

Quantitative wave-particle duality and non-erasing quantum erasure

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The notion of wave-particle duality may be quantified by the inequality $V^2 + K^2 \leq 1$, relating interference fringe visibility V and path knowledge K . With a single-photon interferometer in which polarization is used to label the paths, we have investigated the relation for various situations, including pure, mixed, and partially-mixed input states. A quantum eraser scheme has been realized that recovers interference fringes even when no which-way information is available to erase.

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Wave-particle duality (WPD) dates back to Einstein's seminal paper on the photoelectric effect [1], and is a striking manifestation of Bohr's complementarity principle [2] (for a formal definition see Ref. [3]). The familiar phrase "each experiment must be described either in terms of particles or in terms of waves" emphasizes the extreme cases and disregards intermediate situations, in which particle and wave aspects coexist. Theoretical investigations [4,5], supplemented by a few experimental studies [6,7], have led to a quantitative formulation of WPD [Eq. (1) below]. Here we report an experiment using a single-photon Mach-Zehnder interferometer in which polarization marks the path. We investigated the entire scope of the duality relation, for pure, mixed, and partially-mixed input states, and found *absolute* agreement at the percent level [8]. We also realized a novel quantum eraser scheme, whereby interference is recoverable although no which way (WW) information was available to erase. In view of the kinematical equivalence of all binary degrees of freedom, our results are directly applicable whenever an interfering particle is entangled with a 2-state quantum system.

To quantify WPD, one needs quantitative, measurable characteristics for the wave-like and particle-like behavior of quanta. In an interferometer, the former is naturally quantified by the *Visibility* V of the observed interference fringes. The quantification of the latter is based on the *Likelihood* L of correctly guessing the path taken by a particular quantum — the better one can guess, the more pronounced are the particle aspects. A random guess gives $L = \frac{1}{2}$, whereas $L = 1$ indicates that the way is known with certainty. The actual WW *Knowledge* K is given by $K = 2L - 1$, with $0 \leq K \leq 1$. In an *asymmetric* interferometer one way is more likely than the other to begin with ($L_{\text{a priori}} > \frac{1}{2}$); we call WW knowledge of this kind *Predictability* ($P = 2L_{\text{a priori}} - 1$). The statement $V^2 + P^2 \leq 1$ has been known for some time, implicitly or explicitly, in various physical contexts [4,6]. Since one cannot lose *a priori* knowledge, $P \leq K$; in fact, $P \approx 0$ in our experiments. Nevertheless, owing to an *entanglement* of the system wave function with the wave function of some WW marker (WWM), the Knowledge can still be as large as 1.

The actual value of K depends on the "betting strategy" employed; the optimal strategy maximizes K and identifies the *Distinguishability* $D = \max\{K\}$ — it is the maximum amount of WW Knowledge available, although a non-optimal measurement may yield less, or even zero. (Experimental inaccessibility of some crucial degrees of freedom may force the experimenter to settle for a non-optimal measurement; see Ref. [9] for further remarks.) Except where noted, our measurements were suitably optimized to maximize K . The *Duality Relation* accessible to experimental test then becomes [5,9]:

$$V^2 + K^2 \leq 1. \quad (1)$$

The equality holds for pure initial states of the WWM, while the inequality applies to (partially-)mixed states.

I. EXPERIMENTAL SETUP AND PROCEDURE

In our experiments single photons (at 670 nm) were directed into a compressed Mach-Zehnder interferometer [10] (see Fig. 1). An adjustable half waveplate (HWP) in path 1 was used to entangle the photon's path with its polarization (i.e., with the WWM) thus yielding WW Knowledge [11]. Our adjustable analysis system — quarter waveplate (QWP), HWP, and calcite prism (PBS) — allowed the polarization WWM to be measured in any arbitrary basis. The photons were detected using geiger-mode avalanche photodiodes — Single Photon Counting Modules (EG&G #SPCM-AQ, efficiency $\sim 60\%$). The input source, described below, was greatly attenuated so that the maximum detection rates were always less than $50,000\text{s}^{-1}$; for the interferometer passage time of 1ns, this means that on average fewer than 10^{-4} photons were in the interferometer at any time.

