Universal Thermal Radiation Drag on Neutral Objects

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Abstract

We compute the force on a small neutral polarizable object moving at velocity $\vec{v}$ relative to a photon gas equilibrated at a temperature T We find a drag force linear in $\vec{v}$. Its physical basis is identical to that in recent formulations of the dissipative component of the Casimir force. We estimate the strength of this universal Casimir drag force for different dielectric response functions and comment on its relevance in various contexts.

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The residual drag force on an AFM tip close to, but not in direct contact with a substrate *in vacuo* raises an important and fundamental question on the origin of non-contact friction [1]. Other experimental techniques also are sensitive to non-contact friction [2]. Since it became clear that the Casimir effect, being *par excellence* a non-contact phenomenon, can lead to a dissipative drag (see [4] and references therein), it has become a primary focus of theoretical research [3]. Such Casimir dissipative drag occurs when electromagnetic field fluctuations equilibrate in a specific reference frame, relative to which another system (e.g. a dielectric or a conducting body) is in uniform motion [4, 5, 6]. The difference between the frames of reference in the dissipative Casimir effect can be due to relative motion of *different* bodies, as for two conducting plates with relative motion in the parallel direction, or for a neutral body moving relative to a conducting plate [7, 8, 9, 10, 4, 11]. In these cases the radiation equilibrates in one of the plates, and the friction depends upon the proximity of the other one.

We show here that such friction can also come about when a *single* body moves relative to a thermal bath of the electromagnetic field excitations, such as those between the walls of an oven or in the cosmic microwave background. The friction has no position dependence, *i.e.* it is spatially homogeneous. The consequence is a universal dissipative drag acting on all matter in relative motion with respect to a thermalized photon gas. To estimate the magnitude of this universal drag we evaluate it as a function of the dominant frequency of the electromagnetic response of the body for dielectrics and conductors.

Consider the Lorentz force

$$\vec{F} = \int d^3\vec{r} \left(\rho\vec{E} + \vec{j} \times \vec{B}\right). \tag{1}$$

on a dielectric in a field $\vec{E}(\vec{r},t)$, $\vec{B}(\vec{r},t)$. The charge and the current densities in the dielectric are set by the polarization, $\vec{P}(\vec{r},t)$, such that $\rho(\vec{r},t) = -\vec{\nabla}\cdot\vec{P}$ and $\vec{j}(\vec{r},t) = \partial_t\vec{P}$, where $\vec{P}$, $\vec{j}$ obviously obey the continuity equation $\partial_t\rho + \vec{\nabla}\cdot\vec{j} = 0$. Assume that the response of the matter to the field is both linear as well as spatially local. Thus we write

$$\vec{P}(\vec{r},t) = \epsilon_0 \int \chi_e(t-t')\vec{E}(\vec{r},t')dt', \tag{2}$$

in MKS units, where the dielectric susceptibility χ_e is dimensionless. In the frequency domain $\chi_e(\omega)$ is equal to $(\epsilon(\omega)-1)/\epsilon_0$ for condensed matter. For weakly interacting molecules or atoms $\chi_e = \rho_N\alpha_m$, with ρ_N the atomic or molecular number density of the medium and α_m the polarizability of the single molecule or atom.

With the polarization proportional to the field, the force is bilinear in the field. This bilinearity holds for objects moving at arbitrary non-relativistic velocities relative to the frame of reference of the thermal bath. We require the thermal average of the force acting on a moving body in unbounded space filled with radiation at rest, working in the reference frame where the particle is instantaneously at rest and the photon gas moves at velocity $\vec{v}$. The average force is obtained in terms of the thermal averages of the Fourier components of the field correlations. In unbounded space the Fourier components of the polarization and electric fields are

$$\vec{P}(\vec{k},\omega), \vec{E}(\vec{k},\omega) = \int dt \int d^3\vec{r}\ \vec{P}(\vec{r},t), \vec{E}(\vec{r},t)\ e^{-i(\vec{k}\cdot\vec{r}-i\omega t)} \tag{3}$$

leading to the following form of the thermal average $\langle..\rangle$ of the Lorentz force

$$\begin{aligned}\left\langle\vec{F}\right\rangle = -i\epsilon_0 \int d^3\vec{r} \int \frac{d^3\vec{k}d\omega}{(2\pi)^4} \int \frac{d^3\vec{k}'d\omega'}{(2\pi)^4} e^{-i(\vec{k}+\vec{k}')\cdot\vec{r}+i(\omega+\omega')t}\ \chi_e(\omega) \times \\ \times \left\langle \frac{\omega}{\omega'} \left(\vec{k}' \left(\vec{E}(\vec{k},\omega)\vec{E}(\vec{k}',\omega')\right) - \left(\vec{k}'\cdot\vec{E}(\vec{k},\omega)\right)\vec{E}(\vec{k}',\omega')\right)\right\rangle,\end{aligned} \tag{4}$$

