Confronting Electroweak Fine-tuning with No-Scale Supergravity

Tristan Leggett,[1] Tianjun Li,[2,3] James A. Maxin,[4,5] Dimitri V. Nanopoulos,[1,6,7] and Joel W. Walker[8]

[1]*George P. and Cynthia W. Mitchell Institute for Fundamental Physics and Astronomy,*
Texas A&M University, College Station, TX 77843, USA
[2]*State Key Laboratory of Theoretical Physics and Kavli Institute for Theoretical Physics China (KITPC),*
Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China
[3]*School of Physical Electronics, University of Electronic Science and Technology of China, Chengdu 610054, P. R. China*
[4]*Department of Physics and Engineering Physics,*
The University of Tulsa, Tulsa, OK 74104 USA
[5]*Department of Physics and Astronomy, Ball State University, Muncie, IN 47306 USA*
[6]*Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, TX 77381, USA*
[7]*Academy of Athens, Division of Natural Sciences,*
28 Panepistimiou Avenue, Athens 10679, Greece
[8]*Department of Physics, Sam Houston State University, Huntsville, TX 77341, USA*

Applying No-Scale Supergravity boundary conditions at a heavy unification scale to the Flipped $SU(5)$ grand unified theory with extra TeV-scale vector-like multiplets, *i.e.* No-Scale $\mathcal{F}$-$SU(5)$, we express the Z-boson mass M_Z as an explicit function of the boundary gaugino mass $M_{1/2}$, $M_Z^2 = M_Z^2(M_{1/2}^2)$, with implicit dependence upon a dimensionless ratio c of the supersymmetric Higgs mixing parameter μ and $M_{1/2}$. Setting the top Yukawa coupling consistent with $m_t = 174.3$ GeV at $M_Z = 91.2$ GeV, the value of c naturally tends toward $c \simeq 1$, which indirectly suggests underlying action of the Giudice-Masiero mechanism. Proportional dependence of all model scales upon the unified gaugino mass $M_{1/2}$ in the No-Scale $\mathcal{F}$-$SU(5)$ model suggests one possible mechanism of confronting the electroweak fine tuning problem.

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INTRODUCTION

Supersymmetry (SUSY) represents a solution to the *big* gauge hierarchy problem, logarithmically sequestering the reference to ultra-heavy (Grand Unification, Planck, String) scales of new physics. However, there is a residual *little* hierarchy problem, implicit in the gap separating TeV-scale collider bounds on (strong production of) yet elusive colored superpartner fields from the observation of a 125 GeV light CP-even Higgs; indeed, this same heaviness of the SUSY scale appears equally requisite to sufficient elevation of loop contributions to the physical Higgs mass itself. One possible mechanism for reconciliation of these considerations without an unnatural invocation of fine tuning, *vis-à-vis* unmotivated cancellation of more than (say) a few parts *per centum* between contributions to physics at the electroweak (EW) scale, could be the providence of a unified framework wherein the entire physical spectrum (Standard Model + SUSY) may be expressed as functions of a single parameter.

The SUSY framework naturally provides for interplay between quartic and quadratic field strength terms in the scalar potential of the type essential to spontaneous destabilization of the null vacuum, the former emerging with dimensionless gauge-squared coupling coefficients from the D-term, and the latter with dimensionful mass-squared coefficients referencing the bilinear Higgs mixing scale μ from the chiral F-term. Crucially though, this radiative electroweak symmetry breaking (EWSB) event, as driven by largeness of the top-quark Yukawa coupling, is not realizable without the supplementary inclusion of soft mass terms $m_{H_{u,d}}$ and the analog B_μ of μ, which herald first the breaking of SUSY itself. In a supergravity (SUGRA) context, these terms may be expected to appear in proportion to the order parameter of SUSY breaking in the visible sector, as gravitationally suppressed from higher scale effects in an appropriately configured hidden sector, namely the gravitino mass $M_{3/2}$. The gravitino mass may itself be exponentially radiatively suppressed relative to the high scale, plausibly and naturally taking a value in the TeV range. The Giudice-Masiero (GM) mechanism may be invoked to address the parallel "μ problem", suggesting that this SUSY-preserving coupling may likewise be of the same order, and likewise generated as a consequence of SUSY breaking, as evaluated at the high scale.

Minimization of the Higgs scalar potential with respect to the H_u and H_d field directions yields two conditions on the pair of resulting vacuum expectation values (VEVs) (v_u, v_d). The overall scale $(v_u^2 + v_d^2)^{1/2}$, in product with the gauge coefficients $(g_L^2 + g_Y'^2)^{1/2}/2$, is usually traded for the physical Z-boson mass M_Z, whereas the relative VEV strengths are parameterized ($\tan\beta \equiv v_u/v_d$) by an angle β. This allows one to solve for μ and B_μ at the electroweak scale in terms of M_Z, $\tan\beta$, and the soft masses $m_{H_{u,d}}$. When addressing the question of fine tuning, the solution for μ^2 is typically inverted as follows in Eq. (1), and an argument is made regarding the permissible fraction of cancellation between terms on the right-hand side,

whose individual scales may substantially exceed M_Z^2.