The probability for having no photon at all is close to unity at any arbitrary instant, but state reduction removes this part a posteriori as soon as a detector "clicks." The reduced state is virtually indistinguishable from a one-photon Fock state because the probability for two or more photons is negligibly small. This one-photon-at-a-time operation is essential to allow sensible discussion of the likely path taken by an individual light quantum [12].

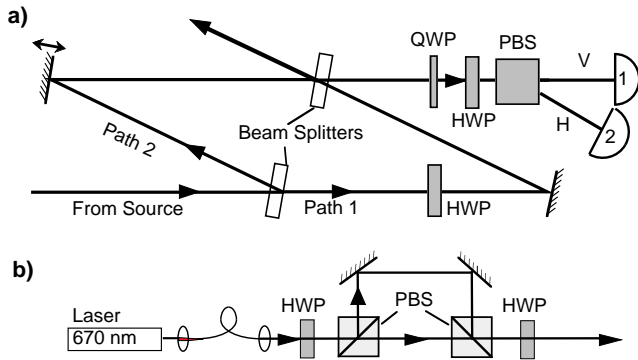


FIG. 1. (a) Experiment to quantify the wave-particle duality. (b) An asymmetric Mach-Zehnder interferometer with polarizing beam splitters separates the horizontal and vertical components of the input light by much more than the ~ 1 -cm coherence length; one can thus make arbitrary quantum states (pure, mixed, and partially-mixed) by varying the input polarization. A single-mode fiber filters the spatial mode.

Perhaps unnecessarily, we emphasize that our experiment is not intended to be a direct proof of the quantum nature of light. Rather, we accept the existence of photons as an established experimental fact [13]. The quantized electromagnetic field has a classical limit as a field (unlike other quantum fields that have, at best, a limit in terms of particles), and some properties of the quantum field have close classical analogs. In particular, the counting rates of single-photon interferometers, such as the one used in our experiment, are proportional to the intensities of the corresponding classical electromagnetic field. But there is no allowance for individual detector clicks in Maxwell’s equations [14], nor for the quantum entanglement of photonic degrees of freedom that we exploit. And clearly, the trajectory of a light quantum through the interferometer is a concept alien to classical electrodynamics, as is the experimenter’s knowledge K about this trajectory.

For Visibility measurements the polarization analyzer was lowered out of the beam path, and the maximum and minimum count rates on detector 1 were measured as the length of path 2 was varied slightly (via a piezoelectric transducer). After subtracting out the separately-measured detector background (i.e., the count rate when the input to the interferometer was blocked, typically $100\text{--}400\text{s}^{-1}$), the visibility was calculated in the standard manner: $V = (\text{Max} - \text{Min})/(\text{Max} + \text{Min})$.

For the determination of the Likelihood, and hence the Knowledge, the following procedure was used. With the polarization analyzer in place, and path 2 blocked, the counts on the two detectors were measured. Detector 1 (2) looked at polarization λ (λ^\perp), determined by the analysis settings. After subtracting the backgrounds measured for each detector, the count rates from detector 1 were scaled by the relative efficiency of the two detectors: $\eta_2/\eta_1 = 1.11 \pm 0.01$. (In this way

our calculated value of the Knowledge corresponds to what would have been measured if our detectors had been identical and noiseless.) Call the resulting scaled rates $R_{1\lambda} \equiv R(\text{path 1, polarization } \lambda)$ and $R_{1\lambda^\perp} \equiv R(\text{path 1, polarization } \lambda^\perp)$. Next, we measure the corresponding quantities for path 2: $R_{2\lambda}$ and $R_{2\lambda^\perp}$. The betting strategy is the one introduced by Wootters and Zurek [4] and optimized in Ref. [5]: Pick the path which contributes most to the probability of triggering the detector that has actually fired. The Likelihood is then

$$L = \frac{\max\{R_{1\lambda}, R_{2\lambda}\} + \max\{R_{1\lambda^\perp}, R_{2\lambda^\perp}\}}{R_{1\lambda} + R_{2\lambda} + R_{1\lambda^\perp} + R_{2\lambda^\perp}}. \quad (2)$$