where we took note of the fact that in empty space the electric field has no sources. A slight generalization of the standard expression ([12] Eq. 77.12) for the thermal average of the correlator of the electric field vectors yields

$$\left\langle E_i(\vec{k},\omega)E_j(\vec{k}',\omega')\right\rangle = (2\pi)^4\delta(\vec{k}+\vec{k}')\delta(\omega+\omega')\left\langle E_iE_j\right\rangle_{\vec{k},\omega}. \tag{5}$$

with

$$< E_i E_j >_{\vec{k},\omega} = \frac{2\pi^2\hbar}{\epsilon_0 k}\left(\frac{\omega^2}{c^2}\delta_{ij} - k_i k_j\right)\left[\delta(\frac{\omega}{c} - k) - \delta(\frac{\omega}{c} + k)\right]\left(1 + 2n(\omega, \vec{k})\right), \qquad (6)$$

where $k = |\vec{k}|$ and

$$n(\omega, \vec{k}) = \frac{1}{e^{\beta\hbar(\omega - \vec{k}\cdot\vec{v})} - 1}$$

is the Bose occupation number for a photon distribution at temperature T, moving with velocity $\vec{v}$, and $\beta = (k_B T)^{-1}$. Eq. 6 is obtained by generalizing the fluctuation-dissipation theorem to a translationally invariant system, and taking a Gibbs distribution corresponding to a photon gas moving with velocity $\vec{v}$, just as done for excitations in superfluidity [13]. From Eq. 6 we end up with the following average of the Lorentz force

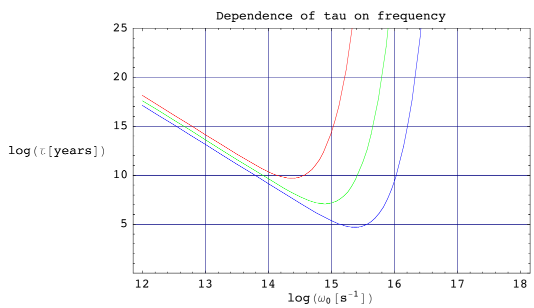

Figure 1: Dependence of the logarithm of the relaxation time in years on the logarithm of the primary relaxation frequency (ω_0 in Eq. 14) for three different temperatures, $T = 300$ K, $T = 1000$ K, $T = 3000$ K (upper, middle and lower curve). The frequency of the minimum is given by $\hbar\omega_0 = 5.9694\ k_B T$.

$$\left\langle \vec{F} \right\rangle = -i\epsilon_0 \int d^3\vec{r} \int \frac{d^3\vec{k}\ d\omega}{(2\pi)^4}\ \chi_e(\omega)\ \vec{k} \left\langle \vec{E}\cdot\vec{E} \right\rangle_{\vec{k},\omega}. \qquad (7)$$

Substitution of (4) in (5) with $\vec{v} = 0$ gives an integral over $\vec{k}$ that, by symmetry, is zero. To obtain a non-zero result it is convenient to first eliminate the delta functions by integrating over ω and then expanding in powers of $\vec{v}$ (as done to obtain the normal fluid density in the theory of superfluidity). Integration over ω then yields

$$\begin{aligned} < \vec{F} > &= -i\ 2\pi\hbar c\ V \int \frac{d^3\vec{k}}{(2\pi)^3}\ k\vec{k}\ \chi_e(ck)\left(1 + 2n(ck, \vec{k})\right) + \\ & i\ 2\pi\hbar c\ V \int \frac{d^3\vec{k}}{(2\pi)^3}\ k\vec{k}\ \chi_e(-ck)\left(1 + 2n(-ck, \vec{k})\right), \end{aligned} \qquad (8)$$

where we have taken the dielectric to be homogeneous so the integral over its volume simply gives V.

The same equation also can be obtained from energy loss considerations, following the arguments of Volokitin and Persson [10]. One starts from the dissipation rate of the energy in the rest frame of the fluctuating field,

$$\frac{dW}{dt} = \int d^3\vec{r} \left(\vec{j} + \rho\vec{v}\right) \cdot \vec{E} = \frac{dW_0}{dt} - \vec{F} \cdot \vec{v}. \tag{9}$$

Here we take into account that in the rest frame of the thermalized photon gas the total electric current is given *via* the linearized Lorentz form $\vec{j} \longrightarrow \vec{j} + \rho\vec{v}$. Apart from the heat production $\frac{dW_0}{dt}$ in the frame of the body, Eq. 8 is reproduced immediately.

