

Four-Dimensional Einstein Yang-Mills De Sitter Gravity From Eleven Dimensions

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ABSTRACT

We obtain $D = 4$ de Sitter gravity coupled to $SU(2)$ Yang-Mills gauge fields from an explicit and consistent truncation of $D = 11$ supergravity *via* Kaluza-Klein dimensional reduction on a non-compact space. The “internal” space is a smooth hyperbolic 7-space (H^7) written as a foliation of two 3-spheres, on which the $SU(2)$ Yang-Mills fields reside. The positive cosmological constant is completely fixed by the $SU(2)$ gauge coupling constant. The explicit reduction ansatz enables us to lift any of the $D = 4$ solutions to $D = 11$. In particular, we obtain dS_2 in M-theory, where the nine-dimensional transverse space is an H^7 bundle over S^2 . We also obtain a new smooth embedding of dS_3 in $D = 6$ supergravity.

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1 Introduction

The embedding of Anti-de Sitter (AdS) spacetimes in M-theory and string theories is rather straightforward. In fact, gauged supergravities in diverse dimensions with AdS vacuum have either been shown or are expected to be obtainable from consistent Kaluza-Klein sphere reductions of $D = 11$ or $D = 10$ supergravities. Notable examples include the simple embedding of $D = 4$, $\mathcal{N} = 2$, $SU(2)$ Yang-Mills AdS supergravity in $D = 11$ [1], the significantly more complicated S^7 [2, 3] and S^4 [4, 5, 6] reductions of M-theory, and the warped S^4 reduction of massive type IIA theory [7]. Although the S^5 reduction of type IIB theory has yet to be fully established,¹ the reduction of a certain truncation of the theory has been constructed [9, 10].

On the other hand, there is less known regarding embedding de Sitter (dS) spacetime in M-theory or string theories, mostly because this is quite a bit more complicated than the case of AdS. With recent experimental evidence suggesting that our universe might be de Sitter [11, 12], there is increasing interest in de Sitter gravity in cosmology and the dS/CFT correspondence [13, 14, 15, 16, 17]. Thus, it is of importance to obtain the embedding of a non-trivial de Sitter gravity theory in M-theory or string theories.

The first de Sitter solution within the context of an extended supergravity theory was found in [18]. While no-go theorems [19, 20] imply that de Sitter spacetime cannot arise from a compactification of a supergravity theory, it can arise from a supergravity theory with a non-compact “internal” space [21]. Explicit embeddings of dS_4 and dS_5 in M-theory and type IIB supergravity, respectively, were obtained in [22]. These arise as ten or eleven-dimensional solutions that have a non-compact hyperbolic internal space.

In this paper, we obtain four-dimensional de Sitter gravity with $SU(2)$ Yang-Mills gauge fields from a less constrained truncation of $D = 11$ supergravity *via* Kaluza-Klein dimensional reduction on the non-compact space. In this construction, a consistent truncation of the higher-dimensional theory is required in which there are no modes which depend on the internal space. The $SU(2)$ fields arise from modes on the S^3 portions of the internal space. The Yang-Mills gauge coupling constant

¹The full metric ansatz was conjectured in [8].

is completely fixed by the cosmological constant. The kinetic terms for the $SU(2)$ gauge fields have the correct sign, implying that the theory is not merely an analytical continuation of $SU(2)$ AdS supergravity.

This paper is organized as follows. In section 2, we rederive the embedding of dS_4 spacetime in M-theory, with the transverse space being an H^7 written as a foliation of two 3-spheres. In section 3, we propose a reduction ansatz for obtaining $D = 4$ $SU(2)$ Yang-Mills de Sitter gravity from $D = 11$. We show that the reduction ansatz is indeed consistent with the $D = 11$ equations of motion, and hence obtain the Lagrangian for $D = 4$ $SU(2)$ Yang-Mills de Sitter gravity. The consistency of the reduction ansatz enables us to lift any $D = 4$ solution back to $D = 11$. In section 4, we discuss this in detail. In particular, we embed dS_2 and $(\text{Minkowski})_2$ spacetimes in $D = 11$. The internal space is an H^7 bundle over S^2 . We also embed a cosmological solution which smoothly interpolates between $dS_2 \times S^2$ at the infinite past in the co-moving time to a dS_4 -type geometry at the infinite future. In section 5, we obtain an embedding of dS_3 in $D = 6$ supergravity with the transverse space being an H^3 written as a foliation of two circles. We conclude our paper in section 6.

