

# Limits on SUSY Particle Spectra from Proton Stability and Dark Matter Constraints

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## Abstract

It is shown that the combined constraints on the amount of cold dark matter and of proton stability produce a stringent upper limit on the gluino mass  $m_{\tilde{g}}$  (and hence on the lightest neutralino mass  $m_{\chi_1^0} \simeq m_{\tilde{g}}/7$  for a large class of gravity mediated supergravity unified models. One finds that for the minimal SU(5) model current data (Kamiokande) restricts  $m_{\tilde{g}} \leq 400$  GeV for a scalar soft breaking mass  $m_0 \leq 1$  TeV. Expected future data from Super Kamionkande and ICARUS will be sensitive to the entire range of gluino mass for  $m_0 \leq 1$  TeV, and be able to exclude the region  $m_{\tilde{g}} \geq 500$  GeV for  $m_0 \leq 5$  TeV. Effects of quark mass textures are studied and one finds that the bound  $m_{\tilde{g}} \leq 500$  GeV holds when the experimental proton lifetime for the  $p \rightarrow \bar{\nu}K^+$  mode becomes  $\geq 5 \times 10^{32}$  yr. Implications of these results for a test of these models at the Tevatron and at the LHC are discussed. The effects of non-universal soft breaking in the Higgs and the third generation squark sectors are also examined, and it is found that the proton lifetime is sensitive to these non-universal effects. The current data already eliminates some regions of non-universalities. The constraints of proton stability on the direct detection of dark matter are seen to reduce the maximum event rates by as much as a factor of  $10^3$ .

**Introduction:** In this Letter we study the combined constraints of proton stability and supersymmetric dark matter on supergravity unified models with R-parity invariance. We consider models where gravity breaks supersymmetry in a hidden sector at scale  $\geq M_G$  (the GUT scale) with gravity as the messenger to the physical sector. Models with R parity invariance eliminate in a natural fashion dimension four operators responsible for rapid proton decay. However, as is well known one can have proton decay proceeding via dimension five operators[1, 2]. Simultaneously, with R parity invariance one finds that the lightest supersymmetric particle (LSP) is absolutely stable and further in supergravity unification[3, 4] the LSP is seen to be the

lightest neutralino ( $\chi_1^0$ ) in almost all of the parameter space of the model[5], and thus the  $\chi_1^0$  is a candidate for cold dark matter (CDM). In this Letter we show that the combined constraints of proton stability and dark matter put an upper limit on the gluino mass of 500 GeV for the case of the minimal supersymmetric SU(5) model as well as constraining the masses of the other SUSY particles. We argue that this result would hold for a much wider class of models if the current lower limit on the proton lifetime is increased by a factor of 4. We also show that p stability is sensitive to the nature of non-universal soft breaking[6, 7, 8] and that GUT models with certain type of non-universal soft breaking can already be eliminated on the basis of p stability and dark matter constraints.

**Proton Stability Constraint:** It is well known that even with R parity invariance, instability of the proton can arise in most SUSY and string models via dimension five operators due to the exchange of color triplet Higgsinos. In general the Higgsino mediated p decay is governed by the interaction  $\bar{H}_1 J + \bar{K} H_1 + \bar{H}_i M_{ij} H_j$  where J and K are matter currents defined in terms of quark and lepton fields and we have carried out a field redefinition so that  $H_1$  and  $\bar{H}_1$  are the Higgs combinations that couple to the currents. Then it is easily seen that the condition needed to suppress proton decay from dimension five operators is given by  $(M^{-1})_{11}=0$ . A constraint such as this can arise either as a consequence of discrete symmetries or as a consequence of non-standard choice of matter representations. However, most of the SUSY/string models do not exhibit a constraint of this type and consequently in such models one will have proton decay induced by dimension five operators[9]. One needs a doublet-triplet splitting to achieve a heavy Higgs triplet to suppress dimension five proton decay and a number of mechanisms have been discussed in the literature to accomplish this. These include fine tuning, the sliding singlet mechanism valid for SU(N) with  $N \geq 6$ , the missing partner mechanism, VEV alignment, Higgs as a pseudo-Goldstone particle and the choice of more than one adjoint to break the GUT symmetry. For a wide class of models, i.e., SU(5), SO(10), E(6), etc where proton decay proceeds via dimension five operators, the dominant decay mode of the proton is given by  $p \rightarrow \bar{\nu} K^+$  and the decay width may be parametrized by[2]

