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A Supersymmetric Flipped SU(5) Intersecting Brane World

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ABSTRACT

We construct an $N = 1$ supersymmetric three-family flipped $SU(5)$ model from type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D6-branes intersecting at general angles. The spectrum contains a complete grand unified and electroweak Higgs sector. In addition, it contains extra exotic matter both in bi-fundamental and vector-like representations as well as two copies of matter in the symmetric representation of $SU(5)$.

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1 Introduction

The intersecting D-brane world approach [1, 2, 3, 4] plays a prominent role in the attempts of string phenomenologists to reproduce the standard model physics in a convincing way from type II string theory.

A number of consistent non-supersymmetric three-generation standard-like models have been constructed in [5, 6] (for a complete set of references the reader should consult the excellent reviews [7, 8, 9, 10, 11]). Open strings that begin and end on a stack of M D-branes generate the gauge bosons of the group $U(M)$ living in the world volume of the D-branes. So the standard approach is to start with one stack of 3 D-branes, another of 2, and n other stacks each having just 1 D-brane, thereby generating the gauge group $U(3) \times U(2) \times U(1)^n$. The D4-, D5- or D6-branes wrap the three large spatial dimensions and respectively 1-, 2- or 3-cycles of the six-dimensional internal space (typically a torus T^6 or a Calabi-Yau 3-fold). Fermions in bi-fundamental representations of the corresponding gauge groups can arise at the multiple intersections of such stacks [1]. For D4- and D5-branes, to get $D = 4$ *chiral* fermions the intersecting branes should sit at a singular point in the space transverse to the branes, an orbifold fixed point, for example. In general, intersecting-brane configurations yield a non-supersymmetric spectrum, so to avoid the hierarchy problem the string scale associated with such models must be no more than a few TeV. Gravitational interactions occur in the bulk ten-dimensional space, and to ensure that the Planck mass has its observed large value, it is necessary that there are large dimensions transverse to the branes [12]. Thus getting the correct Planck scale effectively means that only D4- and D5-brane models are viable, since for D6-branes there is no dimension transverse to all of the intersecting branes. However, a generic feature of these models is that flavour changing neutral currents are generated by four-fermion operators induced by string instantons. Although such operators allow the emergence of a realistic pattern of fermion masses and mixing angles, the severe experimental limits on flavour changing neutral currents require that the string scale is rather high , of order 10^4 TeV [13]. In a non-supersymmetric theory the cancellation of the closed-string (twisted) Ramond-Ramond (RR) tadpoles does *not* ensure the cancellation of the Neveu-Schwarz-Neveu-Schwarz (NSNS) tadpoles. There is a resulting instability in the complex structure moduli [14]. One way to stabilise some of the (complex structure) moduli is to use an orbifold, rather than a torus, for the space wrapped by the D-branes.

If the embedding is supersymmetric, then the instabilities including the gauge hierachy problem are removed. In this case, one in general has to introduce in addition to D6-branes orientifold O6-planes, which can be regarded as branes of negative RR-charge and tension. For a general Calabi-Yau compact space these orientifold planes wrap special Lagrangian 3-cycles calibrated with respect to the real part of the holomorphic 3-form Ω_3 of the Calabi-Yau compact space [6].

[6] In this case, the gauge hierarchy problem can be addressed with soft supersymmetry-breaking terms.

This has been studied [15], using D6-branes and a $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, but it has so far proved difficult to get realistic phenomenology consistent with experimental data from such models. Further progress has been achieved using D6-branes and a $\mathbb{Z}_4$ [16], $\mathbb{Z}_4 \times \mathbb{Z}_2$ [17] or $\mathbb{Z}_6$ [18] orientifold. Although a semi-realistic three-generation model has been obtained this way [18], it has non-minimal Higgs content, so it too will have flavour changing neutral currents [19] (for recent progress in orientifolds of Gepner models see [20, 21]).

An alternative approach in this framework is to start engineering a grand unified gauge symmetry which subsequently breaks down to the standard model gauge group [22]. This possibility is not available in standard type IIB orientifolds, due to the difficulty in getting adjoint representations to break the GUT group to the Standard Model [23]. A well motivated example is the flipped $SU(5) \times U(1)_X$ model [24, 25], which had been extensively studied in the closed string era of the heterotic compactifications [26, 27]. From the theoretical point of view this motivation was coming from the fact that its symmetry breaking requires only $\mathbf{10}$ and $\overline{\mathbf{10}}$ representations at the grand unification scale, as well as $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations at the electroweak scale, and these were consistent with the representations of $SU(5)$ allowed by the unitarity condition with gauge group at level 1 [28, 29] [7]. From the phenomenological point of view flipped $SU(5) \times U(1)_X$ [24, 25] has a number of attractive features in its own right [30]. For example, it has a very elegant missing-partner mechanism for suppressing proton decay via dimension-5 operators [25], and is probably the simplest GUT to survive experimental limits on proton decay [31]. These considerations motivated the derivation of a number of flipped $SU(5)$ models from constructions using fermions on the world sheet [26, 27]. Consistency of the low energy values of the gauge coupling constants with string unification at about 10^{18} GeV (in the absence of large string loop threshold corrections) required the existence of extra matter, besides that of the supersymmetric standard model [32, 33].

