

# $R_b$ in supersymmetric models

XU WANG<sup>1,2</sup>, JORGE L. LOPEZ<sup>1,2</sup>, and D. V. NANOPOULOS<sup>1,2,3</sup>

<sup>1</sup>*Center for Theoretical Physics, Department of Physics, Texas A&M University  
College Station, TX 77843-4242, USA*

<sup>2</sup>*Astroparticle Physics Group, Houston Advanced Research Center (HARC)  
The Mitchell Campus, The Woodlands, TX 77381, USA*

<sup>3</sup>*CERN Theory Division, 1211 Geneva 23, Switzerland*

## Abstract

We compute the supersymmetric contribution to  $R_b \equiv \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$  in a variety of supersymmetric models. In the context of supergravity models with universal soft-supersymmetry-breaking and radiative electroweak breaking we find  $R_b^{\text{susy}} \lesssim 0.0004$ , which does not shift significantly the Standard Model prediction ( $R_b^{\text{SM}} = 0.2157$  for  $m_t = 175$  GeV;  $R_b^{\text{exp}} = 0.2204 \pm 0.0020$ ). We also compute  $R_b$  in the minimal supersymmetric standard model (MSSM), and delineate the region of parameter space which yields interestingly large values of  $R_b$ . This region entails light charginos and top-squarks, but is *strongly* restricted by the *combined* constraints from  $B(b \rightarrow s\gamma)$  and a not-too-large invisible top-quark branching ratio: only a few percent of the points with  $R_b^{\text{susy}} > 0.0020$  ( $1\sigma$ ) are allowed.

# 1 Introduction

Precision tests of the electroweak interactions at LEP have provided the most sensitive checks of the Standard Model of particle physics. The pattern that has emerged is that of consistent agreement with the Standard Model predictions. This pattern seems to have so far only one apparently dissonant note, namely in the measurement of the ratio  $R_b \equiv \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$ , where the latest global fit to the LEP data ( $R_b^{\text{exp}} = 0.2204 \pm 0.0020$  [1]) lies more than two standard deviations above the one-loop Standard Model prediction [2] for all preferred values of the top-quark mass (*e.g.*,  $R_b^{\text{SM}} = 0.2157$  for  $m_t = 175$  GeV). Further experimental statistics will reveal whether this is indeed a breakdown of the Standard Model. In the meantime, it is important to explore what new contributions to  $R_b$  are expected in models of new physics, such as supersymmetry.

The study of supersymmetric contributions to  $\Gamma(Z \rightarrow b\bar{b})$  has proceeded in two phases. Originally the quantity  $\epsilon_b$  [3] was defined as an extension of the  $\epsilon_{1,2,3}$  scheme [4] for model-independent fits to the electroweak data. More recently it has become apparent that the ratio  $R_b$  is more directly calculable [5, 6, 7] and readily measurable. It has been made apparent [6] that supersymmetric contributions to  $R_b$  are not likely to increase the total predicted value for  $R_b$  in any significant manner, as long as typical assumptions about unified supergravity models are made. On the other hand, if these assumptions are relaxed, it is possible for supersymmetry to make a significant contribution to  $R_b$ , if certain conditions on the low-energy supersymmetric spectrum are satisfied [6, 7].

In this paper we reexamine this question in the context of a variety of supergravity models, which include the constrained minimal supergravity model considered in Ref. [6], as well as non-minimal string-inspired supergravity models. In all cases we find  $R_b^{\text{susy}} \lesssim 0.0004$ , which brings the Standard Model prediction at most one-fifth of a standard deviation closer to the experimental result. Moreover,  $R_b^{\text{susy}}$  could well be negative, worsening the Standard Model fit, although only slightly. We then turn to the minimal supersymmetric standard model (MSSM), where the many parameters are *a priori* independent, and seek to delineate the region of parameter space which yields large values of  $R_b$ . This exercise confirms the results of Ref. [6], that light charginos and top-squarks of definite composition are required. However, we take the further step of trying to quantify how large a region of parameter space this is. In other words, what kind of fine-tuning is involved in attaining such large values of  $R_b$ . To this end we apply all known experimental constraints which may affect the preferred region of parameter space: (i) that  $B(b \rightarrow s\gamma)$  be within the allowed CLEO range, (ii) that the lightest supersymmetric particle (LSP) be neutral and colorless, (iii) that the invisible top-quark branching ratio (*i.e.*,  $B(t \rightarrow \tilde{t}_1\chi_{1,2}^0)$ ) be within experimental limits, (iv) that the invisible  $Z$  width be within LEP limits, and (v) that  $B(Z \rightarrow \chi_1^0\chi_2^0)$  be within LEP limits. By sampling a large number of random values for the six-plet of parameters that determine  $R_b^{\text{susy}}$  and the other five experimental observables, we conclude that, of all points in parameter space that yield  $R_b^{\text{susy}} > 0.0020$  ( $1\sigma$ ), only a few percent satisfy the *combined* additional experimental constraints.

We should remark that the region of MSSM parameter space where  $R_b^{\text{susy}}$  is enhanced, has recently become the focus of attention for another reason. It has been argued that the apparent disagreement between the LEP determination of  $\alpha_s$  and the corresponding value obtained from low-energy measurements, may hint to the possibility of new physics [8]. Moreover, light supersymmetric particles seem to fit the bill by shifting downwards the LEP-extracted value of  $\alpha_s$  [9], if observables like  $R_b$  can be brought to better agreement with experiment. Our restrictions on parameter space apply also to this case, to the extent that sufficiently large values of  $R_b$  are called for.

## 2 Supersymmetric contributions to $R_b$

Besides the one-loop Standard Model contributions to  $R_b$ , in supersymmetric models there are four new diagrams, as follows:

- The charged-Higgs–top-quark loop, depends on the charged Higgs mass and the  $t - b - H^\pm$  coupling. For a left-handed  $b$  quark the coupling is  $\propto m_t / \tan \beta$  whereas for a right-handed  $b$  quark it is  $\propto m_b \tan \beta$ . Therefore, for small<sup>1</sup> (large)  $\tan \beta$  left- (right)-handed  $b$ -quark production is dominant. (For  $\tan \beta \gg 1$ , the value of  $m_b$  impacts the contribution significantly.) It has been shown that the  $H^\pm - t$  contribution is always negative [10], a fact which makes the prediction for  $R_b$  in two Higgs-doublet models always in worse agreement with experiment.
- The chargino–top-squark loop, is the supersymmetric counterpart of the  $H^\pm - t$  loop discussed above. The chargino mass eigenstate is a mixture of (charged) Higgsino and wino, and the coupling strength is a complicated matter now because it involves the top-squark mixing matrix and the chargino mixing matrix. However, because only the Higgsino admixture in the chargino eigenstate has a Yukawa coupling to the  $t$ - $b$  doublet, generally speaking a light chargino with a significant Higgsino component, and a light top-squark with a significant right-handed component are required for this diagram to make a non-negligible contribution to  $R_b$  [6].
- The neutralino–bottom-squark loop, is the supersymmetric counterpart of the neutral-Higgs–bottom-quark loop. The coupling strength of  $\chi_1^0 - \tilde{b} - b$  is also rather complicated, since it involves the bottom-squark mixing, the neutralino mixing, and their masses. However, it can be non-negligible since it is proportional to  $m_b \tan \beta$  for the left-handed  $b$ -quark. Therefore in the high- $\tan \beta$  region we have to include this contribution.<sup>2</sup>

---

<sup>1</sup>In supergravity models, the radiative electroweak breaking mechanism requires  $\tan \beta > 1$ .

<sup>2</sup>This term was not included in our previous study in terms of the parameter  $\epsilon_b$  [17], although it was pointed out that its effects were non-negligible for  $\tan \beta \gg 1$ .

- The neutral-Higgs–bottom-quark loop, involves the three neutral scalars, two CP-even ( $h, H$ ) and one CP-odd ( $A$ ). For the  $h$  ( $H$ ) neutral Higgs boson the coupling to the bottom quark is  $\propto m_b \sin \alpha (\cos \alpha)$  which in the absence of a  $\tan \beta$  enhancement makes its contribution negligible. For the  $A$  Higgs boson, the coupling to  $b\bar{b}$  is  $\propto \tan \beta$  and the  $A$ -dependent contribution can be large and positive if  $m_A \lesssim 90$  GeV and  $\tan \beta \gtrsim 30$  [10]. Since we restrict ourselves to  $\tan \beta \lesssim 20$ , and  $m_A \gtrsim 100$  GeV in supergravity models, this contribution is neglected in what follows.