II. EXPERIMENTAL RESULTS

A. Wave-particle duality for pure states

Figure 2 shows the results when a pure vertical-polarization state (V) was input to the interferometer, as a function of the internal HWP’s orientation. As expected, when the HWP is aligned to the vertical ($\theta_{\text{HWP}} = 0$), therefore leaving the polarization unchanged, we see nearly complete Visibility and get no WW Knowledge. The measured values of V are slightly lower than the theoretical curve because the intrinsic visibility of the interferometer (even without the HWP) is only $\sim 98\%$, due to nonideal optics [15]. Conversely, with the HWP set (at $\theta_{\text{HWP}} = 45^\circ$) to rotate the polarization in path 1 to horizontal (H), the Visibility is essentially zero, and the Knowledge nearly equal to 1. Formally, the spatial wave function and the polarization WWM wave function are completely entangled by the HWP: $|\psi\rangle \propto |1\rangle|H\rangle_{\text{WWM}} + e^{i\phi}|2\rangle|V\rangle_{\text{WWM}}$, where ϕ is the relative phase between paths 1 and 2. Tracing over the WWM effectively removes the coherence between the spatial modes. That a small visibility persists in our results can be explained by slight residual polarization transformations by the interferometer mirrors and beam splitters, so that the polarizations from the two paths are not completely orthogonal; and by the remarkable robustness of interference — both theoretically and experimentally, $V > 4.4\%$ even though $L > 99.9\%$!

In Fig. 2 we also display two sets of Knowledge data, one taken in the optimal basis [16], the other fixed in the horizontal-vertical basis. These data demonstrate that Knowledge can depend on the measurement technique. With the optimal basis, the value of $V^2 + K^2$ is always very close to the predicted unit value; our experiment is the first to verify this. The average of all the data points in Fig. 2 gives 0.976 ± 0.017 . The slight discrepancy with the predicted value of 1 is mostly due to the intrinsic visibility of the interferometer — for the minimum-visibility arrangement, $V^2 + K^2 = 0.998$.

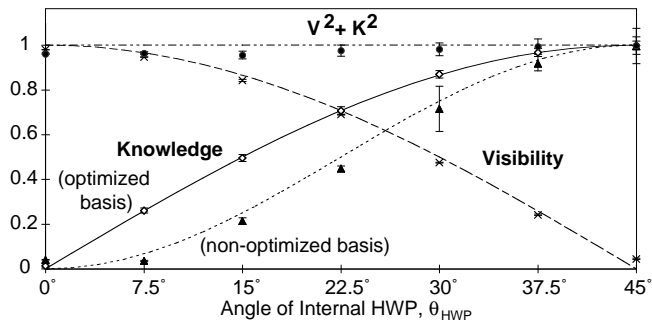


FIG. 2. Experimental data and theoretical curves for a purely vertically-polarized input to the system shown in Fig. 1, as a function of the orientation of a HWP in path 1. The crosses are the measured Visibilities; the dotted line and triangles correspond to K measurements fixed in the horizontal-vertical basis; and the solid line and diamonds correspond to measurements in the optimal basis [16].

