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Cosmic Acceleration and the String Coupling

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Abstract

In the context of a cosmological string model describing the propagation of strings in a time-dependent Robertson-Walker background space-time, we show that the asymptotic acceleration of the Universe can be identified with the square of the string coupling. This allows for a direct measurement of the ten-dimensional string coupling using cosmological data. We conjecture that this is a generic feature of a class of non-critical string models that approach asymptotically a conformal (critical) σ model whose target space is a four-dimensional space-time with a dilaton background that is linear in σ-model time. The relation between the cosmic acceleration and the string coupling does not apply in critical strings with constant dilaton fields in four dimensions.

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String theory [1, 2] was first developed as a theory of the strong interactions, but it soon turned out that mathematical consistency (world-sheet conformal invariance) required the theory to live in higher-dimensional space times. Even target-space supersymmetry was not successful in reducing the number of space-time dimensions below ten. Thus, enormous effort has been expended on the compactification of the extra dimensions, with the eventual aim of accommodating the Standard Model at low energies. Many ways were found to construct low-energy models that could be consistent with the current particle physics phenomenology, but string models of this type had zero predictability, in the sense that they were unable to make predictions for the parameters of the Standard Model, and there were many string models with indistinguishable low-energy limits.

Although in principle string theory has no free parameters, and the ground state corresponding to the observable low-energy world is supposed to be chosen dynamically, a detailed understanding of mechanism for choosing the ground state has not been achieved so far. Lacking a microscopic, dynamical mechanism for specifying the various string model parameters, such as the compactification radii and the four-dimensional gauge couplings, one has had simply to fix them by hand, so as to match the results with experimental observations in particle physics. In this framework, the mechanism whereby one particular model is chosen from among the complicated string 'landscape' [3] is still unclear.

The most important and fundamental string parameter is the string coupling, g_s, which determines the regime of validity of string perturbation theory, and hence the world-sheet σ-model scheme for low-energy computations of the low-energy string effective action. Since g_s is connected to the unified ten-dimensional gauge coupling of the effective supersymmetric low-energy theory, its value is usually inferred from particle phenomenology. The string coupling is not a constant but, like any other dynamical coupling in a supersymmetric field theory, is related to the vacuum expectation value of a field, in this particular case the dilaton field Φ, which belongs to the gravitational multiplet obtained from the string [1]

$$g_s^2 = e^{2\Phi}. \tag{1}$$

Usually, upon compactification the dilaton field is split into a product of two factors, one depending on the compact six-dimensional space coordinates and the other on the four-dimensional space-time coordinates, which are supposed to correspond to the large, uncompactified coordinates of our observable world. In most of the phenomenological approaches to model building, the four-dimensional dilaton field has been assumed to be constant and therefore trivial, since this constant value could be absorbed in an innocuous shift in the field.

In this approach, neither the string coupling nor the unified gauge coupling are accessible directly to experimental measurement. It is consistency of the available phenomenological model with low-energy observational data that leads to an indirect fixing of the string coupling. A popular value is $g_s^2 \simeq 0.52$, which, upon compactification to small dimensions (of the order of a tenth of the four-dimensional Planck mass, $M_P \sim 10^{19}$ GeV), yields a four-dimensional unified gauge coupling strength $g_U^2/4\pi \sim 1/24$ at scales $M_U \sim 10^{16}$ GeV, as suggested by extrapolating the measured gauge couplings to high energies in the context of the minimal supersymmetric extension of the Standard Model.

Modern developments in string theory [2] make possible consistent quantum treatments of domain-wall structures in string theory (D-branes). These have opened up novel ways of looking at both the microcosmos and the macrocosmos, offering new insights into both particle phenomenology and the cosmic evolution of our Universe. In the microcosmos, there are novel ways of compactification, either via the observation [4] that large (compared to the string scale) extra dimensions are consistent both with the foundations of string theory and phenomenology, or by viewing our four-dimensional world as a brane embedded in a bulk space-time. This would allow for large extra bulk dimensions, which could even be infinite in size [5], offering new ways to analyze the large hierarchy between the Planck scale and the electroweak symmetry-breaking scale. In this modern approach, fields in the gravitational (super)multiplet of the (super)string are allowed to propagate in the bulk, but not the gauge fields, which are attached to the brane world. In this way, the weakness of gravity as compared to the rest of the interactions is a result of the large extra dimensions. Their compactification is not necessarily achieved through conventional means, i.e., closing up the extra dimensions in compact spatial manifolds, but might also involve shadow brane worlds with special reflecting properties (such as orientifolds), which bound the bulk dimension [6]. In such approaches, the string scale M_s is not necessarily identical to the four-dimensional Planck mass scale M_P, but instead they are related through the large compactification volume V_6:

$$M_P^2 = \frac{8M_s^8 V_6}{g_s^2}. \tag{2}$$

As for the macrocosmos, there are novel ways of discussing cosmology in brane worlds, which may revolutionize our way of approaching issues such as inflation [8, 7].

Mounting experimental evidence from diverse astrophysical sources presents important cosmological puzzles that string theory must address if it is to provide a realistic description of

Nature. Observations of large-scale structures, distant Type-1asupernovae [9], and the cosmic microwave background fluctuations (by WMAP [10] in particular) have established that the Hubble expansion of our Universe is currently accelerating, and that 70% of its energy density consists of unknown dark energy that appears in 'empty' space and does not clump with ordinary matter.

These observations have great potential significance for string theory, and may even revolutionize the approach to it that has normally been followed so far. If the dark energy leads to an asymptotic de Sitter horizon, as would occur if it turns out to be a true cosmological constant, then the entire concept of the scattering S-matrix breaks down, and hence the conventional approach to string theory. On the other hand, if there is some quintessential mechanism for relaxing the vacuum energy, so that the vacuum energy density vanishes at large cosmic times in a manner consistent with the existence of an S-matrix, there is still the open issue of embedding such models in (perturbative) string theory. One would need, in particular, to develop a consistent σ-model formulation of strings propagating in such time-dependent, relaxing space-time backgrounds.

We here propose a resolution of this dilemma, based on string theory in a time-dependent dilaton background, in which the asymptotic acceleration of the Universe is directly related to the string coupling.

The world-sheet conformal-invariance conditions of critical string theory are equivalent to the target-space equations of motion for the background fields through which the string propagates. These conditions are very restrictive, allowing only for vacuum solutions of (critical) strings to be constructed in this way. The main problem may be expressed as follows. Consider the graviton world-sheet β function, which is nothing but the Ricci tensor of the target space-time background to lowest order in α':

$$\beta_{\mu\nu} = \alpha' R_{\mu\nu} \ , \tag{3}$$

where we ignore the possible presence of other fields, for simplicity. Conformal invariance requires the vanishing condition $\beta_{\mu\nu} = 0$, which implies that the background must be Ricci flat, which is a solution of the vacuum Einstein equations. The issue then arises how to describe in string theory cosmological backgrounds, which are not vacuum solutions, but require the presence of a matter fluid and hence a non-vanishing Ricci tensor. In this respect, we see that a cosmological constant is inconsistent with the conformal invariance of string since, for instance, a de Sitter Universe with a positive cosmological constant $\Lambda > 0$ has a non-zero Ricci tensor

$R_{\mu\nu} = \Lambda g_{\mu\nu}$, where $g_{\mu\nu}$ is the metric tensor.

An interesting proposal for obtaining a non-zero cosmological constant in string theory was made in [11]. It was suggested that dilaton tadpoles in higher-genus world-sheet surfaces, which produce additional modular infinities whose regularization leads to extra world-sheet structures in the σ-model not appearing at the world-sheet level, modify the string β-function in such a way that the Ricci tensor of the space-time background is now that of a de Sitter Universe, with a cosmological constant specified by the dilaton tadpole graph. The problem with this approach is the above-mentioned existence of an asymptotic horizon in the de Sitter case, which prevents the proper definition of asymptotic states, and hence an S-matrix. Since the perturbative world-sheet formalism is based on the existence of such an S-matrix, there is a question of consistency in this approach.