Our computations of  $R_b^{\text{susy}}$  have been performed using the expressions given in Ref. [6]. Even though these formulas are given explicitly, the details are quite complicated by the presence of various Passarino-Veltman loop functions. As a check of our computations, we have verified numerically that the results are independent of the unphysical renormalization scale that appears in the formulas.

### 3 $R_b$ in supergravity models

We consider unified supergravity models with universal soft supersymmetry breaking at the unification scale, and radiative electroweak symmetry breaking (enforced using the one-loop effective potential) at the weak scale [11]. These constraints reduce the number of parameters needed to describe the models to four, which can be taken to be  $m_{1/2}, \xi_0 \equiv m_0/m_{1/2}, \xi_A \equiv A/m_{1/2}, \tan \beta$ , with a specified value for the top-quark mass, which we take to be  $m_t = 175$  GeV. Among these four-parameter supersymmetric models, we consider generic models with continuous values of  $m_{\chi_{1\pm}^0}$  and discrete choices for the other three parameters:

$$\tan \beta = 2, 10, 20 ; \quad \xi_0 = 0, 1, 2, 5 ; \quad \xi_A = 0 . \quad (1)$$

The choices of  $\tan \beta$  are representative; higher values of  $\tan \beta$  are likely to yield values of  $B(b \rightarrow s\gamma)$  in conflict with present experimental limits [12]. The choices of  $\xi_0$  correspond to  $m_{\tilde{q}} \approx (0.8, 0.9, 1.1, 1.9)m_{\tilde{g}}$ . The choice of  $A$  has little impact on the results. We also consider the case of no-scale  $SU(5) \times U(1)$  supergravity [11]. In this class of models the supersymmetry breaking parameters are related in a string-inspired way. In the two-parameter *moduli* scenario  $\xi_0 = \xi_A = 0$  [13], whereas in the *dilaton* scenario  $\xi_0 = \frac{1}{\sqrt{3}}, \xi_A = -1$  [14]. A series of experimental constraints and predictions for these models have been given in Ref. [15]. In particular, the issue of precision electroweak tests in this class of models has been addressed in Refs. [16, 17].

The predictions for  $R_b^{\text{susy}}$  in the generic supergravity models are shown in Fig. 1. Only curves for  $\tan \beta = 2, 10$  are shown, since the corresponding sets of curves for other values of  $\tan \beta$  fall between these two sets of curves. The results in  $SU(5) \times U(1)$  supergravity, for the same values of the parameters, differ little from those in the generic models. To add generality to our result and to compare with the study made below in the case of the MSSM, we have also considered the case where the four supergravity parameters are chosen at random. We have sampled

10,000 random four-plets of parameter values, in the ranges:  $50 \leq m_{1/2} \leq 500$  GeV,  $0 \leq \xi_0 \leq 5$ ,  $-5 \leq \xi_A \leq 5$ , and  $1 \leq \tan \beta \leq 40$ . As expected, we find  $R_b^{\text{susy}} \lesssim 0.0004$ . A histogram of the relative distribution of  $R_b^{\text{susy}}$  values is shown in Fig. 2, which shows that  $R_b^{\text{susy}}$  is equally likely to be positive or negative, and that the “preferred value” is very small.

In almost all cases the largest positive contribution to  $R_b^{\text{susy}}$  comes from the chargino–top-squark loop. As expected, the largest contribution from this diagram happens for points with the lightest chargino masses (which correspond to the lightest  $\tilde{t}_1$  masses) since supersymmetry is a decoupling theory. However, even the largest value ( $\approx 0.0004$ ) is still very small compared with the largest possible result in a generic low-energy supersymmetric model. The reason for this is that while the smallest possible chargino and a top-squark masses are required for an enhanced contribution, it is also necessary that the top-squark be mostly right-handed and that the chargino has a significant Higgsino component [6]. A scatter plot of the values of these couplings is shown in Fig. 3, where the right-handed component of the lightest top-squark ( $T_{12}$ ) is plotted against the higgsino component of the lightest chargino ( $V_{12}$ ). The first requirement is in fact attainable in these models (*i.e.*,  $|T_{12}| \sim 1$ ), but the former is not (*i.e.*,  $|V_{12}| \lesssim 0.4$ ). This behavior is expected in supergravity models with radiative electroweak symmetry breaking since for light charginos  $|\mu| \gg M_2$ , which makes the lightest chargino mainly a wino instead of a Higgsino. (This is also the reason why the results in  $SU(5) \times U(1)$  supergravity differ little from those in the generic models.) Concerning the other contributions to  $R_b^{\text{susy}}$ , the charged-Higgs–top-quark loop is always negative, and is enhanced for either small or large values of  $\tan \beta$ . The neutralino–bottom-squark contribution is almost always smaller than the chargino–top-squark contribution and not of definite sign.

Thus,  $R_b$  in supergravity models could improve the Standard Model fit to the LEP data by at most one-fifth of a standard deviation, and it could well worsen the fit by the same small amount.

## 4 $R_b$ in the MSSM

We now relax the supergravity assumptions that correlate the various supersymmetric parameters, in order to explore the region of the MSSM parameter space that yields large values of  $R_b$ . This region is then subjected to all available experimental constraints, which have the effect of restricting it significantly. The  $R_b^{\text{susy}}$  observable depends on six basic parameters: the elements of the chargino mass matrix ( $M_2, \mu, \tan \beta$ ) which determine the chargino masses and their composition, and the two top-squark ( $\tilde{t}_1, \tilde{t}_2$ ) masses and their mixing angle ( $\theta_{\tilde{t}}$ ). In addition, the Higgs boson masses enter, although for  $\tan \beta < 30$  the neutral Higgs boson contribution is not relevant [6]. The charged Higgs mass is relevant for small values of  $\tan \beta$ , but this contribution to  $R_b$  is always negative and will be neglected in what follows (*i.e.*, we take a large charged Higgs mass). Neglecting altogether the Higgs-boson contribution to  $R_b^{\text{susy}}$  is generally justified when looking for the largest values of  $R_b$ , although some

exceptions exist for rather low values of the pseudoscalar Higgs boson mass ( $m_A$ ) and large values of  $\tan\beta$  [7].

We have sampled a large number of random choices for the six-plet of parameters, in the ranges  $0 \leq M_2, |\mu|, m_{\tilde{t}_1}, m_{\tilde{t}_2} \leq 250$  GeV,  $1 \leq \tan\beta \leq 5$ , and  $0 \leq \theta_{\tilde{t}} \leq \pi$ . We have concentrated on the small- $\tan\beta$  region since for large values of  $\tan\beta$ , where  $R_b^{\text{susy}}$  may also be enhanced,  $B(b \rightarrow s\gamma)$  is likely to exceed significantly the allowed CLEO range [12]. A histogram depicting the relative distribution of calculated values of  $R_b^{\text{susy}}$  is shown in Fig. 4. [Note that small negative values of  $R_b^{\text{susy}}$  are possible (although not very likely).] As expected, there is a steady decline in the likelihood of the larger values of  $R_b^{\text{susy}}$ , with the following relative distributions

Condition	Comment	$\mu > 0$	$\mu < 0$
$R_b^{\text{susy}} > 0.0008$	Ok at 95%CL	12%	8.8%
$R_b^{\text{susy}} > 0.0014$	Ok at 90%CL	3.0%	1.9%
$R_b^{\text{susy}} > 0.0020$	$R_b^{\text{SM}}$ up by $1\sigma$	0.87%	0.52%
$R_b^{\text{susy}} > 0.0027$	Ok at $1\sigma$	0.20%	0.12%

(2)

where for example, only  $\sim (2 - 3)\%$  of the sampled six-plets yield  $R_b^{\text{susy}} > 0.0014$ , which brings the total  $R_b$  prediction inside the 90%CL allowed range for  $m_t = 175$  GeV. Also, to be within the  $1\sigma$  range, and thus to significantly improve the fit to the LEP data, requires considerable fine tuning. If the sampling region were to be extended (*i.e.*, a larger mass interval or larger values of  $\tan\beta$ ) we would find that the new points fall into the lowest bins in Fig. 4, while the population of the bins with large  $R_b$  values would correspondingly decrease. The distributions in Eq. (2) would change accordingly.