B. Wave-particle duality for (partially-)mixed states

Using photons from an attenuated quartz halogen lamp that was spectrally-filtered with a narrow-band interference filter (centered at 670 nm, 1.5 nm FWHM) and spatially-filtered via a single-mode optical fiber, we explored Eq. (1) for *mixed* states (slight polarizing effects from the fiber actually led to $\sim 4\%$ residual net polarization). The measurements of Visibility and Knowledge for this nearly completely-mixed input state have values close to the theoretical prediction of 0 (Fig. 3a). $K \rightarrow 0$ for a completely-mixed WWM state because any unitary transformations on an unpolarized input also yield an unpolarized state (the density matrix is unaffected), so there is no WW information. That $V \rightarrow 0$ can be understood by examining the behavior of orthogonal pure WWM states, with no definite phase relationship between them. In the basis where the HWP rotates the WWM states by 90° , the orthogonal polarizations from paths 1 and 2 cannot interfere; in the basis aligned with the HWP's axes, each polarization individually interferes, but the interference patterns are shifted relatively by 180° (due to the birefringence of the HWP), so the sum is a fringeless constant.

To enable production of an even more mixed input, and to allow generation of arbitrary partially-mixed states, we used a “tunable” diode-laser scheme (see Fig. 1b). By rotating the (pure linear) polarization input to the first polarizing beam splitter, one can control the relative contribution of horizontal and vertical components. For example, for incident photons at 45° , one has equal H and V amplitudes which are then added together with a random and rapidly varying phase to produce an effectively completely mixed state of polarization [17]. With 5 times more vertical than horizontal, the state is then $1/3$ completely-mixed to $2/3$ pure. This case is shown in Fig. 3b. Note that the maximum Visibility (and Knowl-

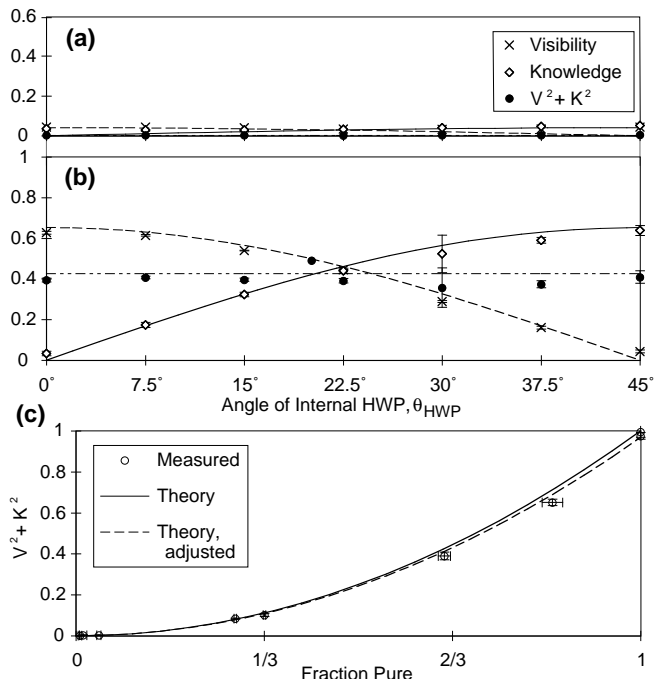


FIG. 3. Visibility and Knowledge measurements (and theory curves) for various mixed and partially-mixed input states. (a) A mixed state input from the filtered white-light source yielded an average value for $V^2 + K^2$ of 0.003 ± 0.001 ; the theoretical prediction based on the measured input state is 0.017 ± 0.003 (the uncertainty in the theory comes from imperfect determination of the state). The slight disparity arises from residual polarization transformations by the empty interferometer. (b) A partially-mixed state (purity = 0.65 ± 0.01) was generated using the tunable source. (c) A summary of all $V^2 + K^2$ data for various input states. The solid curve is the uncorrected theory: $V^2 + D^2 = 2\text{Tr}(\rho^2) - 1$, where ρ is the density matrix of the polarization WWM. The dashed curve is the theory accounting for the maximum visibility and the slight polarization-dependence of the empty interferometer.

edge) is numerically equal to the state purity, as the mixed-component displays no interference (and contains no WW information). The data taken for various input states show excellent agreement with theoretical predictions (Fig. 3c).

