ACT-13-11, MIFPA-11-42

The Lightest CP-Even Higgs Boson Mass in the Testable Flipped $SU(5) \times U(1)_X$ Models from F-Theory

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(Dated: April 25, 2018)

Abstract

We study the lightest CP-even Higgs boson mass in five kinds of testable flipped $SU(5)\times U(1)_X$ models from F-theory. Two kinds of models have vector-like particles around the TeV scale, while the other three kinds also have the vector-like particles at the intermediate scale that can be considered as messenger fields in gauge mediated supersymmetry breaking. We require that the Yukawa couplings for the TeV-scale vector-like particles and the third family of the Standard Model (SM) fermions are smaller than three from the electroweak scale to the $SU(3)_C\times SU(2)_L$ unification scale. With the two-loop renormalization group equation running for the gauge couplings and Yukawa couplings, we obtain the maximal Yukawa couplings between the TeV-scale vector-like particles and Higgs fields. To calculate the lightest CP-even Higgs boson mass upper bounds, we employ the renormalization group improved effective Higgs potential approach, and consider the two-loop leading contributions in the supersymmetric SM and one-loop contributions from the TeV-scale vector-like particles. We assume maximal mixings between the stops and between the TeV-scale vector-like scalars. The numerical results for these five kinds of models are roughly the same. In particular, we show that the lightest CP-even Higgs boson can have mass up to 146 GeV naturally, which is the current upper bound from the CMS and ATLAS collaborations.

PACS numbers: 11.10.Kk, 11.25.Mj, 11.25.-w, 12.60.Jv

I. INTRODUCTION

The Higgs boson mass in the Standard Model (SM) is not stable against qunatum corrections and has quadratic divergences. Because the reduced Planck scale is about 16 order larger than the electroweak (EW) scale, there exists huge fine-tuning to have the EW-scale Higgs boson mass, which is called the gauge hierarchy problem. Supersymmetry is a symmetry between the bosonic and fermionic states, and it naturally solves this problem due to the cancellations between the bosonic and fermionic quantum corrections.

In the Minimal Supersymmetric Standard Model (MSSM) with R parity under which the SM particles are even while the supersymmetric particles (sparticles) are odd, the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge couplings can be unified around 2×10^{16} GeV [1], the lightest supersymmetric particle (LSP) such as the neutralino can be a cold dark matter candidate [2, 3], and the EW precision constraints can be evaded, etc. Especially, the gauge coupling unification strongly suggests Grand Unified Theories (GUTs), which can explain the SM fermion quantum numbers. However, in the supersymmetric $SU(5)$ models, there exist the doublet-triplet splitting problem and dimension-five proton decay problem. Interestingly, these problems can be solved elegantly in the flipped $SU(5) \times U(1)_X$ models via missing partner mechanism [4–6]. Previously, the flipped $SU(5) \times U(1)_X$ models have been constructed systematically in the free fermionic string constructions at the Kac-Moody level one [7, 8]. To solve the little hierarchy problem between the traditional unification scale and the string scale, two of us (TL and DVN) with Jiang have proposed the testable flipped $SU(5) \times U(1)_X$ models, where the TeV-scale vector-like particles are introduced [9]. There is a two-step unifcation: the $SU(3)_C \times SU(2)_L$ gauge couplings are unified at the scale M_{32} around the usual GUT scale, and the $SU(5) \times U(1)_X$ gauge couplings are unified at the final unification scale $M_{\mathcal{F}}$ around 5×10^{17} GeV [9]. Moreover, such kind of models have been constructed locally from the F-theory model building [10, 11], and are dubbed as $\mathcal{F}$-$SU(5)$ [11]. In particular, these models are very interesting from the phenomenological point of view [11]: the vector-like particles can be observed at the Large Hadron Collider (LHC), proton decay is within the reach of the future Hyper-Kamiokande [12] and Deep Underground Science and Engineering Laboratory (DUSEL) [13] experiments [14, 15], the hybrid inflation can be naturally realized, and the correct cosmic primodial density fluctuations can be generated [16].

With No-Scale boundary conditions at $M_{\mathcal{F}}$ [17], two of us (TL and DVN) with Maxin and Walker have described an extraordinarily constrained "golden point" [18] and "golden strip" [19] that satisfied all the latest experimental constraints and has an imminently observable proton decay rate [14]. Especially, the UV boundary condition $B_\mu = 0$ gives very strong constraint on the viable parameter space, where B_μ is the soft bilinear Higgs mass term in the MSSM. In addition, exploiting a "Super-No-Scale" condition, we dynamically determined the universal gaugino mass $M_{1/2}$ and the ratio of the Higgs Vacuum Expectation Values (VEVs) $\tan\beta$. Since $M_{1/2}$ is related to the modulus field of the internal space in string models, we stabilized the modulus dynamically [20, 21]. Interestingly, the supersymmetric particle (sparticle) spectra generially have a light stop and gluino, which are lighter than all the other squarks. Thus, we can test such kinds of models at the LHC by looking for the ultra high jet signals [22, 23]. Moreover, the complete viable parameter space in no-scale $\mathcal{F}$-$SU(5)$ has been studied by considering a set of "bare minimal" experimental constaints [24]. For the other LHC and dark matter phenomenological studies, see Refs. [25–27].

It is well known that one of main LHC goals is to detect the SM or SM-like Higgs boson. Recently, both the CMS [28] and ATLAS [29] collaborations have presented their combined searches for the SM Higgs boson, base on the integrated luminosities between 1 fb^{-1} and 2.3 fb^{-1} depending on the search channels. For the light SM Higgs boson mass region preferred by the EW precision data, they have excluded the SM Higgs boson with mass larger than 145 GeV and 146 GeV, respectively. In the no-scale $\mathcal{F}$-$SU(5)$, the lightest CP-even Higgs boson mass is generically about 120 GeV if the contributions from the vector-like particles are neglected [30]. Thus, the interesting question is whether the lightest CP-even Higgs boson can have mass up to 146 GeV naturally if we include the contributions from the additional vector-like particles.

In this paper, we consider five kinds of testable flipped $SU(5) \times U(1)_X$ models from F-theoy. Two kinds of models only have vector-like particles around the TeV scale. Because the gauge mediated supersymmetry breaking can be realized naturally in the F-theory GUTs [31], we also introduce vector-like particles with mass around 10^{11} GeV [31], which can be considered as messenger fields, in the other three kinds of models. We require that the Yukawa couplings for the TeV-scale vector-like particles and the third family of the SM fermions are smaller than three from the EW scale to the scale M_{32} from the perturbative bound, *i.e.*, the Yukawa coupling squares are less than 4π. With the two-loop Renormalization Group Equation (RGE) running for the gauge couplings and Yukawa couplings, we obtain the maximal Yukawa couplings for the TeV-scale vector-like particles. To calculate the lightest CP-even Higgs boson mass upper bounds, we employ the Renormalization Group (RG) improved effective Higgs potential approach, and consider the two-loop leading contributions in the MSSM and one-loop contributions from the TeV-scale vector-like particles. For simplicity, we assume that the mixings both between the stops and between the TeV-scale vector-like scalars are maximal. In general, we shall increase the lightest CP-even Higgs boson mass upper bounds if we increase the supersymmetry breaking scale or decrease the TeV-scale vector-like particle masses. The numerical results for our five kinds of models are roughly the same. For the TeV-scale vector-like particles and sparticles with masses around 1 TeV, we show that the lightest CP-even Higgs boson can have mass up to 146 GeV naturally.

This paper is organized as follows. In Section II, we briefly review the testable flipped $SU(5) \times U(1)_X$ models from F-theory and present five kinds of models. We calculate the lightest CP-even Higgs boson mass upper bounds in Section III. Section IV is our conclusion. In Appendices, we present all the RGEs in five kinds of models.

II. TESTABLE FLIPPED $SU(5) \times U(1)_X$ MODELS FROM F-THEORY

We first briefly review the minimal flipped $SU(5)$ model [4–6]. The gauge group for flipped $SU(5)$ model is $SU(5) \times U(1)_X$, which can be embedded into $SO(10)$ model. We define the generator $U(1)_{Y'}$ in $SU(5)$ as

$$T_{\mathrm{U(1)_{Y'}}} = \mathrm{diag}\left(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}\right). \tag{1}$$

The hypercharge is given by

$$Q_Y = \frac{1}{5}\left(Q_X - Q_{Y'}\right). \tag{2}$$

There are three families of the SM fermions whose quantum numbers under $SU(5)\times U(1)_X$ are

$$F_i = (\mathbf{10}, \mathbf{1}),\ \bar{f}_i = (\bar{\mathbf{5}}, \mathbf{-3}),\ \bar{l}_i = (\mathbf{1}, \mathbf{5}), \tag{3}$$

where $i = 1, 2, 3$. The SM particle assignments in F_i, $\bar{f}_i$ and $\bar{l}_i$ are

$$F_i = (Q_i, D_i^c, N_i^c),\ \overline{f}_i = (U_i^c, L_i),\ \bar{l}_i = E_i^c\ , \tag{4}$$

where Q_i and L_i are respectively the superfields of the left-handed quark and lepton doublets, U_i^c, D_i^c, E_i^c and N_i^c are the CP conjugated superfields for the right-handed up-type quarks, down-type quarks, leptons and neutrinos, respectively. To generate the heavy right-handed neutrino masses, we introduce three SM singlets ϕ_i [32].

