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The Golden Point of No-Scale and No-Parameter $\mathcal{F}$-$SU(5)$

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The $\mathcal{F}$-lipped $SU(5)\times U(1)_X$ Grand Unified Theory (GUT) supplemented by TeV-scale vector-like particles from $\mathcal{F}$-theory, together dubbed $\mathcal{F}$-$SU(5)$, offers a natural multi-phase unification process which suggests an elegant implementation of the No-Scale Supergravity boundary conditions at the unification scale $M_{\mathcal{F}} \simeq 7 \times 10^{17}$ GeV. Enforcing the No-Scale boundary conditions, including $B_\mu(M_{\mathcal{F}}) = 0$ on the Higgs bilinear soft term, with the precision 7-year WMAP value on the dark matter relic density isolates a highly constrained "Golden Point" located near $M_{1/2} = 455$ GeV and $\tan\beta = 15$ in the $\tan\beta - M_{1/2}$ plane, which simultaneously satisfies all known experiments, and moreover corresponds to an imminently observable proton decay rate. Because the universal gaugino mass is actually determined from established low energy data via Renormalization Group Equation (RGE) running, there are no surviving arbitrary scale parameters in the present model.

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Introduction – The driving aim of theoretical physics is to achieve maximal efficiency in the correlation of observations. This entails the unification of apparently distinct forces under a master symmetry group, and the successful reinterpretation of experimentally constrained parameters and finely tuned scales as dynamically evolved consequences of the underlying equations of motion.

We propose in this paper a variation of the No-Scale Supergravity [1] scenario which successfully eliminates all extraneously adjustable degrees of freedom while dynamically addressing all fundamental scales and maintaining consistency with all low energy phenomenology constraints, including the precision electroweak scale data [2], the 7-year WMAP constraint on dark matter relic density [3], the experimental limits on the Flavor Changing Neutral Current (FCNC) process $b \to s\gamma$ [4, 5], the anomalous magnetic moment of the muon [6], the process $B_s^0 \to \mu^+\mu^-$ [7], and the LEP limit on the lightest CP-even Higgs boson mass [8], additionally predicting an experimentally safe, yet still imminently observable proton lifetime.

The No-Scale picture inherits an associative weight of motivation from its robustly generic and natural appearance across string-, M-, and $\mathcal{F}$-theory derived model building efforts [9, 10]. It represents moreover a case study in reductionism, wherein the universal scalar mass M_0, universal trilinear soft term A and Higgs bilinear soft term B_μ each vanish at some common high mass boundary, and only the single universal gaugino mass parameter $M_{1/2}$ is left to float free. All low energy scales are dynamically generated by quantum corrections, *i.e.* running under the RGEs, to the classically flat potential.

This appealing perspective however, has historically been undermined by a basic inconsistency of the $M_0 = 0$ condition as applied at a GUT scale of order 10^{16} GeV with precision phenomenology. Attempts [11–13] to reinterpret the No-Scale paradigm as a boundary near the Planck scale have met with some exciting success, but we suggest that these efforts have been missing one most crucial piece of the puzzle. Our prior study, succinctly dubbed $\mathcal{F}$-$SU(5)$ [14–16], of the $\mathcal{F}$-lipped $SU(5)$ GUT [17, 18] supplemented by $\mathcal{F}$-theory derived vector-like multiplets at the TeV scale, provides the essential rationale for the separation of an initial unification of the $SU(3)_C \times SU(2)_L$ gauge symmetry at a mass M_{32} near the traditional GUT scale, from a second phase running up to a point $M_{\mathcal{F}}$ of final unification near the reduced Planck mass [14, 15]. The dual high scales of $\mathcal{F}$-$SU(5)$ fit hand to glove with the proposal for salvaging the no-scale conditions.

Only a small portion of viable parameter space appears to be consistent with the $B_\mu(M_{\mathcal{F}}) = 0$ condition, which thus constitutes a strong constraint. In the narrow region of overlap, we identify a highly confined "Golden Point" at which all phenomenological limits are respected. Moreover, since the boundary value of the universal gaugino mass $M_{1/2}$, and even the unification scale $M_{\mathcal{F}}$ itself, are established by the low energy experiments via RGE running, we are not left with any surviving scale parameters in the present model.