Non-supersymmetric flipped $SU(5)$ models have been produced in [34] using D6-branes wrapping toroidal 3-cycles and also when the wrapping space is the $T^6/\mathbb{Z}_3$ orbifold [8]. It is therefore of interest to search for supersymmetric flipped $SU(5)$ models from type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D6-branes intersecting at general angles.

The wrapping numbers of the various stacks are constrained by the requirement of RR-tadpole cancellation as well as the supersymmetry conditions. Tadpole cancellation ensures the absence of non-abelian anomalies in the emergent low-energy quantum field theory. A generalised Green-Schwarz mechanism ensures that the gauge bosons associated with all anomalous $U(1)$s acquire string-scale masses [35], but the gauge bosons of some non-anomalous $U(1)$s can also acquire string-scale masses [36]; in all such cases the $U(1)$ group survives as a global symmetry. Thus we must also

[7] Thus attempts to embed conventional grand unified theory (GUT) groups such as $SU(5)$ or $SO(10)$ in heterotic string required more complicated compactifications, but none of these has been completely satisfactory. Constructions with the minimal option to embed just the standard model gauge group, were plagued with at least extra unwanted U(1) factors.

[8] The $T^6/\mathbb{Z}_3$ orbifold is not suitable for supersymmetric model building.

ensure the flipped $U(1)_X$ group remains a gauge symmetry by requiring that its gauge boson does *not* get such a mass.

The material of this Letter is organized as follows. In section 2 we provide all the necessary formalism for constructing a consistent string supersymmetric model on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. This formalism includes the RR tadpole consistency conditions and the restrictions placed on each stack of D6-branes for preservation of supersymmetry as well as the generalised Green-Schwarz anomaly cancellation mechanism and the requirements we impose such that the flipped $U(1)_X$ remains a gauge symmetry.

In section 3, for the convenience of the reader, we first provide the minimal field-theory content of the flipped $SU(5)$ model and then we proceed to derive a string model consistent with the rules described in section 2. This is a three-generation model, whose gauge symmetry includes $SU(5) \times U(1)_X$, however with a non-minimal matter content.

Finally, we use section 4 for our discussions and conclusions.

2 Search for Supersymmetric Flipped $SU(5)\times U(1)_X$ Brane Models on a $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifold

We have several choices at our disposal in attempting to build a four-dimensional three-generation GUT flipped $SU(5)$ model. A flipped $SU(5)$ model was successfully built in [34] on a $\mathbb{Z}_3$ orientifold but it was not supersymmetric. So, in this paper we will focus on the supersymmetric type IIA orientifold on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D6-branes intersecting at generic angles. This choice has the feature that $\mathbb{Z}_2$ actions do not constrain the ratio of the radii on any 2-torus. Additionally, the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold has only bulk cycles, contrasting the cases of $\mathbb{Z}_4$ and $\mathbb{Z}_6$ orientifolds where exceptional cycles also necessarily exist and generally increase the difficulty of satisfying the Ramond-Ramond tadpole condition. However, as we shall see only a limited range of ratio of the complex structure moduli is consistent with the supersymmetry conditions.

This $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ structure was first introduced in [15] and further studied in [22] [9], and we will use the same notations here. Consider type IIA theory on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold, where the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ generators θ, ω act on the complex coordinates (z_1, z_2, z_3) of $T^6 = T^2 \times T^2 \times T^2$ as

$$\begin{aligned} \theta &: (z_1, z_2, z_3) \to (-z_1, -z_2, z_3) \\ \omega &: (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \end{aligned} \qquad (1)$$

We implement an orientifold projection ΩR, where Ω is the world-sheet parity, and R acts as

$$R : (z_1, z_2, z_3) \to (\overline{z}_1, \overline{z}_2, \overline{z}_3) \qquad (2)$$

[9] See also [37].

Although the complex structure of the tori is arbitrary under the action of $\mathbb{Z}_2 \times \mathbb{Z}_2$, it must be assigned consistently with the orientifold projection. Crystallographic action of the complex conjugation R restricts consideration to just two shapes. We may take either a rectangular toroidal cell or a very specific tilted variation. Define here a canonical basis of homology cycles ($[a_i]$, $[b_i]$) lying respectively along the $(\hat{x}_i, \mathrm{i}\hat{y}_i)$ coordinate directions, where $i = 1, 2, 3$ labels each of the three 2-tori. Next, consider K different stacks of N_a D6-branes wrapping on ($[a_i]$, $[b_i]$) with integral coefficients (n_a^i, m_a^i), where $a = 1, 2, ...K$. For the tilted complex structure variants the toroidal cell is skewed such that an alternate homology basis is required to close cycles spanning the displaced lattice points. Specifically, we must consider the cycle $[a_i'] \equiv [a_i] + \frac{1}{2}[b_i]$, so that the tilted wrapping is described by $n_a^i[a_i'] + m_a^i[b_i] = n_a^i[a_i] + (n_a^i/2 + m_a^i)[b_i]$. For convenience, define the effective wrapping number l_a^i as $l_a^i \equiv m_a^i$ for rectangular and $l_a^i \equiv 2m_a^i + n_a^i$ for tilted tori.