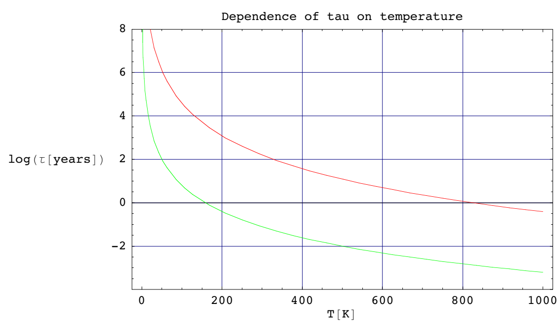

Figure 2: Dependence of the relaxation time in years on temperature. For the dielectric response (upper curve) we plot the minimal relaxation time obtained for the relaxation frequency that is proportional to the first Matsubara frequency. We take ρ_M to be the density of water and the dimensionless measure of atomic polarizability $\alpha_0 = (N/V)\epsilon\alpha_m \approx 1$. Numerically this value corresponds to a single particle of molecular polarizability $\epsilon_0\alpha_m \approx 1.0 \times 10^{-30}$ m^3 and a mass equal to the proton mass 1.67×10^{-27} kg. For the metallic response (lower curve) we take the characteristic time $\tau_0 = \frac{\epsilon_0}{\sigma} \sim 10^{-18}$ s, well within the range of conductivities of simple metals.

We now write, in a standard way, $\chi_e(x) = \chi_e'(x) + i\chi_e''(x)$, noting that the real part is an even and the imaginary part is an odd function of the argument. The expression for the force can now be obtained in the following form

$$< \vec{F} >= -i\,\frac{4\pi\hbar cV}{(2\pi)^3} \int d^3\vec{k}\; k\vec{k}\;\; \Big[\chi_e'(ck)\left(n(ck,\vec{k}) - n(-ck,\vec{k})\right) + \\ + i\;\chi_e''(ck)\left(1 + n(ck,\vec{k}) + n(-ck,\vec{k})\right)\Big]\,. \tag{10}$$

Expanding the Bose function to the lowest order in velocity, we have $n(ck,\vec{k}) - n(-ck,\vec{k}) = \coth\frac{1}{2}\beta\hbar ck + \mathcal{O}(v^2)$ and $1 + n(ck,\vec{k}) + n(-ck,\vec{k}) = \frac{1}{2}\mathrm{csch}^2\frac{1}{2}\beta\hbar ck\;\left(\beta\hbar(\vec{k}\cdot\vec{v})\right) + \mathcal{O}(v^3)$.

Placed in Eq. 8, the $\vec{k}$-space integral over the term in $\chi'_e(ck)$ is zero, by symmetry. The remaining term then yields

$$< \vec{F} >= (8\pi\beta\hbar^2 c)\ V\ \int \frac{d^3\vec{k}}{(2\pi)^3}\ \frac{k\ \vec{k}(\vec{k}\cdot\vec{v})\ \chi''_e(ck)}{\sinh^2\frac{1}{2}\beta\hbar ck}, \tag{11}$$

Performing the angular integral over $d^3\vec{k}$ gives a factor of $4\pi/3$. Reverting again to ω by substituting $k = \omega/c$ gives

$$< \vec{F} >= V\ \vec{v}\ \left(\frac{\beta\hbar^2}{3\pi c^5}\right)\ \int_0^\infty d\omega\ \frac{\omega^5\ \chi''_e(\omega)}{\sinh^2(\frac{1}{2}\beta\hbar\omega)}. \tag{12}$$

This is the fundamental result of our paper. It states that the EM field fluctuations exert a drag, proportional in the lowest order to the velocity, on a particle that moves with respect to the frame of reference in which the EM field fluctuations are thermalized. Setting this force to $M\vec{v}/\tau$, where M is the total mass of the object and $1/\tau$ is the drag time, and using $\rho_M = M/V$, yields the result that

$$\frac{1}{\tau} = \left(\frac{\beta\hbar^2}{3\pi\rho_M c^5}\right)\int_0^\infty d\omega\ \frac{\omega^5\chi''_e(\omega)}{\sinh^2(\frac{1}{2}\beta\hbar\omega)}. \tag{13}$$