$$\Gamma(p \rightarrow \bar{\nu} K^+) = \sum_{i=e,\mu,\tau} \Gamma(p \rightarrow \bar{\nu}_i K^+) = \left(\frac{\beta_p}{M_{\tilde{H}_3}}\right)^2 |A|^2 |B|^2 C \quad (1)$$

where  $M_{\tilde{H}_3}$  is the Higgs triplet mass,  $\beta_p$  is the matrix element of the three quark operator between the vacuum and the the proton state and its numerical value from lattice gauge calculations is given by[10]  $\beta_p = 5.6 \times 10^{-3} \text{ GeV}^3$ , A depends on the quark masses and the K-M matrix elements, B contains the dressing loop integrals which depend on the SUSY mass spectrum, and C contains the chiral current algebra factors which convert the chiral quark Lagrangian into an effective Lagrangian involving mesons and baryons(Chadha et.al. ref.[1]). In the analysis of this paper we constrain  $M_{\tilde{H}_3}$  mass to satisfy the relation  $M_{\tilde{H}_3} \leq 10M_G$  as in ref.[2].

We begin by analysing the proton lifetime within minimal SU(5) supergravity with radiative breaking of the electro-weak symmetry. This model is characterized by the following parameters: the universal scalar mass  $m_0$ , the universal gaugino mass  $m_{1/2}$ , the universal tri-linear coupling  $A_0$ , and  $\tan\beta$  which is the ratio of the two Higgs VEVs

needed in SUSY models to break the electro-weak symmetry. (The extensions to non-minimal models will be discussed below). The maximum lifetime of the proton is then calculated as a function of the gluino mass within a given naturalness constraint on  $m_0$ . The result is given in Fig. 1 where we plot the maximum  $p \rightarrow \bar{\nu}K^+$  lifetime for naturalness limits on  $m_0$  of 1 TeV, 1.5 TeV, and 2 TeV. (We estimate an error perhaps of a factor of 2, from uncertainties in quark masses, CKM factors,  $\beta_p$  etc). One may compare these results with the current experimental limit on this decay mode of  $> 1 \times 10^{32}$  yrs[11] and with the lower limit of  $\tau(p \rightarrow \bar{\nu}K^+) > 2 \times 10^{33}$  yrs expected from Super Kamiokande (Super K)[12] and a lower limit of  $\tau(p \rightarrow \bar{\nu}K^+) > 5 \times 10^{33}$  yrs expected from ICARUS[13]. One finds that the expected sensitivity of the Super K will be able to exhaust essentially all of the parameter space of the minimal model except for gluino masses less than about 400 GeV for the case when  $m_0 \leq 1$  TeV. Similarly for  $m_0 \leq 1.5$  TeV the expected lower limit from Super K will be able to exhaust the parameter space of the model for  $m_{\tilde{g}} < 750$  GeV. For the case of  $m_0 \leq 2$  TeV, Super K will not constrain the gluino mass within its naturalness limits of  $m_{\tilde{g}} \leq 1$  TeV. The constraints from ICARUS will be stronger if it can reach its maximum sensitivity of  $5 \times 10^{33}$  yr. Here one will be able to exhaust the gluino mass range  $m_{\tilde{g}} \leq 1$  TeV for  $m_0 \leq 1$  TeV, the gluino mass range  $m_{\tilde{g}} \leq 450$  GeV for  $m_0 \leq 1.5$  TeV, and the the gluino mass range  $m_{\tilde{g}} \leq 800$  for  $m_0 \leq 2$  TeV.

**Relic Density Constraints:** As mentioned in the Introduction, SUSY unified models with R parity invariance imply the existence of an LSP which is absolutely stable, and that over most of the parameter space the LSP is the lightest neutralino which is generally a combination of the two neutral  $SU(2) \times U(1)$  gauginos and the two neutral  $SU(2)$  higgsino states. There are various models for dark matter. In our analysis the details of these models do not play a significant role. Rather we will show that for a broad class of models where the neutralino is the CDM one finds rather stringent constraints on proton stability analyses. The quantity of theoretical interest is  $\Omega_{\tilde{\chi}_1^0} h^2$  where  $\Omega_{\tilde{\chi}_1^0} = \rho_{\tilde{\chi}_1^0} / \rho_c$ , and  $\rho_{\tilde{\chi}_1^0}$  is the neutralino relic density,  $\rho_c$  is the critical relic density needed to close the universe, and h is the Hubble parameter in units of 100 km/sMpc. The current astrophysical data allows one to impose the constraint