$$\frac{M_Z^2}{2} = \frac{m_{H_d}^2 - \tan^2\beta \; m_{H_u}^2}{\tan^2\beta - 1} - \mu^2 \qquad (1)$$

For moderately large $\tan\beta$, Eq. (1) reduces to

$$\frac{M_Z^2}{2} \simeq -m_{H_u}^2 - \mu^2 \; . \qquad (2)$$

At the outset, it must be noted that some level of cancellation should here be permitted, and indeed expected, within Eq. (2) as an unavoidably natural consequence of the EWSB event itself. Specifically, destabilization of the symmetric vacuum occurs in conjunction with (the falsely tachyonic) negation of the quadratic H_u^2 field coefficient, provided by $m_{H_u}^2 + \mu^2 \simeq -M_Z^2/2$, which is dynamically driven to vanish and then flow negative under the renormalization group. However, the proverbial devil may lurk in the details, an element of which are loop-level radiative corrections $V_{\text{eff}} \Rightarrow V_{\text{tree}} + V_{1-\text{loop}}$ to the effective scalar potential, and likewise via derivatives of $V_{1-\text{loop}}$ to the minimization condition expressed in Eq. (1), which is recast with $m_{H_u}^2 \Rightarrow m_{H_u}^2 + \Sigma_u^u$, and $m_{H_d}^2 \Rightarrow m_{H_d}^2 + \Sigma_d^d$.

VARIOUS MEASURES OF FINE-TUNING

Several approaches to quantifying the amount of fine tuning implicit in Eq. (1) have been suggested, one of the oldest being that Δ_{EENZ} [1, 2] first prescribed some 30 years ago by Ellis, Enqvist, Nanopoulos, and Zwirner (EENZ), consisting of the maximal logarithmic M_Z derivative with respect to all fundamental parameters φ_i, evaluating at some high unification scale Λ as is fitting for gravity-mediated SUSY breaking. In this treatment, low fine-tuning mandates that heavy mass scales only weakly influence M_Z, whereas strongly correlated scales should be light.

$$\Delta_{\text{EENZ}} = \text{Max} \left\{ \left| \frac{\partial \ln(M_Z^n)}{\partial \ln(\varphi_i^n)} \right| \equiv \left| \frac{\varphi_i}{M_Z} \frac{\partial M_Z}{\partial \varphi_i} \right| \right\}_\Lambda \qquad (3)$$

Lately, a prescription Δ_{EW} emphasizing evaluation directly at the electroweak scale [3, 4] has attracted attention, isolating contributions to the (loop-modified) right-hand side of Eq. (1) as a ratio with the left.

$$\Delta_{\text{EW}} = \text{Max} \left\{ \left| \frac{m_{H_d}^2}{\tan^2\beta - 1} \right|, \left| \frac{\Sigma_d^d}{\tan^2\beta - 1} \right|, \right.$$
$$\left. \left| \frac{m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} \right|, \left| \frac{\Sigma_u^u \tan^2\beta}{\tan^2\beta - 1} \right|, \mu^2 \right\}_{\text{EW}} \div \frac{M_Z^2}{2} \qquad (4)$$

More strictly [3], individual contributions to $\Sigma_{u,d}^{u,d}$ may be compared to $M_Z^2/2$ in isolation, without allowance for cancellation even within the internal summation.

A variation [4, 5] of the prior, dubbed Δ_{HS}, would further split the electroweak evaluation of the soft masses and μ-term into the sum of their corresponding values at the high scale (HS) Λ plus a logarithmic correction from the renormalization group running, specifically $m_{H_{u,d}}^2 \Rightarrow m_{H_{u,d}}^2(\Lambda) + \delta m_{H_{u,d}}^2$, and $\mu^2 \Rightarrow \mu^2(\Lambda) + \delta\mu^2$.

$$\Delta_{\text{HS}} = \text{Max} \left\{ \left| \frac{m_{H_d}^2}{\tan^2\beta - 1} \right|, \left| \frac{\delta m_{H_d}^2}{\tan^2\beta - 1} \right|, \right.$$
$$\left. \left| \frac{m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} \right|, \left| \frac{\delta m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} \right|, \mu^2, \delta\mu^2 \right\}_\Lambda \div \frac{M_Z^2}{2} \qquad (5)$$

The scale Λ referenced in Eq. (5) is not necessarily of GUT or Planck order, but may rather contextually refer to a SUSY breaking messenger as low as some tens of TeV [6], although this somewhat blurs distinction from the $\Sigma_{u,d}^{u,d}$ terms comprising Eq. 4. In any event, leading top-stop loops generate corrections $\delta m_{H_u}^2 \sim m_{\tilde{t}_1}^2 \times \ln(m_{\tilde{t}_1}/\Lambda)$ that have been used [3, 6] to infer naturalness bounds on the light stop mass (thereby also limiting vital parallel contributions to the Higgs mass), and similarly on the gluino mass. It has been suggested [3, 6, 7] that a large negative trilinear soft term A_t, which may be generated radiatively at the intermediate scale, may engender cancellations in the light stop $\tilde{t}_1$ tuning contribution while simultaneously lifting m_h.

