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Four-Dimensional Einstein Yang-Mills De Sitter Gravity From Eleven Dimensions

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ABSTRACT

We obtain $D = 4$ de Sitter gravity coupled to $SU(2)$ Yang-Mills gauge fields from an explicit and consistent truncation of $D = 11$ supergravity *via* Kaluza-Klein dimensional reduction on a non-compact space. The "internal" space is a smooth hyperbolic 7-space (H^7) written as a foliation of two 3-spheres, on which the $SU(2)$ Yang-Mills fields reside. The positive cosmological constant is completely fixed by the $SU(2)$ gauge coupling constant. The explicit reduction ansatz enables us to lift any of the $D = 4$ solutions to $D = 11$. In particular, we obtain dS_2 in M-theory, where the nine-dimensional transverse space is an H^7 bundle over S^2. We also obtain a new smooth embedding of dS_3 in $D = 6$ supergravity.

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1 Introduction

The embedding of Anti-de Sitter (AdS) spacetimes in M-theory and string theories is rather straightforward. In fact, gauged supergravities in diverse dimensions with AdS vacuum have either been shown or are expected to be obtainable from consistent Kaluza-Klein sphere reductions of $D = 11$ or $D = 10$ supergravities. Notable examples include the simple embedding of $D = 4$, $\mathcal{N} = 2$, $SU(2)$ Yang-Mills AdS supergravity in $D = 11$ [1], the significantly more complicated S^7 [2, 3] and S^4 [4, 5, 6] reductions of M-theory, and the warped S^4 reduction of massive type IIA theory [7]. Although the S^5 reduction of type IIB theory has yet to be fully established,[1] the reduction of a certain truncation of the theory has been constructed [9, 10].

On the other hand, there is less known regarding embedding de Sitter (dS) spacetime in M-theory or string theories, mostly because this is quite a bit more complicated than the case of AdS. With recent experimental evidence suggesting that our universe might be de Sitter [11, 12], there is increasing interest in de Sitter gravity in cosmology and the dS/CFT correspondence [13, 14, 15, 16, 17]. Thus, it is of importance to obtain the embedding of a non-trivial de Sitter gravity theory in M-theory or string theories.

While no-go theorems [18, 19] imply that de Sitter spacetime cannot arise from a compactification of a supergravity theory, it can arise from a supergravity theory with a non-compact "internal" space [20]. Explicit embeddings of dS_4 and dS_5 in M-theory and type IIB supergravity, respectively, were obtained in [21]. These arise as ten or eleven-dimensional solutions that have a non-compact hyperbolic internal space.

In this paper, we obtain four-dimensional de Sitter gravity with $SU(2)$ Yang-Mills gauge fields from a less constrained truncation of $D = 11$ supergravity *via* Kaluza-Klein dimensional reduction on the non-compact space. In this construction, a consistent truncation of the higher-dimensional theory is required in which there are no modes which depend on the internal space. The $SU(2)$ fields arise from modes on the S^3 portions of the internal space. The Yang-Mills gauge coupling constant is completely fixed by the cosmological constant. The kinetic terms for the $SU(2)$

[1] The full metric ansatz was conjectured in [8].

gauge fields have the correct sign, implying that the theory is not merely an analytical continuation of $SU(2)$ AdS supergravity.

This paper is organized as follows. In section 2, we rederive the embedding of dS_4 spacetime in M-theory, with the transverse space being an H^7 written as a foliation of two 3-spheres. In section 3, we propose a reduction ansatz for obtaining $D=4$ $SU(2)$ Yang-Mills de Sitter gravity from $D=11$. We show that the reduction ansatz is indeed consistent with the $D=11$ equations of motion, and hence obtain the Lagrangian for $D=4$ $SU(2)$ Yang-Mills de Sitter gravity. The consistency of the reduction ansatz enables us to lift any $D=4$ solution back to $D=11$. In section 4, we discuss this in detail. In particular, we obtain the embedding of dS_2 spacetime in $D=11$, with the internal space being an H^7 bundle over S^2. In section 5, we obtain an embedding of dS_3 in $D=6$ supergravity with the transverse space being an H^3 written as a foliation of two circles. We conclude our paper in section 6.

