(1)}^{56} = A_{(1)}^3, \quad (10)$$

with all other gauge fields (except for those implied by antisymmetry in  $ij$ ) set to zero. The scalar fields that we retain are given by

$$T_{ij} = \text{diag}(X_1 e^{-\varphi_1}, X_1 e^{\varphi_1}, X_2 e^{-\varphi_2}, X_2 e^{\varphi_2}, X_3 e^{-\varphi_3}, X_3 e^{\varphi_3}). \quad (11)$$

The Lagrangian that describes this truncated system is given by

$$\begin{aligned} \mathcal{L}_5 = & R * \mathbf{1} - \frac{1}{2} \sum_{i=1}^3 *d\varphi_i \wedge d\varphi_i - \frac{1}{2} \sum_{\alpha=1}^2 *d\phi_\alpha \wedge d\phi_\alpha - \frac{1}{2} \sum_{i=1}^3 X_i^{-2} *F_{(2)}^i \wedge F_{(2)}^i \\ & - 2g^2 \sum_{i=1}^3 \sinh^2 \varphi_i *A_{(1)}^i \wedge A_{(1)}^i - V * \mathbf{1} + F_{(2)}^1 \wedge F_{(2)}^2 \wedge A_{(1)}^3, \end{aligned} \quad (12)$$

where we write

$$X_1 = e^{-\frac{1}{\sqrt{6}} \phi_1 - \frac{1}{\sqrt{2}} \phi_2}, \quad X_2 = e^{-\frac{1}{\sqrt{6}} \phi_1 + \frac{1}{\sqrt{2}} \phi_2}, \quad X_3 = e^{\frac{2}{\sqrt{6}} \phi_2}. \quad (13)$$

Note that  $X_1 X_2 X_3 = 1$ . The scalar potential following from (3) and (11) is

$$\begin{aligned} V = & 2g^2 (X_1^2 \sinh^2 \varphi_1 + X_2^2 \sinh^2 \varphi_2 + X_3^2 \sinh^2 \varphi_3 - 2X_2 X_3 \cosh \varphi_2 \cosh \varphi_3 \\ & - 2X_3 X_1 \cosh \varphi_3 \cosh \varphi_1 - 2X_1 X_2 \cosh \varphi_1 \cosh \varphi_2). \end{aligned} \quad (14)$$

Based on the previously-known black-hole solutions in five-dimensional gauged supergravity [7], and the seven-dimensional single-charge AdS bubble metrics of [1] and their 2-charge generalisation that we found in section 2, we are led to the following ansatz for 3-charge AdS bubble solutions of five-dimensional gauged supergravity:

$$\begin{aligned} ds_5^2 &= -(H_1 H_2 H_3)^{-2/3} f dt^2 + (H_1 H_2 H_3)^{1/3} (f^{-1} dr^2 + r^2 d\Omega_3^2), \\ A_{(1)}^i &= -H_i^{-1} dt, \quad X_i = (H_1 H_2 H_3)^{1/3} H_i^{-1}, \quad \cosh \varphi_i = (RH_i)', \\ f &= 1 + g^2 r^2 H_1 H_2 H_3, \end{aligned} \quad (15)$$

where  $R \equiv r^2$  and a prime denotes a derivative with respect to  $R$ . Substituting into the equations of motion following from (12), we find that they are indeed solutions, provided that the functions  $H_i$  obey the equations

$$f (RH_i)'' = -g^2 [(RH_i)'^2 - 1] (H_1 H_2 H_3) H_i^{-1}, \quad (16)$$

where the prime denotes the derivative with respect to  $R$ .

These solutions in general preserve  $\frac{1}{8}$  of the maximal supersymmetry in five dimensions. In general, if only  $N$  of the maximum of 3 charges is turned on, the solution preserves a fraction  $2^{-N}$  of the supersymmetry. The solutions can be lifted to give solutions of the type IIB theory in ten dimensions, using the formulae obtained in [6]. If, in particular, only one of the three gauge fields is taken to be non-zero (achieved by setting two of the three  $H_i$  functions to 1), the resulting  $\frac{1}{2}$ -supersymmetric configurations will coincide with solutions obtained in [1].

## 4 $\frac{1}{8}$ -BPS Geometries in Four Dimensions

The discussion of the previous two sections can be extended also to  $SO(8)$  gauged supergravity in four dimensions. In this case, we can construct AdS bubble solutions that generalise the 4-charge BPS solutions that are contained within the AdS black hole solutions constructed in [12].