$$0.1 \leq \Omega_{\tilde{\chi}_1^0} \leq 0.4 \quad (2)$$

Theoretically one computes the neutralino relic density using the Boltzman equation for the neutralino number density in the early universe, and  $\Omega_{\tilde{\chi}_1^0} h^2$  is obtained by integrating the number density from the current temperature to the freezeout temperature. One has[14]

$$\Omega_{\tilde{\chi}_1^0} h^2 \cong 2.48 \times 10^{-11} \left( \frac{T_{\tilde{\chi}_1^0}}{T_\gamma} \right)^3 \left( \frac{T_\gamma}{2.73} \right)^3 \frac{N_f^{1/2}}{J(x_f)} \quad (3)$$

Here  $x_f = kT_f/m_{\tilde{\chi}_1^0}$ , where k is the Boltzman constant,  $T_f$  is the freezeout temperature,  $T_\gamma$  is the current micro-wave background temperature,  $(T_{\tilde{\chi}_1^0}/T_\gamma)^3$  is the reheating factor,  $N_f$  is the number of degrees of freedom at freezeout, and  $J(x_f)$  is given by

$$J(x_f) = \int_0^{x_f} dx \langle \sigma v \rangle (x) GeV^{-2} \quad (4)$$

Here  $\langle \sigma v \rangle$  is the thermal average,  $\sigma$  is the annihilation cross-section for the neutralinos and  $v$ , their relative velocity.  $\sigma$  involves cross channel sfermion exchanges and direct channel exchanges of Z and Higgs bosons. The annihilation of the neutralinos via direct channel Z and Higgs poles will be seen to play an important role in our analysis. It is thus important that one carries out the correct thermal averaging over these poles. In the analysis of this work we have employed the accurate method for this purpose[15, 16].

We discuss next the implications of imposing simultaneously the proton stability and dark matter constraints. The result of the analysis for the case of the minimal SU(5) unification is exhibited in Fig2. One finds that with the naturalness constraint of  $m_0 = 1$  TeV (solid curve) and with the relic density constraint of  $0.1 \leq \Omega_{\chi_1} h^2 \leq 0.4$  the upper limit on the gluino mass falls below 400 GeV when the current experimental limit on p lifetime of  $\tau(p \rightarrow \bar{\nu} K^+) > 1 \times 10^{32}$  is imposed. In Fig.2 we have also considered the case when the naturalness limit on  $m_0$  is raised to 5 TeV (dashed curve). We find that even in this case the upper limit on the gluino mass does not exceed 500 GeV. This remarkable result arises because the relic density constraint of Eq.(4) requires that  $m_0$  be small, i.e.,  $\leq 200$  GeV, for gluino masses  $m_{\tilde{g}} > 450$  GeV. The reason for this is that in this region, the t-channel sfermion exchanges dominate the neutralino annihilation cross section, and large  $m_0$  (i.e., large sfermion mass) will not give sufficient annihilation to satisfy the upper bound of Eq.(2). However, since the proton decay rate has a rough dependence on  $m_{\tilde{g}}$  and  $m_0$  of the form  $m_{\tilde{g}}/m_0^2$ [2], large values of the gluino mass and small values of  $m_0$  tend to destabilize the proton. Thus unified models in the neutralino annihilation region beyond the Z pole and the Higgs pole region, i.e., in the region  $m_{\tilde{g}} \geq 450$  GeV, tend to act like certain no-scale models ( $m_0 = 0$ ) which also exhibits proton instability[17]. In Table 1 we give the factor by which the proton lifetime is reduced due to the relic density constraint of Eq.(4) as a function of the gluino mass. This reduction factor lies in the range 10-30 and is thus very significant. We note that the reduction factor is largely independent of the GUT physics which enters via the Higgs triplet mass and also is independent of the naturalness assumption on  $m_0$  in this region of the gluino mass as already discussed above. An upper limit of 450 GeV for the minimal supergravity model may be testable at the upgraded Tevatron if it can reach its optimum energy and luminosity, and the minimal model will be fully tested at the LHC. The simultaneous constraints of p stability and relic density constraints also effect the supersymmetric particle spectrum. An example of this is shown in Fig.3 where the minimum and the maximum of the Higgs mass is plotted as a function of the gluino mass. One finds that the Higgs mass has a lower limit of 75 GeV which is consistent with the current LEP data but a significant amount of the parameter space could be tested at LEP190.