To break the GUT and electroweak gauge symmetries, we introduce two pairs of Higgs representations

$$H = (\mathbf{10}, \mathbf{1}),\ \overline{H} = (\overline{\mathbf{10}}, -\mathbf{1}),\ h = (\mathbf{5}, -\mathbf{2}),\ \overline{h} = (\bar{\mathbf{5}}, \mathbf{2}). \tag{5}$$

We label the states in the H multiplet by the same symbols as in the F multiplet, and for $\overline{H}$ we just add "bar" above the fields. Explicitly, the Higgs particles are

$$H = (Q_H, D_H^c, N_H^c)\ ,\ \overline{H} = (\overline{Q}_{\overline{H}}, \overline{D}^c_{\overline{H}}, \overline{N}^c_{\overline{H}})\ , \tag{6}$$

$$h = (D_h, D_h, D_h, H_d)\ ,\ \overline{h} = (\overline{D}_{\overline{h}}, \overline{D}_{\overline{h}}, \overline{D}_{\overline{h}}, H_u)\ , \tag{7}$$

where H_d and H_u are one pair of Higgs doublets in the MSSM. We also add one SM singlet Φ.

To break the $SU(5) \times U(1)_X$ gauge symmetry down to the SM gauge symmetry, we introduce the following Higgs superpotential at the GUT scale

$$W_{\rm GUT} = \lambda_1 HHh + \lambda_2 \overline{H}\overline{H}\overline{h} + \Phi(\overline{H}H - M_{\rm H}^2)\ . \tag{8}$$

There is only one F-flat and D-flat direction, which can always be rotated along the N_H^c and $\overline{N}^c_{\overline{H}}$ directions. So, we obtain that $< N_H^c > = < \overline{N}^c_{\overline{H}} > = M_{\rm H}$. In addition, the superfields H and $\overline{H}$ are eaten and acquire large masses via the supersymmetric Higgs mechanism, except for D_H^c and $\overline{D}^c_{\overline{H}}$. The superpotential $\lambda_1 HHh$ and $\lambda_2 \overline{H}\overline{H}\overline{h}$ couple the D_H^c and $\overline{D}^c_{\overline{H}}$ with the D_h and $\overline{D}_{\overline{h}}$, respectively, to form the massive eigenstates with masses $2\lambda_1 < N_H^c >$ and $2\lambda_2 < \overline{N}^c_{\overline{H}} >$. So, we naturally have the doublet-triplet splitting due to the missing partner mechanism [6]. Because the triplets in h and $\overline{h}$ only have small mixing through the μ term, the Higgsino-exchange mediated proton decay are negligible, *i.e.*, we do not have the dimension-5 proton decay problem.

The SM fermion masses are from the following superpotential

$$W_{\rm Yukawa} = y_{ij}^{D} F_i F_j h + y_{ij}^{U\nu} F_i \overline{f}_j \overline{h} + y_{ij}^{E} \overline{l}_i \overline{f}_j h + \mu h \overline{h} + y_{ij}^{N} \phi_i \overline{H} F_j\ , \tag{9}$$

where y_{ij}^D, $y_{ij}^{U\nu}$, y_{ij}^E and y_{ij}^N are Yukawa couplings, and μ is the bilinear Higgs mass term.

After the $SU(5) \times U(1)_X$ gauge symmetry is broken down to the SM gauge symmetry, the above superpotential gives

$$\begin{aligned} W_{SSM} &= y_{ij}^D D_i^c Q_j H_d + y_{ji}^{U\nu} U_i^c Q_j H_u + y_{ij}^E E_i^c L_j H_d + y_{ij}^{U\nu} N_i^c L_j H_u \\ &\quad + \mu H_d H_u + y_{ij}^N \langle \overline{N_{\overline{H}}^c} \rangle \phi_i N_j^c + \cdots (\text{decoupled below } M_{GUT}). \end{aligned} \tag{10}$$

Similar to the flipped $SU(5) \times U(1)_X$ models with string-scale gauge coupling unification [9, 33], we introduce vector-like particles which form complete flipped $SU(5) \times U(1)_X$ multiplets. The quantum numbers for these additional vector-like particles under the $SU(5) \times U(1)_X$ gauge symmetry are

$$XF = (\mathbf{10}, \mathbf{1}) \ , \ \overline{XF} = (\overline{\mathbf{10}}, -\mathbf{1}) \ , \tag{11}$$

$$Xf = (\mathbf{5}, \mathbf{3}) \ , \ \overline{Xf} = (\overline{\mathbf{5}}, -\mathbf{3}) \ , \tag{12}$$

$$Xl = (\mathbf{1}, -\mathbf{5}) \ , \ \overline{Xl} = (\mathbf{1}, \mathbf{5}) \ , \tag{13}$$

$$Xh = (\mathbf{5}, -\mathbf{2}) \ , \ \overline{Xh} = (\overline{\mathbf{5}}, \mathbf{2}) \ , \tag{14}$$

$$XT = (\mathbf{10}, -\mathbf{4}) \ , \ \overline{XT} = (\overline{\mathbf{10}}, \mathbf{4}) \ . \tag{15}$$

Moreover, the particle contents from the decompositions of XF, $\overline{XF}$, Xf, $\overline{Xf}$, Xl, $\overline{Xl}$, Xh, $\overline{Xh}$, XT, and $\overline{XT}$, under the SM gauge symmetry are

$$XF = (XQ, XD^c, XN^c) \ , \ \overline{XF} = (XQ^c, XD, XN) \ , \tag{16}$$

$$Xf = (XU, XL^c) \ , \ \overline{Xf} = (XU^c, XL) \ , \tag{17}$$

$$Xl = XE \ , \ \overline{Xl} = XE^c \ , \tag{18}$$

$$Xh = (XD, XL) \ , \ \overline{Xh} = (XD^c, XL^c) \ , \tag{19}$$

$$XT = (XY, XU^c, XE) \ , \ \overline{XT} = (XY^c, XU, XE^c) \ . \tag{20}$$

Under the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry, the quantum numbers for the extra vector-like particles are

$$XQ = (\mathbf{3}, \mathbf{2}, \frac{\mathbf{1}}{\mathbf{6}}) \ , \ XQ^c = (\overline{\mathbf{3}}, \mathbf{2}, -\frac{\mathbf{1}}{\mathbf{6}}) \ , \tag{21}$$

$$XU = (\mathbf{3}, \mathbf{1}, \frac{\mathbf{2}}{\mathbf{3}}) \ , \ XU^c = (\overline{\mathbf{3}}, \mathbf{1}, -\frac{\mathbf{2}}{\mathbf{3}}) \ , \tag{22}$$

$$XD = (\mathbf{3}, \mathbf{1}, -\frac{\mathbf{1}}{\mathbf{3}}) \ , \ XD^c = (\overline{\mathbf{3}}, \mathbf{1}, \frac{\mathbf{1}}{\mathbf{3}}) \ , \tag{23}$$

$$XL = (\mathbf{1}, \mathbf{2}, -\frac{\mathbf{1}}{\mathbf{2}}) \ , \ \ XL^c = (\mathbf{1}, \mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}}) \ , \tag{24}$$

$$XE = (\mathbf{1}, \mathbf{1}, -\mathbf{1}) \ , \ \ XE^c = (\mathbf{1}, \mathbf{1}, \mathbf{1}) \ , \tag{25}$$

$$XN = (\mathbf{1}, \mathbf{1}, \mathbf{0}) \ , \ \ XN^c = (\mathbf{1}, \mathbf{1}, \mathbf{0}) \ , \tag{26}$$

$$XY = (\mathbf{3}, \mathbf{2}, -\frac{\mathbf{5}}{\mathbf{6}}) \ , \ \ XY^c = (\bar{\mathbf{3}}, \mathbf{2}, \frac{\mathbf{5}}{\mathbf{6}}) \ . \tag{27}$$

To separate the mass scales M_{32} and $M_{\mathcal{F}}$ in our F-theory flipped $SU(5) \times U(1)_X$ models, we need to introduce sets of vector-like particles around the TeV scale or intermediate scale whose contributions to the one-loop beta functions satisfy $\Delta b_1 < \Delta b_2 = \Delta b_3$. To avoid the Landau pole problem, we have shown that there are only five possible such sets of vector-like particles as follows due to the quantizations of the one-loop beta functions [9]

$$Z0 : XF + \overline{XF} \ ; \tag{28}$$

$$Z1 : XF + \overline{XF} + Xl + \overline{Xl} \ ; \tag{29}$$

$$Z2 : XF + \overline{XF} + Xf + \overline{Xf} \ ; \tag{30}$$

$$Z3 : XF + \overline{XF} + Xl + \overline{Xl} + Xh + \overline{Xh} \ ; \tag{31}$$

$$Z4 : XF + \overline{XF} + Xh + \overline{Xh} \ . \tag{32}$$

We have systematically constructed flipped $SU(5) \times U(1)_X$ models with generic sets of vector-like particles around the TeV scale and/or around the intermediate scale from the F-theory. In addition, gauge mediated supersymmetry breaking can be realized naturally in the F-theory GUTs [31], and there may exist vector-like particles as the messenger fields at the intermediate scale around 10^{11} GeV [31]. Therefore, in this paper, we shall calculate the lightest CP-even Higgs boson mass in five kinds of the flipped $SU(5) \times U(1)_X$ models from F-theory: (i) In Model I, we introduce the $Z0$ set of vector-like particles (XF, $\overline{XF}$) at the TeV scale, and we shall add superheavy vector-like particles around M_{32} so that the $SU(5) \times U(1)_X$ unification scale is smaller than the reduced Planck scale; (ii) In Model II, we introduce the vector-like particles (XF, $\overline{XF}$) at the TeV scale and the vector-like particles (Xf, $\overline{Xf}$) at the intermediate scale which can be considered as the messenger fields; (iii) In Model III, we introduce the vector-like particles (XF, $\overline{XF}$) at the TeV scale and the vector-like particles (Xf, $\overline{Xf}$) and (Xl, $\overline{Xl}$) at the intermediate scale; (iv) In Model IV, we introduce the $Z1$ set of vector-like particles (XF, $\overline{XF}$) and (Xl, $\overline{Xl}$) at the TeV scale; (v) In Model V, we introduce the vector-like particles (XF, $\overline{XF}$) and (Xl, $\overline{Xl}$) at the

TeV scale, and the vector-like particles $(Xf,\ \overline{Xf})$ at the intermediate scale. In particular, we emphasize that the vector-like particles at the intermediate scale in Models II, III, and V will give us the generalized gauge medidated supersymmetry breaking if they are the messenger fields [34]. By the way, if we introduce the vector-like particles $(Xh,\ \overline{Xh})$ at the intermediate scale which are the traditional messenger fields in gauge mediation, the discussions are similar and the numerical results are almost the same. Thus, we will not study such kind of models here.