With these definitions the homology three-cycles for a stack a of D6-branes and its orientifold image a' are given by

$$[\Pi_a] = \prod_{i=1}^{3} (n_a^i [a_i] + 2^{-\beta_i} l_a^i [b_i]), \quad [\Pi_{a'}] = \prod_{i=1}^{3} (n_a^i [a_i] - 2^{-\beta_i} l_a^i [b_i]) \tag{3}$$

where $\beta_i = 0$ if the ith torus is not tilted and $\beta_i = 1$ if it is tilted.

There are four kinds of orientifold 6-planes associated with the actions of ΩR, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$. The homology three-cycles which they wrap are [22]

$$\begin{array}{rr} \Omega R : [\Pi_1] = 2^3 [a_1][a_2][a_3], & \Omega R\omega : [\Pi_2] = -2^{3-\beta_2-\beta_3} [a_1][b_2][b_3] \\ \Omega R\theta\omega : [\Pi_3] = -2^{3-\beta_1-\beta_3} [b_1][a_2][b_3], & \Omega R\theta : [\Pi_4] = -2^{3-\beta_1-\beta_2} [b_1][b_2][a_3] \end{array} \tag{4}$$

This represents the fact that 180° rotation *plus* conjugate reflection produce 'vertical', i.e. $[b_i]$-oriented, invariant cycles, while the operator R alone preserves certain cycles along the 'horizontal', or $[a_i]$ axis. Each two-torus yields always a pair of such cycles, with the exception of the $[b_i]$-type tilted scenario where only a single invariant wrapping exists. This explains then the normal counting of $8 = 2^3$ distinct combinations, halved for each application of tilting in the vertically aligned case.

The total effect of these four planes should be combined, so we define $[\Pi_{O6}] = \sum_i [\Pi_i]$ [22]. In addition, a set of new parameters which are convenient in the following discussion are introduced [22]:

$$\begin{array}{l} A_a = -n_a^1 n_a^2 n_a^3, \ B_a = n_a^1 l_a^2 l_a^3, \ C_a = l_a^1 n_a^2 l_a^3, \ D_a = l_a^1 l_a^2 n_a^3 \\ \tilde{A}_a = -l_a^1 l_a^2 l_a^3, \ \tilde{B}_a = l_a^1 n_a^2 n_a^3, \ \tilde{C}_a = n_a^1 l_a^2 n_a^3, \ \tilde{D}_a = n_a^1 n_a^2 l_a^3 \end{array} \tag{5}$$

With the basic definitions in hand, we can continue working on the global constraints of this model.

2.1 RR-tadpole Consistency Conditions

The Ramond-Ramond tadpole cancellation requires the total homology cycle charge of D6-branes and O6-planes to vanish [3]. The resulting equation

$$\sum_a N_a[\Pi_a] + \sum_a N_a[\Pi_{a'}] - 4[\Pi_{O6}] = 0 \tag{6}$$

can be expressed in terms of the parameters defined in (5) as

$$\sum_a N_a A_a = \sum_a N_a B_a = \sum_a N_a C_a = \sum_a N_a D_a = -16 \tag{7}$$

It should be stressed that the tadpole condition is independent of the selected tilting. However, these coupled constraints are generally quite difficult to satisfy. The introduction of so called 'filler branes' [22] which wrap along the O6-planes can help somewhat. Such branes automatically preserve supersymmetry, so that they can be selected with only an eye for independent saturation of each RR-tadpole condition. If $N^{(i)}$ branes wrap along the i^{th} O6-plane, (7) is updated to

$$\begin{aligned} -2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a = \\ -2^k N^{(3)} + \sum_a N_a C_a = -2^k N^{(4)} + \sum_a N_a D_a = -16 \end{aligned} \tag{8}$$

Here $k = \beta_1 + \beta_2 + \beta_3$ is the total number of tilted tori.

2.2 Conditions for Supersymmetric Brane Configurations

The condition to preserve $N = 1$ supersymmetry in four dimensions is that the rotation angle of any D-brane with respect to the orientifold plane is an element of $SU(3)$ [1, 15]. Consider the angles between each brane and the R-invariant axis of i^{th} torus θ_a^i, we require $\theta_a^1 + \theta_a^2 + \theta_a^3 = 0 \bmod 2\pi$. This means $\sin(\theta_a^1 + \theta_a^2 + \theta_a^3) = 0$ and $\cos(\theta_a^1 + \theta_a^2 + \theta_a^3) = 1 > 0$. We define

$$\tan\theta_a^i = \frac{2^{-\beta_i} l_a^i R_2^i}{n_a^i R_1^i} \tag{9}$$

where R_2^i and R_1^i are the radii of the i^{th} torus. Then the above supersymmetry conditions can be recast in terms of the parameters defined in (5) as follows [22]:

$$\begin{aligned} x_A\tilde{A}_a + x_B\tilde{B}_a + x_C\tilde{C}_a + x_D\tilde{D}_a = 0 \\ A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0 \end{aligned} \tag{10}$$

where x_A, x_B, x_C, x_D are complex structure parameters, all of which share the same sign. These parameters are given in terms of the complex structure moduli $\chi_i = (R_2^i/R_1^i)$ by

$$x_A = \lambda, \quad x_B = \lambda 2^{\beta_2+\beta_3}/\chi_2\chi_3, \quad x_C = \lambda 2^{\beta_1+\beta_3}/\chi_1\chi_3, \quad x_D = \lambda 2^{\beta_1+\beta_2}/\chi_1\chi_2 \tag{11}$$

The positive parameter λ was introduced in [22] to put all the variables A, B, C, D on an equal footing. However, among the x_i only three are independent.