In Fig. 5 we show the maximum attainable value of  $R_b^{\text{susy}}$  as a function of the chargino mass, which makes apparent the need for a light chargino if  $R_b^{\text{susy}}$  is to be enhanced. In fact, the maximum value is obtained for  $m_{\chi_1^\pm} \rightarrow \frac{1}{2}M_Z$  (and also  $m_{\tilde{t}_1} \rightarrow \frac{1}{2}M_Z$ ). This phenomenon has been observed before in the context of the  $\epsilon_b$  approach to the problem [3, 16], and corresponds to a wavefunction renormalization effect when the particles in the loop go on shell.

For concreteness, in what follows we concentrate on the  $\lesssim 1\%$  of the points that increase  $R_b$  by at least  $1\sigma$ ; the other cases in Eq. (2) depend on the choice of  $m_t$ . By design, this sample contains 1000 points for each sign of  $\mu$ . First let us show that the region of parameter space which leads to enhanced values of  $R_b^{\text{susy}}$  is indeed characterized by light higgsino-like charginos and light right-handed-like top-squarks. In Fig. 6 we show the distribution of points in the  $(m_{\chi_1^\pm}, m_{\tilde{t}_1})$  plane that lead to  $R_b^{\text{susy}} > 0.0020$ , and in Fig. 7 we show the corresponding distribution in  $(|V_{12}|, |T_{12}|)$  space. If  $R_b^{\text{susy}} > 0.0020$  is indeed required, then LEP II should see the lightest chargino and possibly also the lightest top-squark. The observed marked preference for large values of the  $V_{12}, T_{12}$  admixtures becomes evident when one considers the top-quark Yukawa coupling

$$\lambda_t Q_3 \widetilde{H} \widetilde{t}_R \rightarrow \lambda_t b_L \widetilde{H}^\pm \widetilde{t}_R, \quad \lambda_t t_L \widetilde{H}^0 \widetilde{t}_R, \quad (3)$$

Table 1: The 95%CL upper limit on the invisible top-quark branching ratio, obtained from a comparison of D0 data with theoretical expectations. All masses in GeV, all cross sections in pb.

$m_t$	$\sigma_{\min}^{\text{D0}}$	$\sigma_{\max}^{\text{th}}$	$B_{\min}^{t \rightarrow bW}$	$B_{\max}^{t \rightarrow \text{inv}}$
150	3.6	13.8	0.51	0.49
160	3.4	9.53	0.60	0.40
170	3.1	6.68	0.68	0.32
180	2.5	4.78	0.72	0.28
190	2.2	3.44	0.80	0.20
200	2.1	2.52	0.91	0.09

which picks out  $\tilde{t}_1 \rightarrow \tilde{t}_R$ ,  $\chi_1^\pm \rightarrow \tilde{H}^\pm$ ,  $\chi_1^0 \rightarrow \tilde{H}^0$ . This interaction also leads to an enhanced coupling between the top-quark, a higgsino-like neutralino, and a right-handed top-squark, and may lead to enhanced exotic decays of the top-quark, as we discuss below. Comparing Fig. 3 with Fig. 7 we see why  $R_b^{\text{susy}}$  is always suppressed in supergravity models: the regions in  $(|V_{12}|, |T_{12}|)$  space are completely non-overlapping. We should note that for  $\mu > 0$  it is easier to obtain large- $R_b^{\text{susy}}$  solutions since the chargino can have a larger higgsino component (as Fig. 7 shows), *i.e.*,  $|V_{12}| = |\cos \phi_+|$  with  $\tan(2\phi_+) \propto -\mu \cos \beta + M_2 \sin \beta$  [18].

The parameters that determine  $R_b^{\text{susy}}$  also determine the following observables:

1.  $B(b \rightarrow s\gamma)$ , as measured by CLEO to be  $(1 - 4) \times 10^{-4}$  [19], has been computed as in Ref. [12]. The charged-Higgs loop is negligible in the limit of a large charged Higgs mass (as assumed above) and in general its contribution is not large and does not significantly affect whether  $B(b \rightarrow s\gamma)$  is in agreement with experiment or not.
2.  $m_{\chi_1^0} < \{m_{\chi_1^\pm}, m_{\tilde{t}_1}\}$ , is the cosmological requirement of a neutral and colorless lightest supersymmetric particle (LSP). The lightest neutralino mass follows from the inputs to the chargino mass matrix and the usual assumption relating  $M_1$  to  $M_2$ . This constraint has already been included in Eq. (2) and in Figs. 4,6,7.
3.  $B(t \rightarrow \tilde{t}_1 \chi_{1,2}^0)$  leads to an “invisible” decay of the top quark if the  $\tilde{t}_1 \rightarrow b\chi_1^\pm \rightarrow b(q\bar{q}', \ell\nu_\ell)\chi_1^0$  decay products do not pass the standard top-quark cuts, as may likely be the case given the larger amount of missing energy and the softer leptons that accompany this decay. Taking the 95% CL lower bound on the top-quark cross section from D0 (see Fig. 3 in Ref. [20]), and dividing it by the “upper” estimate of the theoretical cross section (see Table 1 in Ref. [21]), one can obtain a 95%CL lower bound on  $[B(t \rightarrow Wb)]^2$ . As a function of  $m_t$ , this exercise is carried out in Table 1. For  $m_t = 175 \pm 15$  GeV we obtain  $B_{\max}^{t \rightarrow \text{inv}} = 0.3 \mp 0.1$ .

Table 2: The fraction of MSSM parameter space with  $R_b^{\text{susy}} > 0.0020$ , where the five experimental constraints are satisfied individually, and the combined allowed region, for  $B_{\text{max}}^{\text{t} \rightarrow \text{inv}} = 0.3 \pm 0.1$ .

	$B_{\text{max}}^{\text{t} \rightarrow \text{inv}}$	$B^{b \rightarrow s\gamma}$	LSP	$B^{\text{t} \rightarrow \text{inv}}$	$\Gamma_Z^{\text{inv}}$	$B^{Z \rightarrow \chi_1^0 \chi_2^0}$	All
$\mu > 0$	0.4	20%	98%	41%	88%	72%	5.3%
	0.3	20%	98%	19%	88%	72%	1.7%
	0.2	20%	98%	8.0%	88%	72%	0.9%
	$B_{\text{max}}^{\text{t} \rightarrow \text{inv}}$	$B^{b \rightarrow s\gamma}$	LSP	$B^{\text{t} \rightarrow \text{inv}}$	$\Gamma_Z^{\text{inv}}$	$B^{Z \rightarrow \chi_1^0 \chi_2^0}$	All
$\mu < 0$	0.4	37%	97%	31%	98%	95%	8.9%
	0.3	37%	97%	14%	98%	95%	1.9%
	0.2	37%	97%	8.4%	98%	95%	0.6%

4. The invisible  $Z$  width  $\Gamma(Z \rightarrow \chi_1^0 \chi_1^0)$  follows from the neutralino mass and composition. It should not exceed 7.6 MeV [22].
5. The branching ratio  $B(Z \rightarrow \chi_1^0 \chi_2^0)$  also follows from the neutralino mass matrix, and should not exceed  $10^{-4}$  [23].