IMPACT OF DYNAMICS ON TUNING

In the SUGRA context, M_Z^2 is generically bound to dimensionful inputs φ_i at the high scale Λ via a bilinear functional, as shown following. The parameters φ_i may include scalar and gaugino soft SUSY breaking masses (whether universal or not), the bi- and tri-linear soft terms B_μ and A_i, as well as the μ-term. The coefficients C_i and C_{ij} are calculable, in principle, under the renormalization group dynamics.

$$M_Z^2 = \sum_i C_i \varphi_i^2(\Lambda) + \sum_{ij} C_{ij} \varphi_i(\Lambda) \varphi_j(\Lambda) \qquad (6)$$

Applying the Eq. (3) prescription, a typical contribution to the fine tuning takes the subsequent form.

$$\frac{\partial \ln(M_Z^2)}{\partial \ln(\varphi_i)} = \frac{\varphi_i}{M_Z^2} \times \left\{ 2\, C_i \varphi_i + \sum_j C_{ij} \varphi_j \right\} \qquad (7)$$

Comparing with Eq. (6), each individual term in the Eq. (7) sum is observed, modulo a possible factor of 2, to be simply the ratio of one contribution to the unified M_Z^2 mass, divided by $M_Z^2/2$. The structural similarity of the Δ_{EENZ} and Δ_{HS} prescriptions is therefore clear.

Differing conclusions drawn with respect to the supposed naturalness of a given SUSY model construction by the various fine tuning measures described are implicit within underlying assumptions that each makes regarding what may constitute a natural cancellation. For

example, Eq. (3) permits two types of dynamic cancellation that are not recognized by Eq. (5). The first of these is between multiple Eq. (6) terms in which a single parameter φ_i may appear. The second is between each high-scale parameter and its running correction, for example between $\mu^2(\Lambda)$ and $\delta\mu^2$, as represented by, and absorbed into, the numerical coefficients C_i and C_{ij}. Δ_{HS} is therefore a harsher metric of tuning than Δ_{EENZ}. It is sometimes argued [4, 8] that Δ_{EW}, which adopts a perfectly *a priori* view of only the low energy particle spectrum, sets an absolute lower bound on fine tuning via holistic recombination of high scale mass parameters and their potentially large running logarithms. Proponents of this opinion acknowledge [4] that the suggested bound is only as strong as an assumption that the SUSY breaking soft masses $m^2_{H_{u,d}}$ and the SUSY preserving μ-term harken from wholly disparate origins, such that cancellations amongst the two classes are *inherently* unnatural.

The positions of the authors in the current work are that (i) precisely the prior mode of cancellation is *made* natural by its essential role in promoting the electroweak symmetry breaking event, and that (ii) solid theoretical motivation exists (GM mechanism) for suspecting the SUSY preserving μ scale to have yet likewise been forged in the supersymmetry breaking event. If the former precept is granted, then the limit offered by Δ_{EW} is potentially spurious. If the latter precept is additionally granted, then the very notion of electroweak fine-tuning may be moot.

TUNING IN NO-SCALE $\mathcal{F}$-$SU(5)$

Bottom up support for the prior opinions is provided here by consideration of a specific model of low energy physics named $\mathcal{F}$-$SU(5)$ (see Ref. [9] and references therein), which combines (i) field content of the Flipped $SU(5)$ grand unified theory (GUT), with (ii) a pair of hypothetical TeV-scale vector-like supermultiplets ("flippons") of mass M_V derivable within local F-Theory model building, and (iii) the boundary conditions of No-Scale Supergravity (SUGRA). The latter amount to vanishing of the scalar soft masses and the tri-/bi-linear soft couplings $M_0 = A_0 = B_\mu = 0$ at the ultimate $\mathcal{F}$-$SU(5)$ gauge unification scale $M_{\mathcal{F}} \simeq M_{\mathrm{Pl}}$, and are enforced dynamically by invocation of a minimal Kähler potential [10, 11]. Non-zero boundary values may be applied solely to the universal gaugino mass $M_{1/2}$, as necessarily implied by SUSY breaking, and to the μ-parameter. In this perspective, the value of μ is actually that evolved *up* from the scale dynamically established in EWSB; its retrospective similarity to (and proportional scaling with) $M_{1/2}$ is interpreted as a *deeply suggestive* accident. The model is highly constrained by the need to likewise dynamically tether (via the Renormalization Group Equations (RGEs)) the value of B_μ generated in EWSB to

its mandated vanishing at $M_{\mathcal{F}}$; in fact, this releases the constraint typically exhausted by the fixing of B_μ to instead determine $\tan\beta \simeq 20$, which incidentally supports the approximation adopted by Eq. (2), while not being so large that impact of the bottom quark Yukawa coupling is substantially heightened.

The consequence (at fixed Z-boson M_Z and top-quark m_t masses) is an effectively *one-parameter* model, with all leading dynamics established by just the single degree of freedom allocated to $M_{1/2}$. Inclusion of the flippon multiplets provides a vital modification to the β-function RGE coefficients, most notably resulting in nullification of the color-charge running ($b_3 = 0$) at the first loop; however, dynamic dependence on the mass scale M_V is quite weak, affecting gauge unification only via logarithmic feedback from a threshold correction term. Some vestigial freedom remains for the preservation of $B_\mu(M_{\mathcal{F}}) = 0$ by compensating adjustments to $\tan\beta$ and M_V at fixed $M_{1/2}$, at the price of disrupting the natural tendency of this model to supply a suitable thermal dark matter (over 99% Bino) candidate; to be precise, there are associated fluctuations induced in the lightest neutralino (Bino) vs. the next to the LSP (stau) mass gap that synchronously affect the dark matter coannihilation rate.