It was proposed in [12] that a way out of this difficulty would be to assume specific time-dependent dilaton backgrounds, with a linear dependence on time in the so-called σ-model frame:

$$\Phi = \text{const} - Q\ t \tag{4}$$

where Q is a constant, and $Q^2 > 0$ is a deficit in the σ-model central charge. Such backgrounds, even when the σ-model metric is flat, lead to exact solutions (in all orders in α') of the conformal-invariance conditions of the pertinent stringy σ-model, thereby constituting acceptable solutions from a perturbative string viewpoint. The appearance of Q allowed this supercritical string theory [12] to be formulated in spaces with numbers of dimensions different from the critical case. This was actually the first example of a non-critical string, with the target-space coordinates X^i, $i = 1, \ldots D-1$, playing the rôles of the pertinent σ-model fields. This non-critical string was not conformally invariant, and hence required Liouville dressing [13]. The Liouville field had time-like signature in target space, since the central charge deficit Q^2 was positive in the model of [12], and its zero mode played the rôle of target time.

As a result of the existence of a non-trivial dilaton field, the Einstein term in the effective D-dimensional low-energy field theory action is conformally rescaled by $e^{-2\Phi}$. This requires a specific redefinition of target time in order that the metric acquires the standard Robertson-Walker (RW) form in the normalized Einstein frame for the effective action:

$$ds_E^2 = -dt_E^2 + a_E(t_E)^2 \left(dr^2 + r^2 d\Omega^2\right), \tag{5}$$

where we have only exhibited a spatially-flat RW metric for definiteness, and $a_E(t_E)$ is an appropriate scale factor, which is simply a function of the Einstein-frame time t_E in the homogeneous cosmological backgrounds that we assume throughout.

The Einstein-frame time is related to the σ-model-frame time [12] by:

$$dt_E = e^{-\Phi} dt \qquad \to \qquad t_E = \int^t e^{-\Phi(t)} dt \ . \tag{6}$$

The linear dilaton background (4) yields then the following relation between the Einstein- and σ-model-frame times:

$$t_E = c_1 + \frac{c_0}{Q} e^{Qt}, \tag{7}$$

where $c_{1,0}$ are appropriate (positive) constants. Thus, a dilaton background that is linear in σ-model-frame time (4) will scale logarithmically with the Einstein-frame time t_E, which is just the Robertson-Walker cosmic time:

$$\Phi(t_E) = \text{const.}' - \ln(\frac{Q}{c_0} t_E). \tag{8}$$

In this regime, the string coupling (1) varies with the cosmic time t_E as:

$$g_s^2(t_E) \propto \frac{1}{t_E^2}, \tag{9}$$

implying that the effective string coupling vanishes asymptotically in cosmic time. In the linear-dilaton background of [12], the asymptotic space-time metric in the Einstein frame reads:

$$ds^2 = -dt_E^2 + a_0^2 t_E^2 \left(dr^2 + r^2 d\Omega^2 \right), \tag{10}$$

where a_0 a constant, which is a linearly-expanding Universe. Clearly, there is no acceleration in the Universe (10).

In [14] we went one step further than the analysis in [12], and considered more complicated σ-model metric backgrounds, which did not satisfy the σ-model conformal-invariance conditions, and therefore were in need of Liouville dressing in order to restore conformal invariance. Such backgrounds were also allowed to be time-dependent, and the target time was identified with the Liouville world-sheet zero mode, thereby not increasing the target space-time dimensionality. We have provided several justifications and checks of this identification [14], which is possible only when the initial σ-model is supercritical, so that the Liouville mode has time-like signature [12, 13]. For example, in certain models [15, 16], such an identification was energetically preferable from a target-space viewpoint, since it minimized certain effective potentials in the low-energy field theory corresponding to the string theory at hand.

Such non-critical σ models relax asymptotically in cosmic Liouville time to conformal σ models, the latter viewed as equilibrium points in string theory space. In some interesting cases of relevance to cosmology, which were particularly generic, the asymptotic conformal field theory was that of [12], with a linear dilaton and a flat Minkowski target-space metric in the σ-model frame.

One such model was considered in detail in [17]. The model was originally formulated within a specific string theory, namely ten-dimensional Type-0 [18], which leads to a non-supersymmetric target-space spectrum, as a result of a special projection of the supersymmetric partners out of the spectrum. However, the basic properties of its cosmology, which are those interest to us in in this work, are sufficiently generic that they can be extended to the bosonic sector of any other effective low-energy supersymmetric field theory of supersymmetric strings, including those relevant to unified particle physics phenomenology.