C. Quantum erasure (erasing and non-erasing)

In contrast to many interference situations where the WW information may be inaccessible, the quantum state of our WWM is easily manipulated. One can then in fact “erase” the distinguishing information and recover interference [3,18] (though this simple physical picture fails when non-pure WWM states are considered). In our experiments, such an erasure consists of using a polarization analysis to reduce or remove the WW labeling. For example, if the path 1 and 2 polarizations are horizontal

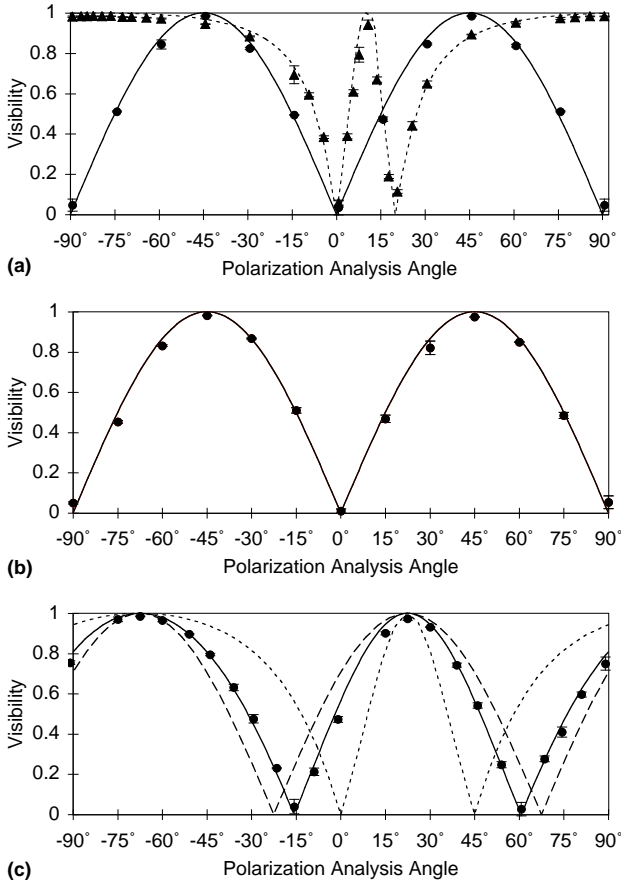


FIG. 4. “Quantum eraser” data and theory curves for various input states. The minima on the curves correspond to analysis that transmits light from only one or the other path; the maxima fall midway between these minima. (a) A purely vertically-polarized input ($\equiv 90^\circ$), with the polarization rotated by the HWP in path 1 by 90° (circles, solid line; $\theta_{\text{HWP}} = 45^\circ$) or 20° (triangles, dashed line; $\theta_{\text{HWP}} = 10^\circ$); (b) a completely-mixed state, with $\theta_{\text{HWP}} = 45^\circ$; and (c) a partially-mixed state (1:2 pure to mixed; circles, solid line), with the HWP at $\theta_{\text{HWP}} = 22.5^\circ$ — the dotted and dashed curves show the corresponding theoretical predictions for pure and completely-mixed states, respectively.

and vertical, respectively, analysis at $\pm 45^\circ$ will recover complete fringes; any photon transmitted through a 45° polarizer is equally likely to have come from either path.

Figure 4a shows quantum eraser data under the condition that a pure vertical photon is input to the interferometer, and rotated by the HWP in path 1 by either 90° or 20° . The visibility on detector 1 *after* the analyzer can assume any value from 0 to 1, the latter case being a complete quantum erasure. Even for a completely-mixed state, it is still possible to recover interference (Fig. 4b). With no WW information to erase, this *non-erasing quantum erasure* may seem quite remarkable at first. However, the essential feature of quantum erasure is not that it destroys the possibly available WW infor-

mation, but that it sorts the photons into sub-ensembles (depending on the quantum state of the WWM) each exhibiting high-visibility fringes. Complete interference is recoverable by analyzing along the eigenmodes of the internal HWP — along one axis we see fringes, along the other we see “anti-fringes,” shifted by 180° [19]. More generally, one post-selects one of the WWM eigenstates as determined by the interaction Hamiltonian of the interfering quantum system and the WWM [20].