Numerical analysis of the parameter interdependencies in No-Scale $\mathcal{F}$-$SU(5)$ is conducted with `SuSpect 2.34` [12], utilizing a proprietary codebase modification that incorporates the flippon-enhanced RGEs. Applying the Δ_{EW} measure of Eq. (4) to the $\mathcal{F}$-$SU(5)$ model is thereby found to indicate a level of tuning that may indeed be considered large. Down-type contributions to Eq. (4) are $\tan\beta$-suppressed, and Σ_u^u is computed to be rather small, possibly reflecting the presence of large, negative trilinear couplings. Dominant contributions to $M_Z^2/2$ are thus restricted to solely the pair of terms $m^2_{H_u}$ and μ^2 appearing directly in Eq. (2). Each term, or its absolute square root evaluated at the EWSB scale, is larger than and roughly proportional to the boundary value of $M_{1/2}$, with a ratio around 1.8 for $M_{1/2} \sim 400$ GeV that drops to about 1.3 for $M_{1/2} \sim 1500$ GeV. The corresponding contributions to Δ_{EW} therefore increase from about 140 to about 1000 over the same range of inputs. One interpretation of this circumstance, taking $|m_{H_u}|$ and $|\mu|$ to arise from disparate mechanisms, is that narrow tracking and cancellation of the two terms indicates fine tuning. However, we make the case here for a very different point of view: that it is *precisely* the close tracking of $|m_{H_u}|$ and $|\mu|$, as *dynamically* induced by electroweak symmetry breaking, and *preserved* under projection onto the boundary scale $M_{\mathcal{F}}$ by the renormalization group, in the *context* of a single parameter construction, that suggests a *natural* underlying interdependence.

It is not the purpose of this work to present a functioning hidden sector that is capable of producing at some

high scale the requisite SUSY breaking, along with the associated soft term and μ-parameter boundary values. It is the purpose of this work to investigate, primarily by numerical means [1], the dependency of all contextual low energy physics upon the single input scale $M_{1/2}$ (and by extension its dependence in turn upon $M_{3/2}$). Insofar as this may be demonstrated, the considered model can successfully attack the fine-tuning problem, *i.e.* no large cancellations enforced coincidentally, without a common dynamic origin. Insofar as the relation $\mu(M_{\mathcal{F}}) \simeq M_{1/2}$ may be validated, the providence of an underlying GM mechanism is supported through identification of the "fingerprints" which it has impressed upon the low energy phenomenology.

The most remarkable element of this construction may be the capacity to so severely constrain freedom of input parameterization (with all concomitant benefits to the interrelated questions of the little hierarchy and the μ-problem) while retaining consistency (even at the purely thermal level) with dark matter observations, limits on rare processes, and collider bounds.

GAUGINO PARAMETERIZATION OF $\mathcal{F}$-$SU(5)$

The EW fine-tuning was numerically computed for No-Scale $\mathcal{F}$-$SU(5)$ according to the Eq. (3) prescription in Ref. [13], yielding result of $\mathcal{O}(1)$. This absence of fine-tuning is equivalent to a statement that the Z-boson mass M_Z can be predicted in $\mathcal{F}$-$SU(5)$ as a parameterized function of $M_{1/2}$; clarifying and rationalizing this intuition in a more quantitative manner is a key intention of the present section. First, we define a dimensionless ratio c of the supersymmetric Higgs mixing parameter μ at the unification scale $M_{\mathcal{F}}$ with the gaugino mass $M_{1/2}$.

$$c = \frac{\mu(M_{\mathcal{F}})}{M_{1/2}} \tag{8}$$

This parameter c is a fixed constant if the μ term is generated via the Giudice-Masiero mechanism [14], which can, in principle, be computed from string theory. Its invocation implicitly addresses the need to otherwise explicitly refer to the μ parameter as an independent high scale input. We shall numerically scan over arbitrary values of this parameter, although the No-Scale $\mathcal{F}$-$SU(5)$ construction will be demonstrated to prefer a narrow range near $c \simeq 1$. The vector-like flippon mass parameter M_V is expected to develop an exponential compensation (for

$B_\mu = 0$ and all else constant)

$$M_V \sim A(\lambda) e^{M_{1/2}/R(\lambda)} \tag{9}$$

of the fundamental scale $M_{1/2}$ due to its previously described appearance within a logarithmic threshold correction, where the undetermined dimensionful parameters $A(\lambda)$ and $R(\lambda)$ may be sensitive (among other things) to the top quark Yukawa coupling λ. In the same vein, sensitivity to $\tan\beta$ is weak, and it is expected that any residual dependencies upon either parameter within the region of interest may be Taylor-expanded for absorption into a generic quadratic function of $M_{1/2}$.