2 Embedding dS_4 in $D=11$

We now show how the dS_4 embedding in $D=11$ supergravity found in [21] can be obtained directly from the eleven-dimensional equations of motion. We start with the Lagrangian of the bosonic sector of $D=11$ supergravity, given by

$$\mathcal{L} = R\,{*\mathbb{1}} - \tfrac{1}{2}{*F_{(4)}}\wedge F_{(4)} - \tfrac{1}{6}A_{(3)}\wedge F_{(4)}\wedge F_{(4)}\,, \tag{1}$$

where $F_{(4)}=dA_{(3)}$. We consider the ansatz

$$\begin{aligned} ds^2 &= H^2\,ds_4^2 + d\rho^2 + a^2\,d\Omega_3^2 + b^2\,d\widetilde{\Omega}_3^2\,,\\ F_4 &= q\,\epsilon_{(4)}\,, \end{aligned} \tag{2}$$

where H, a and b are functions of ρ, ds_4^2 is four-dimensional de Sitter spacetime with cosmological constant $\Lambda = 6\lambda^2$, *i.e.* $R_{\mu\nu} = 3\lambda^2\,g_{\mu\nu}$, and $\epsilon_{(4)}$ is the corresponding volume-form. $d\Omega_3^2$ and $d\widetilde{\Omega}_3^2$ are the metrics of the two unit 3-spheres. The Einstein equations of motion are given by

$$\begin{aligned} \frac{4\ddot H}{H} + \frac{3\ddot a}{a} + \frac{3\ddot b}{b} &= -\frac{q^2}{6H^8}\,,\\ \frac{\ddot H}{H} + \frac{3\dot H^2}{H^2} + \frac{3\dot H}{H}\Big(\frac{\dot a}{a} + \frac{\dot b}{b}\Big) &= \frac{q^2}{3H^8} + \frac{3\lambda^2}{H^2}\,, \end{aligned}$$

$$
\begin{aligned}
\frac{\ddot a}{a} + \frac{2\dot a^2}{a^2} + \frac{3\dot a\,\dot b}{a\,b} + \frac{4\dot a\,\dot H}{a\,H} - \frac{2}{a^2} &= -\frac{q^2}{6H^8}\,,\\
\frac{\ddot b}{b} + \frac{2\dot b^2}{b^2} + \frac{3\dot a\,\dot b}{a\,b} + \frac{4\dot b\,\dot H}{b\,H} - \frac{2}{b^2} &= -\frac{q^2}{6H^8}\,,
\end{aligned}
\tag{3}
$$

where a dot represents a derivative with respect to ρ. If the metric ds_4^2 is AdS instead of dS, then one can have a solution with H being a constant. In this case, ρ becomes an angular coordinate with $a \sim \cos\rho$ and $b \sim \sin\rho$, so that the metric becomes the direct product of AdS_4 and S^7, with the seven sphere written as a foliation of two three spheres. Inspired by the sphere reduction ansatz [3], we consider the following redefinition of variables

$$
H^2 = \Delta^{2/3}\,, \qquad a^2 = \Delta^{-1/3}\,\tilde a^2\,, \qquad b^2 = \Delta^{-1/3}\,\tilde b^2\,. \tag{4}
$$

Following the analogous relation in the case of $\mathrm{AdS}_4 \times S^7$, we have

$$
\tilde a^2 = \tfrac12 \ell^2\,(\Delta + 1)\,, \qquad \tilde b^2 = \tfrac12 \ell^2\,(\Delta - 1)\,, \tag{5}
$$

where ℓ is a constant scale parameter. Substituting (4) and (5) into (3), we find that the constants q and λ must satisfy

$$
q^2\,\ell^2 = 4\,, \qquad \lambda^2\,\ell^2 = \tfrac43\,. \tag{6}
$$

The equations (3) reduce to a single first-order differential equation

$$
\ell^2\,\Delta^{\frac23}\,\dot\Delta^2 - 4\Delta^2 + 4 = 0\,. \tag{7}
$$

Making a coordinate change $d\rho = \ell\,\Delta^{1/3}\,d\theta$, we can easily solve for Δ, which is given by