No Scale Supergravity – Supersymmetry (SUSY) naturally solves the gauge hierarchy problem in the Standard Model (SM), and suggests, along with R parity conservation, the lightest supersymmetric particle (LSP) as a suitable cold dark matter candidate. Since we do not see mass degeneracy of the superpartners however, SUSY must be broken around the TeV scale. In GUTs with gravity mediated supersymmetry breaking, called the supergravity models, we can fully characterize the supersymmetry breaking soft terms by four universal parameters (gaugino mass $M_{1/2}$, scalar mass M_0, trilinear soft term A, and the low energy ratio of Higgs vacuum expectation values (VEVs) $\tan\beta$), plus the sign of the Higgs bilinear mass term μ.

No-Scale Supergravity was proposed [1], to address the cosmological flatness problem, as the subset of supergravity models which satisfy the following three constraints: (i) The vacuum energy vanishes automatically due to the suitable Kähler potential; (ii) At the minimum of the scalar potential, there are flat directions which leave the gravitino mass $M_{3/2}$ undetermined; (iii) The quantity $\mathrm{Str}\mathcal{M}^2$ is zero at the minimum. If the third condition were not true, large one-loop corrections would force $M_{3/2}$ to be either identically zero or of the Planck scale. A simple Kähler potential which satisfies the first two conditions is [1]

$$K \;=\; -3\mathrm{ln}(T+\overline{T}-\sum_i \overline{\Phi}_i \Phi_i) \;, \tag{1}$$

where T is a modulus field and Φ_i are matter fields. The third condition is model dependent and can always be satisfied in principle [19]. For the simple Kähler potential in Eq. (1) we automatically obtain the no-scale boundary condition $M_0 = A = B_\mu = 0$ while $M_{1/2}$ is allowed, and indeed required for SUSY breaking. Because the minimum of the electroweak (EW) Higgs potential $(V_{EW})_{min}$ depends on $M_{3/2}$, the gravitino mass is determined by the equation $d(V_{EW})_{min}/dM_{3/2} = 0$. Thus, the supersymmetry breaking scale is determined dynamically. No-scale supergravity can be realized in the compactification of the weakly coupled heterotic string theory [9] and the compactification of M-theory on S^1/Z_2 at the leading order [10].

Models –In the flipped $SU(5)$ GUT [17] there are three families of SM fermions whose quantum numbers under the $SU(5)\times U(1)_X$ gauge group are

$$F_i = (\mathbf{10}, \mathbf{1}),\; \bar{f}_i = (\bar{\mathbf{5}}, -\mathbf{3}),\; \bar{l}_i = (\mathbf{1}, \mathbf{5}), \tag{2}$$

where $i = 1, 2, 3$.

To break the GUT and electroweak gauge symmetries, we introduce two pairs of Higgs fields

$$H = (\mathbf{10}, \mathbf{1}),\; \overline{H} = (\overline{\mathbf{10}}, -\mathbf{1}),\; h = (\mathbf{5}, -\mathbf{2}),\; \overline{h} = (\bar{\mathbf{5}}, \mathbf{2}). \tag{3}$$

To separate the M_{32} and $M_{\mathcal{F}}$ scales and obtain true string-scale gauge coupling unification in free fermionic string models [14, 18] or the decoupling scenario in F-theory models [15], we introduce vector-like particles which form complete flipped $SU(5)\times U(1)_X$ multiplets. In order to avoid the Landau pole problem for the strong coupling constant, we can only introduce the following two sets of vector-like particles around the TeV scale [14]

$$Z1 : XF = (\mathbf{10}, \mathbf{1}) \;,\; \overline{XF} = (\overline{\mathbf{10}}, -\mathbf{1}) \;; \tag{4}$$

$$Z2 : XF \;,\; \overline{XF} \;,\; Xl = (\mathbf{1}, -\mathbf{5}) \;,\; \overline{Xl} = (\mathbf{1}, \mathbf{5}) \;. \tag{5}$$

In this paper, we only consider the flipped $SU(5)\times U(1)_X$ models with $Z2$ set of vector-like particles. The discussions for the models with $Z1$ set and heavy threshold corrections [15] are similar.

The Golden Point – In the No-Scale context, we impose $M_0 = A = B_\mu = 0$ at the unification scale $M_{\mathcal{F}}$, and allow distinct inputs for the single parameter $M_{1/2}(M_{\mathcal{F}})$ to translate under the RGEs to distinct low scale outputs of B_μ and the Higgs mass-squares $M^2_{H_u}$ and $M^2_{H_d}$. This continues until the point of spontaneous breakdown of the electroweak symmetry at $M^2_{H_u} + \mu^2 = 0$, at which point minimization of the broken potential establishes the physical low energy values of μ and $\tan\beta$. In practice however, this procedure is at odds with the existing `SuSpect 2.34` code [20] base from which our primary routines have been adapted. In order to impose the minimal possible refactoring, we have instead opted for an inversion wherein $M_{1/2}$ and $\tan\beta$ float as two effective degrees of freedom. Thus, we do not fix $B_\mu(M_{\mathcal{F}})$. We take $\mu > 0$ as suggested by the results of $g_\mu - 2$ for the muon, and use 1 TeV for the universal vector-like particle mass [21].