2.3 Intersection Numbers

The initial $U(N_a)$ gauge group supported by a stack of N_a identical D6-branes is broken down by the $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry to a subgroup $U(N_a/2)$ [15]. Chiral matter particles are formed from open strings with two ends attaching on different stacks. By using Grassmann algebra $[a_i][b_j] = -[b_j][a_i] = \delta_{ij}$ and $[a_i][a_j] = -[b_j][b_i] = 0$ we can calculate the intersection numbers between stacks a and b and provide the multiplicity ($\mathcal{M}$) of the corresponding bi-fundamental representation:

$$\mathcal{M}(\frac{N_a}{2}, \frac{\overline{N_b}}{2}) = I_{ab} = [\Pi_a][\Pi_b] = 2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i - n_b^i l_a^i) \tag{12}$$

Likewise, stack a paired with the orientifold image b' of b yields

$$\mathcal{M}(\frac{N_a}{2}, \frac{N_b}{2}) = I_{ab'} = [\Pi_a][\Pi_{b'}] = -2^{-k} \prod_{i=1}^{3} (n_a^i l_b^i + n_b^i l_a^i) \tag{13}$$

Strings stretching between a brane in stack a and its mirror image a' yield chiral matter in the antisymmetric and symmetric representations of the group $U(N_a/2)$ with multiplicities

$$\mathcal{M}((\mathrm{A}_a)_L) = \frac{1}{2} I_{aO6}, \quad \mathcal{M}((\mathrm{A}_a + \mathrm{S}_a)_L) = \frac{1}{2}(I_{aa'} - \frac{1}{2} I_{aO6}) \tag{14}$$

so that the net total of antisymmetric and symmetric representations are given by

$$\begin{aligned}
\mathcal{M}(\mathrm{Anti}_a) &= \frac{1}{2}(I_{aa'} + \frac{1}{2} I_{aO6}) = -2^{1-k}[(2A_a - 1)\tilde{A}_a - \tilde{B}_a - \tilde{C}_a - \tilde{D}_a] \\
\mathcal{M}(\mathrm{Sym}_a) &= \frac{1}{2}(I_{aa'} - \frac{1}{2} I_{aO6}) = -2^{1-k}[(2A_a + 1)\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a]
\end{aligned} \tag{15}$$

where

$$I_{aa'} = [\Pi_a][\Pi_{a'}] = -2^{3-k} \prod_{i=1}^{3} n_a^i l_a^i \tag{16}$$

$$I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k}(\tilde{A}_a + \tilde{B}_a + \tilde{C}_a + \tilde{D}_a) \tag{17}$$

This distinction is critical, as we require independent use of the paired multiplets such as $(\mathbf{10}, \overline{\mathbf{10}})$ which are masked in expression (15). In what follows we consider the case $k = 0$.

2.4 Generalized Green-Schwarz Mechanism

Although the total non-Abelian anomaly in intersecting brane world models cancels automatically when the RR-tadpole conditions are satisfied, there may be additional mixed anomalies present. For instance, the mixed gravitational anomalies which generate massive fields are not trivially zero [15]. These anomalies are cancelled by a generalized Green-Schwarz (G-S) mechanism which involves untwisted Ramond-Ramond forms. The couplings of the four untwisted Ramond-Ramond forms B_2^i to the $U(1)$ field strength F_a of each stack a are

$$
\begin{aligned}
&N_a l_a^1 n_a^2 n_a^3 \int_{M4} B_2^1 \wedge \mathrm{tr} F_a, \quad N_a n_a^1 l_a^2 n_a^3 \int_{M4} B_2^2 \wedge \mathrm{tr} F_a \\
&N_a n_a^1 n_a^2 l_a^3 \int_{M4} B_2^3 \wedge \mathrm{tr} F_a, \quad -N_a l_a^1 l_a^2 l_a^3 \int_{M4} B_2^4 \wedge \mathrm{tr} F_a
\end{aligned} \tag{18}
$$

These couplings determine the linear combinations of $U(1)$ gauge bosons that acquire string scale masses via the G-S mechanism. In flipped $SU(5) \times U(1)_X$, the symmetry $U(1)_X$ must remain a gauge symmetry so that it may remix to help generate the standard model hypercharge after the breaking of $SU(5)$. Therefore, we must ensure that the gauge boson of the flipped $U(1)_X$ group does not receive such a mass. The $U(1)_X$ is a linear combination (to be identified in section 3.2) of the $U(1)$s from each stack :

$$
U(1)_X = \sum_a C_a U(1)_a \tag{19}
$$

The corresponding field strength must be orthogonal to those that acquire G-S mass. Thus we demand :

$$
\begin{aligned}
&\sum_a C_a N_a \tilde{B}_a = 0, \quad \sum_a C_a N_a \tilde{C}_a = 0 \\
&\sum_a C_a N_a \tilde{D}_a = 0, \quad \sum_a C_a N_a \tilde{A}_a = 0
\end{aligned} \tag{20}
$$