$$
\Delta = \cosh(2\theta)\,. \tag{8}
$$

Thus we have an explicit embedding of dS_4 in $D = 11$ given by

$$
\begin{aligned}
ds^2 &= \cosh^{\frac23}(2\theta)\,ds_4^2 + \ell^2\,\cosh^{\frac23}(2\theta)\,d\theta^2\\
&\quad + \cosh^{-\frac13}(2\theta)\,\Big(\cosh^2\theta\,d\Omega_3^2 + \sinh^2\theta\,d\widetilde\Omega_3^2\Big)\,,\\
F_4 &= \frac{2}{\ell}\,\epsilon_{(4)}\,.
\end{aligned}
\tag{9}
$$

This solution was obtained in [21].

If we consider (9) as a reduction ansatz from $D = 11$ to $D = 4$, then the resulting four-dimensional theory is Einstein gravity with a positive cosmological constant, and with a corresponding Lagrangian given by

$$e^{-1}\mathcal{L} = R - \frac{8}{\ell^2}\,. \qquad (10)$$

3 Embedding Yang-Mills de Sitter gravity

The metric of the 3-spheres in (9) can be written as

$$d\Omega_3^2 = \tfrac{1}{4}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2)\,, \qquad d\widetilde{\Omega}_3^2 = \tfrac{1}{4}(\tilde\sigma_1^2 + \tilde\sigma_2^2 + \tilde\sigma_3^2)\,, \qquad (11)$$

where σ_i and $\tilde\sigma_i$ are $SU(2)$ left-invariant 1-forms satisfying

$$d\sigma_i = -\tfrac{1}{2}\epsilon_{ijk}\,\sigma_j \wedge \sigma_k\,, \qquad d\tilde\sigma_i = -\tfrac{1}{2}\epsilon_{ijk}\,\tilde\sigma_j \wedge \tilde\sigma_k\,. \qquad (12)$$

Thus, we can introduce $SU(2)$ Yang-Mills fields $A^i_{(1)}$ to the vielbein

$$h^i = \sigma_i - g\,A^i_{(1)}\,, \qquad \tilde h^i = \tilde\sigma_i - g\,A^i_{(1)}\,. \qquad (13)$$

With these preliminaries, we propose the reduction ansatz

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{2}{3}}\,ds_4^2 + g^{-2}\,\Delta^{\frac{2}{3}}\,d\theta^2 + \tfrac{1}{4}g^{-2}\,\Delta^{-\frac{1}{3}}\Big[c^2\sum_i (h^i)^2 + s^2\sum_i (\tilde h^i)^2\Big]\,, \qquad (14)\\ F_{(4)} &= 2g\,\epsilon_{(4)} - \tfrac{1}{4}g^{-2}\Big(s\,c\,d\theta\wedge h^i\wedge {*F^i_{(2)}} - s\,c\,d\theta\wedge \tilde h^i\wedge {*F^i_{(2)}}\\ &\quad -\tfrac{1}{4}c^2\,\epsilon_{ijk}\,h^i\wedge h^j\wedge {*F^k_{(2)}} + \tfrac{1}{4}s^2\,\epsilon_{ijk}\,\tilde h^i\wedge\tilde h^j\wedge {*F^k_{(2)}}\Big)\,, \qquad (15)\end{aligned}$$

where $c = \cosh\theta$, $s = \sinh\theta$, $\Delta = \cosh(2\theta)$, and $*$ denotes the four-dimensional Hodge dual. Note that we have rewritten the scale parameter ℓ of section 2 in terms of $g = \ell^{-1}$. The $SU(2)$ Yang-Mills field strengths $F^i_{(2)}$ are given by

$$F^i_{(2)} = dA^i_{(1)} + \tfrac{1}{2}g\,\epsilon_{ijk}A^j_{(1)}\wedge A^k_{(1)}\,. \qquad (16)$$