The relevant truncation of the full four-dimensional  $SO(8)$  gauged supergravity, containing the fields utilised in the AdS bubble solutions, is described again by the Lagrangian (1), where now the indices  $i, j$  lie in the vector representation of  $SO(8)$ . Since the reduction ansatz for obtaining this truncation via a reduction from  $D = 11$  on  $S^7$  has not appeared explicitly in the literature in a coherent form, here we draw together various presentations of parts of the ansatz, including results in [8, 9, 10], to give the full expressions. The reduction ansatz for the eleven-dimensional metric is

$$d\hat{s}_{11}^2 = \Delta^{2/3} ds_4^2 + g^{-2} \Delta^{-1/3} T_{ij}^{-1} D\mu^i D\mu^j, \quad (17)$$

where  $\mu^i \mu^i = 1$  defines the unit 7-sphere, and

$$\Delta \equiv T_{ij} \mu^i \mu^j, \quad D\mu^i \equiv d\mu^i + g A_{(1)}^{ij} \mu^j. \quad (18)$$

The 4-form field strength is reduced according to the ansatz

$$\hat{F}_{(4)} = -g U \epsilon_{(4)} + g^{-1} T_{ij}^{-1} *DT_{jk} \wedge (\mu^k D\mu^i) - \frac{1}{2} g^{-2} T_{ik}^{-1} T_{j\ell}^{-1} *F_{(2)}^{ij} \wedge D\mu^k \wedge D\mu^\ell, \quad (19)$$

where  $U \equiv 2T_{ij} T_{jk} \mu^i \mu^k - \Delta T_{ii}$ , and  $\epsilon_{(4)}$  is the volume form in the lower-dimensional metric  $ds_4^2$ . It should be noted that this reduction is fully consistent *provided* that one restricts the field-strengths to configurations for which  $F_{(2)}^{[ij} \wedge F_{(2)}^{k\ell]} = 0$ , as indeed we shall be doing in the AdS bubble solutions obtained below, which carry only electric charges. For generic configurations where the quadratic products of field strengths are non-vanishing,

these products would be acting as sources for the 35 additional pseudo-scalar fields of  $\mathcal{N} = 8$  supergravity, and the reduction ansatz would be immeasurably more complicated. Fortunately, this complication need not concern us here.

We then make a further consistent truncation that retains just the metric, the four gauge fields of the  $U(1)^4$  maximal torus in  $SO(8)$ , and seven scalar fields:

$$\begin{aligned} A_{(1)}^{12} &= A_{(1)}^1, & A_{(1)}^{34} &= A_{(1)}^2, & A_{(1)}^{56} &= A_{(1)}^3, & A_{(1)}^{78} &= A_{(1)}^4, \\ T_{ij} &= \text{diag}(X_1 e^{-\varphi_1}, X_1 e^{\varphi_1}, X_2 e^{-\varphi_2}, X_2 e^{\varphi_2}, X_3 e^{-\varphi_3}, X_3 e^{\varphi_3}, X_4 e^{-\varphi_4}, X_4 e^{\varphi_4}). \end{aligned} \quad (20)$$

The scalars  $X_i$ , which satisfy  $X_1 X_2 X_3 X_4 = 1$ , can be parameterised canonically in the form

$$X_1 = e^{\frac{1}{2}(-\phi_1 - \phi_2 - \phi_3)}, \quad X_2 = e^{\frac{1}{2}(-\phi_1 + \phi_2 + \phi_3)}, \quad X_3 = e^{\frac{1}{2}(\phi_1 - \phi_2 + \phi_3)}, \quad X_4 = e^{\frac{1}{2}(\phi_1 + \phi_2 - \phi_3)}. \quad (21)$$

Substituting the above into the expression (3) for the scalar potential, we find that for this truncated system it is given by

$$V = 2g^2 \sum_{i=1}^4 X_i^2 \sinh^2 \varphi_i - 2g^2 \sum_{i \neq j} X_i X_j \cosh \varphi_i \cosh \varphi_j. \quad (22)$$

The Lagrangian for the truncated system is given by

$$\begin{aligned} \mathcal{L}_4 &= R * \mathbf{1} - \frac{1}{2} \sum_{i=1}^4 *d\varphi_i \wedge d\varphi_i - \frac{1}{2} \sum_{\alpha=1}^3 *d\phi_\alpha \wedge d\phi_\alpha - \frac{1}{2} \sum_{i=1}^4 X_i^{-2} *F_{(2)}^i \wedge F_{(2)}^i \\ &\quad - 2g^2 \sum_{i=1}^4 \sinh^2 \varphi_i *A_{(1)}^i \wedge A_{(1)}^i - V * \mathbf{1}, \end{aligned} \quad (23)$$