3 Flipped $SU(5) \times U(1)_X$ Model Building

In the previous section we have outlined all the necessary machinery for constructing an intersecting-brane model on the $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. Our goal now is to realize a supersymmetric $SU(5) \times U(1)_X$ gauge theory with three generations and a complete GUT and electroweak Higgs sector in the four-dimensional spacetime. We also try to avoid as much extra matter as possible.

3.1 Basic Flipped $SU(5)$ Phenomenology

In a flipped $SU(5) \times U(1)_X$ [24, 25] unified model, the electric charge generator Q is only partially embedded in $SU(5)$, *i.e.*, $Q = T_3 - \frac{1}{5}Y' + \frac{2}{5}\tilde{Y}$, where Y' is the $U(1)$

internal $SU(5)$ and $\tilde{Y}$ is the external $U(1)_X$ factor. Essentially, this means that the photon is 'shared' between $SU(5)$ and $U(1)_X$. The Standard Model (SM) plus right handed neutrino states reside within the representations $\bar{\mathbf{5}}$, $\mathbf{10}$, and $\mathbf{1}$ of $SU(5)$, which are collectively equivalent to a spinor $\mathbf{16}$ of $SO(10)$. The quark and lepton assignments are flipped by $u_L^c \leftrightarrow d_L^c$ and $\nu_L^c \leftrightarrow e_L^c$ relative to a conventional $SU(5)$ GUT embedding:

$$\bar{f}_{\bar{\mathbf{5}},-\frac{\mathbf{3}}{\mathbf{2}}} = \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \\ e \\ \nu_e \end{pmatrix}_L \; ; \quad F_{\mathbf{10},\frac{\mathbf{1}}{\mathbf{2}}} = \left(\begin{pmatrix} u \\ d \end{pmatrix}_L d_L^c \;\; \nu_L^c \right); \quad l_{\mathbf{1},\frac{\mathbf{5}}{\mathbf{2}}} = e_L^c \tag{21}$$

In particular this results in the $\mathbf{10}$ containing a neutral component with the quantum numbers of ν_L^c. We can break spontaneously the GUT symmetry by using a $\mathbf{10}$ and $\overline{\mathbf{10}}$ of superheavy Higgs where the neutral components provide a large vacuum expectation value, $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle$,

$$H_{\mathbf{10},\frac{\mathbf{1}}{\mathbf{2}}} = \{Q_H,\ d_H^c,\ \nu_H^c\}\,; \quad \bar{H}_{\overline{\mathbf{10}},-\frac{\mathbf{1}}{\mathbf{2}}} = \{Q_{\bar{H}},\ d_{\bar{H}}^c,\ \nu_{\bar{H}}^c\}\,. \tag{22}$$

The electroweak spontaneous breaking is generated by the Higgs doublets H_2 and $\bar{H}_{\bar{2}}$

$$h_{\mathbf{5},-\mathbf{1}} = \{H_2, H_3\}\,; \quad \bar{h}_{\bar{\mathbf{5}},\mathbf{1}} = \left\{\bar{H}_{\bar{2}}, \bar{H}_{\bar{3}}\right\} \tag{23}$$

Flipped $SU(5)$ model building has two very nice features which are generally not found in typical unified models: (i) a natural solution to the doublet (H_2)-triplet(H_3) splitting problem of the electroweak Higgs pentaplets $h, \bar{h}$ through the trilinear coupling of the Higgs fields: $H_{\mathbf{10}} \cdot H_{\mathbf{10}} \cdot h_{\mathbf{5}} \rightarrow \langle \nu_H^c \rangle\, d_H^c H_3$, and (ii) an automatic see-saw mechanism that provide heavy right-handed neutrino mass through the coupling to singlet fields ϕ, $F_{\mathbf{10}} \cdot \bar{H}_{\overline{\mathbf{10}}} \cdot \phi \rightarrow \left\langle \nu_{\bar{H}}^c \right\rangle \nu^c \phi$.

The generic superpotential W for a flipped $SU(5)$ model will be of the form :

$$\lambda_1 FFh + \lambda_2 F\bar{f}\bar{h} + \lambda_3 \bar{f} l^c h + \lambda_4 F\bar{H}\phi + \lambda_5 HHh + \lambda_6 \bar{H}\bar{H}\bar{h} + \cdots \in W \tag{24}$$

the first three terms provide masses for the quarks and leptons, the fourth is responsible for the heavy right-handed neutrino mass and the last two terms are responsible for the doublet-triplet splitting mechanism [25].