We thus adopt an *ansätz*

$$M_Z^2 = f_1 + f_2 M_{1/2} + f_3 M_{1/2}^2 \tag{10}$$

consistent with Eq. (6), where the undetermined coefficients f_i represent implicit functions of dimensionless quantities including c and λ. Some evidence suggests that the dimensionful coefficients f_1 and f_2 may additionally be sensitive to B_μ, particularly to any potential deviations from the null No-Scale boundary value. If $(f_1 \lll M_{1/2}^2)$ and $(f_2 \ll M_{1/2})$, then a linearized approximation of the prior is applicable:

$$M_Z = f_a + f_b M_{1/2} \quad . \tag{11}$$

The form of Eq. (10) must now be verified with explicit RGE calculations. This is accomplished via a numerical sampling, wherein the Z-boson mass is floated within $20 \leq M_Z \leq 500$ GeV, and the top quark mass (equivalently its Yukawa coupling) within $125 \leq m_t \leq 225$ GeV. The region scanned for the gaugino mass boundary is within $100 \leq M_{1/2} \leq 1500$ GeV. In order to truncate the scanning dimension, M_V and $\tan\beta$ are explicitly parameterized functions of $M_{1/2}$ (consistent with the prior description) such that the physical region of the model space corresponding to $M_Z = 91.2$ GeV and $m_t = 174.3$, along with a valid thermal relic density, is continuously intersected[2]; this may be considered equivalent to fixing the top quark Yukawa coupling (and associated higher-order feedback) within just this subordinate parameterization. The range of the ratio c from Eq. (8) is an output of this analysis, which is run from the EWSB scale up to $M_{\mathcal{F}}$ under the RGEs.

As an initial phase of the analysis, the only constraints applied are the No-Scale SUGRA boundary conditions $M_0 = A_0 = 0$, along with correct EWSB, a convergent μ term, and no tachyonic sfermion or pseudoscalar

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[1] A parallel analytical treatment, making directly formal, albeit approximate, application of the No-Scale $\mathcal{F}$-$SU(5)$ renormalization group and boundary conditions is underway, with results intended to follow in a separate publication.

[2] The No-Scale $\mathcal{F}$-$SU(5)$ model space favors a top quark mass of $m_t = 174.3 - 174.4$ GeV in order to compute a Higgs boson mass of $m_h \sim 125$ GeV [15–17]. The central world average top quark mass has recently ticked upward (along with an increase in precision) to $m_t = 174.34$ GeV [18], affirming this preference.

Higgs boson masses, these latter conditions being the required minimum for proper RGE evolution. These constructionist elements carve out a narrow viable parameter space between $83 \lesssim M_Z \lesssim 93$ GeV, which is illustrated as a function of the dimensionless parameter c in FIG. 1. The blue region represents those points that cannot satisfy the minimum evolution requirements. The band of points clustered on the M_Z axis with $c = 0$ have no RGE solution for μ at all. The red sliver within $83 \lesssim M_Z \lesssim 93$ GeV, narrowly bounding the physical region around $M_Z \sim 91$ GeV, represents the sole surviving region of model space satisfying the boundary condition $M_0 = A_0 = 0$ with a convergent RGE solution. The gauge couplings being held fixed, the fluctuation of M_Z is attributable to fluctuation of the Higgs VEV magnitude, as compensated by differential contributions to the right-hand side of Eq. (1) induced by variation of μ, $M_{1/2}$, and the top quark Yukawa λ.
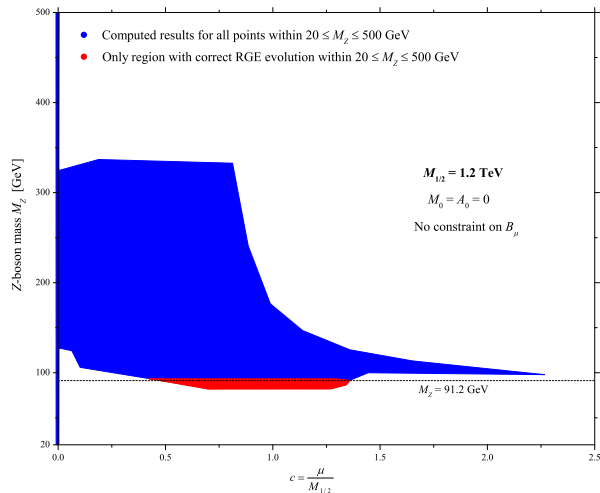

FIG. 1: Representation of the region of the model space for $M_{1/2} = 1.2$ TeV that cannot sustain correct RGE evolution (blue) versus the region that can compute a SUSY spectrum with no RGE errors, EWSB, convergent μ term, and no tachyonic masses (red). All other $M_{1/2}$ produce identical results.

As a second phase of the analysis, the final No-Scale SUGRA constraint $B_\mu = 0$ must be applied at the $M_{\mathcal{F}}$ unification scale. The vanishing $B_\mu = 0$ requirement is enforced numerically with a width $|B_\mu| \leq 1$ GeV that is comparable to the scale of EW radiative corrections. The effect is to carve out a simply connected string of points from the narrow red region, depicted in FIG. 2 for seven values of $M_{1/2}$ in the model space.