**Non-minimal Models:** We discuss now the effects that non-universalities may have on the results of our analysis. As is well known non-universalities in the soft SUSY breaking sector introduce flavor changing neutral currents (FCNC) which are strongly constrained by experiment. One kind of non-universality which is not very stringently constrained by FCNC is the non-universality in the Higgs sector. One may parametrize it at the GUT scale by  $m_{H_1}^2(M_G) = m_0^2(1 + \delta_1)$ ,  $m_{H_2}^2(M_G) = m_0^2(1 + \delta_2)$

with a reasonable range of  $\delta_i$  given by  $|\delta_i| \leq 1$  ( $i=1,2$ ). However, it was pointed out in ref.[8] that one should also consider the non-universalities in the third generation sector along with the non-universalities in the Higgs sector as the non-universalities in the Higgs sector and in the third generation sector give contributions of the same size and may enhance or cancel each other. We therefore parametrize non-universalities in the third generation sector by  $m_{Q_L}^2 = m_0^2(1 + \delta_3)$ ,  $m_{U_R}^2 = m_0^2(1 + \delta_4)$  where as before one limits  $|\delta_i| \leq 1$  ( $i=3,4$ ). The effects of non-universalities in the third generation sector are similar to those in the Higgs sector and we will not consider them in detail here. Instead we will focus on the non-universalities in the Higgs sector. Fig.4 gives the result of the analysis for the case of  $\delta_1 = 1 = -\delta_2$ . We find that the limit on the gluino mass consistent with the proton lifetime constraints falls below 400 GeV and  $\tau(p \rightarrow \bar{\nu}K)$  generally lies below the value of the universal case. We have carried out a similar analysis for the case of  $\delta_1 = -1 = -\delta_2$ . Here, however, we find that in the region of  $m_{\tilde{g}} < 500$  GeV, radiative breaking generally gives rise to a large  $\tan\beta$  and a small  $m_0$  resulting in a maximum proton lifetime already below the current Kamiokande proton lifetime upper limit. For values of  $m_{\tilde{g}} > 500$  GeV the relic density constraint leads to theoretical upper limits on the p-lifetime which also fall below the current experimental limits for p-decay lifetime except in a couple of narrow domains of the gluino mass. These narrow corridors will also be tested when the proton lifetime limit on the  $p \rightarrow \bar{\nu}K$  mode improves by a factor of about 4 over the current limit.

The analysis we presented above was for the SU(5) model. Similar analyses should be valid for all unified models where one has low  $\tan\beta$ , i.e.,  $\tan\beta \leq 25$  because the constraints of the relic density on  $m_0$  would not be substantially modified in this case. The SO(10) models typically have  $b - t - \tau$  unification which implies large  $\tan\beta$ , i.e.,  $\tan\beta \sim 50$ . A large  $\tan\beta$  tends to destabilize the proton requiring a large effective mass  $M_{11}^{-1} \simeq 10^{17-18}$  GeV which tends to ruin the success of the unification of couplings[18, 19, 20] unless one has large threshold corrections in the GUT sector. For these reasons the SO(10) case requires a separate treatment which is outside the scope of this Letter.

Next we discuss the effects that inclusion of textures[21, 22, 23] will have on the analysis. An appropriate treatment of textures requires inclusion of higher dimensional operators in the effective potential such as those due to Planck scale corrections. A rough analysis shows that the  $p \rightarrow \bar{\nu}K$  lifetime is modified by a factor  $\sim (\frac{9}{8} \frac{m_s}{m_\mu})^2$ . Effectively the p-decay lifetime is enhanced by a small numerical factor ( $\sim 3 - 5$ ). The relative suppression due to the relic density constraint remains essentially unchanged when the enhancement factor is included. Including the relative suppression due to relic density constraints as given in Table 1 one finds that the limit on the gluino mass,  $m_{\tilde{g}} \leq 500$  GeV will occur when the experimental lower limit on the proton decay lifetime for the  $p \rightarrow \bar{\nu}K$  mode approaches  $\sim 5 \times 10^{32}$  yr. We expect the results of our analysis also to hold for the Calabi-Yau Models of the type discussed in Ref.[24].