Following analogous considerations to those in the previous two sections we are led to make the following ansatz for 4-charge AdS bubble solutions of  $SO(8)$  gauged four-dimensional supergravity:

$$\begin{aligned} ds_4^2 &= -(H_1 H_2 H_3 H_4)^{-1/2} f dt^2 + (H_1 H_2 H_3 H_4)^{1/2} (f^{-1} dr^2 + r^2 d\Omega_2^2), \\ A_{(1)}^i &= -H_i^{-1} dt, \quad X_i = (H_1 H_2 H_3 H_4)^{1/4} H_i^{-1}, \quad \cosh \varphi_i = (RH_i)', \\ f &= 1 + 4g^2 r^2 H_1 H_2 H_3 H_4, \end{aligned} \quad (24)$$

Substituting into the equations of motion following from (23), we find that they are indeed satisfied, provided that the functions  $H_i$  obey the equations

$$R^{-1} f (RH_i)'' = -g^2 [(RH_i)'^2 - 1] (H_1 H_2 H_3 H_4) H_i^{-1}, \quad (25)$$

where in this case we have defined  $R = 2r$ , and a prime denotes  $\partial/\partial R$ . These solutions in general preserve  $\frac{1}{8}$  of the supersymmetry.<sup>3</sup> In special cases where three of the  $H_i$  are set to 1, the solutions will generically preserve  $\frac{1}{2}$  supersymmetry; if two of the  $H_i$  are set to 1, the solutions will generically preserve  $\frac{1}{4}$  supersymmetry; and if one of the  $H_i$  is set to 1 the solutions will generically preserve  $\frac{1}{8}$  supersymmetry.

## 5 $\frac{1}{8}$ -BPS Geometries in Six Dimensions

Gauged supergravity in  $D = 6$  [13] follows a different pattern from those in other dimensions. The maximum number of supercharges in the  $D = 6$  gauged supergravity is 16, rather than the 32 in dimensions 7, 5 and 4. This is because its  $\text{AdS}_6$  vacuum is related to the D4-D8 system [14]. The pure gauged supergravity theory can be obtained from a consistent Pauli sphere reduction, starting from massive type IIA supergravity in  $D = 10$ , and reducing on a warped  $S^4$  hemisphere [15]. As discussed in [16], one can presumably augment the theory with  $\mathcal{N} = 2$  matter. We are interested in such a theory with  $U(1)^2$  gauge fields and four scalars. The consistent reduction of such a four-scalar system, and its resulting six-dimensional scalar potential, were obtained in [16]. Following a pattern analogous to those for the other theories discussed in the paper, we expect that the relevant Lagrangian for our purpose should be given by

$$\begin{aligned} \mathcal{L}_6 = & R * \mathbf{1} - \frac{1}{2} \sum_{i=1}^2 *d\varphi_i \wedge d\varphi_i - \frac{1}{2} \sum_{\alpha=1}^2 *d\phi_\alpha \wedge d\phi_\alpha - \frac{1}{2} \sum_{i=1}^2 X_i^{-2} *F_{(2)}^i \wedge F_{(2)}^i \\ & - 2g^2 \sum_{i=1}^2 \sinh^2 \varphi_i *A_{(1)}^i \wedge A_{(1)}^i - V * \mathbf{1}, \end{aligned} \quad (26)$$

where

$$X_1 = e^{-\frac{1}{2\sqrt{2}}\phi_1 - \frac{1}{\sqrt{2}}\phi_2}, \quad X_2 = e^{-\frac{1}{2\sqrt{2}}\phi_1 + \frac{1}{\sqrt{2}}\phi_2}. \quad (27)$$

The scalar potential is given by [16]

$$V = -\frac{1}{2}g^2 \left( \left( \sum_{i=1}^4 Y_i \right)^2 - 2 \sum_{i=1}^4 Y_i^2 + \frac{8}{3}Y_0 \sum_{i=1}^4 Y_i - \frac{8}{9}Y_0^2 \right), \quad (28)$$

where

$$\begin{aligned} Y_1 &= X_1 e^{-\varphi_1}, & Y_2 &= X_1 e^{\varphi_1}, & Y_3 &= X_2 e^{-\varphi_2}, & Y_4 &= X_2 e^{\varphi_2}, \\ Y_0 &\equiv (Y_1 Y_2 Y_3 Y_4)^{-3/4} = (X_1 X_2)^{-3/2}. \end{aligned} \quad (29)$$

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<sup>3</sup>To be precise, the 4-charge solutions will preserve either  $\frac{1}{8}$  of the supersymmetry or none, depending on a sign choice in the expressions for the gauge potentials. This is the same phenomenon as is seen in standard 4-charge BPS black holes in four dimensions.