For simplicity, we assume that the masses for the vector-like particles around the TeV scale or the intermediate scale are universal. Also, we denote the universal mass for the vector-like particles at the TeV scale as M_V, and the universal mass for the vector-like particles at the intermediate scale as M_I. With this convention, we present the vector-like particle contents of our five kinds of models in Table I. In the following discussions, we shall choose $M_I = 1.0 \times 10^{11}$ GeV. Moreover, we will assume universal supersymmetry breaking at low energy and denote the universal supersymmetry breaking scale as M_S.

Models	Vector-Like Particles at M_V	Vector-Like Particles at M_I
Model I	$(XF,\ \overline{XF})$	
Model II	$(XF,\ \overline{XF})$	$(Xf,\ \overline{Xf})$
Model III	$(XF,\ \overline{XF})$	$(Xf,\ \overline{Xf}),\ (Xl,\ \overline{Xl})$
Model IV	$(XF,\ \overline{XF}),\ (Xl,\ \overline{Xl})$	
Model V	$(XF,\ \overline{XF}),\ (Xl,\ \overline{Xl})$	$(Xf,\ \overline{Xf})$

TABLE I: The vector-like particle contents in Model I, Model II, Model III, Model IV, and Model V.

It is well known that there exists a few pecent fine-tuning for the lightest CP-even Higgs boson mass in the MSSM to be larger than 114.4 GeV. In all the above five kinds of models, we have the vector-like particles XF and $\overline{XF}$ at the TeV scale. Then we can introduce the following Yukawa interaction terms between the MSSM Higgs fields and these vector-like particles in the superpotential in the flipped $SU(5) \times U(1)_X$ models:

$$W \ = \ \frac{1}{2} Y_{xd} XF XF h + \frac{1}{2} Y_{xu} \overline{XF} \overline{XF} \overline{h} \ , \tag{33}$$

where Y_{xd} and Y_{xu} are Yukawa couplings. After the gauge symmetry $SU(5) \times U(1)_X$ is broken down to the SM gauge symmetry, we have the following relevant Yukawa coupling terms in the superpotential

$$W \ = \ Y_{xd} XQXD^c H_d + Y_{xu} XQ^c XDH_u \ . \tag{34}$$

To have the upper bounds on the lightest CP-even Higgs boson mass, we first need to calculate the upper bounds on the Yukawa couplings Y_{xu} and Y_{xd}. In this paper, employing the two-loop RGE running, we will require that all the Yukawa couplings including Y_{xu} and Y_{xd} are smaller than three (perturbative bound) below the $SU(3)_C \times SU(2)_L$ unification scale M_{32} for simplicity since M_{32} is close to the $SU(5) \times U(1)_X$ unification scale $M_{\mathcal{F}}$. The other point is that above the scale M_{32}, there might exist other superheavy threshold corrections and then the RGE running for the gauge couplings and Yukawa couplings might be very complicated. Moreover, we will not give the two-loop RGEs in the SM and the MSSM, which can be easily found in the literatures, for example, in the Refs. [35, 36]. We shall present the RGEs in the SM with vector-like particles, and Models I to V in the Appendices A, B, C, D, E, and F, respectively.

III. THE LIGHTEST CP-EVEN HIGGS BOSON MASS

In our calculations, we employ the RG improved effective Higgs potential approach. The two-loop leading contributions to the lightest CP-even Higgs boson mass m_h in the MSSM are [37, 38]

$$\begin{aligned}[m_h^2]_{\rm MSSM} \ = & \ M_Z^2 \cos^2 2\beta (1 - \frac{3}{8\pi^2}\frac{m_t^2}{v^2} t) \\ & + \frac{3}{4\pi^2}\frac{m_t^4}{v^2}[t + \frac{1}{2}X_t + \frac{1}{(4\pi)^2}(\frac{3}{2}\frac{m_t^2}{v^2} - 32\pi\alpha_s)(X_t t + t^2)],\end{aligned} \tag{35}$$

where M_Z is the Z boson mass, m_t is the $\overline{MS}$ top quark mass, v is the SM Higgs VEV, and α_S is the strong coupling constant. Also, t and X_t are given as follows

$$t = \log \frac{M_S^2}{M_t^2}, \quad X_t = \frac{2\tilde{A}_t^2}{M_S^2}(1 - \frac{\tilde{A}_t^2}{12M_S^2}), \quad \tilde{A}_t = A_t - \mu \cot\beta, \tag{36}$$

where M_t is the top quark pole mass, and A_t denotes the trilinear soft term for the top quark Yukawa coupling term.

Moreover, we use the RG-improved one-loop effective Higgs potential approach to calculate the contributions to the lightest CP-even Higgs boson mass from the vector-like particles [39, 40]. Such contributions in our models are

$$\begin{aligned}
\Delta m_h^2 \;=\; & -\frac{N_c}{8\pi^2}M_Z^2\cos^2 2\beta(\hat{Y}_{xu}^2+\hat{Y}_{xd}^2)t_V+\frac{N_c v^2}{4\pi^2}\times\{\hat{Y}_{xu}^4[t_V+\frac{1}{2}X_{xu}] \\
& +\hat{Y}_{xu}^3\hat{Y}_{xd}[-\frac{2M_S^2(2M_S^2+M_V^2)}{3(M_S^2+M_V^2)^2}-\frac{\tilde{A}_{xu}(2\tilde{A}_{xu}+\tilde{A}_{xd})}{3(M_S^2+M_V^2)}] \\
& +\hat{Y}_{xu}^2\hat{Y}_{xd}^2[-\frac{M_S^4}{(M_S^2+M_V^2)^2}-\frac{(\tilde{A}_{xu}+\tilde{A}_{xd})^2}{3(M_S^2+M_V^2)}] \\
& +\hat{Y}_{xu}\hat{Y}_{xd}^3[-\frac{2M_S^2(2M_S^2+M_V^2)}{3(M_S^2+M_V^2)^2}-\frac{\tilde{A}_{xd}(2\tilde{A}_{xd}+\tilde{A}_{xu})}{3(M_S^2+M_V^2)}]+\hat{Y}_{xd}^4[t_V+\frac{1}{2}X_{xd}]\}, \qquad (37)
\end{aligned}$$

where

$$\begin{aligned}
& \hat{Y}_{xu}=Y_{xu}\sin\beta, \qquad \hat{Y}_{xd}=Y_{xd}\cos\beta, \qquad t_V=\log\frac{M_S^2+M_V^2}{M_V^2}, \\
& X_{xu}=-\frac{2M_S^2(5M_S^2+4M_V^2)-4(3M_S^2+2M_V^2)\tilde{A}_{xu}^2+\tilde{A}_{xu}^4}{6(M_V^2+M_S^2)^2}, \\
& X_{xd}=-\frac{2M_S^2(5M_S^2+4M_V^2)-4(3M_S^2+2M_V^2)\tilde{A}_{xd}^2+\tilde{A}_{xd}^4}{6(M_V^2+M_S^2)^2}, \\
& \tilde{A}_{xu}=A_{xu}-\mu\cot\beta, \qquad \tilde{A}_{xd}=A_{xd}-\mu\tan\beta, \qquad (38)
\end{aligned}$$

where A_{xu} and A_{xd} denote the supersymmetry breaking trilinear soft terms for the superpotential Yukawa terms $Y_{xu}XQ^cXDH_u$ and $Y_{xd}XQXD^cH_d$, respectively.

The third, fourth, fifth, and sixth terms in Eq. (37) are suppressed by the inverses of $\tan\beta$, $\tan^2\beta$, $\tan^3\beta$, and $\tan^4\beta$, respectively. To have the lightest CP-even Higgs boson mass upper bounds, we usually need $\tan\beta\sim 22$ from the numerical calculations. Especially, in order to increase the lightest CP-even Higgs boson mass, we should choose relatively large Y_{xu} and small Y_{xd} [39, 40]. Thus, for simplicity, we only employ the first and second terms in our calculations, *i.e.*, the first line of Eq. (37). In order to have larger corrections to the lightest CP-even Higgs boson mass, we consider the maximal mixings X_t and X_{xu} respectively for both the stops and the TeV-scale vector-like scalars, *i.e.*, $X_t=6$ with $\tilde{A}_t^2=6M_S^2$, and $X_{xu}=\frac{8}{3}+\frac{M_S^2(5M_S^2+4M_V^2)}{3(M_S^2+M_V^2)}$ with $\tilde{A}_{xu}^2=6M_S^2+4M_V^2$.