**Implications for Dark Matter Detection:** Finally we discuss implications of the constraints of proton stability on the direct detection of neutralino dark matter, e.g., in the scattering of neutralinos from target nuclei. The result of the analysis is given in Fig.5 for the minimal SU(5) model. One finds that the effect of p stability constraint is to significantly reduce the maximum event rates curves, by as much as

a factor of  $10^3$ . The reason for this is because, as discussed above, proton stability generally requires a large  $m_0$ , which reduces significantly the dark matter detection event rate. (The maximum neutralino mass of about 65 GeV for the dashed curve corresponds to the bound  $m_{\tilde{g}} \leq 450$  GeV in Fig 2.) We expect this typical suppression to hold also in other unified models under the simultaneous constraints of p stability and relic density constraints. The results imply that p stability constraint renders the detection of dark matter significantly more difficult. Thus SUSY unified models which allow p decay via dimension five operators will require significantly more sensitive dark matter detectors, more sensitive by a factor of  $10^3$  or more than those currently available for the detection of supersymmetric dark matter[25].

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Table 1: Reduction of  $\tau(p \rightarrow \bar{\nu}K)_{max}$  from CDM constraint

gluino mass (GeV)	reduction factor when $0.1 < \Omega h^2 < 0.4$
500	29.5
550	23.2
600	18.6
650	15.3
700	12.9
750	11.3
800	9.8

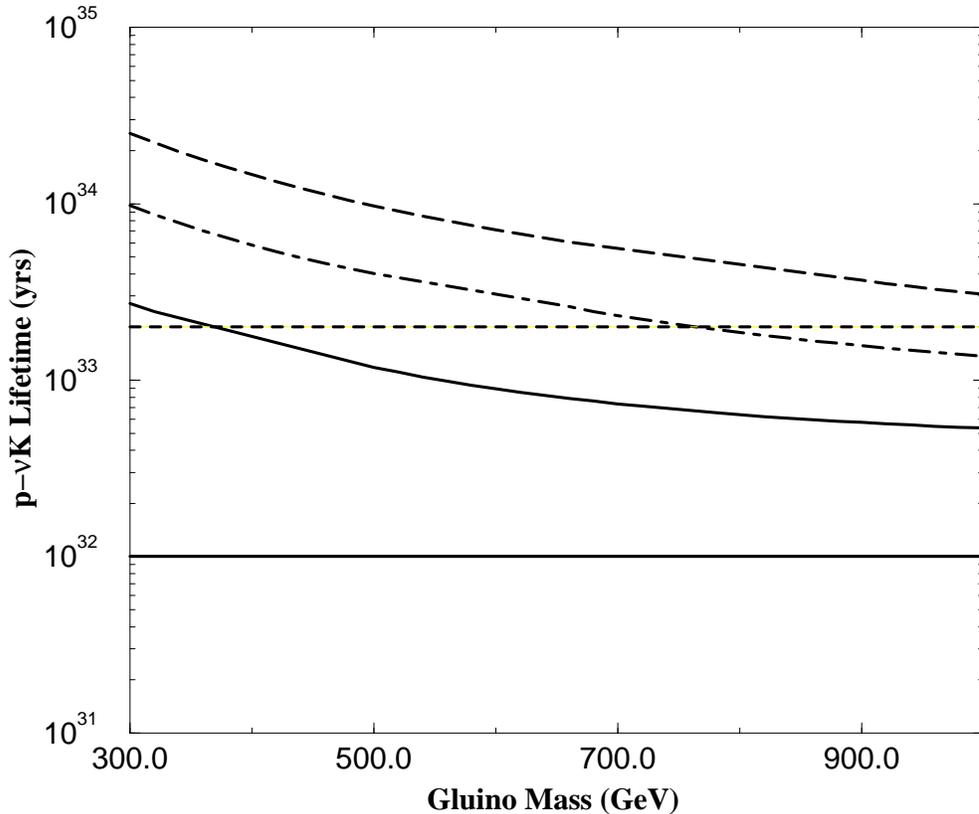


Fig.1. The maximum  $\tau(p \rightarrow \bar{\nu}K)$  lifetime in minimal SU(5) supergravity unification for universal soft breaking as a function of the gluino mass for naturalness limits on  $m_0$  of 1 TeV (solid), 1.5 TeV (dashed-dot) and 2 TeV (dashed). The solid horizontal line is the current experimental lower limit for this mode and the horizontal dashed line is the lower limit expected from Super K.