The $D = 11$ Hodge dual of the 4-form is given by

$$\begin{aligned} \hat{*}F_{(4)} &= \tfrac{1}{32}\,g^{-6}\,\Delta^{-2}\,c^3\,s^3\,d\theta\wedge\epsilon_{(3)}\wedge\tilde\epsilon_{(3)}\\ &\quad -\tfrac{1}{128}\,g^{-5}\,\Delta^{-1}\,c^2\,s^4\,\epsilon_{ijk}\,h^i\wedge h^j\wedge F^k_{(2)}\wedge\tilde\epsilon_{(3)}\\ &\quad -\tfrac{1}{128}\,g^{-5}\,\Delta^{-1}\,s^2\,c^4\,\epsilon_{ijk}\,\tilde h^i\wedge\tilde h^j\wedge F^i_{(2)}\wedge\epsilon_{(3)}\\ &\quad +\tfrac{1}{32}\,g^{-5}\,c\,s^3\,d\theta\wedge h^i\wedge F^i_{(2)}\wedge\tilde\epsilon_{(3)} + \tfrac{1}{32}\,g^{-5}\,s\,c^3\,d\theta\wedge\tilde h^i\wedge F^i_{(2)}\wedge\epsilon_{(3)}\,. \qquad (17)\end{aligned}$$

It is straightforward to verify that the Bianchi identity $dF_{(4)} = 0$ is satisfied provided that the $SU(2)$ Yang-Mills fields $A^i_{(2)}$ satisfy the lower-dimensional equations of motion

$$D{*}F^i_{(2)} = 0\,, \tag{18}$$

where the covariant derivative D is defined by $DV^i = dV^i + g\,\epsilon_{ijk}\,A^j \wedge V^k$, for any vector V^i. The following identities are useful in verifying the equations of motion

$$DF^i_{(2)} = 0\,, \quad Dh^i = -\tfrac{1}{2}\epsilon_{ijk}\,h^j \wedge h^k - g\,F^i_{(2)} \quad D\tilde{h}^i = -\tfrac{1}{2}\epsilon_{ijk}\,\tilde{h}^j \wedge \tilde{h}^k - g\,F^i_{(2)}\,. \tag{19}$$

The following formulae are also useful

$$\begin{aligned}
d(h^i \wedge {*}F^i_{(2)}) &= Dh^i \wedge {*}F^i_{(2)} - h^i \wedge D{*}F^i_{(2)} \\
&= -\tfrac{1}{2}\epsilon_{ijk}\,h^j \wedge h^k \wedge {*}F^i_{(2)} - g\,F^i_{(2)} \wedge {*}F^i_{(2)}\,, \\
\epsilon_{ijk}\,d(h^i \wedge h^j \wedge {*}F^k_{(2)}) &= \epsilon_{ijk}\,D(h^i \wedge h^j) \wedge {*}F^k_{(2)} = 0\,.
\end{aligned} \tag{20}$$

The verification of $d\hat{*}F_{(4)} = \frac{1}{2}F_4 \wedge F_{(4)}$ requires the following identity

$$\begin{aligned}
\Delta^{-2}\,c^3\,s^3 - \tfrac{1}{2}(\Delta^{-1}\,c^2\,s^4)' + c\,s^3 &= 0\,, \\
-\Delta^{-2}\,c^3\,s^3 - \tfrac{1}{2}(\Delta^{-1}\,s^2\,c^4)' + s\,c^3 &= 0\,.
\end{aligned} \tag{21}$$

The evaluation of the $D = 11$ Einstein equations of motion are much more complicated, and we have not performed the calculation in full detail. However, we have verified that the equations of motion work for the $U(1)$ subsector of the $SU(2)$ gauge fields. Combining the result, the lower-dimensional equations of motion can be obtained from the Lagrangian

$$e^{-1}\mathcal{L} = R - \tfrac{1}{4}(F^i_{(2)})^2 - 8g^2\,. \tag{22}$$

The cosmological constant $8g^2$ is totally fixed by the gauge coupling constant g. Thus, we have obtained four-dimensional Einstein $SU(2)$ Yang-Mills de Sitter gauged gravity from $D = 11$ by consistent Kaluza-Klein reduction on a hyperbolic 7-space.

It is worth remarking that $D = 11$ supergravity can also give rise to $D = 4$ $SU(2)$ AdS supergravity [1]. Also, de Sitter gravity with the wrong sign in the kinetic terms for gauge fields can arise from hyperbolic reduction of * variations of M-theory, type IIB or massive type IIA theories [22, 23]. In our reduction of M-theory, however, the sign of the kinetic term for $A^i_{(1)}$ is the right one.