The relic LSP neutralino density, WIMP-nucleon direct detection cross sections and photon-photon annihilation cross sections are computed with `MicrOMEGAs 2.1` [22] wherein the revised `SuSpect` RGEs have also implemented. We use a top quark mass of $m_t = 173.1$ GeV [2] and employ the following experimental constraints: (1) The WMAP 7-year measurements of the cold dark matter density [3], $0.1088 \leq \Omega_\chi \leq 0.1158$. We allow Ω_χ to be larger than the upper bound due to a possible $\mathcal{O}(10)$ dilution factor [23] and to be smaller than the lower bound due to multicomponent dark matter. (2) The experimental limits on the FCNC process, $b \to s\gamma$. We use the limits $2.86\times10^{-4} \leq Br(b\to s\gamma) \leq 4.18\times10^{-4}$ [4, 5]. (3) The anomalous magnetic moment of the muon, $g_\mu - 2$. We use the 2σ level boundaries, $11\times10^{-10} < \Delta a_\mu < 44\times10^{-10}$ [6]. (4) The process $B^0_s \to \mu^+\mu^-$ where we take the upper bound to be $Br(B^0_s \to \mu^+\mu^-) < 5.8\times10^{-8}$ [7]. (5) The LEP limit on the lightest CP-even Higgs boson mass, $m_h \geq 114$ GeV [8].

In the $\tan\beta - M_{1/2}$ plane, $B_\mu(M_{\mathcal{F}})$ is then calculated along with the low energy supersymmetric particle spectrum and checks on various experimental constraints. The subspace corresponding to a No-Scale model is clearly then a one dimensional slice of this manifold, as demonstrated in Fig. 1. It is quite remarkable that the $B_\mu(M_{\mathcal{F}}) = 0$ contour so established runs sufficiently perpendicular to the WMAP strip that the point of intersection effectively absorbs our final degree of freedom, creating what we have labeled as a No-Parameter

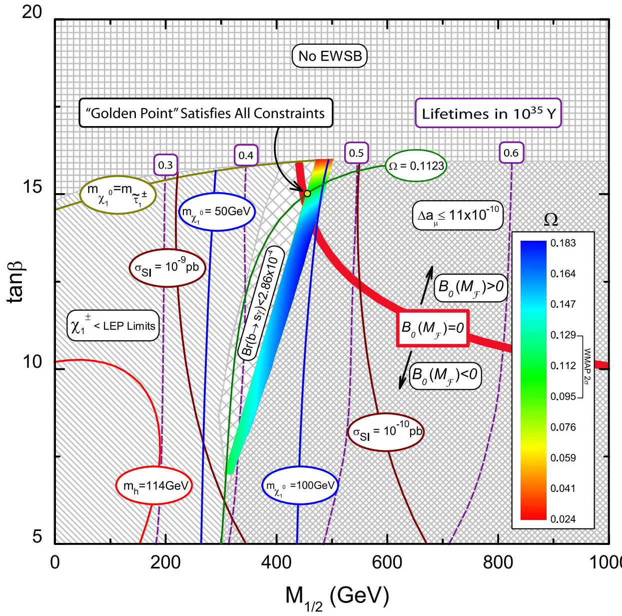

FIG. 1: Viable parameter space in the $\tan\beta - M_{1/2}$ plane. The Golden Point is annotated. The thin, dark green line denotes the WMAP 7-year central value $\Omega_\chi = 0.1123$. The dashed purple contours label $p \to (e|\mu)^+\pi^0$ proton lifetime predictions, in units of 10^{35} years.

Model. It is truly extraordinary however that this intersection occurs exactly at the centrally preferred relic density, that being our strongest experimental constraint. We emphasize again that there did not have to be an experimentally viable $B_\mu(M_{\mathcal{F}}) = 0$ solution, and that the consistent realization of this scenario depended crucially on several uniquely identifying characteristics of the underlying proposal. Specifically again, it appears that the No-Scale condition comes into its own only when applied at near the Planck mass, and that this is naturally identified as the point of the final $\mathcal{F}$-$SU(5)$ unification, which is naturally extended and decoupled from the primary GUT scale only via the modification to the RGEs from the TeV scale $\mathcal{F}$-theory vector-like multiplet content. The union of our top-down model based constraints with the bottom-up experimental data exhausts the available freedom of parameterization in a uniquely consistent and predictive manner, phenomenologically defining a truly Golden Point near $M_{1/2} = 455$ GeV and $\tan\beta = 15$.