The ten-dimensional metric configuration considered in [17] was:

$$G_{MN} = \begin{pmatrix} g^{(4)}_{\mu\nu} & 0 & 0 \\ 0 & e^{2\sigma_1} & 0 \\ 0 & 0 & e^{2\sigma_2} I_{5\times 5} \end{pmatrix}, \tag{11}$$

where lower-case Greek indices are four-dimensional space-time indices, and $I_{5\times 5}$ denotes the 5×5 unit matrix. We have chosen two different scales for the internal space. The field σ_1 sets the scale of the fifth dimension, while σ_2 parametrizes a flat five-dimensional space. In the context of the cosmological models we treat here, the fields $g^{(4)}_{\mu\nu}$, σ_i, $i = 1, 2$ are assumed to depend on the time t only.

Type-0 string theory, as well as its supersymmetric versions appearing in other scenarios including brane models, contains appropriate form fields with non-trivial gauge fluxes (flux-form fields), which live in the higher-dimensional bulk space. In the specific model of [18], one such field was considered to be non-trivial. As was demonstrated in [17], a consistent background choice for the flux-form field has the flux parallel to to the fifth dimension σ_1. This implies that the internal space is crystallized (stabilized) in such a way that this dimension is much larger than the remaining five dimensions σ_2, demonstrating the physical importance of the flux fields for large radii of compactification.

Considering the fields to be time-dependent only, i.e., considering spherically-symmetric homogeneous backgrounds, restricting ourselves to the compactification (11), and assuming a Robertson-Walker form of the four-dimensional metric with scale factor $a(t)$, the generalized conformal-invariance conditions and the Curci-Pafutti σ-model renormalizability constraint [19] yield a set of differential equations which were solved numerically in [17]. The generic form of these equations reads [13, 14, 17]:

$$\ddot{g}^i + Q(t)\dot{g}^i = -\tilde{\beta}^i, \tag{12}$$

where the $\tilde{\beta}^i$ are the Weyl-anomaly coefficients of the stringy σ-model on the background $\{g^i\}$. In the model of [17], the $\{g^i\}$ include graviton, dilaton, tachyon, flux and moduli fields $\sigma_{1,2}$, whose vacuum expectation values control the sizes of the extra dimensions.

The detailed analysis of [17] indicated that the moduli fields σ_i froze quickly to their equilibrium values. Thus, together with the tachyon field which also decays to a constant value rapidly, they decouple from the four-dimensional fields at very early stages in the evolution of this string Universe [1]. There is an inflationary phase in this scenario and a dynamical exit from it. The important point to guarantee the exit is the fact that the central-charge deficit Q^2 is a time-dependent entity in this approach, obeying specific relaxation laws determined by the underlying conformal field theory [17, 15, 16]. In fact, the central charge runs with the local world-sheet renormalization group scale, namely the zero mode of the Liouville field, which is identified [14] with the target time in the σ-model frame. The supercriticality [12] $Q^2 > 0$ of the underlying σ model is crucial, as already mentioned. Physically, the non-critical string provides a framework for non-equilibrium dynamics, which may be the result of some catastrophic cosmic event, such as a collision of two brane worlds [7, 15, 16], or an initial quantum fluctuation.

In the generic class of non-critical string models considered in this work, the σ model always asymptotes, for long enough cosmic times, to the linear-dilaton conformal σ-model field theory of [12]. But it is important to stress that this is only an asymptotic limit. In this respect, the current era of our Universe is viewed as being close, but still not quite at the relaxation (equilibrium) point, in the sense that the dilaton is almost linear in the σ-model time, and hence varies logarithmically with the Einstein-frame time (8). It is expected that this slight non-equilibrium will lead to a time-dependence of the unified gauge coupling and other constants such as the four-dimensional Planck length (2) that characterize the low-energy effective field theory, mainly through the time-dependence of the string coupling (1) that results from the time-dependent linear dilaton (4).

[1] The presence of the tachyonic instability in the spectrum is due to the fact that in Type-0 strings there is no target-space supersymmetry, by construction. From a cosmological viewpoint the tachyon fields are not necessarily bad features, since they may provide the initial instability leading to cosmic expansion [17].