4 Lifting of solutions

The four-dimensional Lagrangian (22) admits a large class of solutions, including multi-center black holes [24, 22]. Using our reduction ansatz (14) and (15), all of these solutions can be lifted to eleven dimensions. We will first explicitly lift the case of $\mathrm{dS}_2 \times S^2$, which is supported by one of the $SU(2)$ gauge fields, *e.g.* $F^3_{\scriptscriptstyle(2)}$. This solution is given by

$$\begin{aligned} ds_4^2 &= -d\tau^2 + e^{\frac{\tau}{\ell\sqrt{2}}}\, dx^2 + 4\ell^2\, d\Omega_2^2\,, \\ F^3_{\scriptscriptstyle(2)} &= \tfrac{1}{2\ell}\, e^{\frac{\tau}{2\sqrt{2}\,\ell}}\, d\tau\wedge dx\,, \qquad {*F^3_{\scriptscriptstyle(2)}} = 2\ell\,\Omega_{\scriptscriptstyle(2)}\,, \end{aligned} \tag{23}$$

where $\ell^2 = 3/(64g^2)$. Note that the role of $F^3_{\scriptscriptstyle(2)}$ and ${*F^3_{\scriptscriptstyle(2)}}$ are interchangeable, giving rise to either the electric or magnetic solutions, with the metric unchanged. Lifting this solution back to $D = 11$ yields a smooth and regular embedding of dS_2 given by

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{2}{3}}\Big(-d\tau^2 + e^{\frac{\tau}{\ell\sqrt{2}}}\, dx^2 + 4\ell^2\, d\Omega_2^2\Big) + g^{-2}\,\Delta^{\frac{2}{3}}\, d\theta^2 \\ &\quad + \tfrac14 g^{-2}\,\Delta^{-\frac13}\Big[c^2\Big(d\omega_2^2 + (\sigma_3 - \sqrt{2}\,g\,e^{\frac{\tau}{2\sqrt{2}\,\ell}}\, dx)^2\Big) \\ &\qquad\qquad + s^2\Big(d\tilde\omega_2^2 + (\tilde\sigma_3 - \sqrt{2}\,g\,e^{\frac{\tau}{2\sqrt{2}\,\ell}}\, dx)^2\Big)\Big]\,, \\ F_{\scriptscriptstyle(4)} &= 8g\,\ell^2\, e^{\frac{\tau}{2\sqrt{2}\,\ell}}\, d\tau\wedge dx\wedge\Omega_{\scriptscriptstyle(2)} \\ &\quad -\tfrac12 g^{-2}\,\ell\,\Big(s\ c\ d\theta\wedge(\sigma_3 - \tilde\sigma_3) - \tfrac12 c^2\,\omega_{\scriptscriptstyle(2)}\wedge + \tfrac12 s^2\,\tilde\omega_{\scriptscriptstyle(2)}\Big)\wedge\Omega_{\scriptscriptstyle(2)}\,. \end{aligned} \tag{24}$$

We explicitly verified that the above solution satisfies the equations of motion of $D = 11$ supergravity, which serves as a double check of our reduction ansatz (14) and (15). In this smooth embedding of dS_2 in M-theory, the metric can be viewed as a rotating brane. Of course, if we interchange the role of $F^3_{\scriptscriptstyle(2)}$ and ${*F^3_{\scriptscriptstyle(2)}}$ in (23), then the transverse space is a nine-dimensional non-compact space which can be viewed as an H^7 bundle over S^2.