Following analogous considerations to those of the previous sections, and guided by the AdS<sub>6</sub> black hole solutions obtained in  $D = 6$  [15], we find that the 2-charge AdS bubble solutions are given by

$$\begin{aligned} ds_6^2 &= -(H_1 H_2)^{-3/4} f dt^2 + (H_1 H_2)^{1/4} (f^{-1} dr^2 + r^2 d\Omega_4^2), \\ A_i &= -H_i^{-1} dt, \quad X_i = (H_1 H_2)^{3/8} H_i^{-1}, \quad \cosh \varphi_i = (R H_i)', \\ f &= 1 + \frac{4}{9} g^2 r^2 H_1 H_2, \end{aligned} \quad (30)$$

where  $R = (2/3)^{4/9} r^3$ , and a prime denotes a derivative with respect to  $R$ . The full set of equations of motion now reduces to

$$R^{1/3} f (R H_i)'' = -g^2 [(R H_i)'^2 - 1] (H_1 H_2) H_i^{-1}. \quad (31)$$

The solution preserves  $\frac{1}{4}$  of the supersymmetry of six-dimensional gauged supergravity, but since this theory itself has only 16 supercharges, the solution preserves  $\frac{1}{8}$  of the maximal supersymmetry from the point of view of massive ten-dimensional type IIA supergravity.

Having obtained the solutions in  $D = 6$ , we can lift them back to massive ten-dimensional type IIA supergravity, by combining results from [15] and [16]. The reduction ansatz that encompasses the solutions we are considering here can be written as

$$\begin{aligned} ds_{10}^2 &= \mu_0^{1/12} (X_1 X_2)^{1/16} \left( \Delta^{3/8} ds_6^2 + g^{-2} \Delta^{-5/8} T_{\alpha\beta}^{-1} D\mu^\alpha D\mu^\beta \right), \\ e^{\frac{1}{2}\phi} *F_{(4)} &= g \sum_{\alpha} (2Y_\alpha^2 \mu_\alpha^2 - \Delta Y_\alpha) \epsilon_{(6)} - \frac{1}{3} g \Delta X_0 \epsilon_{(6)} \\ &\quad + \frac{1}{2} g^{-2} T_{\alpha\gamma}^{-1} T_{\beta\delta}^{-1} *F_{(2)}^{\gamma\delta} \wedge D\mu^\alpha \wedge D\mu^\beta + \frac{1}{2} g^{-1} T_{\alpha\beta}^{-1} *DT^{\alpha\gamma} \wedge (\mu^\gamma D\mu^\beta), \\ e^\phi &= \mu_0^{-5/16} \Delta^{1/4} (X_1 X_2)^{-5/8}, \end{aligned} \quad (32)$$

where  $D\mu^\alpha = d\mu^\alpha + g A_{(1)}^{\alpha\beta} \mu^\beta$ ,  $\Delta = T_{\alpha\beta} \mu^\alpha \mu^\beta$ , and

$$T_{\alpha\beta} \equiv \text{diag}(Y_0, Y_1, Y_2, Y_3, Y_4). \quad (33)$$

Note that the  $\alpha, \beta$  indices range from 0 to 4, that  $\mu^\alpha \mu^\alpha = 1$ , and that  $A_{(1)}^{12} = A_{(1)}^1$  and  $A_{(1)}^{34} = A_{(1)}^2$ , with all other non-symmetry related components of  $A_{(1)}^{\alpha\beta}$  vanishing.

## 6 Conclusions

The  $\frac{1}{2}$ -BPS AdS bubble solutions obtained in [1] can be thought of as generalisations of  $\frac{1}{2}$ -BPS AdS black hole solutions in five or seven dimensional gauged supergravity. In this paper, we have extended these solutions by invoking additional gauge fields and scalar

fields in the supergravity Lagrangians, thereby obtaining AdS bubble generalisations of the previously-known multi-charge AdS black solutions of gauged supergravity. We have also obtained analogous AdS bubble solutions in four-dimensional gauged supergravity. Our generic solutions preserve supersymmetry fractions  $\frac{1}{4}$ ,  $\frac{1}{8}$  and  $\frac{1}{8}$  in seven, five and four dimensions respectively, with larger fractions  $2^N$  arising as special cases when some of the additional gauge fields and scalars are set to zero.

Using formulae for the embeddings via sphere reductions of the four and seven dimensional gauged supergravities in  $D = 11$ , and of the five-dimensional gauged supergravity in type IIB supergravity, the AdS bubble solutions that we have obtained can be lifted to M-theory or string theory. We also obtained AdS bubble solutions in six-dimensional gauged supergravity, and described their lifting to the massive type IIA supergravity.

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