In this Section, we shall calculate the lightest CP-even Higgs boson mass in our five kinds of models. The relevant parameters are the universal supersymmetry breaking scale M_S, the light vector-like particle mass M_V, the intermediate scale M_I, the mixing terms X_t and X_V respectively for the stops and TeV-scale vector-like scalars, and the two new Yukawa couplings for TeV-scale vector-like particles Y_{xu} and Y_{xd}. Because we consider low energy supersymmetry, we choose M_S from 360 GeV to 2 TeV. In order to increase the lightest

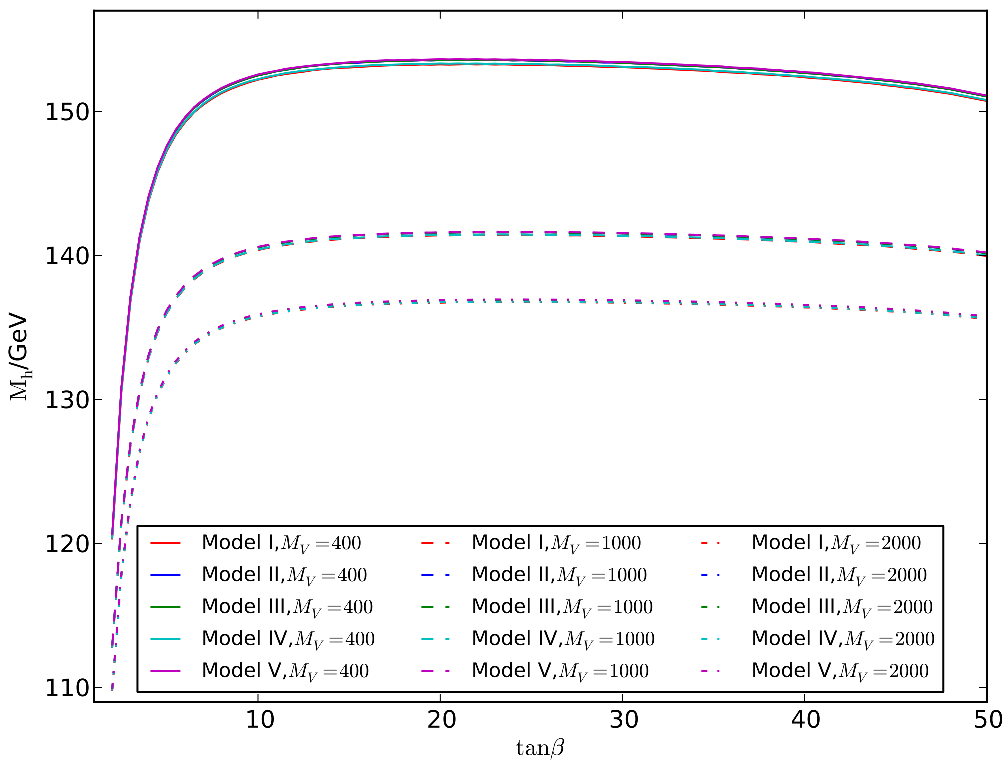

FIG. 1: (color online). The upper bounds on the lightest CP-even Higgs boson mass versus $\tan\beta$ for our five kinds of models with $Y_{xd} = 0$, $M_S = 800$ GeV, and $M_I = 1.0 \times 10^{11}$ GeV. The upper lines, middle lines, and lower lines are for $M_V = 400$ GeV, 1000 GeV, and 2000 GeV, respectively.

CP-even Higgs boson mass, we need to choose small M_V as well. The experimental lower bound on M_V is about 325 GeV [41], so we will choose M_V from 360 GeV to 2 TeV. In our numerical calculations, we will use the SM input parameters at scale M_Z from Particle Data Group [42]. In particular, we use the updated top quark pole mass $M_t = 172.9$ GeV, and the corresponding $\overline{MS}$ top quark mass $m_t = 163.645$ GeV [42].

In this paper, we require that all the Yukawa couplings for both the TeV-scale vector-like particles and the third family of SM fermions are less than three (perturbative bound) from the EW scale to the scale M_{32}. To obtain the upper bounds on the Yukawa couplings Y_{xu} and Y_{xd} at low energy, we consider the two-loop RGE running for both the SM gauge couplings and all the Yukawa couplings. The only exception is that when $M_V < M_S$, we use the two-loop RGE running for the SM gauge couplings and one-loop RGE running for all the Yukawa couplings from M_V to M_S, see Appendix A for details. Because in this case M_V

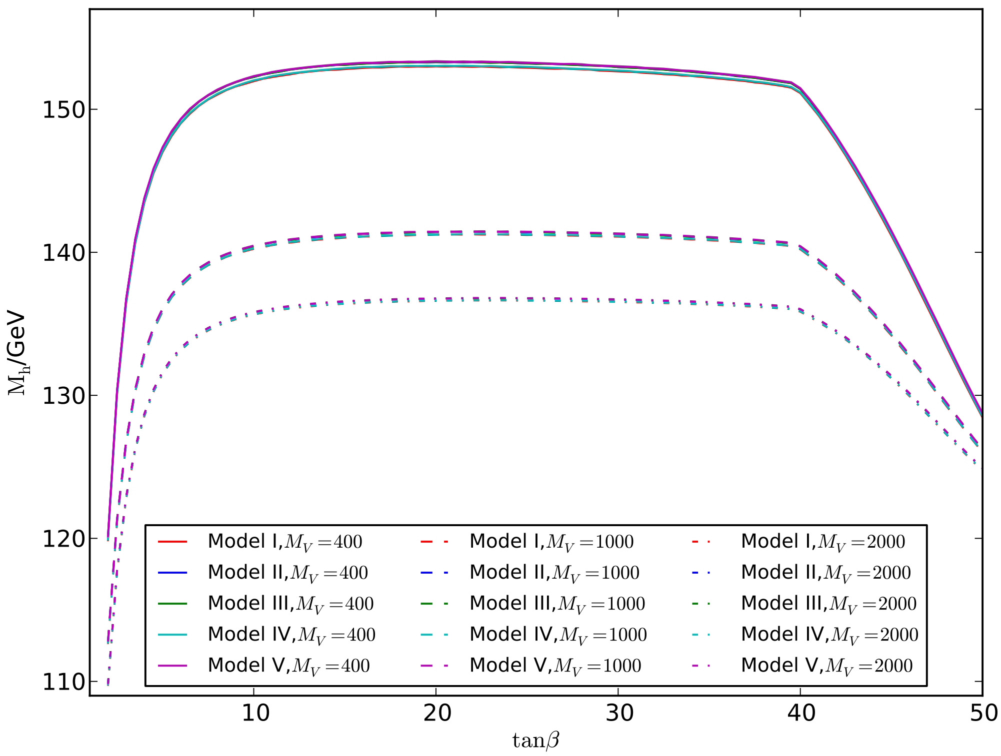

FIG. 2: (color online). The upper bounds on the lightest CP-even Higgs boson mass versus $\tan\beta$ for our five kinds of models with $Y_{xd}(M_V) = Y_{xu}(M_V)$, $M_S = 800$ GeV, and $M_I = 1.0 \times 10^{11}$ GeV. The upper lines, middle lines, and lower lines are for $M_V = 400$ GeV, 1000 GeV, and 2000 GeV, respectively.

is still close to M_S, such small effects are negligible. After we obtain the upper bounds on Y_{xu} and Y_{xd}, we use the maximal Y_{xu} to calculate the upper bounds on the lightest CP-even Higgs boson mass with the maximal mixings for stops and TeV-scale vector-like scalars.

First, we consider $Y_{xd} = 0$, $M_S = 800$ GeV, and $M_I = 1.0 \times 10^{11}$ GeV. We choose three values for M_V: $M_V = 400$ GeV, 1000 GeV, and 2000 GeV. In Fig. 1, we present the upper bounds on the lightest CP-even Higgs boson mass by varying $\tan\beta$ from 2 to 50. We find that for the same M_V, the upper bounds on the lightest CP-even Higgs boson mass are almost the same for five kinds of models. In particular, the small differences are less than 0.4 GeV. Because the gauge couplings will give negative contributions to the Yukawa coupling RGEs, we will have a little bit larger maximal Yukawa couplings Y_{xu} if the vector-like particles contribute more to the gauge coupling RGE running. Thus, the Model order for the lightest

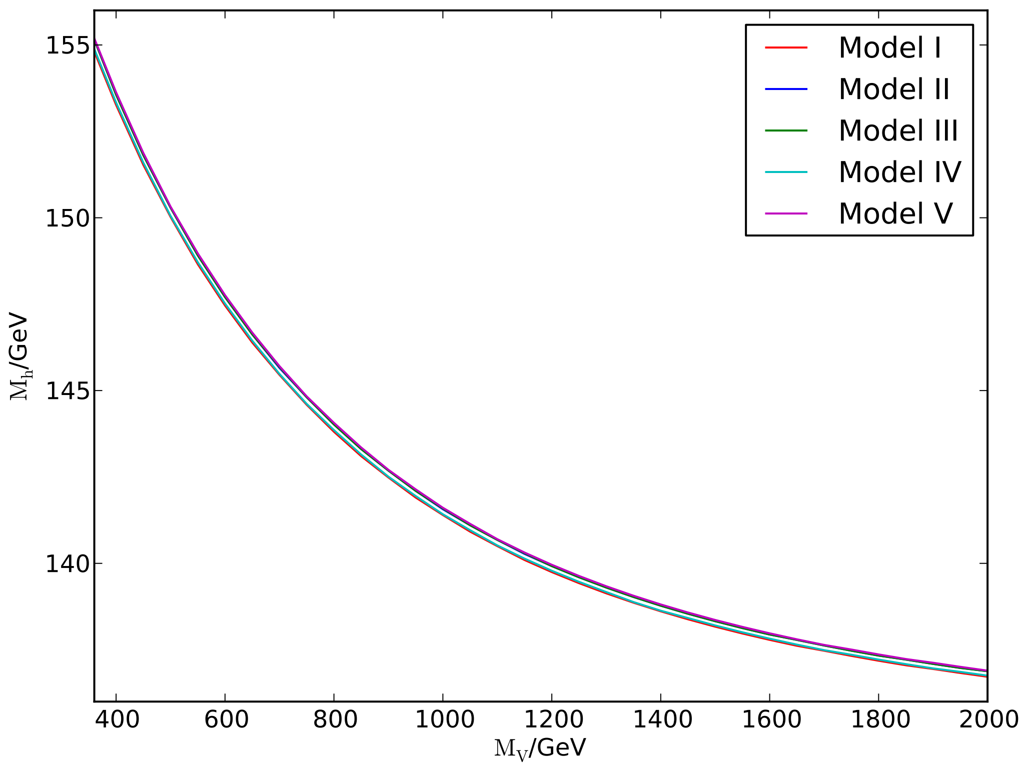

FIG. 3: (color online). The upper bounds on the lightest CP-even Higgs boson mass versus M_V for our five kinds of models with $Y_{xd} = 0$, $\tan\beta = 20$, $M_S = 800$ GeV, and $M_I = 1.0 \times 10^{11}$ GeV.