We now impose these additional constraints on the points in MSSM parameter space with  $R_b^{\text{susy}} > 0.0020$ . First let us give the fraction of parameter space where these constraints are satisfied individually, as well as the combined allowed region. In Table 2 we carry out this exercise for  $B^{\text{t} \rightarrow \text{inv}} = 0.3 \pm 0.1$ , which corresponds to  $m_t = 175 \mp 15$  GeV. One can see that only a few percent of the points with interestingly large values of  $R_b^{\text{susy}}$  are experimentally allowed. Table 2 also shows that the  $b \rightarrow s\gamma$  and  $B^{\text{t} \rightarrow \text{inv}}$  constraints are the most restricting ones, and that they are both satisfied only in a small region of parameter space. This trend is shown more concretely in Fig. 8, where the distribution in  $[B^{\text{t} \rightarrow \text{inv}}, B(b \rightarrow s\gamma)]$  space is shown, together with the present experimental limits on these observables. In this figure, for  $\mu > 0$  ( $\mu < 0$ ):  $\sim 20$  (37)% of the points fall inside the allowed  $B(b \rightarrow s\gamma)$  region,  $\sim 41$  (31)% of the points fall inside the allowed  $B^{\text{t} \rightarrow \text{inv}}$  area, and  $\sim 5$  (9)% fall in the combined allowed area. As remarked above, in regions of parameter space with enhanced values of  $R_b^{\text{susy}}$ , we may also expect enhanced values of  $B^{\text{t} \rightarrow \text{inv}}$ . In Fig. 9 we show the distribution of  $R_b^{\text{susy}}$  versus  $B^{\text{t} \rightarrow \text{inv}}$  calculated values (with a cutoff of  $R_b^{\text{susy}} > 0.0020$ ), which indeed shows a distinct correlated enhancement of these two observables.

Finally, we consider the case where *all* experimental constraints are applied simultaneously. The resulting allowed points in the  $(m_{\chi_1^\pm}, m_{\tilde{t}_1})$  plane, for  $B_{\text{max}}^{\text{t} \rightarrow \text{inv}} = 0.4$ , are shown in Fig. 10. This figure is to be contrasted with Fig. 6, where none of the experimental constraints were applied (except for the rather unrestricting LSP constraint). We see that the extent of the allowed area is reduced:  $m_{\chi_1^\pm}^{\text{max}} : (70 \rightarrow$



60) GeV and  $m_{\tilde{t}_1}^{\max} : (110 \rightarrow 80)$  GeV. This restricted parameter space should be even more straightforwardly tested at LEP2, with now guaranteed sensitivity to the lightest top-squark. If we strengthen the  $B^{t \rightarrow \text{inv}}$  constraint to  $B_{\max}^{t \rightarrow \text{inv}} = 0.2$ , the more relevant shrinking of parameter space occurs in the  $m_{\tilde{t}_2}$  and  $\tan \beta$  directions:  $m_{\tilde{t}_2} < 100$  GeV and  $\tan \beta \lesssim 1.5$  are now required. Such strong restrictions would also allow detection of the lightest Higgs boson at LEP2, and both top-squarks at the Tevatron and LEP2.

## 5 Conclusions

We have studied the contributions to  $R_b$  that are expected in low-energy supersymmetric models. In the case of supergravity models with radiative electroweak symmetry breaking,  $R_b^{\text{susy}}$  could improve the Standard Model fit to the LEP data by only a small fraction of a standard deviation, and it could well worsen the fit by the same small amount. In the MSSM, light higgsino-like charginos and light right-handed-like top-squarks are required to obtain a sizeable  $R_b^{\text{susy}}$  value. Such values occur in a small fraction of the parameter space, which is largely accessible to LEP2 searches. This region of parameter space is further constrained by five experimental observables, most importantly  $B(b \rightarrow s\gamma)$  and the invisible top-quark branching fraction. Imposing these additional constraints reduces the allowed parameter space to a few percent of its unconstrained size, and makes experimental exploration of this scenario in both chargino and top-squarks assured at LEP2.

## Acknowledgements

This work has been supported in part by DOE grant DE-FG05-91-ER-40633. The work of X. W. has been supported by the World Laboratory. We would like to thank Chris Kolda for helpful communications. X. W. would like to thank J. T. Liu for helpful discussions. J. L. would like to thank James White and Teruki Kamon for helpful discussions.

## References

- [1] The LEP Electroweak Working Group, *A combination of Preliminary Results for the 1995 Winter Conferences*, LEPEWWG/95-01.
- [2] J. Bernabeu, A. Pich, and A. Santamaria, Phys. Lett. B **200** (1988) 569; W. Beenaker and W. Hollik, Z. Phys. C40, 141(1988); A. Akhundov, D. Bardin, and T. Riemann, Nucl. Phys. B **276** (1986) 1; F. Boudjema, A. Djouadi, and C. Verzegnassi, Phys. Lett. B **238** (1990) 423; A. Blondel and C. Verzegnassi, Phys. Lett. B **311** (1993) 346.
- [3] G. Altarelli, R. Barbieri, and F. Caravaglios, Nucl. Phys. B **405** (1993) 3.
- [4] G. Altarelli and R. Barbieri, Phys. Lett. B **253** (1990) 161; G. Altarelli, R. Barbieri, and S. Jadach, Nucl. Phys. B **369** (1992) 3; G. Altarelli, R. Barbieri, and F. Caravaglios, Phys. Lett. B **314** (1993) 357.
- [5] M. Bouwens, D. Finnel, Phys. Rev. D **44** (1991) 2054; A. Djouadi, G. Girardi, C. Verzegnassi, W. Hollik and F. Renard, Nucl. Phys. B **349** (1991) 48.
- [6] J. D. Wells, C. Kolda, and G. L. Kane, Phys. Lett. B **338** (1994) 219.
- [7] D. Garcia, R. Jimenez, and J. Sola, Phys. Lett. B **347** (1995) 321; D. Garcia and J. Sola, UAB-FT-358 (hep-ph/9502317).
- [8] M. Shifman, Mod. Phys. Lett. A **10** (1995) 605; L. Roszkowski and M. Shifman, TPI-MINN-95-04-T (hep-ph/9503358).
- [9] G. Kane, R. Stuart, and J. Wells, UM-TH-95-16 (hep-ph/9505207); P. Chankowski and S. Pokorski, IFT-95/5 (hep-ph/9505304); P. Barmert, C. Burgess, and I. Maksymyk, McGill-95/18 (hep-ph/9505339); D. Garcia and J. Sola, UAB-FT-365 (hep-ph/9505350).
- [10] W. Hollik, Int. J. Mod. Phys. A **5** (1990) 1909; A. Denner, R. Guth, W. Hollik, and J. Kühn, Z. Phys. **C51** (1991) 695.
- [11] For a recent review see *e.g.*, J. L. Lopez, D. V. Nanopoulos, and A. Zichichi, Prog. Part. Nucl. Phys. **33** (1994) 303.
- [12] See *e.g.*, S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, Nucl. Phys. B **353** (1991) 591; J. L. Lopez, D. V. Nanopoulos, and G. T. Park, Phys. Rev. D **48** (1993) 974; R. Garisto and J. N. Ng, Phys. Lett. B **315** (1993) 372; F. Borzumati, Z. Phys. C63 (1994) 291; J. L. Lopez, D. V. Nanopoulos, X. Wang, and A. Zichichi, Phys. Rev. D **51** (1995) 147.
- [13] J. L. Lopez, D. V. Nanopoulos, and A. Zichichi, Phys. Rev. D **49** (1994) 343.
- [14] J. L. Lopez, D. V. Nanopoulos, and A. Zichichi, Phys. Lett. B **319** (1993) 451.

- [15] J. L. Lopez, D. V. Nanopoulos, G. Park, X. Wang, and A. Zichichi, Phys. Rev. D **50** (1994) 2164.
- [16] J. L. Lopez, D. V. Nanopoulos, G. T. Park, H. Pois, and K. Yuan, Phys. Rev. D **48** (1993) 3297; J. L. Lopez, D. V. Nanopoulos, G. T. Park, and A. Zichichi, Phys. Rev. D **49** (1994) 355.
- [17] J. L. Lopez, D. V. Nanopoulos, G. T. Park, and A. Zichichi, Phys. Rev. D **49** (1994) 4835.
- [18] H. Haber and G. Kane, Phys. Rep. **117** (1985) 75.
- [19] M. S. Alam, *et. al.* (CLEO Collaboration), Phys. Rev. Lett. **74** (1995) 2885.
- [20] S. Abachi, *et. al.* (D0 Collaboration), Phys. Rev. Lett. **74** (1995) 2632.
- [21] E. Laanen, J. Smith, and W. L. van Neerven, Phys. Lett. B **321** (1994) 254.
- [22] See *e.g.*, A. Sopczak, CERN-PPE/94-188 (hep-ph/9504299), Mod. Phys. Lett. A (to appear).
- [23] M. Acciarri, *et. al.* (L3 Collaboration), CERN-PPE-95-14 (February 1995).