For a partially-mixed WWM state, just as the value of $V^2 + K^2$ lies partway between 1 (pure state) and 0 (completely-mixed state), the analysis angles yielding zero visibility also fall between those for pure and mixed states (Fig. 4c). Quantitatively, for a fractional purity s , the angles are at $\theta_{\text{HWP}} \pm 1/2 \arccos(s \cos(2\theta_{\text{HWP}}))$; consult Ref. [21] for further details.

A convenient geometrical visualization of our results can be had by considering the polarization analysis in the Poincaré sphere representation [22], in which all linear polarizations lie on the equator of the sphere, circular polarizations lie on the poles, and arbitrary elliptical states lie in between. Any two orthogonal states lie diametrically opposed on the sphere. For pure polarization input states to our interferometer, there are in general exactly two points on the sphere for which the interference visibility will be exactly equal to zero. These correspond to the polarizations where a detector sees light from only one of the interferometer paths. Along the entire great circle that bisects the line connecting these two points, the quantum eraser will yield perfect visibility. Curiously, the situation for a completely-mixed input state is reversed. Here there are in general exactly two polarization states for which the quantum eraser recovers unit visibility, corresponding to the eigenmodes of the polarization elements inside the interferometer; on the great circle equidistant from these two points the Visibility vanishes. For example, in some mixed-state experiments described in Ref. [21], the eigenmodes are the poles on the Poincaré sphere, and the great circle corresponds to the equator — no visibility is observed for any linear polarization analysis.

III. DISCUSSION

Our results demonstrate the validity of Eq. (1) at the percent level. Moreover, they highlight some features associated with mixed states, which may not have been widely appreciated. Namely, that it is possible for both the interference visibility and the path distinguishability to equal zero. We have also seen that in some cases where the visibility is intrinsically equal to zero, it is possible to perform quantum erasure on the photons, and recover the interference. Remarkably, this is true even when the input state is completely mixed, and there exists no WW information to erase. The operation of the polarizer is essentially to *select* a sub-ensemble of photons. Depend-

ing on how this selection is performed, we may recover fringes, anti-fringes, no fringes, or any intermediate case.

The WW labeling in our experiment arose from an *entanglement* between the photon’s spatial mode and polarization state. It could just as well have been with another photon altogether, as in the experiments in [23], or even with a different kind of quantum system [24]. The same results are predicted, as long as the WW information is stored in a 2-state quantum system (e.g., internal energy states, polarization, spin, etc.). More generally, our findings are extendible to analogous experiments with quanta of different kinds such as, for example, interferometers with electrons [25], neutrons [26], or atoms [7,8].

To counter a possible misunderstanding let us note that, quite generally, entanglement concerns different degrees of freedom (DoF’s), not different particles. For certain purposes, such as quantum dense coding [27] or quantum teleportation [28], it is essential that the entangled DoF’s be carried by different particles and can thus be manipulated at a distance. For other purposes, however, one can just as well entangle an internal DoF of the interfering particle itself with its center-of-mass DoF [29]. In our experiment the photon’s polarization DoF is entangled with the spatial mode DoF represented by the binary alternative “reflected at the entry beam splitter, or transmitted?” Analogously, hyperfine levels of an atom were used to mark its path in the experiments of Refs. [7,8].