CP-even Higgs boson mass upper bounds from small to large is Model I, Model IV, Model II, Model III, Model V. Also, the upper bounds on the lightest CP-even Higgs boson mass will decrease when we increase M_V, which is easy to understand from physics point of view. Moreover, the maximal Yukawa couplings Y_{xu} are about 0.96, 1.03, and 1.0 for $\tan\beta = 2$, $\tan\beta \sim 23$, and $\tan\beta = 50$, respectively. In addition, for $M_V = 400$ GeV and $\tan\beta \simeq 21$, $M_V = 1000$ GeV and $\tan\beta \simeq 23.5$, and $M_V = 2000$ GeV and $\tan\beta \simeq 24.5$, we obtain the lightest CP-even Higgs boson mass upper bounds around 153.5 GeV, 141.6 GeV, and 136.8 GeV, respectively.

Second, we consider $Y_{xd} = Y_{xu}$ at the scale M_V, $M_S = 800$ GeV, and $M_I = 1.0 \times 10^{11}$ GeV. We choose three values for M_V: $M_V = 400$ GeV, 1000 GeV, and 2000 GeV. In Fig. 2, we present the upper bounds on the lightest CP-even Higgs boson mass by varying $\tan\beta$ from 2 to 50. For $\tan\beta < 40$, we find that the lightest CP-even Higgs boson mass upper bounds are almost the same as those in Fig. 1. However, for $\tan\beta > 40$, we find that the lightest

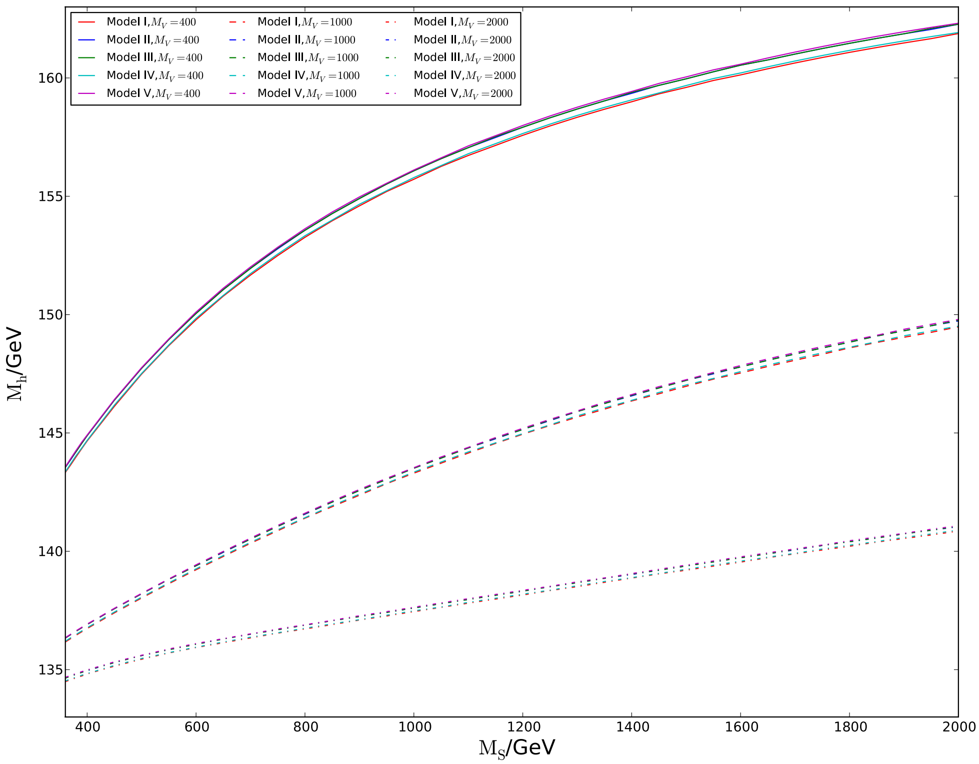

FIG. 4: (color online). The upper bounds on the lightest CP-even Higgs boson mass versus M_S for our five kinds of models with $Y_{xd} = 0$, $\tan\beta = 20$, and $M_I = 1.0 \times 10^{11}$ GeV. The upper lines, middle lines, and lower lines are for $M_V = 400$ GeV, 1000 GeV, and 2000 GeV, respectively.

CP-even Higgs boson mass upper bounds decrease fast when $\tan\beta$ increases. At $\tan\beta = 50$, the upper bounds on the lightest CP-even Higgs boson mass are smaller than 130 GeV for all our scenarios. The reasons are the following: for $\tan\beta < 40$, the Yukawa couplings Y_{xu} and Y_t are easy to run out of the perturbative bound, while for $\tan\beta > 40$, the Yukawa couplings Y_{xd}, Y_b, and especially Y_τ are easy to run out, where Y_t, Y_b and Y_τ are Yukawa couplings for the top quark, bottom quark, and tau lepton, respectively. In particular, for $\tan\beta = 50$, the maximal Yukawa couplings $Y_{xd} = Y_{xu}$ are as small as 0.67 while they are about 1.025 for $\tan\beta < 40$.

Third, we consider $Y_{xd} = 0$, $\tan\beta = 20$, $M_S = 800$ GeV, and $M_I = 1.0 \times 10^{11}$ GeV. In Fig. 3, we present the upper bounds on the lightest CP-even Higgs boson mass by varying M_V from 360 GeV to 2 TeV. We can see that as the value of M_V increases from 360 GeV to 2 TeV, the upper bounds on the lightest CP-even Higgs boson mass decrease from 155 GeV to 137 GeV. In particular, to have the lightest CP-even Higgs boson mass upper bounds larger than 146 GeV, we obtain that M_V is smaller than about 700 GeV. Moreover, the maximal Yukawa couplings Y_{xu} vary only a little bit, decreasing from about 1.029 to 1.016 for M_V from 360 GeV to 2 TeV.

Fourth, we consider $Y_{xd} = 0$, $\tan\beta = 20$, and $M_I = 1.0 \times 10^{11}$ GeV. We choose three values for M_V: $M_V = 400$ GeV, 1000 GeV, and 2000 GeV. In Fig. 4, we present the upper bounds on the lightest CP-even Higgs boson mass by varying M_S from 360 GeV to 2 TeV. As the value of M_S increases, the upper bounds on the lightest CP-even Higgs boson mass increase from about 143 GeV to 162 GeV, from about 136 GeV to 150 GeV, and from about 134 GeV to 141 GeV, for $M_V = 400$ GeV, 1000 GeV, and 2000 GeV, repectively. Especially, to have the lightest CP-even Higgs boson mass upper bounds larger than 146 GeV, we obtain that M_S is larger than about 430 GeV and 1260 GeV for $M_V = 400$ GeV and 1000 GeV, respectively. Moreover, the maximal Yukawa couplings Y_{xu} decrease from about 1.049 to 1.007 for M_S from 360 GeV to 2 TeV.

Fifth, we consider $Y_{xd} = 0$, $\tan\beta = 20$, and $M_S = 800$ GeV. Also, we choose three values for M_V: $M_V = 400$ GeV, 1000 GeV, and 2000 GeV, and three values for X_t: $X_t = 0$, 3, and 6. For simplicity, we only consider Model I here. In Fig. 5, we present the upper bounds on the lightest CP-even Higgs boson mass by varying $\tilde{A}_{xu}$. As we expected, they behave just like the variations of the lightest CP-even Higgs boson mass upper bounds with varying stop mixing X_t, which have been studied extensively before in Refs. [43–46].

IV. CONCLUSION

We calculated the lightest CP-even Higgs boson mass in five kinds of testable flipped $SU(5)\times U(1)_X$ models from F-theory. Two kinds of models have vector-like particles around the TeV scale, while the other three kinds also have the vector-like particles at the intermediate scale as the messenger fields in gauge mediation. The Yukawa couplings for the TeV-scale vector-like particles and the third family of the SM fermions are required to be smaller than three from the EW scale to the scale M_{32}. With the two-loop RGE running for both the gauge couplings and Yukawa couplings, we obtained the maximal Yukawa couplings between the TeV-scale vector-like particles and Higgs fields. To calculate the lightest CP-even Higgs boson mass upper bounds, we used the RG improved effective Higgs poten-

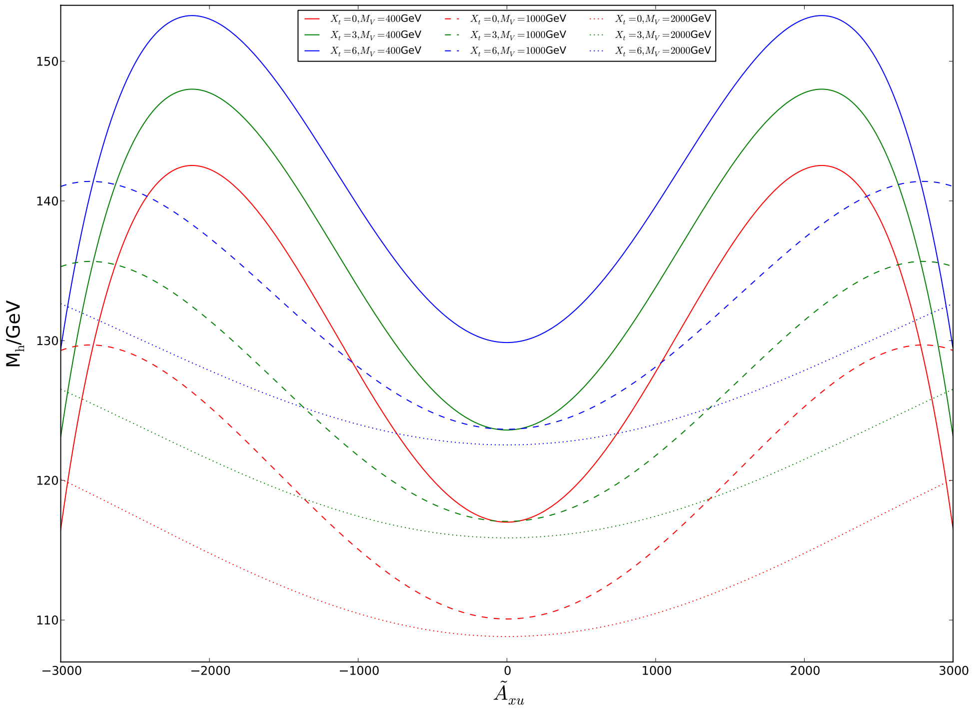

FIG. 5: (color online). The upper bounds on the lightest CP-even Higgs boson mass versus X_{xu} in Model I with $Y_{xd} = 0$, $\tan\beta = 20$, $M_S = 800$ GeV, $M_V = 400$ GeV, 1000 GeV, and 2000 GeV, and $X_t = 0,\ 3,$ and 6.

tial approach, and considered the two-loop leading contributions in the MSSM and one-loop contributions from the TeV-scale vector-like particles. For simplicity, we assumed that the mixings both between the stops and between the TeV-scale vector-like scalars are maximal. The numerical results for these five kinds of models are roughly the same. With M_V and M_S around 1 TeV, we showed that the lightest CP-even Higgs boson mass can be close to 146 GeV naturally, which is the upper bound from the current CMS and ATLAS collaborations.