2 Embedding dS_4 in $D = 11$

We now show how the dS_4 embedding in $D = 11$ supergravity found in [22] can be obtained directly from the eleven-dimensional equations of motion. We start with the Lagrangian of the bosonic sector of $D = 11$ supergravity, given by

$$\mathcal{L} = R * \mathbb{1} - \frac{1}{2} * F_{(4)} \wedge F_{(4)} - \frac{1}{6} A_{(3)} \wedge F_{(4)} \wedge F_{(4)}, \quad (1)$$

where $F_{(4)} = dA_{(3)}$. We consider the ansatz

$$\begin{aligned} ds^2 &= H^2 ds_4^2 + d\rho^2 + a^2 d\Omega_3^2 + b^2 d\tilde{\Omega}_3^2, \\ F_4 &= q \epsilon_{(4)}, \end{aligned} \quad (2)$$

where H , a and b are functions of ρ , ds_4^2 is four-dimensional de Sitter spacetime with cosmological constant $\Lambda = 6\lambda^2$, *i.e.* $R_{\mu\nu} = 3\lambda^2 g_{\mu\nu}$, and $\epsilon_{(4)}$ is the corresponding volume-form. $d\Omega_3^2$ and $d\tilde{\Omega}_3^2$ are the metrics of the two unit 3-spheres. The Einstein

equations of motion are given by

$$\begin{aligned}
\frac{4\ddot{H}}{H} + \frac{3\ddot{a}}{a} + \frac{3\ddot{b}}{b} &= -\frac{q^2}{6H^8}, \\
\frac{\ddot{H}}{H} + \frac{3\dot{H}^2}{H^2} + \frac{3\dot{H}}{H} \left(\frac{\dot{a}}{a} + \frac{\dot{b}}{b} \right) &= \frac{q^2}{3H^8} + \frac{3\lambda^2}{H^2}, \\
\frac{\ddot{a}}{a} + \frac{2\dot{a}^2}{a^2} + \frac{3\dot{a}\dot{b}}{ab} + \frac{4\dot{a}\dot{H}}{aH} - \frac{2}{a^2} &= -\frac{q^2}{6H^8}, \\
\frac{\ddot{b}}{b} + \frac{2\dot{b}^2}{b^2} + \frac{3\dot{a}\dot{b}}{ab} + \frac{4\dot{b}\dot{H}}{bH} - \frac{2}{b^2} &= -\frac{q^2}{6H^8},
\end{aligned} \tag{3}$$

where a dot represents a derivative with respect to ρ . If the metric ds_4^2 is AdS instead of dS, then one can have a solution with H being a constant. In this case, ρ becomes an angular coordinate with $a \sim \cos \rho$ and $b \sim \sin \rho$, so that the metric becomes the direct product of AdS_4 and S^7 , with the seven sphere written as a foliation of two three spheres. Inspired by the sphere reduction ansatz [3], we consider the following redefinition of variables

$$H^2 = \Delta^{2/3}, \quad a^2 = \Delta^{-1/3} \tilde{a}^2, \quad b^2 = \Delta^{-1/3} \tilde{b}^2. \tag{4}$$

Following the analogous relation in the case of $\text{AdS}_4 \times S^7$, we have

$$\tilde{a}^2 = \frac{1}{2}\ell^2 (\Delta + 1), \quad \tilde{b}^2 = \frac{1}{2}\ell^2 (\Delta - 1), \tag{5}$$

where ℓ is a constant scale parameter. Substituting (4) and (5) into (3), we find that the constants q and λ must satisfy

$$q^2 \ell^2 = 4, \quad \lambda^2 \ell^2 = \frac{4}{3}. \tag{6}$$

The equations (3) reduce to a single first-order differential equation

$$\ell^2 \Delta^{\frac{2}{3}} \dot{\Delta}^2 - 4\Delta^2 + 4 = 0. \tag{7}$$

Making a coordinate change $d\rho = \ell \Delta^{1/3} d\theta$, we can easily solve for Δ , which is given by