The No-Scale SUGRA constraint on the B_μ parameter naturally parameterizes all the particle and sparticle masses as a function of the dimensionless parameter c of Eq. (8). This is clearly shown in FIG. 3 for the Z-boson mass M_Z, top quark mass m_t, Higgs boson mass m_h, and gluino mass $m_{\widetilde{g}}$. We use the gluino mass as an example, though the entire SUSY spectrum can also
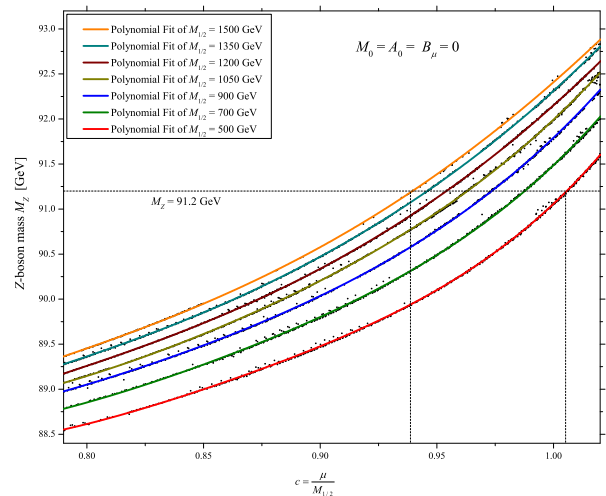

FIG. 2: The Z-boson mass is shown as a function of the dimensionless parameter c for seven different values of $M_{1/2}$. The black points are the results of the RGE calculations, while the curves are polynomial fits. The curves are only comprised of points with a vanishing B_μ parameter at the $M_{\mathcal{F}}$ unification scale.

thusly be parameterized as a function of c via the $B_\mu = 0$ condition. The point chosen in FIG. 3 to exhibit the correlation between the particle and sparticle masses is $M_{1/2} = 1200$ GeV. TABLE I itemizes certain numerical results from the RGE calculations for four of the FIG. 2 curves. The Higgs boson mass m_h in TABLE I includes both the tree level+1-loop+2-loop+3-loop+4-loop contributions [9] and the additional flippon contribution [19]. Sensitivity is observed to fluctuation of the VEV scale with M_Z.

The dimensionless parameter c is expected to be a fixed constant if the μ term is generated by the GM mechanism. On any single-valued slice of the present parameterization with respect to c, particle and sparticle masses will residually be dependent upon just $M_{1/2}$, as is visible for variation of the Z-boson mass in FIG. 2. This is made explicit for $c = 0.80, 0.85, 0.90, 0.95, 1.00$ in FIG. 4, where each curve is well fit by a quadratic in the form of Eq. (10). FIG. 5 demonstrates a fit against the linear approximation in Eq. (11). As the c parameter decreases, FIG. 5 illustrates that the linear fit approaches the precision of the quadratic fit. The dimensionful intercept f_a is a function of c, but is observed generically to take a value in the vicinity of 89 GeV. As seen in FIG. 2, larger values of $M_{1/2}$ correlate with smaller values of c at fixed Z-boson mass; it is the region $M_{1/2} \gtrsim 900$ GeV that remains viable for probing a prospective SUSY signal at the 13–14 TeV LHC in 2015–16.

The relationship between the μ term and $M_{1/2}$ at the $M_{\mathcal{F}}$ unification scale is linear for fixed M_Z, with a slope given by the ratio c from Eq. (8). This is expanded in FIG. 6 for $M_Z = 91.2$. Parameterization of the flippon

TABLE I: Results of RGE calculations for four different values of $M_{1/2}$. These are only points with a vanishing B_μ parameter at the $M_\mathcal{F}$ unification scale. The entries highlighted in red are those that compute the observed experimental measurements for M_Z, m_t, and m_h.

$M_{1/2} = 500$ GeV				
c	M_Z	m_t	m_h	$m_{\widetilde{g}}$
0.955	90.189	164.47	118.88	655
0.965	90.349	166.29	120.03	659
0.975	90.543	168.34	121.30	664
0.985	90.720	170.11	122.48	669
1.005	91.181	174.31	125.28	681
1.015	91.444	176.49	126.84	687
1.025	91.741	178.81	128.45	694
1.035	92.082	181.31	130.21	702
$M_{1/2} = 900$ GeV				
c	M_Z	m_t	m_h	$m_{\widetilde{g}}$
0.933	90.496	167.69	119.46	1182
0.943	90.659	169.38	120.70	1190
0.953	90.818	170.95	121.85	1197
0.963	91.011	172.76	123.20	1205
0.973	91.189	174.35	124.39	1212
0.983	91.390	176.06	125.71	1220
0.993	91.652	178.17	127.28	1230
1.003	91.849	179.67	128.53	1238
$M_{1/2} = 1200$ GeV				
c	M_Z	m_t	m_h	$m_{\widetilde{g}}$
0.913	90.525	167.96	120.31	1574
0.923	90.672	169.49	121.48	1583
0.933	90.832	171.06	122.66	1591
0.943	90.999	172.63	123.88	1601
0.953	91.180	174.24	125.09	1610
0.963	91.371	175.87	126.41	1619
0.970	91.502	176.94	127.26	1626
0.983	91.780	179.11	129.04	1640
$M_{1/2} = 1350$ GeV				
c	M_Z	m_t	m_h	$m_{\widetilde{g}}$
0.905	90.537	168.06	120.76	1773
0.915	90.691	169.66	121.92	1782
0.925	90.850	171.21	123.12	1792
0.935	91.017	172.77	124.35	1802
0.945	91.188	174.29	125.55	1811
0.955	91.367	175.81	126.77	1822
0.969	91.644	178.03	128.59	1838
0.977	91.806	179.28	129.62	1847