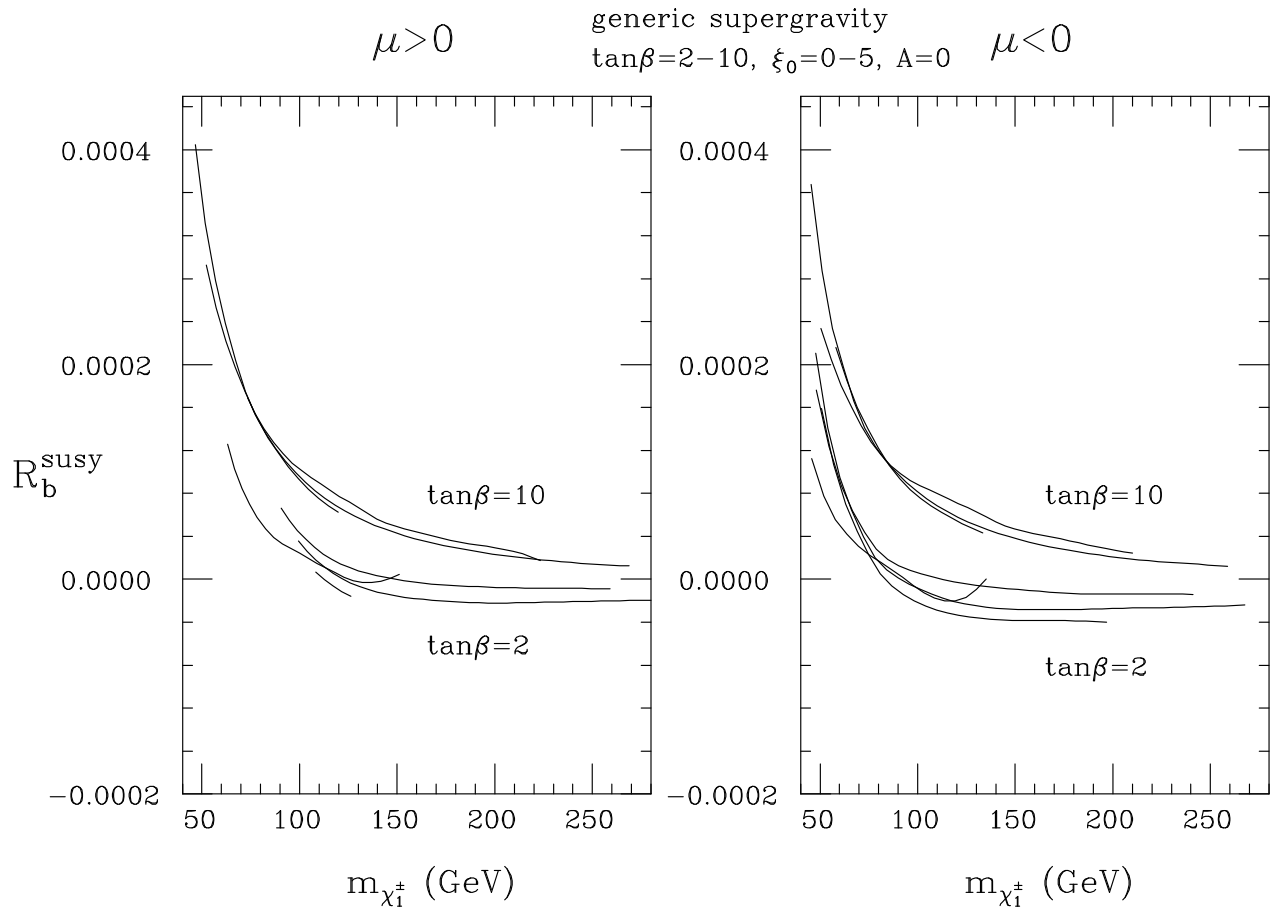


Figure 1: The supersymmetric contribution to  $R_b$  as a function of the chargino mass in generic supergravity models with  $\tan\beta = 2, 10$ ,  $\xi_0 = 0 - 5$ , and  $A = 0$ . Curves for other values of  $\tan\beta$  fall between the two sets shown.

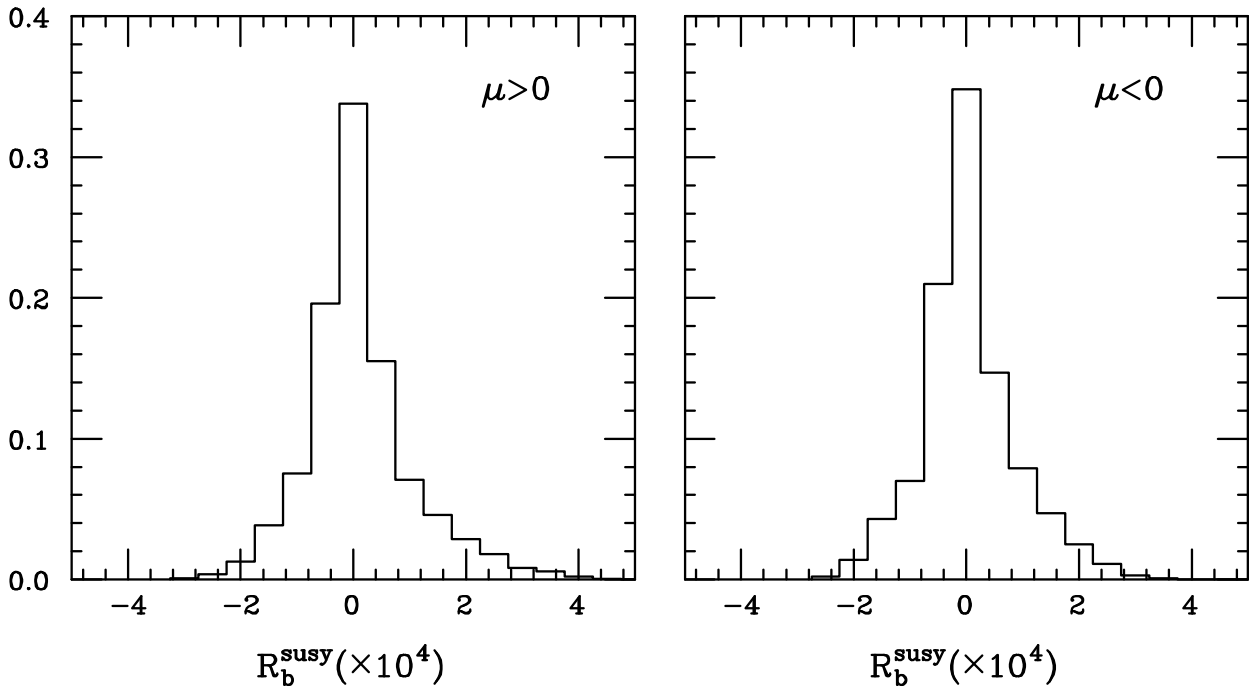


Figure 2: Relative distribution of calculated values of  $R_b^{\text{susy}}$  in a sample of 10,000 random choices for the four-plet of supergravity parameters.