$$\Delta = \cosh(2\theta). \tag{8}$$

Thus we have an explicit embedding of dS_4 in $D = 11$ given by

$$\begin{aligned}
ds^2 &= \cosh^{\frac{2}{3}}(2\theta) ds_4^2 + \ell^2 \cosh^{\frac{2}{3}}(2\theta) d\theta^2 \\
&\quad + \cosh^{-\frac{1}{3}}(2\theta) \left(\cosh^2 \theta d\Omega_3^2 + \sinh^2 \theta d\tilde{\Omega}_3^2 \right), \\
F_4 &= \frac{2}{\ell} \epsilon_{(4)}.
\end{aligned} \tag{9}$$

This solution was obtained in [22].

Each S^3 in (9) can be replaced by a three-dimensional lens space. A Kaluza-Klein reduction and Hopf T-duality transformation on the fibre coordinate of the two lens spaces yields an embedding of dS_4 in type IIB theory. This procedure was used for the case of AdS solutions in [23].

If we consider (9) as a reduction ansatz from $D = 11$ to $D = 4$, then the resulting four-dimensional theory is Einstein gravity with a positive cosmological constant, and with a corresponding Lagrangian given by

$$e^{-1}\mathcal{L} = R - \frac{8}{\ell^2}. \quad (10)$$

3 Embedding Yang-Mills de Sitter gravity

The metric of the 3-spheres in (9) can be written as

$$d\Omega_3^2 = \frac{1}{4}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2), \quad d\tilde{\Omega}_3^2 = \frac{1}{4}(\tilde{\sigma}_1^2 + \tilde{\sigma}_2^2 + \tilde{\sigma}_3^2), \quad (11)$$

where σ_i and $\tilde{\sigma}_i$ are $SU(2)$ left-invariant 1-forms satisfying

$$d\sigma_i = -\frac{1}{2}\epsilon_{ijk}\sigma_j \wedge \sigma_k, \quad d\tilde{\sigma}_i = -\frac{1}{2}\epsilon_{ijk}\tilde{\sigma}_j \wedge \tilde{\sigma}_k. \quad (12)$$

Thus, we can introduce $SU(2)$ Yang-Mills fields $A_{(1)}^i$ to the vielbein

$$h^i = \sigma_i - g A_{(1)}^i, \quad \tilde{h}^i = \tilde{\sigma}_i - g A_{(1)}^i. \quad (13)$$

With these preliminaries, we propose the reduction ansatz

$$ds_{11}^2 = \Delta^{\frac{2}{3}} ds_4^2 + g^{-2} \Delta^{\frac{2}{3}} d\theta^2 + \frac{1}{4}g^{-2} \Delta^{-\frac{1}{3}} \left[c^2 \sum_i (h^i)^2 + s^2 \sum_i (\tilde{h}^i)^2 \right], \quad (14)$$

$$F_{(4)} = 2g \epsilon_{(4)} - \frac{1}{4}g^{-2} \left(s c d\theta \wedge h^i \wedge *F_{(2)}^i - s c d\theta \wedge \tilde{h}^i \wedge *F_{(2)}^i - \frac{1}{4}c^2 \epsilon_{ijk} h^i \wedge h^j \wedge *F_{(2)}^k + \frac{1}{4}s^2 \epsilon_{ijk} \tilde{h}^i \wedge \tilde{h}^j \wedge *F_{(2)}^k \right), \quad (15)$$

where $c = \cosh \theta$, $s = \sinh \theta$, $\Delta = \cosh(2\theta)$, and $*$ denotes the four-dimensional Hodge dual. Note that we have rewritten the scale parameter ℓ of section 2 in terms of $g = \ell^{-1}$. The $SU(2)$ Yang-Mills field strengths $F_{(2)}^i$ are given by

$$F_{(2)}^i = dA_{(1)}^i + \frac{1}{2}g \epsilon_{ijk} A_{(1)}^j \wedge A_{(1)}^k. \quad (16)$$