We will now consider a regular cosmological solution of (22) given by

$$\begin{aligned} ds_4^2 &= H^2\,(-f^{-1}\, dt^2 + f\, dx^2 + t^2\, d\Omega_2^2) \\ F^3_{\scriptscriptstyle(2)} &= \frac{2\ell}{(t\,H)^2}\, dt\wedge dx\,, \qquad {*F^3_{\scriptscriptstyle(2)}} = 2\ell\ \Omega_{\scriptscriptstyle(2)}\,, \\ H &= 1 + \frac{\ell}{t}\,, \qquad f = \tfrac43 g^2 t^2\, H^4 - 1\,. \end{aligned} \tag{25}$$

This solution is, in fact, nothing but the BPS AdS Reissner-Nordstrøm black hole with $g \to \mathrm{i}\, g$ [25]. When $g^2\,\ell^2 = \frac{3}{64}$, the solution interpolates between $\mathrm{dS}_2\times S^2$ at the infinite past in the co-moving time to a dS_4-type geometry at the infinite future with the boundary of $S^2\times S^1$ [25]. It is straightforward to lift the solution back to $D=11$ and obtain a regular cosmological solution in M-theory. The corresponding metric is given by

$$\begin{aligned} ds_{11}^2 &= \Delta^{\frac{2}{3}}\, H^2 \Big(- f^{-1}\, dt^2 + f\, dx^2 + t^2\, d\Omega_2^2\Big) + g^{-2}\,\Delta^{\frac{2}{3}}\, d\theta^2 \\ &\quad +\tfrac{1}{4} g^{-2}\,\Delta^{-\frac{1}{3}}\Big[c^2\Big(d\omega_2^2 + (\sigma_3 - \frac{2g}{H}\, dx)^2\Big) + s^2\Big(d\tilde{\omega}_2^2 + (\tilde{\sigma}_3 - \frac{2g}{H}\, dx)^2\Big)\Big]\,. \end{aligned} \tag{26}$$

5 Further embeddings of dS spacetime

In [21], there are further embeddings of dS spacetime in M-theory or type IIB supergravities. In our example of a dS_4 embedding in M-theory, the H^7 is a foliation of $S^3\times S^3$. The embeddings of dS_4 in M-theory with a squashed H^7 being a foliation of $S^2\times S^4$ and dS_5 in type IIB theory with the H^5 being a foliation of $S^2\times S^2$ were obtained in [21]. In this section, we obtain an new embedding of dS_3 in $D=6$ supergravity. The relevant Lagrangian is given by

$$e^{-1}\,\mathcal{L} = R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{12} e^{\sqrt{2}\phi}\, F_{(3)}^2\,. \tag{27}$$

The solution is given by

$$\begin{aligned} ds_6^2 &= \Delta\, ds_3^2 + g^{-2}\,\Delta\, d\theta^2 + g^{-2}\,\Delta^{-1}\,(c^2\, d\phi_1^2 + s^2\, d\phi_2^2)\,, \\ F_{(3)} &= 2g\,(\epsilon_{(3)} + \hat{*}\epsilon_{(3)})\,, \qquad \phi = 0\,, \end{aligned} \tag{28}$$

where ds_3^2 is dS_3 spacetime with cosmological constant $\Lambda = 2g^2$. Note that the transverse space is an H^3 written as a foliation of two circles. This theory can be reduced to pure de Sitter gravity in three dimensions, with the corresponding Lagrangian given by

$$e^{-1}\mathcal{L}_3 = R - 2g^2\,. \tag{29}$$

6 Conclusions

We have obtained $D=4$ de Sitter gravity coupled to $SU(2)$ Yang-Mills fields from a consistent Kaluza-Klein reduction and truncation of $D=11$ supergravity on a hyperbolic 7-space. The hyperbolic space is written as a foliation of two 3-spheres, on which the $SU(2)$ fields are embedded. Unlike in the case of * theories, the kinetic terms of the gauge fields have the correct sign. The four-dimensional cosmological constant is completely fixed by the gauge coupling constant. Although our reduction procedure fits within the general pattern of non-compact gaugings and their higher-dimensional origins, described in [20], our result provides the first explicit embedding of a non-trivial de Sitter gauge gravity in M-theory.

The reduction ansatz enables us to lift any solution of the four-dimensional theory to eleven dimensions. We discuss the embeddings of smooth cosmological solutions. In particular, we obtain the embedding of dS_2 in M-theory, as well as a cosmological solution which smoothly interpolates between $\mathrm{dS}_2 \times S^2$ at the infinite past in the co-moving time to a dS_4-type geometry at the infinite future. We also obtain a new embedding of dS_3 in $D = 6$ supergravity. Our results provide important tools with which to study the dS/CFT correspondence from the point of view of string and M-theory.

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