Regard this fundamental result in three different contexts: molecules, dielectric and conducting condensed matter. First assume that the dielectric response of the medium can be characterized by a single sharp absorption line at ω_0. Because $\frac{1}{\tau}$ is proportional to $\chi''_e(\omega)$, obviously each of the absorption lines for a molecule or a dielectric will contribute additively to the integral in Eq. 13. We set $\chi''_e(\omega) = \alpha_0\delta(\omega/\omega_0 - 1)$, where α_0 is a constant describing the strength of the response. This assumption gives

$$\tau = \left(\frac{3\pi n c^5}{\beta\hbar^2}\right)\ \frac{\sinh^2(\frac{\beta\hbar\omega_0}{2})}{\alpha_0\omega_0^6} = \left(\frac{3\pi\rho_M c^5\hbar^4}{2^6\alpha_0(k_BT)^5}\right)\ \frac{\sinh^2(x)}{x^6}, \tag{14}$$

after introducing $x = \frac{1}{2}\beta\hbar\omega_0$. In this form the relaxation time depends strongly on the absorption frequency ω_0. It has a minimum at a temperature dependent frequency (see Fig. 1) that coincides with the minimum of the function $f(x) = \frac{\sinh^2(x)}{x^6}$, at $x_m = 2.98$, where $f(x_m) = 0.137$. Because $f(x)$ is the square of the function $\frac{\sinh(x)}{x^3}$, it has a broad minimum with a quartic, rather than a quadratic, dependence upon the deviation from x_m. Taking this minimum at x_m into account, the smallest possible relaxation time can thus be obtained from the above equation in the form

$$\tau = \tau_0\left(\frac{T_0}{T}\right)^5, \tag{15}$$

where $\tau_0 T_0^5 = \left(\frac{3\pi f(x_m)}{2^6}\right)\ \frac{\rho_M c^5\hbar^4}{\alpha_0 k_B^5}$. At this minimum the absorption frequency $\hbar\omega_0 =\ 2\,x_m\ k_BT$ is proportional to the first Matsubara frequency. The temperature dependence of this minimal possible relaxation time is given in Fig. 2.

A different formula for τ is obtained for metals, which have constant conductivity at frequencies below the collision time of their charge-carriers. In this case $\chi_e(\omega) \approx -\frac{\sigma}{i\epsilon_0\omega}$. Inserting this into Eq. 13, the inverse relaxation time for drag now takes the form

$$\frac{1}{\tau} = \left(\frac{\beta\hbar^2\sigma}{3\pi\rho_M c^5\epsilon_0}\right)\int_0^\infty d\omega\ \frac{\omega^4}{\sinh^2(\frac{1}{2}\beta\hbar\omega)}, \tag{16}$$

so that

$$\tau = \tau_0\left(\frac{T_0}{T}\right)^4, \tag{17}$$

with $\tau_0 = \frac{\epsilon_0}{\sigma}$ and $T_0^4 = \frac{45c^5\hbar^3\rho_M}{16\pi^2k_B^4}$. For most common metals the value of τ_0 is between $10^{-19} - 10^{-17}$ s. Taking its geometrical average we obtain the temperature dependence of the relaxation time as given in Fig. 2.

The times given in the figures are relatively long, corresponding to the general weakness of the Casimir interaction and the even lower strength of the dissipative Casimir effect. Two circumstances under which such long times might be observable are ovens and the cosmos.

For ovens, on trapping molecules with unusually large polarizabilities it might be possible to observe a resonance in an ion or atom trap with a quality factor $\mathcal{Q}$ that is determined by the dissipative Casimir interaction. Small metal particles in a high temperature oven should also be susceptible to the effects of the Casimir drag.

For the cosmos, it is believed that hydrogen atoms condensed from protons and electrons at about 3000 K, and that the coupling of radiation and matter due to Compton scattering becomes ineffective below this condensation temperature [14]. However, as is clear from Fig. 2, it should not be difficult for molecules to remain coupled to the cosmic microwave background when the temperature was between about 300 K (and perhaps a bit less) and the 3000 K condensation temperature. This coupling could have an influence on the structure and anisotropies observed in recent experiments on the cosmic microwave background. It could also influence the behavior of molecules formed from the residue of novas and supernovas, and then subject to drag from a still-hot cosmic microwave (i.e., electromagnetic) background.

The universal thermal drag calculated in this contribution is yet another important facet of the Casimir effect, which in this context appears to play a role not only in the static interactions between dielectrically inhomogeneous bodies but also in the dissipative dynamics of particles in homogeneous space.

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