Acknowledgments

This research was supported in part by the Natural Science Foundation of China under grant numbers 10821504 and 11075194 (YH, TL, CT), and by the DOE grant DE-FG03-95-Er-40917 (TL and DVN).

Appendix A: Renormalization Group Equations in the SM with Vector-Like Particles

When $M_V < M_S$, at the renormalization scale between them, we have the Standard Model plus vector-like particles, with the RGE's for the gauge couplings and Yukawa couplings as follows [47–50]:

$$(4\pi)^2 \frac{d}{dt} g_i = b_i g_i^3 + \frac{g_i^3}{(4\pi)^2} \left[\sum_{j=1}^{3} B_{ij} g_j^2 - \sum_{\alpha=u,d,e,xu,xd} d_i^\alpha \mathrm{Tr}(Y_\alpha^\dagger Y_\alpha) \right], \tag{A1}$$

where $t = \ln \mu$ and μ is the renormalization scale. The g_1, g_2 and g_3 are the gauge couplings for $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$, respectively, where we use the $SU(5)$ normalization $g_1^2 \equiv (5/3) g_Y^2$. The beta-function coefficients are

$$b = \left(\tfrac{41}{10},\ -\tfrac{19}{6},\ -7 \right), \qquad B = \begin{pmatrix} \tfrac{199}{50} & \tfrac{27}{10} & \tfrac{44}{5} \\ \tfrac{9}{10} & \tfrac{35}{6} & 12 \\ \tfrac{11}{10} & \tfrac{9}{2} & -26 \end{pmatrix}, \tag{A2}$$

$$d^u = \left(\tfrac{17}{10},\ \tfrac{3}{2},\ 2 \right), \quad d^d = d^{xu} = d^{xd} = \left(\tfrac{1}{2},\ \tfrac{3}{2},\ 2 \right), \quad d^e = \left(\tfrac{3}{2},\ \tfrac{1}{2},\ 2 \right). \tag{A3}$$

And

$$\frac{d}{dt} Y_{u,d,e,xu,xd} = \frac{1}{16\pi^2} Y_{u,d,e,xu,xd} \beta^{(1)}_{u,d,e,xu,xd}, \tag{A4}$$

where

$$\beta_u^{(1)} = \frac{3}{2}(Y_u^\dagger Y_u - Y_d^\dagger Y_d) + Y_2 - (\frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2), \tag{A5}$$

$$\beta_d^{(1)} = \frac{3}{2}(Y_d^\dagger Y_d - Y_u^\dagger Y_u) + Y_2 - (\frac{1}{4} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2), \tag{A6}$$

$$\beta_e^{(1)} = \frac{3}{2} Y_e^\dagger Y_e + Y_2 - \frac{9}{4}(g_1^2 + g_2^2), \tag{A7}$$

$$\beta_{xu}^{(1)} = \frac{3}{2} Y_{xu}^\dagger Y_{xu} + Y_2 - (\frac{1}{4} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2), \tag{A8}$$

$$\beta_{xd}^{(1)} = \frac{3}{2} Y_{xd}^\dagger Y_{xd} + Y_2 - (\frac{1}{4} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2), \tag{A9}$$

with

$$Y_2 = Tr\{3 Y_u^\dagger Y_u + 3 Y_d^\dagger Y_d + Y_e^\dagger Y_e\} + 3 Y_{xu}^\dagger Y_{xu} + 3 Y_{xd}^\dagger Y_{xd}. \tag{A10}$$

Appendix B: Renormalization Group Equations in Model I

In the Model I, the two-loop renormalization group equations for the gauge couplings are

$$(4\pi)^2 \frac{d}{dt}\, g_i \;=\; b_i g_i^3 + \frac{g_i^3}{(4\pi)^2}\left[\,\sum_{j=1}^{3} B_{ij} g_j^2 - \sum_{\alpha=u,d,e,xu,xd} d_i^{\alpha} \mathrm{Tr}\left(Y_\alpha^\dagger Y_\alpha\right)\right] \;, \tag{B1}$$

where Y_u, Y_d, Y_e, Y_{xu}, and Y_{xd} are the Yukawa couplings for the up-type quark, down-type quark, lepton, vector-like particles $\overline{XF}$, and vector-like particles XF, respectively. The beta-function coefficients are

$$b = \left(\frac{33}{5}, 1, -3\right) + \left(\frac{3}{5}, 3, 3\right) \;, \tag{B2}$$

$$B = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} + \begin{pmatrix} \frac{3}{25} & \frac{3}{5} & \frac{16}{5} \\ \frac{1}{5} & 21 & 16 \\ \frac{2}{5} & 6 & 34 \end{pmatrix} \;, \tag{B3}$$

$$d^u = \left(\frac{26}{5}, 6, 4\right) \;,\; d^d = \left(\frac{14}{5}, 6, 4\right) \;,\; d^e = \left(\frac{18}{5}, 2, 0\right) \;, \tag{B4}$$

$$d^{xu} = \left(\frac{14}{5}, 6, 4\right) \;,\; d^{xd} = \left(\frac{14}{5}, 6, 4\right) \;. \tag{B5}$$

The two-loop renormalization group equations for Yukawa couplings are

$$(4\pi)^2 \frac{d}{dt}\, Y_\alpha \;=\; \frac{1}{16\pi^2}\beta_{Y_\alpha}^{(1)} + \frac{1}{(16\pi^2)^2}\beta_{Y_\alpha}^{(2)} \;, \tag{B6}$$

where $\alpha = u,\ d,\ e,\ xu,\ xd$. In addition, $\beta_{Y_\alpha}^{(1)}$ and $\beta_{Y_\alpha}^{(2)}$ are given as follows

$$\beta_{Y_u}^{(1)} \;=\; Y_u \left(3\mathrm{Tr}(Y_u Y_u^\dagger) + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d + 3Y_{xu}^\dagger Y_{xu} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\right) \;, \tag{B7}$$

$$\begin{aligned}\beta_{Y_u}^{(2)} \;=\; Y_u \Big(& -3\mathrm{Tr}(3Y_u Y_u^\dagger Y_u Y_u^\dagger + Y_u Y_d^\dagger Y_d Y_u^\dagger) - 9Y_{xu}^\dagger Y_{xu} Y_{xu}^\dagger Y_{xu} - 9Y_u^\dagger Y_u \mathrm{Tr}(Y_u Y_u^\dagger) \\ & -9Y_u^\dagger Y_u Y_{xu}^\dagger Y_{xu} - Y_d^\dagger Y_d \mathrm{Tr}(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) - 3Y_d^\dagger Y_d Y_{xd} Y_{xd}^\dagger \\ & -4Y_u^\dagger Y_u Y_u^\dagger Y_u - 2Y_d^\dagger Y_d Y_d^\dagger Y_d - 2Y_d^\dagger Y_d Y_u^\dagger Y_u + (16g_3^2 + \frac{4}{5}g_1^2)\mathrm{Tr}(Y_u Y_u^\dagger) \\ & +(16g_3^2 - \frac{2}{5}g_1^2)Y_{xu}^\dagger Y_{xu} + (6g_2^2 + \frac{2}{5}g_1^2)Y_u^\dagger Y_u + \frac{2}{5}g_1^2 Y_d^\dagger Y_d + \frac{128}{9}g_3^4 + 8g_3^2 g_2^2 \\ & +\frac{136}{45}g_3^2 g_1^2 + \frac{33}{2}g_2^4 + g_2^2 g_1^2 + \frac{2977}{450}g_1^4\Big) \;, \end{aligned} \tag{B8}$$

$$\beta_{Y_d}^{(1)} \;=\; Y_d \left(\mathrm{Tr}(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) + 3Y_d^\dagger Y_d + Y_u^\dagger Y_u + 3Y_{xd}^\dagger Y_{xd} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right) \;, \tag{B9}$$

$$\begin{aligned}
\beta^{(2)}_{Y_d} \;=\; & Y_d \Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger) \\
& -3Y_u^\dagger Y_uY_{xu}Y_{xu}^\dagger - 3Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_d^\dagger Y_dY_{xd}^\dagger Y_{xd} - 4Y_d^\dagger Y_dY_d^\dagger Y_d \\
& -2Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_u^\dagger Y_uY_d^\dagger Y_d + (16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) \\
& +(16g_3^2 - \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd} + (6g_2^2 + \frac{4}{5}g_1^2)Y_d^\dagger Y_d + \frac{4}{5}g_1^2Y_u^\dagger Y_u + \frac{128}{9}g_3^4 + 8g_3^2g_2^2 \\
& +\frac{8}{9}g_3^2g_1^2 + \frac{33}{2}g_2^4 + g_2^2g_1^2 + \frac{1561}{450}g_1^4\Big) \ , \qquad \text{(B10)}
\end{aligned}$$