FIG. 3: Depiction of the correlation between the Z-boson mass M_Z, top quark mass m_t, Higgs boson mass m_h, and gluino mass $m_{\widetilde{g}}$, as a function of c, for $M_{1/2} = 1.2$ TeV. All other SUSY particles can be expressed similarly. The curves are only comprised of points with a vanishing B_μ parameter at the $M_\mathcal{F}$ unification scale. All other $M_{1/2}$ produce comparable correlations.
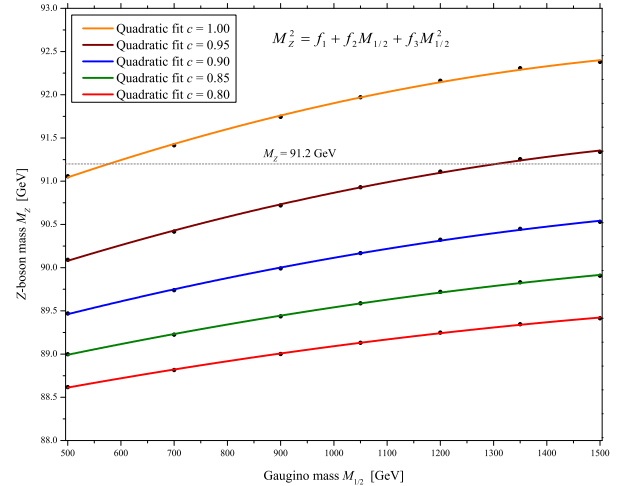
FIG. 4: Simple quadratic fits for M_Z^2 as a function of $M_{1/2}^2$. Five different cases of c are shown. The curves are only comprised of points with a vanishing B_μ parameter at the $M_\mathcal{F}$ unification scale. The black points are sampled from FIG. 2.

mass M_V and $\tan\beta$ as functions of $M_{1/2}$ (with the top quark Yukawa and approximate relic density fixed) are illustrated in FIG. 7.

Having established a (family in c of) quadratic expression(s) for M_Z^2 in the Eq. (6) form, the Z-boson mass is extracted by reference only to $M_{1/2}$ and c at the high scale Λ, and fine tuning may be evaluated. Adopting the linear Eq. (11) form, we first consider tuning with respect to $M_{1/2}$ at fixed c, as prescribed by Eq. (3).
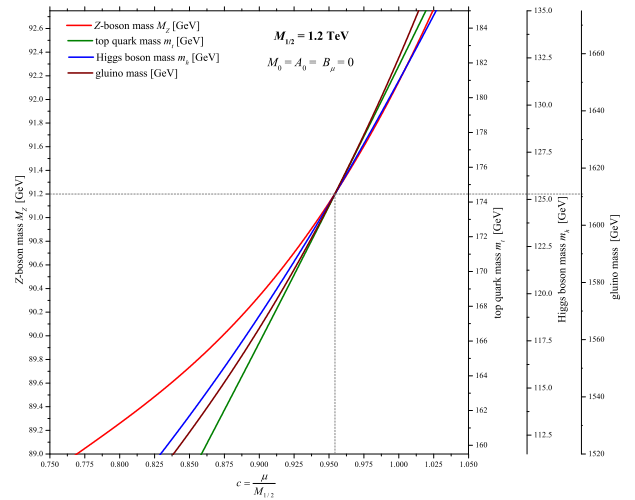
$$\Delta^{\Lambda}_{M_{1/2}} = \left| \frac{\partial \ln(M_Z)}{\partial \ln(M_{1/2})} \right| = \left| \frac{M_{1/2}}{M_Z} \frac{\partial M_Z}{\partial M_{1/2}} \right| \tag{12}$$

$$= \frac{1}{M_Z} \left(\frac{M_Z - f_a}{f_b} \right) f_b = 1 - \frac{f_a}{M_Z} \simeq 1 - \frac{89}{M_Z}$$