In the extreme situation of perfect WW distinguishability, the entangled state is of the form stated in Sec. II A, namely $|\psi\rangle \propto |1\rangle|H\rangle + e^{i\phi}|2\rangle|V\rangle$. Appropriate measurements on the spatial DoF (defined by $|1\rangle$ and $|2\rangle$) and the polarization DoF ($|H\rangle$ and $|V\rangle$) would show that the entanglement is indeed so strong that Bell’s inequality [30] is violated — clear evidence that a description based solely on classical electrodynamics cannot account for all features of our experiment. Of course, inasmuch as one cannot satisfy the implicit assumption that the measurements on the entangled subsystems be space-like separated, this violation of Bell’s inequality implies nothing about the success or failure of local-hidden-variable theories; however, this is not relevant here.

Finally we’d like to mention that further progress was made since the completion of the work reported here. Experimental tests of more sophisticated inequalities than (1) were performed [31], and there was progress in theory as well [32]. In particular, the quantitative aspects of quantum erasure were investigated beyond the initial stage reached in Ref. [9].

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- [1] A. Einstein, *Ann. Physik* **17**, 132 (1905); English translation in *The World of the Atom*, edited by H. A. Boorse and L. Motz (Basic Books, New York, 1966).
 - [2] N. Bohr, *Nature (London)* **121**, 580 (1928).
 - [3] M. O. Scully, B.-G. Englert, and H. Walther, *Nature (London)* **351**, 111 (1991).
 - [4] W. Wootters and W. Zurek, *Phys. Rev. D* **19**, 473 (1979); R. Glauber, *Ann. N. Y. Acad. Sci.* **480**, 336 (1986); D. Greenberger and A. Yasin, *Phys. Lett.* **A128**, 391 (1988); L. Mandel, *Opt. Lett.* **16**, 1882 (1991); G. Jaeger, A. Shimony, and L. Vaidman, *Phys. Rev. A* **51**, 54 (1995).
 - [5] B.-G. Englert, *Phys. Rev. Lett.* **77**, 2154 (1996).
 - [6] H. Rauch and J. Summhammer, *Phys. Lett.* **A104**, 44 (1984); J. Summhammer, H. Rauch, and D. Tuppinger, *Phys. Rev. A* **36**, 4447 (1987); P. Mittelstaedt, A. Prieur, and R. Schieder, *Found. Phys.* **17**, 891 (1987); F. De Martini *et al.* *Phys. Rev. A* **45**, 5144 (1992).
 - [7] S. Dürr, T. Nonn, and G. Rempe, *Nature (London)* **395**, 33 (1998).
 - [8] With internal atomic degrees of freedom employed for the path marking, a recent atom-interferometry experiment investigated the *equality* of (1), with *scaled* results constant to within 10%, but achieving *unscaled* values of only ~ 0.6 [S. Dürr, T. Nonn, and G. Rempe, *Phys. Rev. Lett.* **81**, 5705 (1998)]. For the record we note that our work was simultaneous with and independent of Dürr, Nonn, and Rempe’s although their published account appeared earlier.
 - [9] G. Björk and A. Karlsson, *Phys. Rev. A* **58**, 3477 (1998).
 - [10] The angle of incidence on the beam splitter was set to 10° , in order to minimize polarization variations in the reflection and transmission amplitudes — the resulting beamsplitter reflectivities and transmittivities were found to lie in the range 0.49 to 0.51 for all polarizations.
 - [11] A HWP *reflects* linear polarization about the optic axis direction, effectively rotating the polarization by twice the angle between the incident polarization and this axis.
 - [12] Note that this same feature enables quantum cryptography to be performed with attenuated coherent states; see, for instance, B. Huttner, N. Imoto, N. Gisin, and T. Mor, *Phys. Rev. A* **51**, 1863 (1995).
 - [13] J. F. Clauser, *Phys. Rev. D* **9**, 853 (1974); P. Grangier, G. Roger, and A. Aspect, *Europhys. Lett.* **1**, 173 (1986).
 - [14] It is sometimes claimed that the quantization of matter is all one needs to explain detector clicks and that a semiclassical theory (quantized matter interacting with classical Maxwell fields) is capable of giving a full account. However, experiments (like those in [13]) have proved that such a model incorrectly predicts physical results in some situations. Rather than adopting a semiclassical approach (which we know eventually fails) for some experiments and the quantum approach (which *always*