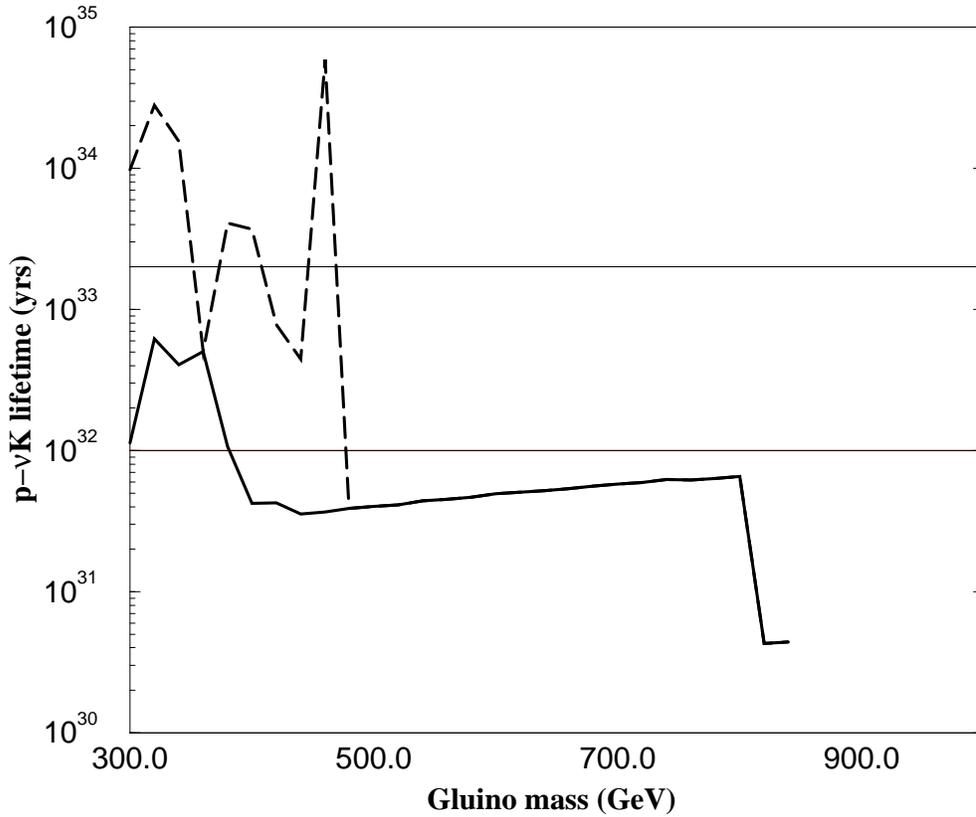


Fig.2. The maximum  $\tau(p \rightarrow \bar{\nu}K)$  lifetime in minimal SU(5) supergravity unification for universal soft breaking as a function of the gluino mass with the constraint  $0.1 < \Omega h^2 < 0.4$  for naturalness limits on  $m_0$  of 1 TeV (solid), and 5 TeV(dashed). The horizontal lines are as in Fig.1.