The asymptotic regime of the Type-0 cosmological string model of [17] has been obtained analytically, by solving the pertinent equations (12) for the various fields. As already mentioned, at late times the theory becomes four-dimensional, and the only non-trivial information is contained in the scale factor and the dilaton, given that the topological flux field remains conformal in this approach, and the moduli and initial tachyon fields decouple very fast during the initial stages after inflation in this model. For times that are long after the initial fluctuations, such as the present epoch when the linear approximation is valid, the solution for the dilaton in the σ-model frame, as derived from the equations (12), takes the form:

$$\Phi(t) = -\ln\left[\frac{\alpha A}{F_1}\cosh(F_1 t)\right] , \tag{13}$$

where F_1 is a positive constant, α is a numerical constant of order one, and

$$A = \frac{C_5 e^{s_{01}}}{\sqrt{2}V_6} , \tag{14}$$

where s_{01} is the equilibrium value of the modulus field σ_1 associated with the large bulk dimension, and C_5 is the corresponding flux of the five-form field. Notice that A is independent of this large bulk dimension.

For very large times $F_1 t \gg 1$ (in string units), one therefore approaches a linear solution for the dilaton: $\Phi \sim \text{const} - F_1 t$. From (13), (1) and (2), we then see that the asymptotic weakness of gravity in this Universe [17] is due to the smallness of the internal space V_6 as compared with the flux C_5 of the five-form field. The constant F_1 is related to the central-charge deficit of the underlying the non-conformal σ-model by [17]:

$$Q = q_0 + \frac{q_0}{F_1}(F_1 + \frac{d\Phi}{dt}), \tag{15}$$

where q_0 is a constant, the parenthesis vanishes asymptotically, and the numerical solution of (12) studied in [17]) requires that $q_0/F_1 = (1+\sqrt{17})/2 \simeq 2.53$. For this behaviour of Φ, the central-charge deficit (15) tends to a constant value q_0. In this way, F_1 is related to the asymptotic constant value of the central-charge deficit, up to an irrelevant proportionality factor of order one, in agreement with the conformal model of [12], to which this model asymptotes. This value should be, for consistency of the underlying string theory [12], some discrete value for which the factorization property (unitarity) of the string scattering amplitudes is valid. Notice that this asymptotic string theory, with a constant (time-independent) central-charge deficit, $q_0^2 \propto c^* - 25$ (or $c^* - 9$ for superstring) is considered as an *equilibrium* situation, and an S matrix can be defined for specific (discrete) values of the central charge c^*. The standard critical (super)string corresponds to a central charge $c^* = 25$ ($c^* = 9$), but in our case c^* differs from that critical value.

Defining the Einstein-frame time t_E through (6), we obtain in the case (13)

$$t_E = \frac{\alpha A}{F_1^2} \sinh(F_1 t). \tag{16}$$

In terms of the Einstein-frame time, one obtains a logarithmic time-dependence [12] for the dilaton

$$\Phi_E = \text{const} - \ln(\gamma t_E) \ , \tag{17}$$

where

$$\gamma \equiv \frac{F_1^2}{\alpha A} \ . \tag{18}$$

For large t_E, e.g., present or later cosmological time values, one has

$$a_E(t_E) \simeq \frac{F_1}{\gamma} \sqrt{1 + \gamma^2 t_E^2}. \tag{19}$$

At very large (future) times, $a(t_E)$ scales linearly with the Einstein-frame cosmological time t_E [17], and hence the cosmic horizon expands logarithmically. From a field-theory viewpoint, this would allow for a proper definition of asymptotic states and thus a scattering matrix. As we mentioned briefly above, however, from a stringy point of view, there are restrictions on the asymptotic values of the central-charge deficit q_0, and it is only a discrete spectrum of values of q_0 which allow for a full stringy S-matrix to be defined, respecting modular invariance [12].

Asymptotically in time, therefore, the Universe relaxes to its ground-state equilibrium situation and the non-criticality of the string, caused by the initial quantum fluctuation or other initial condition, disappears, giving way to a critical (equilibrium) string Universe with a Minkowski metric and a linear-dilaton background. These are the generic features of the models we consider here, which can include strings with target-space supersymmetry as well as the explicit bosonic Type-0 string considered here for simplicity.