- works) for others, we feel compelled to use the latter picture throughout. Moreover, a semiclassical description is unavoidably inconsistent for theoretical reasons. Either the charged quantized matter would have to act as a source for the classical electromagnetic field or, if this is avoided by construction, action would not be properly paired with reaction.
- [15] This distinguishing WW knowledge in the *spatial* wave functions could possibly be extracted using a suitable measurement that included spatial-mode information.
- [16] For linear polarization states, the optimal Knowledge-measurement axes lie exactly between the axes which would equalize the amplitudes from the two paths (and for which the Visibility is maximum), i.e., if the light coming from paths 1 and 2 is polarized at ϕ_1 and ϕ_2 , then the optimal Knowledge basis is at $(\phi_1 + \phi_2)/2 \pm 45^\circ$.
- [17] *Individual* photons thus manipulated are unpolarized; consult J. Lehner, U. Leonhard, and H. Paul, Phys. Rev. A **53**, 2727 (1996), for unpolarized *multiphoton* states.
- [18] M. O. Scully and K. Drühl, Phys. Rev. A **25**, 2208 (1982).
- [19] In another series of measurements [21], the internal HWP was replaced by quartz rotators, relying only on optical activity, whose net effect was to rotate the relative polarizations in the two paths by 90° . When a linear polarization at θ was input, there was basically never any interference without quantum erasure. Complete visibility could be restored using a linear analysis at θ , or a circular-polarization analysis. For a completely mixed state input, however, *only* the circular analysis (i.e., along the eigenmodes of the quartz) recovered complete visibility.
- [20] B.-G. Englert, Z. Naturforsch. **54a**, 11 (1999).
- [21] P. G. Kwiat, P. D. D. Schwindt, and B.-G. Englert, in *Mysteries, Puzzles, and Paradoxes in Quantum Mechanics*, edited by R. Bonifacio (CP461, American Institute of Physics, Woodbury, 1999), p. 69.
- [22] G. N. Ramachandran and S. Ramaseshan, in *Handbuch der Physik*, Vol. 25, Part I (Springer, Berlin, 1961).
- [23] A. G. Zajonc *et al.* Nature (London) **353**, 507 (1991); P. G. Kwiat, A. M. Steinberg, and R. Y. Chiao, Phys. Rev. A **45**, 7729 (1992); T. Herzog, *et al.* Phys. Rev. Lett. **75**, 3034 (1995).
- [24] U. Eichmann *et al.* Phys. Rev. A **57**, 2359 (1993).
- [25] E. Buks *et al.* Nature (London) **391**, 871 (1998).
- [26] G. Badurek, H. Rauch, and J. Summhammer, Physica B&C **151**, 82 (1988); G. Badurek *et al.* Nucl. Instrum. Methods A (to be published).
- [27] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992); K. Mattle, H. Weinfurter, P. G. Kwiat, and A. Zeilinger, Phys. Rev. Lett. **76**, 4656 (1996).
- [28] C. H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993); D. Bouwmeester *et al.*, Nature (London) **390**, 575 (1997); D. Boschi *et al.*, Phys. Rev. Lett. **80**, 1121 (1998); A. Furusawa *et al.*, Science **282**, 706 (1998).
- [29] For example, such single-particle entanglement recently allowed the realization of a quantum search algorithm in an optical system with only passive linear elements [P. G. Kwiat, J. R. Mitchell, P. D. D. Schwindt, and A. G. White, J. Mod. Opt. (to be published)].
- [30] J. Bell, Physics **1**, 195 (1964).
- [31] S. Dürr and G. Rempe, Opt. Commun. (submitted).
- [32] B.-G. Englert and J. A. Bergou, Opt. Commun. (submitted).