The $D = 11$ Hodge dual of the 4-form is given by

$$\begin{aligned}
\hat{*}F_{(4)} &= \frac{1}{32} g^{-6} \Delta^{-2} c^3 s^3 d\theta \wedge \epsilon_{(3)} \wedge \tilde{\epsilon}_{(3)} \\
&\quad - \frac{1}{128} g^{-5} \Delta^{-1} c^2 s^4 \epsilon_{ijk} h^i \wedge h^j \wedge F_{(2)}^k \wedge \tilde{\epsilon}_{(3)} \\
&\quad - \frac{1}{128} g^{-5} \Delta^{-1} s^2 c^4 \epsilon_{ijk} \tilde{h}^i \wedge \tilde{h}^j \wedge F_{(2)}^i \wedge \epsilon_{(3)} \\
&\quad + \frac{1}{32} g^{-5} c s^3 d\theta \wedge h^i \wedge F_{(2)}^i \wedge \tilde{\epsilon}_{(3)} + \frac{1}{32} g^{-5} s c^3 d\theta \wedge \tilde{h}^i \wedge F_{(2)}^i \wedge \epsilon_{(3)}. \quad (17)
\end{aligned}$$

It is straightforward to verify that the Bianchi identity $dF_{(4)} = 0$ is satisfied provided that the $SU(2)$ Yang-Mills fields $A_{(2)}^i$ satisfy the lower-dimensional equations of motion

$$D*F_{(2)}^i = 0, \quad (18)$$

where the covariant derivative D is defined by $DV^i = dV^i + g \epsilon_{ijk} A^j \wedge V^k$, for any vector V^i . The following identities are useful in verifying the equations of motion

$$DF_{(2)}^i = 0, \quad Dh^i = -\frac{1}{2} \epsilon_{ijk} h^j \wedge h^k - g F_{(2)}^i, \quad D\tilde{h}^i = -\frac{1}{2} \epsilon_{ijk} \tilde{h}^j \wedge \tilde{h}^k - g F_{(2)}^i. \quad (19)$$

The following formulae are also useful

$$\begin{aligned}
d(h^i \wedge *F_{(2)}^i) &= Dh^i \wedge *F_{(2)}^i - h^i \wedge D*F_{(2)}^i \\
&= -\frac{1}{2} \epsilon_{ijk} h^j \wedge h^k \wedge *F_{(2)}^i - g F_{(2)}^i \wedge *F_{(2)}^i, \\
\epsilon_{ijk} d(h^i \wedge h^j \wedge *F_{(2)}^k) &= \epsilon_{ijk} D(h^i \wedge h^j) \wedge *F_{(2)}^k = 0. \quad (20)
\end{aligned}$$

The verification of $d\hat{*}F_{(4)} = \frac{1}{2} F_4 \wedge F_{(4)}$ requires the following identity

$$\begin{aligned}
\Delta^{-2} c^3 s^3 - \frac{1}{2} (\Delta^{-1} c^2 s^4)' + c s^3 &= 0, \\
-\Delta^{-2} c^3 s^3 - \frac{1}{2} (\Delta^{-1} s^2 c^4)' + s c^3 &= 0. \quad (21)
\end{aligned}$$

The evaluation of the $D = 11$ Einstein equations of motion are much more complicated, and we have not performed the calculation in full detail. However, we have verified that the equations of motion work for the $U(1)$ subsector of the $SU(2)$ gauge fields. Combining the result, the lower-dimensional equations of motion can be obtained from the Lagrangian

$$e^{-1} \mathcal{L} = R - \frac{1}{4} (F_{(2)}^i)^2 - 8g^2. \quad (22)$$

The cosmological constant $8g^2$ is totally fixed by the gauge coupling constant g . Thus, we have obtained four-dimensional Einstein $SU(2)$ Yang-Mills de Sitter gauged gravity from $D = 11$ by consistent Kaluza-Klein reduction on a hyperbolic 7-space.

It is worth remarking that $D = 11$ supergravity can also give rise to $D = 4$ $SU(2)$ AdS supergravity [1]. Also, de Sitter gravity with the wrong sign in the kinetic terms for gauge fields can arise from hyperbolic reduction of * variations of M-theory, type IIB or massive type IIA theories [27, 24]. In our reduction of M-theory, however, the sign of the kinetic term for $A_{(1)}^i$ is the right one.