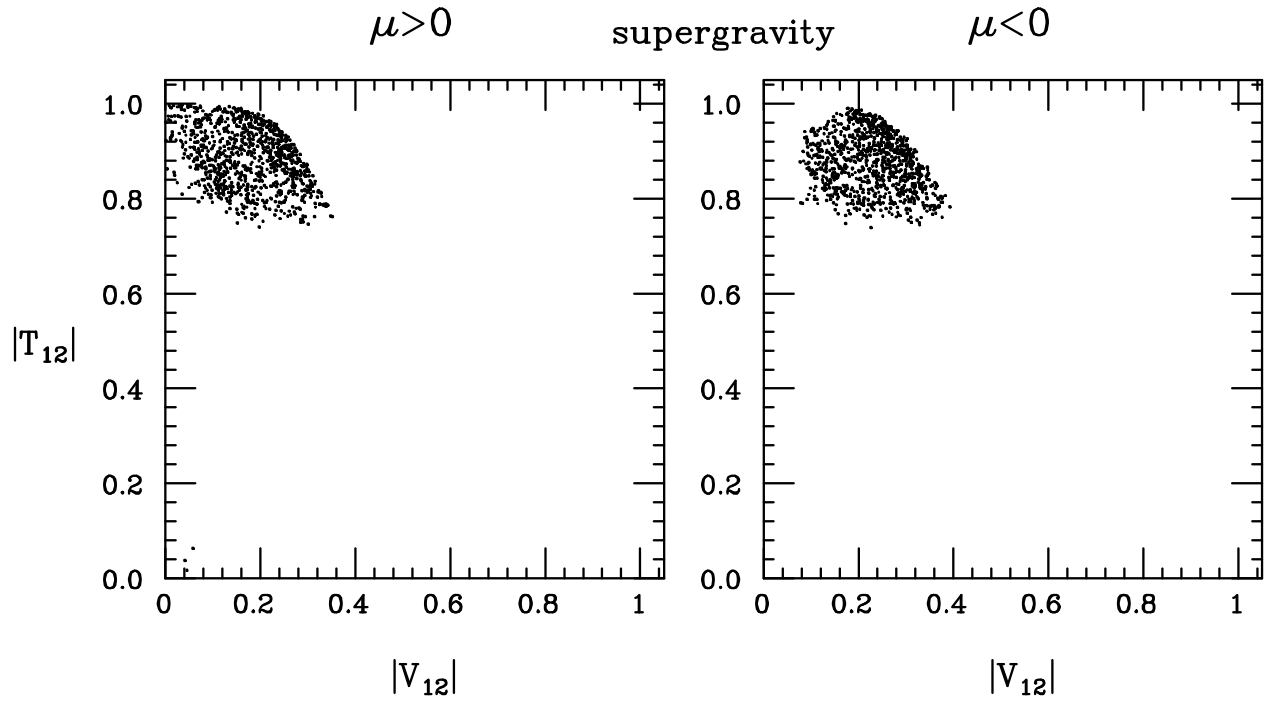


Figure 3: The right-handed component of the lightest top-squark ( $T_{12}$ ) versus the higgsino component of the lightest chargino ( $V_{12}$ ) in a random sample of supergravity models. Large values of both  $T_{12}$  and  $V_{12}$  are required for an enhanced value of  $R_b^{\text{susy}}$ .

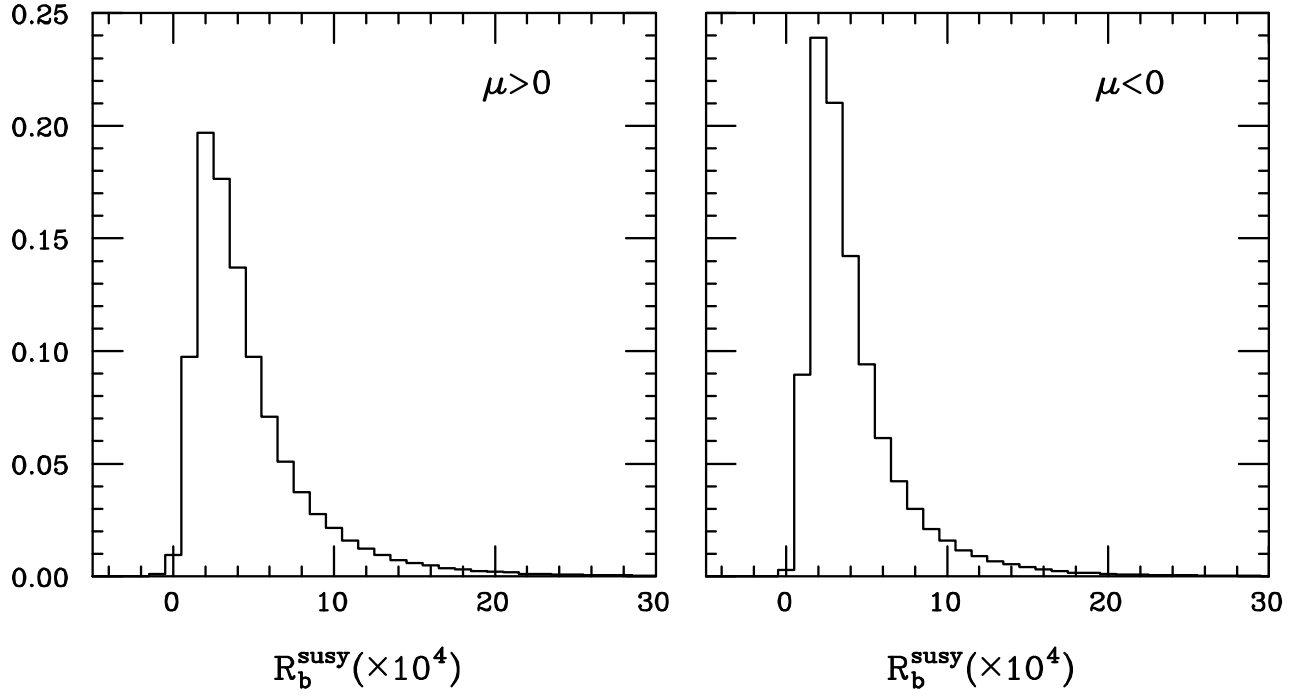


Figure 4: Relative distribution of calculated values of  $R_b^{\text{susy}}$  in a large sample of random choices for the six-plet of relevant MSSM parameters. For  $\mu > 0$  ( $\mu < 0$ ), only  $\approx 0.9\%$  ( $0.5\%$ ) of the points yield  $R_b^{\text{susy}} > 0.0020$  ( $1\sigma$ ).

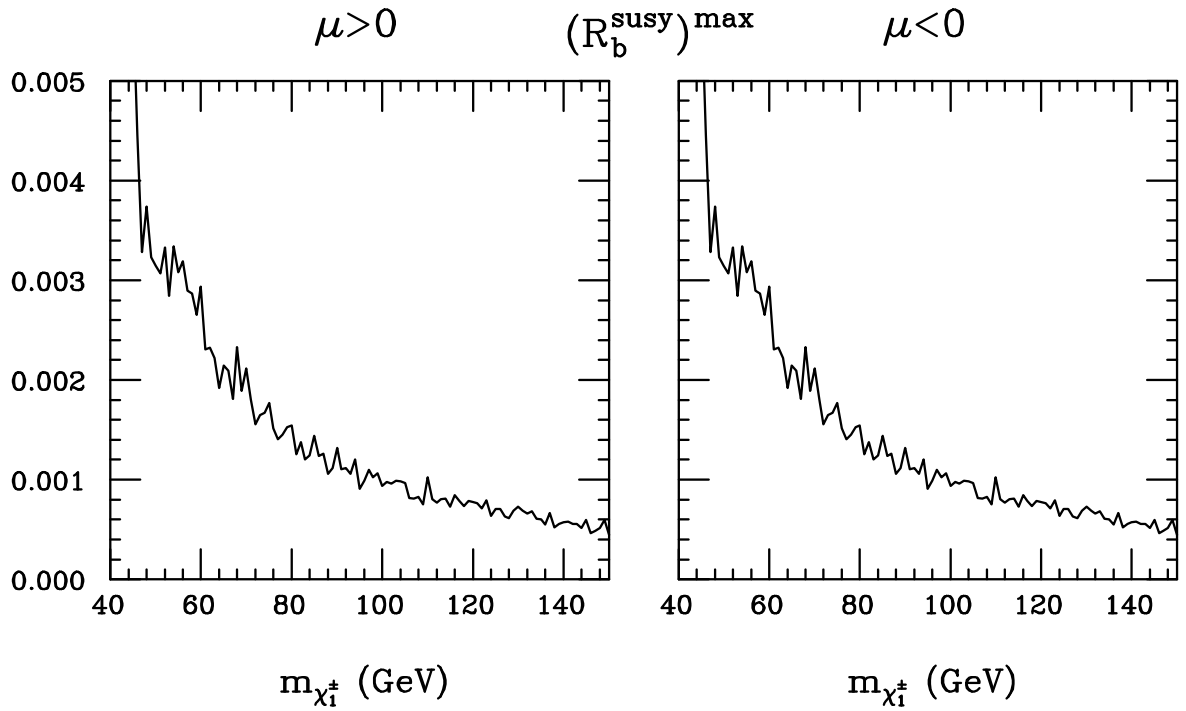


Figure 5: Maximum attainable value of  $R_b^{\text{susy}}$  in the MSSM as a function of the lightest chargino mass. All other variables have been integrated out. The broken line is a figment of the finite sample size.