TABLE I: Spectrum (in GeV) for the Golden Point in Fig. 1. Here, $\Omega_\chi = 0.1123$, $\sigma_{SI} = 1.9 \times 10^{-10}$ pb, and $\langle\sigma v\rangle_{\gamma\gamma} = 1.7 \times 10^{-28}$ cm^3/s. The central prediction for the $p \to (e|\mu)^+\pi^0$ proton lifetime is 4.6×10^{34} years.

$\widetilde{\chi}_1^0$	95	$\widetilde{\chi}_1^\pm$	185	$\widetilde{e}_R$	150	$\widetilde{t}_1$	489	$\widetilde{u}_R$	951	m_h	120.1
$\widetilde{\chi}_2^0$	185	$\widetilde{\chi}_2^\pm$	826	$\widetilde{e}_L$	507	$\widetilde{t}_2$	909	$\widetilde{u}_L$	1036	$m_{A,H}$	920
$\widetilde{\chi}_3^0$	821	$\widetilde{\nu}_{e/\mu}$	501	$\widetilde{\tau}_1$	104	$\widetilde{b}_1$	859	$\widetilde{d}_R$	992	$m_{H^\pm}$	925
$\widetilde{\chi}_4^0$	824	$\widetilde{\nu}_\tau$	493	$\widetilde{\tau}_2$	501	$\widetilde{b}_2$	967	$\widetilde{d}_L$	1039	$\widetilde{g}$	620

The Golden Point parameters are $M_{1/2} = 455.3$ GeV, $\tan\beta = 15.02$, and the point is in full compliance with the WMAP 7-year results with $\Omega_\chi = 0.1123$. It also satisfies the CDMSII [24], Xenon100 [25], and FERMI-LAT space telescope constraints [26], with $\sigma_{SI} = 1.9 \times 10^{-10}$ pb and $\langle\sigma v\rangle_{\gamma\gamma} = 1.7 \times 10^{-28}$ cm^3/s. The proton lifetime is about 4.6×10^{34} years, which is well within reach of the upcoming Hyper-Kamiokande [27] and DUSEL [28] experiments. Inspecting the supersymmetric particle and Higgs spectrum for the Golden Point in Table I reveals that the additional contribution of the 1 TeV vector-like particles lowers the gluino mass quite dramatically. The gluino mass M_3 runs flat from the M_{32} unification scale to 1 TeV as shown in Fig. 2, though, due to supersymmetric radiative corrections, the physical gluino mass at the EW scale is larger than M_3 at the M_{32} scale. This is true for the full parameter space. For the Golden Point, the LSP neutralino is 99.8% Bino. Similarly to the mSUGRA picture, the point is in the stau-neutralino coannihilation region, but the gluino is lighter than the squarks in our models, with the exception of the lightest stop.

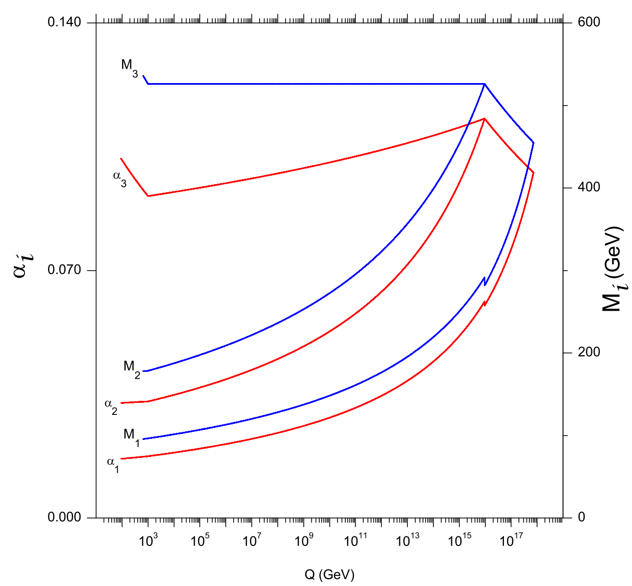

FIG. 2: RGE Running of the SM gauge couplings and gaugino masses from the EW scale to the unification scale $M_{\mathcal{F}}$. Notice the discontinuity of $U(1)_Y$ as it remixes between the $U(1)_X$ and that which emerges out of broken SU(5) at the scale $M_{32} \simeq 1 \times 10^{16}$ GeV. Advancing from this interim stage, $SU(5) \times U(1)_X$ is unified at a higher scale $M_{\mathcal{F}} \simeq 7 \times 10^{17}$ GeV.