4 Lifting of solutions

The four-dimensional Lagrangian (22) admits a large class of solutions, including multi-center black holes [25, 26, 27]. Using our reduction ansatz (14) and (15), all of these solutions can be lifted to eleven dimensions. We will first explicitly lift the case of $dS_2 \times S^2$, which is supported by one of the $SU(2)$ gauge fields, *e.g.* $F_{(2)}^3$. This solution is given by

$$\begin{aligned} ds_4^2 &= -d\tau^2 + e^{\frac{\tau}{\ell\sqrt{2}}} dx^2 + 4\ell^2 d\Omega_2^2, \\ F_{(2)}^3 &= \frac{1}{2\ell} e^{\frac{\tau}{2\sqrt{2}\ell}} d\tau \wedge dx, \quad *F_{(2)}^3 = 2\ell \Omega_{(2)}, \end{aligned} \quad (23)$$

where $\ell^2 = 3/(64g^2)$. Note that the role of $F_{(2)}^3$ and $*F_{(2)}^3$ are interchangeable, giving rise to either the electric or magnetic solutions, with the metric unchanged. Lifting this solution back to $D = 11$ yields a smooth and regular embedding of dS_2 given by

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{2}{3}} \left(-d\tau^2 + e^{\frac{\tau}{\ell\sqrt{2}}} dx^2 + 4\ell^2 d\Omega_2^2 \right) + g^{-2} \Delta^{\frac{2}{3}} d\theta^2 \\ &\quad + \frac{1}{4} g^{-2} \Delta^{-\frac{1}{3}} \left[c^2 \left(d\omega_2^2 + (\sigma_3 - \sqrt{2} g e^{\frac{\tau}{2\sqrt{2}\ell}} dx)^2 \right) \right. \\ &\quad \left. + s^2 \left(d\tilde{\omega}_2^2 + (\tilde{\sigma}_3 - \sqrt{2} g e^{\frac{\tau}{2\sqrt{2}\ell}} dx)^2 \right) \right], \\ F_{(4)} &= 8g \ell^2 e^{\frac{\tau}{2\sqrt{2}\ell}} d\tau \wedge dx \wedge \Omega_{(2)} \\ &\quad - \frac{1}{2} g^{-2} \ell \left(s c d\theta \wedge (\sigma_3 - \tilde{\sigma}_3) - \frac{1}{2} c^2 \omega_{(2)} \wedge + \frac{1}{2} s^2 \tilde{\omega}_{(2)} \right) \wedge \Omega_{(2)}. \end{aligned} \quad (24)$$

We explicitly verified that the above solution satisfies the equations of motion of $D = 11$ supergravity, which serves as a double check of our reduction ansatz (14) and (15). In this smooth embedding of dS_2 in M-theory, the metric can be viewed as a

rotating brane. Of course, if we interchange the role of $F_{(2)}^3$ and $*F_{(2)}^3$ in (23), then the transverse space is a nine-dimensional non-compact space which can be viewed as an H^7 bundle over S^2 .

We will now consider a regular cosmological solution of (22) given by

$$\begin{aligned} ds_4^2 &= H^2 (-f^{-1} dt^2 + f dx^2 + t^2 d\Omega_2^2) \\ F_{(2)}^3 &= \frac{2\ell}{(tH)^2} dt \wedge dx, \quad *F_{(2)}^3 = 2\ell \Omega_{(2)}, \\ H &= 1 + \frac{\ell}{t}, \quad f = \frac{4}{3}g^2 t^2 H^4 - 1. \end{aligned} \quad (25)$$

This solution is, in fact, nothing but the BPS AdS Reissner-Nordström black hole with $g \rightarrow ig$ [28]. When $g^2 \ell^2 = \frac{3}{64}$, the solution interpolates between $dS_2 \times S^2$ at the infinite past in the co-moving time to a dS_4 -type geometry at the infinite future with the boundary of $S^2 \times S^1$ [28]. It is straightforward to lift the solution back to $D = 11$ and obtain a regular cosmological solution in M-theory. The corresponding metric is given by

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{2}{3}} H^2 (-f^{-1} dt^2 + f dx^2 + t^2 d\Omega_2^2) + g^{-2} \Delta^{\frac{2}{3}} d\theta^2 \\ &+ \frac{1}{4}g^{-2} \Delta^{-\frac{1}{3}} \left[c^2 (d\omega_2^2 + (\sigma_3 - \frac{2g}{H} dx)^2) + s^2 (d\tilde{\omega}_2^2 + (\tilde{\sigma}_3 - \frac{2g}{H} dx)^2) \right]. \end{aligned} \quad (26)$$