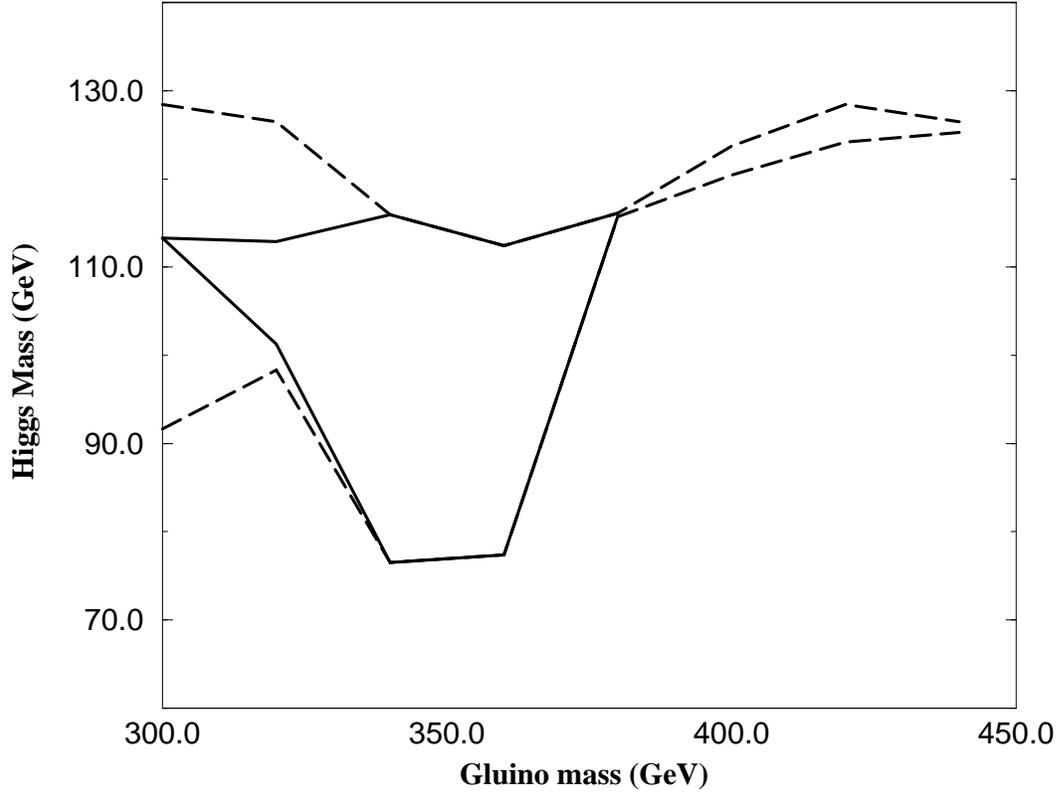


Fig.3. The maximum and the minimum of the Higgs mass as a function of the gluino mass when the p-decay lifetime constraint  $\tau(p \rightarrow \bar{\nu}K) > 1 \times 10^{32}$  yr and the relic density constraint  $0.1 < \Omega h^2 < 0.4$  are imposed in minimal supergravity with naturalness constraints on  $m_0$  of 1 TeV (solid) and 2 TeV(dashed).