3.2 Model Building

We first consider a stack with ten D6-branes to form the desired $U(5)$ group, and then determine additional stacks of two branes which provide $U(1)$ group factors and are compatible with the supersymmetry conditions of the 10-brane stack. To have enough but not too many copies of the antisymmetric and symmetric representation in the first stack a to satisfy the tadpole conditions, it is reasonable to consider the case of no tilted tori ($k = 0$) and we choose a set of proper wrapping numbers to make $\mathcal{M}((\mathrm{A}_a)_L) = 4$ and $\mathcal{M}((\mathrm{A}_a + \mathrm{S}_a)_L) = -2$. Under this setting, one wrapping number is zero and it makes two of the RR-tadpole parameters A, B, C, D zero with the remaining two negative, which forces the structure parameters x_A, x_B, x_C, x_D to be all positive by the SUSY conditions. Then the rest of the 2-brane stacks are chosen in accordance with our requirements.

Because of the combined constraints from RR-tadpole and SUSY conditions, it is harder to get negative values than to get positive values or zero for I_{ab} and $I_{ab'}$ to generate the required bi-fundamental representations. Generally when a negative number is needed, the absolute value cannot be large enough to alone provide three generations of chiral matter. This suggests the consideration of multiple two-brane stacks to share the burden of this task.

Next we turn to the question of the number of stacks we need. Generally speaking a case with three stacks is enough to provide all the required matter to construct a normal $SU(5)$ GUT model. However, as we mentioned we have to ensure that the $U(1)_X$ remains a gauge symmetry after the application of the G-S mechanism. It is clear that at least two more stacks are needed if all the couplings to the four RR forms are present.

The pentaplet $\bar{f}$ which contains Standard Model fermions is different from the Higgs pentaplet $\bar{h}$ resulting from the 'flipped' nature of the model as we saw in section 3.1. For example, if we take $U(1)_X$ for $(\mathbf{10}, \mathbf{1})$ in both SM and Higgs spectrum as $1/2$, then it is $-3/2$ for $(\bar{\mathbf{5}}, \mathbf{1})$ in SM, $5/2$ for $(\mathbf{1}, \mathbf{1})$ in SM, $-1/2$ for $(\overline{\mathbf{10}}, \mathbf{1})$ in Higgs, 1 or -1 for $(\bar{\mathbf{5}}, \mathbf{1})$ and $(\mathbf{5}, \mathbf{1})$ in Higgs, and 0 for $(\mathbf{1}, \mathbf{1})$ in Higgs. These constrain some coefficients of $U(1)$s from the stacks involving the SM and Higgs spectra, and may require more stacks in addition to the five mentioned above for obtaining the correct $U(1)_X$ charge for all the matter and Higgs representations. In this paper we present an example with seven stacks.

However, with seven stacks it was still difficult to find chiral bi-fundamental representations to be identified with the electroweak Higgs pentaplets, $h, \bar{h}$ and at the same time for the $U(1)_X$ group to remain a gauge symmetry. This directed us towards the most natural choice of identifying our Higgs pentaplets as well as some matter representations from intersections which provide non-chiral matter. After all, the Higgs $\mathbf{5}$ and the $\bar{\mathbf{5}}$ construct the vector-like $\mathbf{10}$ representation of $SO(10)$. A zero intersection number between two branes implies that the branes are parallel on at least one torus. At such kind of intersection additional non-chiral (vector-like) multiplet pairs from $ab+ba$, $ab'+b'a$, and $aa'+a'a$ can arise [38][10]. The multiplicity of these non-chiral multiplet pairs is given by the remainder of the intersection product, neglecting the null sector. For example, if $(n_a^1 l_b^1 - n_b^1 l_a^1) = 0$ in $I_{ab} = [\Pi_a][\Pi_b] =$

[10]Representations $(\mathrm{Anti}_a + \overline{\mathrm{Anti}}_a)$ occur at intersection of a with a' if they are parallel on at least one torus.

$2^{-k}\prod_{i=1}^{3}(n_a^i l_b^i - n_b^i l_a^i)$,

$$\mathcal{M}\left[\left(\frac{N_a}{2},\frac{\overline{N_b}}{2}\right)+\left(\frac{\overline{N_a}}{2},\frac{N_b}{2}\right)\right]=\prod_{i=2}^{3}(n_a^i l_b^i - n_b^i l_a^i) \tag{25}$$

This is useful since we can fill the spectrum with this matter without affecting the required global conditions because the total effect of the pairs is zero. For instance in our model, besides the (ae') intersection which provides a vector-like pair of Higgs pentaplets, the intersection (ef') delivers the fermion (singlet under the $SU(5)$ group) $l_{\mathbf{1},\frac{\mathbf{5}}{\mathbf{2}}}$ particles.

In table 1 we present a consistent model compatible with the constraints we described. Note that this is a (7+1)-stack model, with one stack of two filler branes wrapped along the first orientifold plane and two sets of parallel branes; the latter provide several non-chiral pairs. The gauge symmetry associated with the two filler branes is $Usp(2) \cong SU(2)$.