As a final example, we will lift $(\text{Minkowski})_2 \times S^2$ to eleven dimensions. This four-dimensional solution is given by

$$\begin{aligned} ds_4^2 &= -d\tau^2 + dx^2 + \frac{1}{8g^2} d\Omega_2^2, \\ F_{(2)}^3 &= 4g d\tau \wedge dx, \quad *F_{(2)}^3 = \frac{1}{2g} \Omega_{(2)}. \end{aligned} \quad (27)$$

Lifting this solution back to $D = 11$ yields a smooth and regular embedding of M_2 , whose metric is given by

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{2}{3}} \left(-d\tau^2 + dx^2 + \frac{1}{8g^2} d\Omega_2^2 \right) + g^{-2} \Delta^{\frac{2}{3}} d\theta^2 \\ &+ \frac{1}{4}g^{-2} \Delta^{-\frac{1}{3}} \left[c^2 (d\omega_2^2 + (\sigma_3 - 4g^2 dx)^2) \right. \\ &\quad \left. + s^2 (d\tilde{\omega}_2^2 + (\tilde{\sigma}_3 - 4g^2 dx)^2) \right]. \end{aligned} \quad (28)$$

As in the previous examples, one can interchange the role of $F_{(2)}^3$ and $*F_{(2)}^3$ in (27).

5 Further embeddings of dS spacetime

In [22], there are further embeddings of dS spacetime in M-theory or type IIB supergravities. In our example of a dS₄ embedding in M-theory, the H^7 is a foliation of $S^3 \times S^3$. The embeddings of dS₄ in M-theory with a squashed H^7 being a foliation of $S^2 \times S^4$ and dS₅ in type IIB theory with the H^5 being a foliation of $S^2 \times S^2$ were obtained in [22]. In this section, we obtain an new embedding of dS₃ in $D = 6$ supergravity. The relevant Lagrangian is given by

$$e^{-1} \mathcal{L} = R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{12}e^{\sqrt{2}\phi} F_{(3)}^2. \quad (29)$$

The solution is given by

$$\begin{aligned} ds_6^2 &= \Delta ds_3^2 + g^{-2} \Delta d\theta^2 + g^{-2} \Delta^{-1} (c^2 d\phi_1^2 + s^2 d\phi_2^2), \\ F_{(3)} &= 2g(\epsilon_{(3)} + *psilon_{(3)}), \quad \phi = 0, \end{aligned} \quad (30)$$

where ds_3^2 is dS₃ spacetime with cosmological constant $\Lambda = 2g^2$. Note that the transverse space is an H^3 written as a foliation of two circles. This theory can be reduced to pure de Sitter gravity in three dimensions, with the corresponding Lagrangian given by

$$e^{-1} \mathcal{L}_3 = R - 2g^2. \quad (31)$$

6 Conclusions

We have obtained $D = 4$ de Sitter gravity coupled to $SU(2)$ Yang-Mills fields from a consistent Kaluza-Klein reduction and truncation of $D = 11$ supergravity on a hyperbolic 7-space. The hyperbolic space is written as a foliation of two 3-spheres, on which the $SU(2)$ fields are embedded. Unlike in the case of * theories, the kinetic terms of the gauge fields have the correct sign. The four-dimensional cosmological constant is completely fixed by the gauge coupling constant. Although our reduction procedure fits within the general pattern of non-compact gaugings and their higher-dimensional origins, described in [21], our result provides the first explicit embedding of a non-trivial de Sitter gauge gravity in M-theory.

The reduction ansatz enables us to lift any solution of the four-dimensional theory to eleven dimensions. We discuss the embeddings of smooth cosmological solutions.

In particular, we obtain the embeddings of dS_2 and M_2 in M-theory, as well as that of a cosmological solution which smoothly interpolates between $dS_2 \times S^2$ at the infinite past in the co-moving time to a dS_4 -type geometry at the infinite future. We also obtain a new embedding of dS_3 in $D = 6$ supergravity. Our results provide important tools with which to study the dS/CFT correspondence from the point of view of string and M-theory.

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