Curiously, this expression evaluates very close to zero. It would appear this result is a consequence of the fact that
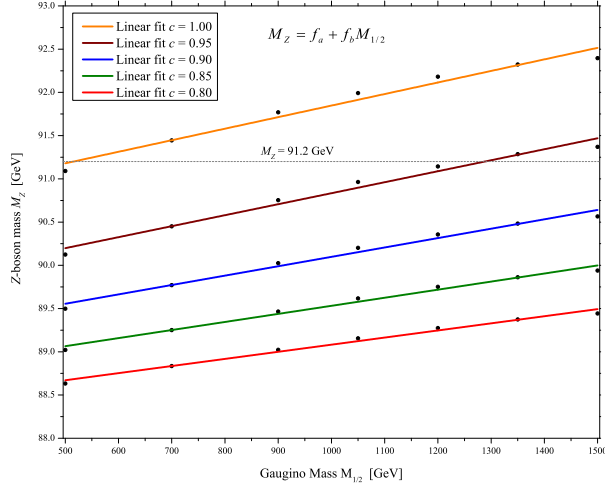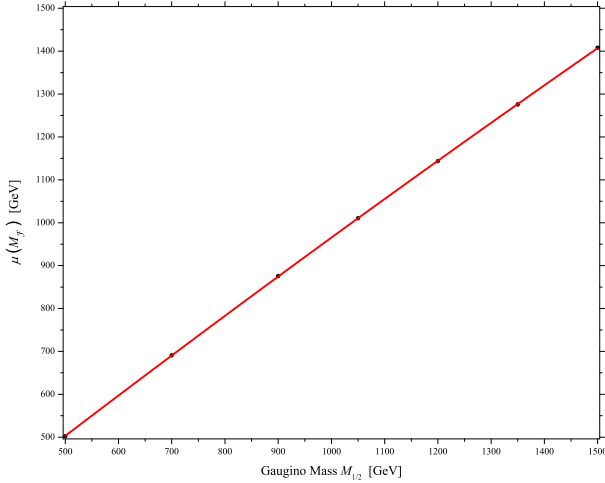
FIG. 5: Linear fits for M_Z as a function of $M_{1/2}$. Five different cases of c are shown. The curves are only comprised of points with a vanishing B_μ parameter at the $M_{\mathcal{F}}$ unification scale. The black points are sampled from FIG. 2.

FIG. 6: Linear relationship between the μ term at the $M_{\mathcal{F}}$ unification scale and $M_{1/2}$ for $M_Z = 91.2$ GeV.

the physical Z-boson mass can in fact be stably realized for a large continuum of $M_{1/2}$ values, at the expense of variation in the ratio c. It may be better understood by attending in turn to the parallel functional dependence on the dimensionless c parameter itself. We have

$$\Delta_c^\Lambda = \left| \frac{\partial \ln(M_Z)}{\partial \ln(c)} \right| = \left| \frac{c}{M_Z} \frac{\partial M_Z}{\partial c} \right| \qquad (13)$$
$$= \left| \frac{c}{M_Z} \left(\frac{\partial f_a}{\partial c} + \frac{\partial f_b}{\partial c} M_{1/2} \right) \right| \sim c \simeq 1 \,,$$

using the numerical observation $\frac{\partial f_a}{\partial c} + \frac{\partial f_b}{\partial c} M_{1/2} \sim M_Z$. Therefore, stipulating the adopted high-scale context, we suggest that the more natural fine-tuning measure for
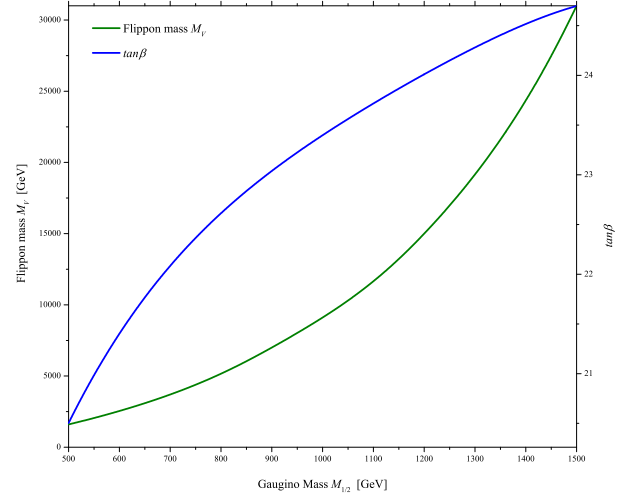

FIG. 7: Vector-like flippon mass M_V and $\tan\beta$ as functions of $M_{1/2}$ with Yukawa couplings fixed.

No-Scale $\mathcal{F}$-$SU(5)$ may be $\Delta_{\text{EENZ}} \sim 1$.

CONCLUSIONS

We have shown here that by implementing only the No-Scale Supergravity boundary conditions of $M_0 = A_0 = B_\mu = 0$ at the unification scale, the Z-boson mass M_Z can be expressed as a simple quadratic function of the unified gaugino mass $M_{1/2}$, i.e. $M_Z^2 = M_Z^2(M_{1/2}^2)$, in the supersymmetric GUT model No-Scale $\mathcal{F}$-$SU(5)$. A top-down string theoretic construction may be expected to fix the Yukawa couplings and a dimensionless boundary ratio c of the supersymmetric Higgs mixing parameter μ with $M_{1/2}$ at some heavy unification scale. The only degree of freedom left to influence M_Z is $M_{1/2}$. Setting the top Yukawa coupling consistent with $m_t = 174.3$ GeV at $M_Z = 91.2$ GeV, the value of c naturally tends toward $c \simeq 1$, which suggests underlying action of the Giudice-Masiero mechanism. The regions of the model space in correspondence with the physical masses $M_Z = 91.2$ GeV and $m_t = 174.3$ GeV are further consistent with the correct Higgs boson mass $m_h \simeq 125$ GeV and dark matter observations, and possess overlap with the limits on rare processes and collider bounds. Proportional dependence of all model scales upon the unified gaugino mass $M_{1/2}$ in the No-Scale $\mathcal{F}$-$SU(5)$ model could suggest one potential mechanism of confronting the electroweak fine tuning problem.

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