The Hubble parameter of such a Universe becomes for large t_E

$$H(t_E) \simeq \frac{\gamma^2 t_E}{1 + \gamma^2 t_E^2} \ . \tag{20}$$

On the other hand, the Einstein-frame effective four-dimensional 'vacuum energy density', which is determined by the running central-charge deficit Q^2 after compactifying to four dimensions the ten-dimensional expression $\int d^{10}x\sqrt{-g}e^{-2\Phi}Q^2(t_E)$, is [17]:

$$\Lambda_E(t_E) = e^{2\Phi-\sigma_1-5\sigma_2}Q^2(t_E) \simeq \frac{q_0^2\gamma^2}{F_1^2(1+\gamma^2 t_E^2)}, \tag{21}$$

where, for large t_E, Q is given in (15), and approaches its equilibrium value q_0. Thus, we see explicitly how the dark energy density relaxes to zero for $t_E \to \infty$.

Finally, and most importantly for our purposes here, the deceleration parameter in the same regime of t_E becomes:

$$q(t_E) = -\frac{(d^2a_E/dt_E^2)\ a_E}{(da_E/dt_E)^2} \simeq -\frac{1}{\gamma^2 t_E^2}. \tag{22}$$

The key point about this expression is that, as is clear from (17) and (1), up to irrelevant proportionality constant factors which by conventional normalization are set to unity, it can be identified with the square of the string coupling:

$$q(t_E) = -\exp[2(\Phi - \text{const})] = -g_s^2. \tag{23}$$

This is our central result.

To guarantee consistency of perturbation theory, one must have $g_s < 1$, which can be achieved in our approach if one defines the present era by the time regime

$$\gamma \sim t_E^{-1} \tag{24}$$

in the Einstein frame. This is compatible with large enough times t_E (in string units) for

$$|C_5|e^{-5s_{02}} \gg 1\ , \tag{25}$$

as becomes clear from (14) and (18). This condition can be guaranteed either for small radii of the five extra dimensions or by a large value of the flux $|C_5|$ of the five-form of the Type-0 string. We recall that the relatively large extra dimension, s_{01}, which extends in the direction of the flux, decouples from this condition. Therefore, effective five-dimensional models with a large uncompactified fifth dimension may be constructed consistently with the condition (24).

We next turn to the equation of state in such a Universe. As discussed in [17], this model resembles quintessence, with the dilaton playing the rôle of the quintessence field. Hence the equation of state for our Type-0 string Universe reads [20]:

$$w_\Phi = \frac{p_\Phi}{\rho_\Phi} = \frac{\frac{1}{2}(\dot{\Phi})^2 - V(\Phi)}{\frac{1}{2}(\dot{\Phi})^2 + V(\Phi)}, \tag{26}$$

where p_Φ is the pressure and ρ_Φ is the energy density, and $V(\Phi)$ is the effective potential for the dilaton, which in our case is provided by the central-charge deficit term. Here the dots denote Einstein-frame differentiation. In the Einstein frame, the potential $V(\Phi)$ is given by Λ_E in (21). In the limit $Q \to q_0$, which we assume characterizes the present era to a good approximation, $V(\Phi)$ is of order $(q_0^2/2F_1^2)t_E^{-2}$, where we recall that q_0/F_1 is of order one, as discussed above. The exact normalization of the dilaton field in the Einstein frame is $\Phi_E = \mathrm{const} - \ln(\gamma t_E)$. We then obtain for the present era:

$$\frac{1}{2}\dot{\Phi}^2 \sim \frac{1}{2t_E^2}, \qquad V(\Phi) \sim \frac{6.56}{2}\frac{1}{t_E^2}, \tag{27}$$

where the numerical factor is a consequence of the numerical result of [17]. This implies an equation of state (26):

$$w_\Phi(t_E \gg 1) \simeq -0.74 \tag{28}$$

for (large) times t_E in string units corresponding to the present era (24). Correspondingly, we have a cosmic deceleration parameter

$$q = \frac{1}{2}(1 + 3w_\Phi) = -0.61. \tag{29}$$

This fixes the string coupling to perturbative values, consistent with naive scenarios for grand unification.