The Result The gauge symmetry of the (7+1)-stack model in table 1 is $U(5) \times U(1)^6 \times Usp(2)$, and the structure parameters of the wrapping space are

$$x_A = 1,\ \ x_B = 2,\ \ x_C = 8,\ \ x_D = 1 \tag{26}$$

which means

$$\frac{R_2^1}{R_1^1}=\frac{1}{2},\ \ \frac{R_2^2}{R_1^2}=2,\ \ \frac{R_2^3}{R_1^3}=\frac{1}{4} \tag{27}$$

The intersection numbers are listed in table 2, and the resulting spectrum in table 3. We have a complete Standard Model sector plus right handed neutrinos in three copies, a complete Higgs spectrum, and in addition extra *exotic* matter which includes two $(\overline{\mathbf{15}},\mathbf{1})$.

The $U(1)_X$ is

$$U(1)_X=\frac{1}{12}(3U(1)_a-20U(1)_b+45U(1)_d-15U(1)_e-15U(1)_f-20U(1)_g) \tag{28}$$

while the other two anomaly-free and massless combinations $U(1)_Y$ and $U(1)_Z$ are

$$\begin{aligned} U(1)_Y &= U(1)_b+U(1)_c-6U(1)_d+3U(1)_e+3U(1)_f+2U(1)_g \\ U(1)_Z &= U(1)_b-U(1)_c+U(1)_e-U(1)_f \end{aligned} \tag{29}$$

These two gauge symmetries can be spontaneously broken by assigning vacuum expectation values to singlets from the intersection (bg). Thus, the final gauge symmetry is $SU(5)\times U(1)_X \times Usp(2)$.

The remaining four global $U(1)$s from the Green-Schwarz mechanism are given respectively by

$$\begin{aligned} U(1)_1 &= -10U(1)_a+2U(1)_b+2U(1)_c-2U(1)_d-8U(1)_g \\ U(1)_2 &= -2U(1)_b-2U(1)_c+2U(1)_g \\ U(1)_3 &= 6U(1)_b+6U(1)_c+4U(1)_d+2U(1)_e+2U(1)_f \\ U(1)_4 &= 20U(1)_a+6U(1)_b+6U(1)_c-2U(1)_e-2U(1)_f. \end{aligned} \tag{30}$$

stack	N_a	(n_1, l_1)	(n_2, l_2)	(n_3, l_3)	A	B	C	D	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$	$\tilde{D}$
a	$N = 10$	(0,-1)	(-1,-1)	(-1,-2)	0	0	-2	-1	2	-1	0	0
b	$N = 2$	(-1,-1)	(-1, 1)	(1, 3)	-1	-3	3	-1	3	1	-1	3
c	$N = 2$	(-1,-1)	(-1, 1)	(1, 3)	-1	-3	3	-1	3	1	-1	3
d	$N = 2$	(-1, 1)	(1, 0)	(-1,-2)	-1	0	-2	0	0	-1	0	2
e	$N = 2$	(-1, 1)	(1,-1)	(0,-1)	0	-1	-1	0	-1	0	0	1
f	$N = 2$	(-1, 1)	(1,-1)	(0,-1)	0	-1	-1	0	-1	0	0	1
g	$N = 2$	(1,-1)	(-4,-1)	(-1, 0)	-4	0	0	-1	0	-4	1	0
filler	$N^{(1)} = 2$	(1, 0)	(1, 0)	(1, 0)	-1	0	0	0	0	0	0	0

Table 1: Wrapping numbers and their consistent parameters.

stk	N	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	f1
a	10	2	-2	-2	0(5)	-2	0(5)	0(1)	4	-2	0(1)	-2	0(1)	6	10	2
b	2	24	0	-	-	0(0)	24	2	0(5)	0(2)	0(2)	0(2)	0(2)	30	0(9)	3
c	2	24	0	-	-	-	-	2	0(5)	0(2)	0(2)	0(2)	0(2)	30	0(9)	3
d	2	2	-2	-	-	-	-	-	-	0(1)	-2	0(1)	-2	0(2)	4	0
e	2	0	0	-	-	-	-	-	-	-	-	0(0)	0(4)	0(5)	-6	-1
f	2	0	0	-	-	-	-	-	-	-	-	-	-	0(5)	-6	-1
g	2	-6	6	-	-	-	-	-	-	-	-	-	-	-	-	0

Table 2: List of intersection numbers. The number in parenthesis indicates the multiplicity of non-chiral pairs.