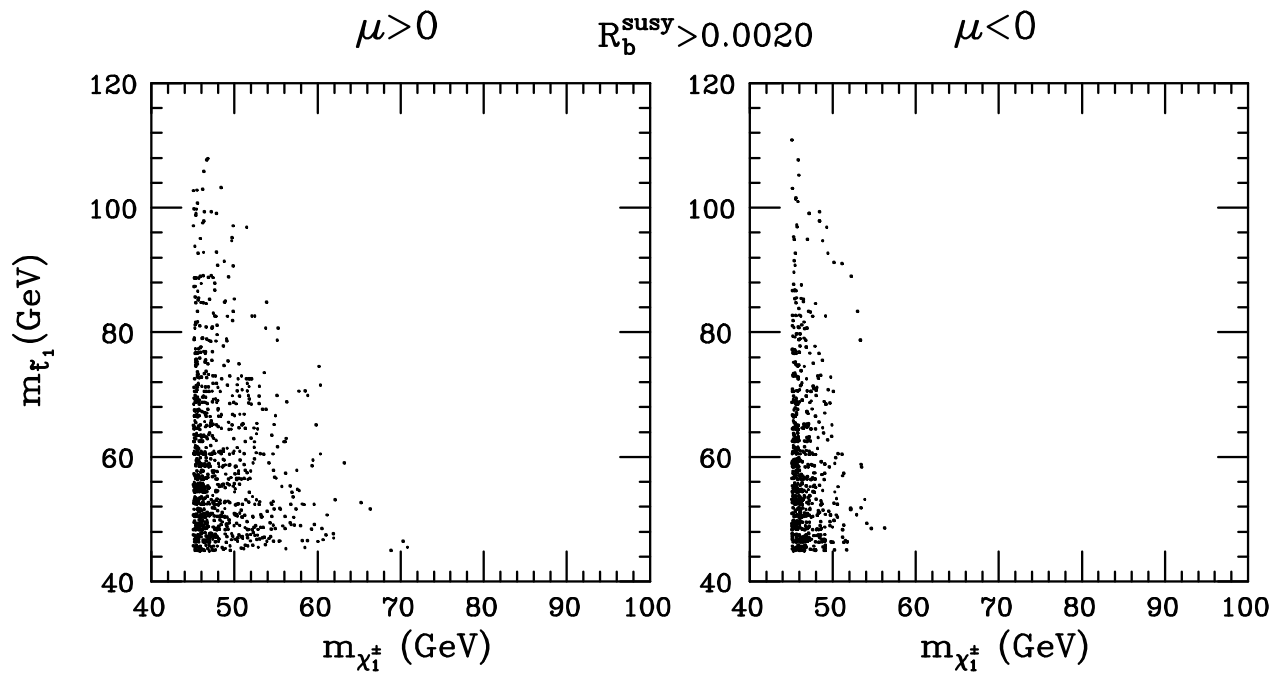


Figure 6: The distribution of lightest chargino and top-squark masses for points in MSSM parameter space with  $R_b^{\text{susy}} > 0.0020$  ( $1\sigma$ ).

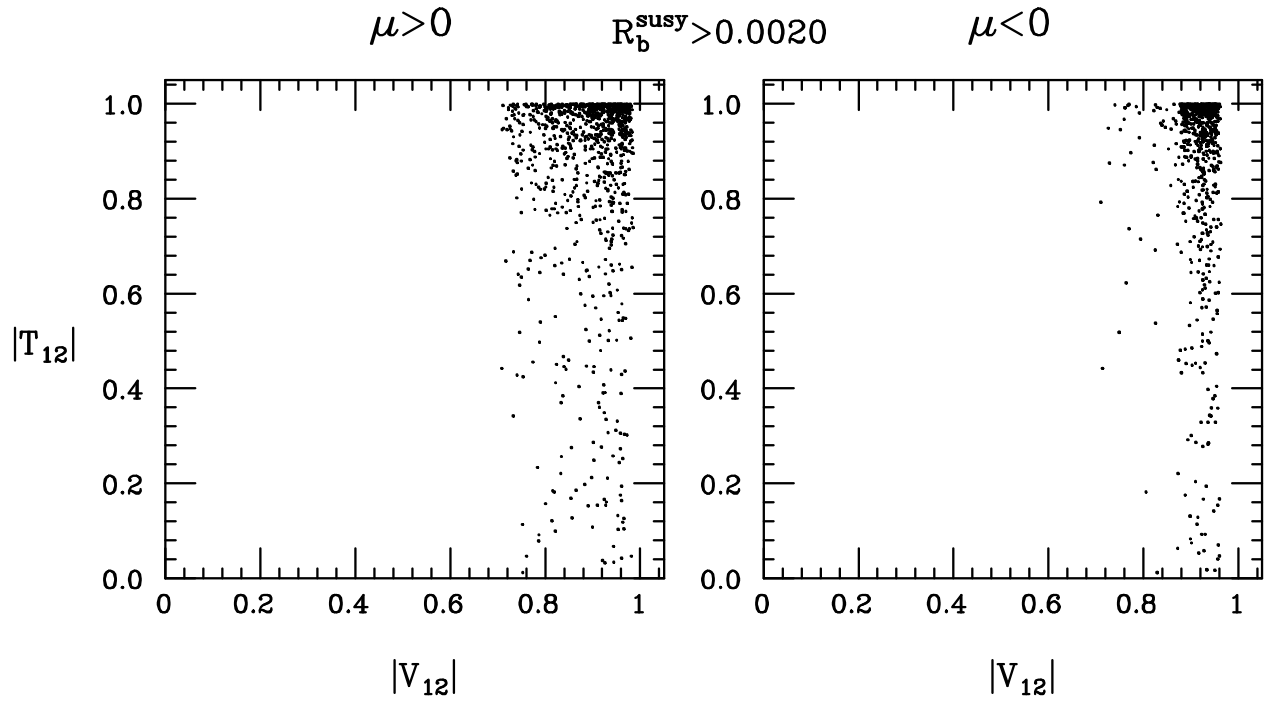


Figure 7: The right-handed component of the lightest top-squark ( $T_{12}$ ) versus the higgsino component of the lightest chargino ( $V_{12}$ ) for the fraction of MSSM sampled points with  $R_b^{\text{susy}} > 0.0020$  ( $1\sigma$ ).