So far the model did not include ordinary matter, as only fields from the gravitational string multiplet have been included. The inclusion of ordinary matter is not expected to change qualitatively the result. We conjecture that the fundamental relation (23) will continue to hold, the only difference being that probably the inclusion of ordinary matter will tend to reduce the string acceleration, due to the fact that matter is subject to attractive gravity, and resists the acceleration of the Universe. In such a case, one has

$$q = \frac{1}{2}\Omega_M - \Omega_\Lambda\ , \tag{30}$$

where $\Omega_M(\Omega_\Lambda)$ denote the matter (vacuum) energy densities, normalized to the critical energy density of a spatially flat Universe.

There is a remarkable coincidence in numbers for this non-supersymmetric Type-0 string Universe with the astrophysical observations, which yield also a q close to this value, since the ordinary matter content of the universe (normalized with respect to the energy density of a flat Universe) is $\Omega_{\text{ordinary matter}} \simeq 0.04$ and the dark matter content is estimated to be $\Omega_{DM} = 0.23$, while the dark energy content is $\Omega_\Lambda \simeq 0.73$. This yields $q = -0.55$, which is only a few per cent away from (29). Conversely, if one used naively in the expression (30) the value (29) for q, obtained in our case where ordinary matter was ignored, one would find $\Omega_\Lambda \simeq 0.74$, indicating that the contribution of the dilaton field to the cosmic acceleration is the dominant one.

If the relation (23) were to hold also upon the inclusion of matter, even in a realistic case with (broken) supersymmetry, one would arrive at a value of the string coupling, $g_s^2 \simeq 0.55$, which would be quite consistent with the unification prediction of the minimal supersymmetric extension of the Standard Model at scales $\sim 10^{16}$ GeV. The only requirement for the asymptotic condition (23) to hold is that the underlying stringy σ model theory is non-critical and asymptotes for large times to the linear-dilaton conformal field theory of [12]. It should be understood, though, that the precise relation of the four-dimensional gauge coupling with the ten-dimensional string coupling depends on the details of compactification, which we did not discuss in this work.

We close this discussion by stressing once more the importance of non-criticality in order to arrive at (23). In critical strings, which usually assume the absence of a four-dimensional dilaton, such a relation cannot be obtained, and the string coupling is not directly measurable. The logarithmic variation of the dilaton field with the cosmic time at late times implies a slow variation of the string coupling (23), $|\dot{g}_s/g_s| = 1/t_E \sim 10^{-60}$ for the present era, and hence a correspondingly slow variation of the gauge couplings.

From a physical point of view, the use of critical strings to describe the evolution of our Universe seems desirable, whilst non-critical strings may be associated with non-equilibrium situations, as undoubtedly occur in the early Universe. The space of non-critical string theories is much larger that of critical strings. Therefore, it is remarkable that the departure from criticality has the potential to enhance the predictability of string theory to such a point that the string coupling may become accessible to experiment. A similar situation arises in a non-critical string approach to inflation, in the scenario where the Big Bang is identified with the collision [7] of two D-branes [16]. In such a scenario, astrophysical observations may place important bounds on the recoil velocity of the brane worlds after the collision, and lead to an estimate of the separation of the branes at the end of the inflationary period.

The approach of identifying target time in such a framework with a world-sheet renormalization-group scale, the Liouville mode [14], provides a novel way of selecting the ground state of the string theory, which may not necessarily be associated with minimization of energy, but could be a matter of cosmic 'chance'. The initial state of our cosmos may correspond to a certain 'random' Gaussian fixed point in the space of string theories, which is then perturbed in the Big Bang by some 'random' relevant (in a world-sheet sense) deformation, making the theory non-critical and taking it out of equilibrium from a target space-time viewpoint. The theory then flows, following a well-defined renormalization-group trajectory, and asymptotes to the specific ground state corresponding to the infrared fixed point of this perturbed world-sheet σ-model theory. This approach allows for many parallel universes to be implemented of course, and our world would be just one of these. Each Universe may flow between a different pair of fixed points, as it may be perturbed by different operators. It seems to us that this scenario is much more attractive (no pun intended) and specific than the static 'landscape' scenario [3], which is currently advocated as an attempt to parametrize our ignorance of the true structure of the string/M theory vacuum and its specification.

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