Rep.	Multi.	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$	$U(1)_e$	$U(1)_f$	$U(1)_g$	$12U(1)_X$	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$	$U(1)_Y$	$U(1)_Z$
$(10,1)$	3	2	0	0	0	0	0	0	6	-20	0	0	40	0	0
$(\bar{5}_a,1_e)$	2	-1	0	0	0	1	0	0	-18	10	0	2	-22	3	1
$(\bar{5}_a,1_f)$	1	-1	0	0	0	0	1	0	-18	10	0	2	-22	3	-1
$(\bar{1}_e,\bar{1}_f)^\star$	3	0	0	0	0	-1	-1	0	30	0	0	-4	4	-6	0
$(10,1)$	1	2	0	0	0	0	0	0	6	-20	0	0	40	0	0
$(\overline{10},1)$	1	-2	0	0	0	0	0	0	-6	20	0	0	-40	0	0
$(5_a,1_e)^\star$	1	1	0	0	0	1	0	0	-12	-10	0	2	18	3	1
$(\bar{5}_a,\bar{1}_e)^\star$	1	-1	0	0	0	-1	0	0	12	10	0	-2	-18	-3	-1
$(1_b,\bar{1}_g)$	4	0	1	0	0	0	0	-1	0	10	-4	6	6	-1	1
$(\overline{15},1)$	2	-2	0	0	0	0	0	0	-6	20	0	0	-40	0	0
$(\overline{10},1)$	1	-2	0	0	0	0	0	0	-6	20	0	0	-40	0	0
$(\bar{5}_a,1_c)$	2	-1	0	1	0	0	0	0	-3	12	-2	6	-14	1	-1
$(5_a,1_d)$	4	1	0	0	1	0	0	0	48	-12	0	4	20	-6	0
$(\bar{5}_a,1_b)$	2	-1	1	0	0	0	0	0	-23	12	-2	6	-14	1	1
$(\bar{5}_a,1_f)$	1	-1	0	0	0	0	1	0	-18	10	0	2	-22	3	-1
$(5_a,\bar{1}_g)$	6	1	0	0	0	0	0	-1	23	-2	-2	0	20	-2	0
$(5_a,1_g)$	10	1	0	0	0	0	0	1	-17	-18	2	0	20	2	0
$(1_b,1_c)$	24	0	1	1	0	0	0	0	-20	4	-4	12	12	2	0
$(1_b,\bar{1}_d)$	2	0	1	0	-1	0	0	0	-65	4	-2	2	6	7	1
$(1_b,\bar{1}_g)$	26	0	1	0	0	0	0	-1	0	10	-4	6	6	-1	1
$(1_c,\bar{1}_d)$	2	0	0	1	-1	0	0	0	-45	4	-2	2	6	7	-1
$(1_c,\bar{1}_g)$	30	0	0	1	0	0	0	-1	20	10	-4	6	6	-1	-1
$(\bar{1}_d,\bar{1}_e)$	2	0	0	0	-1	-1	0	0	-30	2	0	-6	2	3	-1
$(\bar{1}_d,\bar{1}_f)$	2	0	0	0	-1	0	-1	0	-30	2	0	-6	2	3	1
$(1_d,1_g)$	4	0	0	0	1	0	0	1	25	-10	2	4	0	-4	0
$(\bar{1}_e,\bar{1}_g)$	6	0	0	0	0	-1	0	-1	35	8	-2	-2	2	-5	-1
$(\bar{1}_f,\bar{1}_g)$	6	0	0	0	0	0	-1	-1	35	8	-2	-2	2	-5	1
$(\bar{1},\bar{1})$	2	0	0	0	-2	0	0	0	-90	4	0	-8	0	12	0
$(1,1)$	6	0	0	0	0	0	0	2	-40	-16	4	0	0	4	0
$(1_e,1_f)^\star$	4	0	0	0	0	1	1	0	-30	0	0	4	-4	6	0
$(\bar{1}_e,\bar{1}_f)^\star$	1	0	0	0	0	-1	-1	0	30	0	0	-4	4	-6	0
Additional non-chiral Matter															
Usp(2) Matter															

Table 3: The spectrum of $U(5)\times U(1)^6\times Usp(2)$, or $SU(5)\times U(1)_X\times U(1)_Y\times U(1)_Z\times Usp(2)$, with the four global $U(1)$s from the Green-Schwarz mechanism. The $\star' d$ representations stem from vector-like non-chiral pairs.

4 Conclusions

In this Letter we have constructed a particular $N = 1$ supersymmetric three-family model whose gauge symmetry includes $SU(5) \times U(1)_X$, from type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with D6-branes intersecting at general angles. The spectrum contains a complete grand unified theory and electroweak Higgs sector. In addition, it contains extra exotic matter both in bi-fundamental and vector-like representations as well as two copies of matter in the symmetric representation of $SU(5)$. Chiral matter charged under both the $SU(5) \times U(1)_X$ and $USp(2)$ gauge symmetries is also present, as is evident from Table 2. Furthermore, three adjoint ($N = 1$) chiral multiplets are provided from the aa sector [15]. We also note that the low energy spectrum of the model we constructed is free from any $SU(2)$ global anomalies since the number of the corresponding fermion doublets is even [39]. Nevertheless, although the massless spectrum is free from such global anomalies it does not satisfy all the additional constraints arising from the K-theory interpretation of D-branes [40, 41]. This issue will be investigated in a future publication.

The global symmetries, that arise after the G-S anomaly cancellation mechanism, forbid some of the Yukawa couplings required for mass generation, for instance terms like FFh. However, by the same token the term HHh is also forbidden. We note that such a term is essential for the doublet-triplet splitting solution mechanism in flipped $SU(5)$. Nevertheless, it should not escape our notice that while these global $U(1)$ symmetries are exact to all orders in perturbation theory, they can be broken explicitly by non-perturbative instanton effects [7, 42]. Thus, providing us with the possibility of recovering the appropriate superpotential couplings. Another interesting approach toward generating these absent Yukawa couplings may entail the introduction of type IIB flux compactifications [43]. This exceeds the scope of our current Letter, but shall be further investigated in an upcoming publication [44].

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