$$\beta^{(1)}_{Y_e} \;=\; Y_e\left(\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) + 3Y_e^\dagger Y_e + 3Y_{xd}^\dagger Y_{xd} - 3g_2^2 - \frac{9}{5}g_1^2\right) \ , \qquad \text{(B11)}$$

$$\begin{aligned}
\beta^{(2)}_{Y_e} \;=\; & Y_e \Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} \\
& -3Y_e^\dagger Y_e\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_e^\dagger Y_eY_{xd}^\dagger Y_{xd} - 4Y_e^\dagger Y_eY_e^\dagger Y_e \\
& +(16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + (16g_3^2 - \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd} \\
& +6g_2^2Y_e^\dagger Y_e + \frac{33}{2}g_2^4 + \frac{9}{5}g_2^2g_1^2 + \frac{729}{50}g_1^4\Big) \ , \qquad \text{(B12)}
\end{aligned}$$

$$\beta^{(1)}_{Y_{xu}} \;=\; Y_{xu}\left(3\mathrm{Tr}(Y_uY_u^\dagger) + 6Y_{xu}^\dagger Y_{xu} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right) \ , \qquad \text{(B13)}$$

$$\begin{aligned}
\beta^{(2)}_{Y_{xu}} \;=\; & Y_{xu} \Big(-3\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 22Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_{xu}^\dagger Y_{xu}\mathrm{Tr}(Y_uY_u^\dagger) \\
& +(16g_3^2 + \frac{4}{5}g_1^2)\mathrm{Tr}(Y_uY_u^\dagger) + (16g_3^2 + 6g_2^2 + \frac{2}{5}g_1^2)Y_{xu}^\dagger Y_{xu} \\
& +\frac{128}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2 + \frac{33}{2}g_2^4 + g_2^2g_1^2 + \frac{1561}{450}g_1^4\Big) \ , \qquad \text{(B14)}
\end{aligned}$$

$$\beta^{(1)}_{Y_{xd}} \;=\; Y_{xd}\left(\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) + 6Y_{xd}^\dagger Y_{xd} - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\right) \ , \qquad \text{(B15)}$$

$$\begin{aligned}
\beta^{(2)}_{Y_{xd}} \;=\; & Y_{xd} \Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 22Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} \\
& -3Y_{xd}^\dagger Y_{xd}\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) + (16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) \\
& +(16g_3^2 + 6g_2^2 + \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd} + \frac{128}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2 \\
& +\frac{33}{2}g_2^4 + g_2^2g_1^2 + \frac{1561}{450}g_1^4\Big) \ . \qquad \text{(B16)}
\end{aligned}$$

Appendix C: Renormalization Group Equations in Model II

In Model II, below the intermediate scale $M_I = 1.0 \times 10^{11}$ GeV, we have the same RGEs as in Model I. Above M_I, we have additional vector-like particles (Xf, $\overline{Xf}$). Thus, we need to add extra contributions to b and B from the vector-like particles (Xf, $\overline{Xf}$). Comparing to the RGEs in Model I, we also need to change the coefficients of the g_3^4, g_2^4 and g_1^4 terms in $\beta^{(2)}_{Y_u}$, $\beta^{(2)}_{Y_d}$, $\beta^{(2)}_{Y_{xu}}$, and $\beta^{(2)}_{Y_{xd}}$, and change the coefficients of the g_2^4 and g_1^4 terms in $\beta^{(2)}_{Y_e}$. In short, comparing to the RGEs in Model I, the coefficients in the RGEs above M_I, which need to be changed, are the following:

$$b = \left(\frac{33}{5}, 1, -3\right) + \left(\frac{3}{5}, 3, 3\right) + \left(\frac{11}{5}, 1, 1\right) \ , \tag{C1}$$

$$B = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} + \begin{pmatrix} \frac{3}{25} & \frac{3}{5} & \frac{16}{5} \\ \frac{1}{5} & 21 & 16 \\ \frac{2}{5} & 6 & 34 \end{pmatrix} + \begin{pmatrix} \frac{31}{15} & \frac{9}{5} & \frac{128}{15} \\ \frac{3}{5} & 7 & 0 \\ \frac{16}{15} & 0 & \frac{34}{3} \end{pmatrix} \ , \tag{C2}$$

$$\begin{aligned} \beta^{(2)}_{Y_u} \ = \ & Y_u \Big(-3\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 9Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_u^\dagger Y_u \mathrm{Tr}(Y_uY_u^\dagger) \\ & -9Y_u^\dagger Y_uY_{xu}^\dagger Y_{xu} - Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 3Y_d^\dagger Y_dY_{xd}Y_{xd}^\dagger \\ & -4Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_d^\dagger Y_dY_u^\dagger Y_u + (16g_3^2 + \frac{4}{5}g_1^2)\mathrm{Tr}(Y_uY_u^\dagger) \\ & +(16g_3^2 - \frac{2}{5}g_1^2)Y_{xu}^\dagger Y_{xu} + (6g_2^2 + \frac{2}{5}g_1^2)Y_u^\dagger Y_u + \frac{2}{5}g_1^2Y_d^\dagger Y_d + \frac{176}{9}g_3^4 + 8g_3^2g_2^2 \\ & +\frac{136}{45}g_3^2g_1^2 + \frac{39}{2}g_2^4 + g_2^2g_1^2 + \frac{767}{90}g_1^4\Big) \ , \end{aligned} \tag{C3}$$

$$\begin{aligned} \beta^{(2)}_{Y_d} \ = \ & Y_d \Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger) \\ & -3Y_u^\dagger Y_uY_{xu}Y_{xu}^\dagger - 3Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_d^\dagger Y_dY_{xd}^\dagger Y_{xd} - 4Y_d^\dagger Y_dY_d^\dagger Y_d \\ & -2Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_u^\dagger Y_uY_d^\dagger Y_d + (16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) \\ & +(16g_3^2 - \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd} + (6g_2^2 + \frac{4}{5}g_1^2)Y_d^\dagger Y_d + \frac{4}{5}g_1^2Y_u^\dagger Y_u + \frac{176}{9}g_3^4 + 8g_3^2g_2^2 \\ & +\frac{8}{9}g_3^2g_1^2 + \frac{39}{2}g_2^4 + g_2^2g_1^2 + \frac{2023}{450}g_1^4\Big) \ , \end{aligned} \tag{C4}$$

$$\begin{aligned} \beta^{(2)}_{Y_e} \ = \ & Y_e \Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} \\ & -3Y_e^\dagger Y_e\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_e^\dagger Y_eY_{xd}^\dagger Y_{xd} - 4Y_e^\dagger Y_eY_e^\dagger Y_e \\ & +(16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + (16g_3^2 - \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd} \\ & +6g_2^2Y_e^\dagger Y_e + \frac{39}{2}g_2^4 + \frac{9}{5}g_2^2g_1^2 + \frac{927}{50}g_1^4\Big) \ , \end{aligned} \tag{C5}$$

$$\beta^{(2)}_{Y_{xu}} = Y_{xu}\Big(-3\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 22Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_{xu}^\dagger Y_{xu}\mathrm{Tr}(Y_uY_u^\dagger)$$
$$+(16g_3^2 + \frac{4}{5}g_1^2)\mathrm{Tr}(Y_uY_u^\dagger) + (16g_3^2 + 6g_2^2 + \frac{2}{5}g_1^2)Y_{xu}^\dagger Y_{xu}$$
$$+\frac{176}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2 + \frac{39}{2}g_2^4 + g_2^2g_1^2 + \frac{2023}{450}g_1^4\Big) \ , \tag{C6}$$

$$\beta^{(2)}_{Y_{xd}} = Y_{xd}\Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 22Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd}$$
$$-3Y_{xd}^\dagger Y_{xd}\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) + (16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger)$$
$$+(16g_3^2 + 6g_2^2 + \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd} + \frac{176}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2$$
$$+\frac{39}{2}g_2^4 + g_2^2g_1^2 + \frac{2023}{450}g_1^4\Big) \ . \tag{C7}$$

Appendix D: Renormalization Group Equations in Model III

In Model III, below the intermediate scale $M_I = 1.0 \times 10^{11}$ GeV, we have the same RGEs as in Model I. Above M_I, we have additional vector-like particles $(Xf, \overline{Xf})$ and $(Xl, \overline{Xl})$. Thus, we need to add extra contributions to b and B from the vector-like particles $(Xf, \overline{Xf})$ and $(Xl, \overline{Xl})$. Comparing to the RGEs in Model I, we also need to change the coefficients of the g_3^4, g_2^4 and g_1^4 terms in $\beta^{(2)}_{Y_u}$, $\beta^{(2)}_{Y_d}$, $\beta^{(2)}_{Y_{xu}}$, and $\beta^{(2)}_{Y_{xd}}$, and change the coefficients of the g_2^4 and g_1^4 terms in $\beta^{(2)}_{Y_e}$. In short, comparing to the RGEs in Model I, the coefficients in the RGEs above M_I, which need to be changed, are the following:

$$b = \left(\frac{33}{5}, 1, -3\right) + \left(\frac{3}{5}, 3, 3\right) + \left(\frac{17}{5}, 1, 1\right) \ , \tag{D1}$$

$$B = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} + \begin{pmatrix} \frac{3}{25} & \frac{3}{5} & \frac{16}{5} \\ \frac{1}{5} & 21 & 16 \\ \frac{2}{5} & 6 & 34 \end{pmatrix} + \begin{pmatrix} \frac{371}{15} & \frac{9}{5} & \frac{128}{15} \\ \frac{3}{5} & 7 & 0 \\ \frac{16}{15} & 0 & \frac{34}{3} \end{pmatrix} \ , \tag{D2}$$