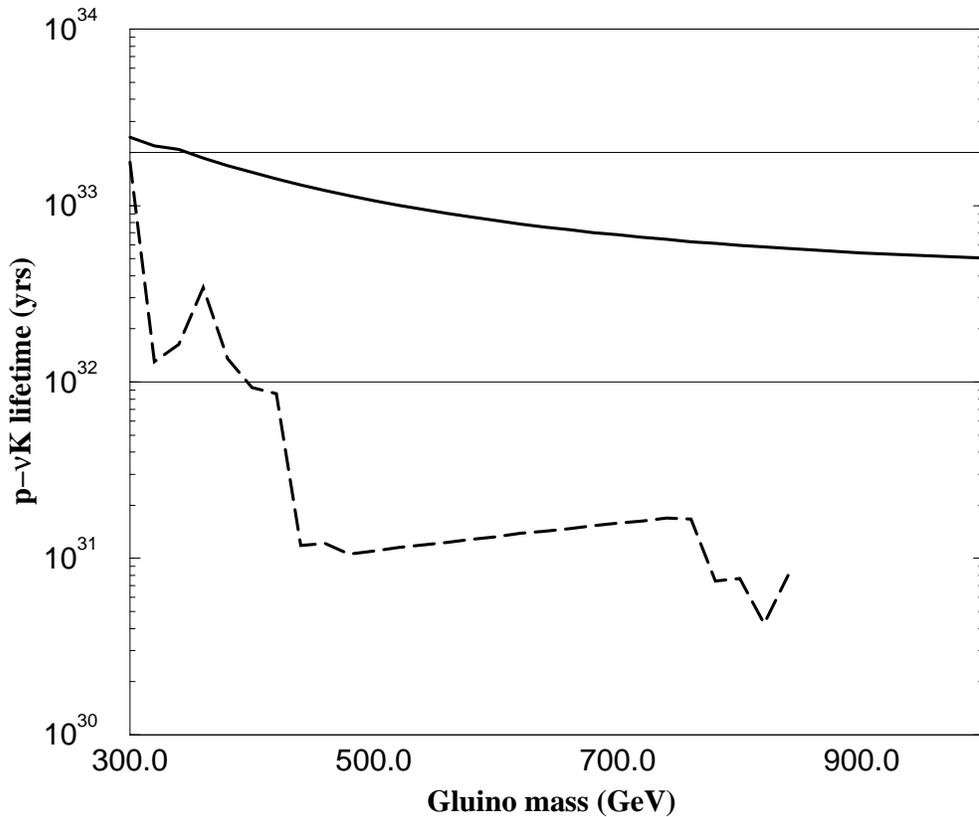


Fig.4. The maximum  $\tau(p \rightarrow \bar{\nu}K)$  lifetime in minimal SU(5) supergravity unification for the non-universal case  $\delta_1 = 1 = -\delta_2$  with limit on  $m_0$  of  $m_0 \leq 1$  TeV as a function of the gluino mass with no constraint on the relic density (solid), and with  $0.1 \leq \Omega h^2 < 0.4$  (dashed). The horizontal lines are as in Fig.1.

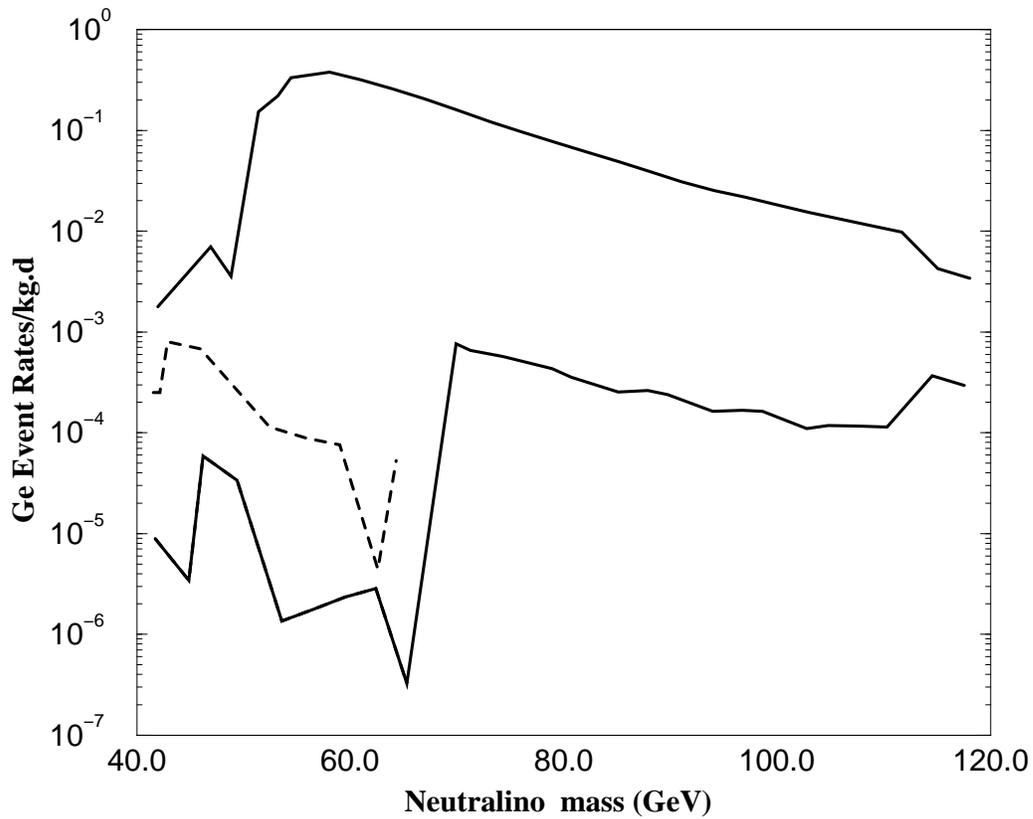


Fig.5. The maximum and minimum of event rates for the scattering of neutralinos off germanium target as a function of the neutralino mass with the relic density constraint  $0.1 < \Omega h^2 < 0.4$  when the naturalness constraint on  $m_0$  is 1 TeV for the cases (1) without p decay constraint (solid), and (2) under the proton lifetime constraint  $\tau(p \rightarrow \bar{\nu}K) > 1 \times 10^{32}$  yr (dashed) (The lower dashed curve coincides with the lower solid curve and is thus not visible).