We plot gauge coupling and gaugino mass unification for the Golden Point in Fig. 2. The figure explicitly demonstrates the two-step unification of flipped $SU(5) \times U(1)_X$. In this work, we consider the two-loop RGE running for the gauge couplings, however, we only consider the one-loop RGE running for the gaugino masses. In $\mathcal{F}$-$SU(5)$ models, the one-loop beta function for $SU(3)_C$ is zero due to the vector-like particle contributions. Therefore, as shown in Fig. 2, M_3 is constant from the electroweak scale to the M_{32} scale since the beta coefficient $b_3 = 0$. In contrast, the gauge couplings and gaugino masses for the $SU(2)_L \times U(1)_Y$ gauge symmetry track each other in Fig. 2 since the gauge couplings for $SU(2)_L \times U(1)_Y$ are weak, thus the two-loop effects are small. In addition, we present the RGE running for the μ term, the SUSY breaking scalar masses, trilinear A-terms, and bilinear B_μ term in Fig. 3. Note in particular that the EW symmetry breaking occurs when $H_u^2 + \mu^2$ goes negative.

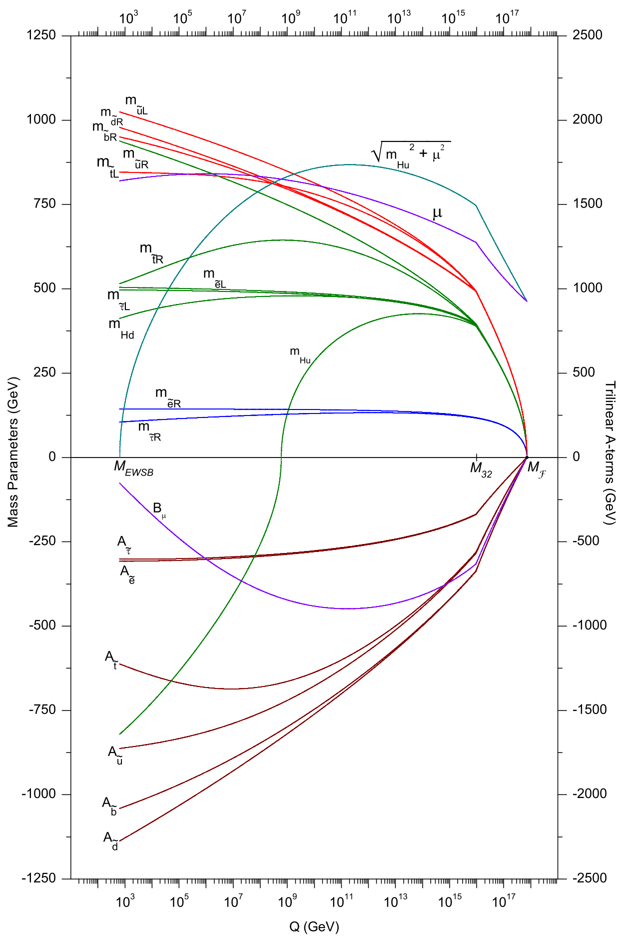

FIG. 3: RGE Running of the μ term and SUSY breaking soft terms from the EW scale to the unification scale $M_{\mathcal{F}}$.

Conclusion – We have studied No-Scale Supergravity in the context of a $\mathcal{F}$-lipped $SU(5) \times U(1)_X$ GUT supplemented with $\mathcal{F}$-theory derived TeV-scale vector-like particles. With the no-scale boundary condition applied at the point of final unification $M_{\mathcal{F}}$, we find a very small "Golden Point" of viable parameter space that is consistent with all known experiments, while fixing all extraneous model parameters. For the Golden Point, we have discussed unification of the SM gauge couplings and gaugino masses, and the RGE running for the μ term and supersymmetry breaking soft terms. Proton decay predictions are well within the range accessible to the future Hyper-Kamiokande and DUSEL experiments. Because the universal gaugino mass is determined by the low energy known experiments via RGE running, we emphasize that there are no surviving arbitrary scale parameters in this model.

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