$$\beta^{(2)}_{Y_u} = Y_u\Big(-3\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 9Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger)$$
$$-9Y_u^\dagger Y_uY_{xu}^\dagger Y_{xu} - Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 3Y_d^\dagger Y_dY_{xd}Y_{xd}^\dagger$$
$$-4Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_d^\dagger Y_dY_u^\dagger Y_u + (16g_3^2 + \frac{4}{5}g_1^2)\mathrm{Tr}(Y_uY_u^\dagger)$$
$$+(16g_3^2 - \frac{2}{5}g_1^2)Y_{xu}^\dagger Y_{xu} + (6g_2^2 + \frac{2}{5}g_1^2)Y_u^\dagger Y_u + \frac{2}{5}g_1^2Y_d^\dagger Y_d + \frac{176}{9}g_3^4 + 8g_3^2g_2^2$$
$$+\frac{136}{45}g_3^2g_1^2 + \frac{39}{2}g_2^4 + g_2^2g_1^2 + \frac{4303}{450}g_1^4\Big) \ , \tag{D3}$$

$$\beta^{(2)}_{Y_d} = Y_d \Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger)$$
$$-3Y_u^\dagger Y_uY_{xu}Y_{xu}^\dagger - 3Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_d^\dagger Y_dY_{xd}^\dagger Y_{xd} - 4Y_d^\dagger Y_dY_d^\dagger Y_d$$
$$-2Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_u^\dagger Y_uY_d^\dagger Y_d + (16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger)$$
$$+(16g_3^2 - \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd} + (6g_2^2 + \frac{4}{5}g_1^2)Y_d^\dagger Y_d + \frac{4}{5}g_1^2Y_u^\dagger Y_u + \frac{176}{9}g_3^4 + 8g_3^2g_2^2$$
$$+\frac{8}{9}g_3^2g_1^2 + \frac{39}{2}g_2^4 + g_2^2g_1^2 + \frac{91}{18}g_1^4\Big) \ , \tag{D4}$$

$$\beta^{(2)}_{Y_e} = Y_e \Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd}$$
$$-3Y_e^\dagger Y_e\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_e^\dagger Y_eY_{xd}^\dagger Y_{xd} - 4Y_e^\dagger Y_eY_e^\dagger Y_e$$
$$+(16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + (16g_3^2 - \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd}$$
$$+6g_2^2Y_e^\dagger Y_e + \frac{39}{2}g_2^4 + \frac{9}{5}g_2^2g_1^2 + \frac{207}{10}g_1^4\Big) \ , \tag{D5}$$

$$\beta^{(2)}_{Y_{xu}} = Y_{xu} \Big(-3\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 22Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_{xu}^\dagger Y_{xu}\mathrm{Tr}(Y_uY_u^\dagger)$$
$$+(16g_3^2 + \frac{4}{5}g_1^2)\mathrm{Tr}(Y_uY_u^\dagger) + (16g_3^2 + 6g_2^2 + \frac{2}{5}g_1^2)Y_{xu}^\dagger Y_{xu}$$
$$+\frac{176}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2 + \frac{39}{2}g_2^4 + g_2^2g_1^2 + \frac{91}{18}g_1^4\Big) \ , \tag{D6}$$

$$\beta^{(2)}_{Y_{xd}} = Y_{xd} \Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 22Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd}$$
$$-3Y_{xd}^\dagger Y_{xd}\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) + (16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger)$$
$$+(16g_3^2 + 6g_2^2 + \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd} + \frac{176}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2$$
$$+\frac{39}{2}g_2^4 + g_2^2g_1^2 + \frac{91}{18}g_1^4\Big) \ . \tag{D7}$$

Appendix E: Renormalization Group Equations in Model IV

In Model IV, we have additional vector-like particles Xl and $\overline{Xl}$. Thus, we need to add extra contributions to b and B from the vector-like particles Xl and $\overline{Xl}$. Comparing to the RGEs in Model I, we also need to change the coefficients of the g_1^4 terms in $\beta^{(2)}_{Y_u}$, $\beta^{(2)}_{Y_d}$, $\beta^{(2)}_{Y_e}$, $\beta^{(2)}_{Y_{xu}}$, and $\beta^{(2)}_{Y_{xd}}$. In short, comparing to the RGEs in Model I, the corresponding coefficients of the RGEs, which need to be changed, are the following:

$$b = \Big(\frac{33}{5}, 1, -3\Big) + \Big(\frac{3}{5}, 3, 3\Big) + \Big(\frac{6}{5}, 0, 0\Big) \ , \tag{E1}$$

$$B = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} + \begin{pmatrix} \frac{3}{25} & \frac{3}{5} & \frac{16}{5} \\ \frac{1}{5} & 21 & 16 \\ \frac{2}{5} & 6 & 34 \end{pmatrix} + \begin{pmatrix} \frac{36}{25} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \tag{E2}$$

$$\begin{aligned} \beta^{(2)}_{Y_u} \; = \; & Y_u \Big(-3\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 9Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger) \\ & -9Y_u^\dagger Y_uY_{xu}^\dagger Y_{xu} - Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 3Y_d^\dagger Y_dY_{xd}Y_{xd}^\dagger \\ & -4Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_d^\dagger Y_dY_d^\dagger Y_d - 2Y_d^\dagger Y_dY_u^\dagger Y_u + (16g_3^2 + \frac{4}{5}g_1^2)\mathrm{Tr}(Y_uY_u^\dagger) \\ & +(16g_3^2 - \frac{2}{5}g_1^2)Y_{xu}^\dagger Y_{xu} + (6g_2^2 + \frac{2}{5}g_1^2)Y_u^\dagger Y_u + \frac{2}{5}g_1^2Y_d^\dagger Y_d + \frac{128}{9}g_3^4 + 8g_3^2g_2^2 \\ & +\frac{136}{45}g_3^2g_1^2 + \frac{33}{2}g_2^4 + g_2^2g_1^2 + \frac{689}{90}g_1^4\Big) , \end{aligned} \tag{E3}$$

$$\begin{aligned} \beta^{(2)}_{Y_d} \; = \; & Y_d \Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} - 3Y_u^\dagger Y_u\mathrm{Tr}(Y_uY_u^\dagger) \\ & -3Y_u^\dagger Y_uY_{xu}Y_{xu}^\dagger - 3Y_d^\dagger Y_d\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_d^\dagger Y_dY_{xd}^\dagger Y_{xd} - 4Y_d^\dagger Y_dY_d^\dagger Y_d \\ & -2Y_u^\dagger Y_uY_u^\dagger Y_u - 2Y_u^\dagger Y_uY_d^\dagger Y_d + (16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) \\ & +(16g_3^2 - \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd} + (6g_2^2 + \frac{4}{5}g_1^2)Y_d^\dagger Y_d + \frac{4}{5}g_1^2Y_u^\dagger Y_u + \frac{128}{9}g_3^4 + 8g_3^2g_2^2 \\ & +\frac{8}{9}g_3^2g_1^2 + \frac{33}{2}g_2^4 + g_2^2g_1^2 + \frac{1813}{450}g_1^4\Big) , \end{aligned} \tag{E4}$$

$$\begin{aligned} \beta^{(2)}_{Y_e} \; = \; & Y_e \Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 9Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} \\ & -3Y_e^\dagger Y_e\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) - 9Y_e^\dagger Y_eY_{xd}^\dagger Y_{xd} - 4Y_e^\dagger Y_eY_e^\dagger Y_e \\ & +(16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) + (16g_3^2 - \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd} \\ & +6g_2^2Y_e^\dagger Y_e + \frac{33}{2}g_2^4 + \frac{9}{5}g_2^2g_1^2 + \frac{837}{50}g_1^4\Big) , \end{aligned} \tag{E5}$$

$$\begin{aligned} \beta^{(2)}_{Y_{xu}} \; = \; & Y_{xu} \Big(-3\mathrm{Tr}(3Y_uY_u^\dagger Y_uY_u^\dagger + Y_uY_d^\dagger Y_dY_u^\dagger) - 22Y_{xu}^\dagger Y_{xu}Y_{xu}^\dagger Y_{xu} - 9Y_{xu}^\dagger Y_{xu}\mathrm{Tr}(Y_uY_u^\dagger) \\ & +(16g_3^2 + \frac{4}{5}g_1^2)\mathrm{Tr}(Y_uY_u^\dagger) + (16g_3^2 + 6g_2^2 + \frac{2}{5}g_1^2)Y_{xu}^\dagger Y_{xu} \\ & +\frac{128}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2 + \frac{33}{2}g_2^4 + g_2^2g_1^2 + \frac{1813}{450}g_1^4\Big) , \end{aligned} \tag{E6}$$

$$\begin{aligned} \beta^{(2)}_{Y_{xd}} \; = \; & Y_{xd} \Big(-3\mathrm{Tr}(3Y_dY_d^\dagger Y_dY_d^\dagger + Y_dY_u^\dagger Y_uY_d^\dagger + Y_eY_e^\dagger Y_eY_e^\dagger) - 22Y_{xd}^\dagger Y_{xd}Y_{xd}^\dagger Y_{xd} \\ & -3Y_{xd}^\dagger Y_{xd}\mathrm{Tr}(3Y_dY_d^\dagger + Y_eY_e^\dagger) + (16g_3^2 - \frac{2}{5}g_1^2)\mathrm{Tr}(Y_dY_d^\dagger) + \frac{6}{5}g_1^2\mathrm{Tr}(Y_eY_e^\dagger) \\ & +(16g_3^2 + 6g_2^2 + \frac{2}{5}g_1^2)Y_{xd}^\dagger Y_{xd} + \frac{128}{9}g_3^4 + 8g_3^2g_2^2 + \frac{8}{9}g_3^2g_1^2 \\ & +\frac{33}{2}g_2^4 + g_2^2g_1^2 + \frac{1813}{450}g_1^4\Big) . \end{aligned} \tag{E7}$$

Appendix F: Renormalization Group Equations in Model V

In Model V, below the intermediate scale $M_I = 1.0 \times 10^{11}$ GeV, we have the same RGEs as in Model IV. Above M_I, we have extra vector-like particles $(Xf, \overline{Xf})$, and we have the same RGEs as in Model III.

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