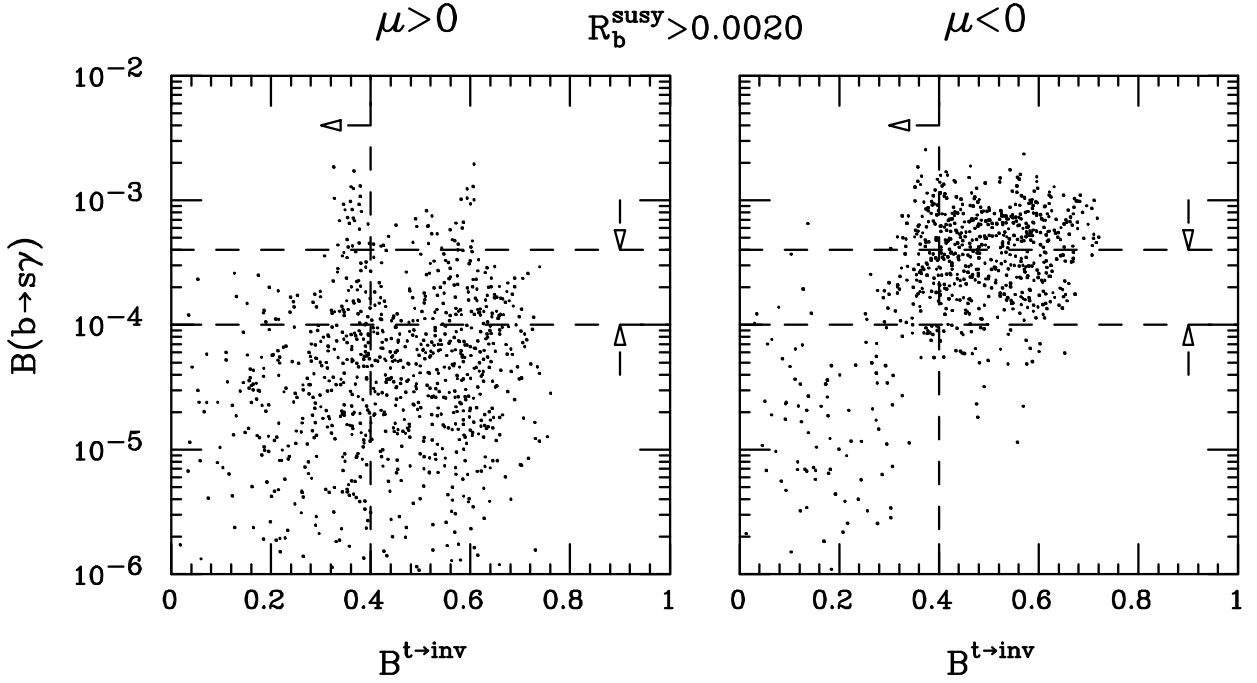


Figure 8: The calculated value of  $B(b \rightarrow s\gamma)$  versus  $B^{t \rightarrow \text{inv}}$  for points in the MSSM parameter space with  $R_b^{\text{susy}} > 0.0020$ . The arrows point into the experimentally allowed region. For  $\mu > 0$  ( $\mu < 0$ ) only  $\approx 5\%$  ( $9\%$ ) of the points fall inside the combined allowed region.

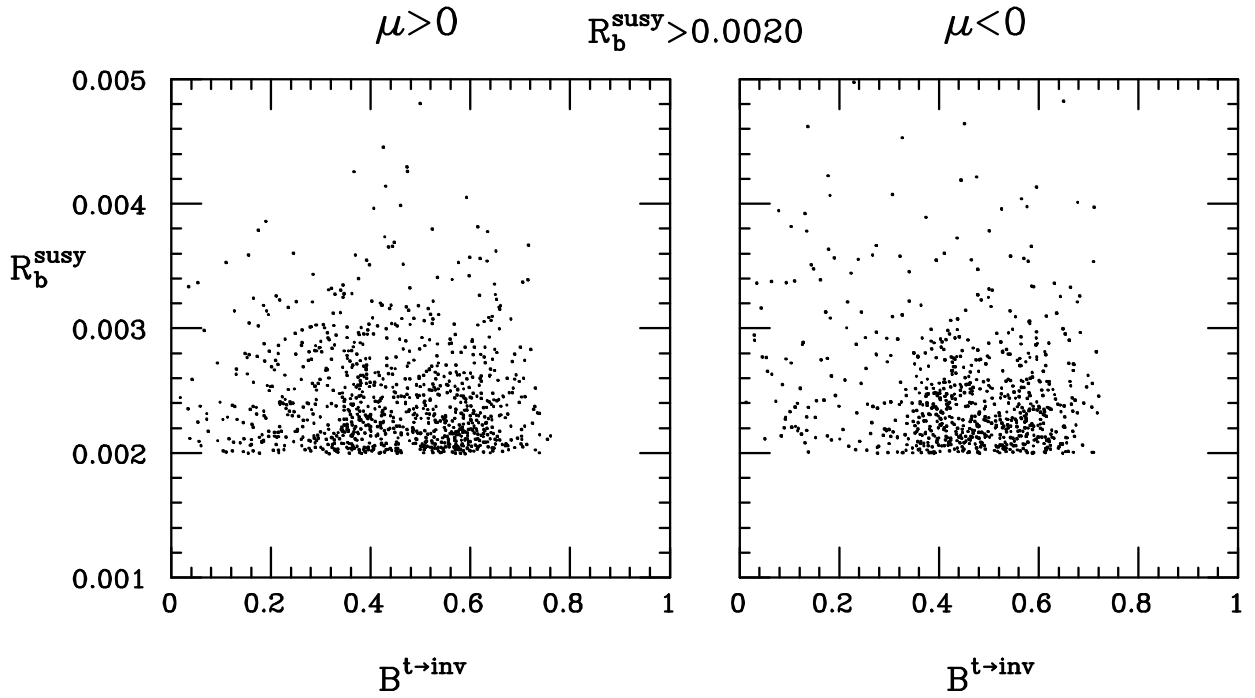


Figure 9: The calculated value of  $R_b^{\text{susy}}$  versus  $B^{t \rightarrow \text{inv}}$  for points in the MSSM parameter space with  $R_b^{\text{susy}} > 0.0020$ . Note the correlated enhancement of these two observables.

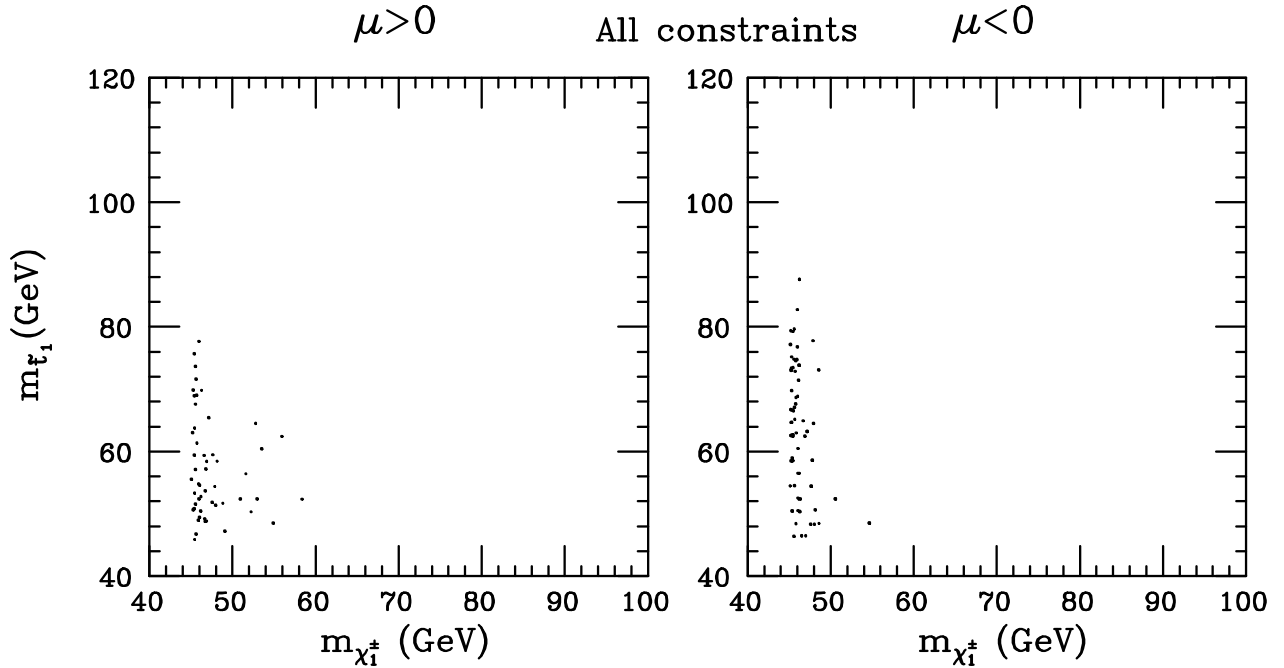


Figure 10: The distribution of lightest chargino and top-squark masses for points in MSSM parameter space with  $R_b^{\text{susy}} > 0.0020$  ( $1\sigma$ ), when *all* experimental constraints are imposed (with  $B_{\text{max}}^{\text{t} \rightarrow \text{inv}} = 0.4$ ). Because of the finite sample size, the whole region spanned by the shown points should be considered viable.