

# Superconducting Vortices and Elliptical Ferromagnetic Textures.

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In this article an analytical and numerical study of superconducting thin film with ferromagnetic textures of elliptical geometries in close proximity is presented. The screening currents induced in the superconductor due to the magnetic texture are calculated. Close to the superconducting transition temperature  $T_c$  the spontaneous creation of superconducting vortices becomes energy favorable depending on the value of the magnetization and the geometrical quantities of the magnetic texture. The creation of vortices by elliptic dots is more energy favorable than those created by circular ones. The superconductor covered by elliptic dots array exhibits anisotropic transport properties.

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The interaction between superconducting vortices and ferromagnetic textures in heterogeneous ferromagnetic-superconducting systems has been under intensive study in the past few years. The static and dynamical phases of these systems have shown richness and promise for superconductor based technology. Basically, such a system consists of a superconducting thin film placed either on the top or under a ferromagnet and a layer of insulator oxide is sandwiched between them to suppress proximity effects and spin diffusion. Different settings and geometries for both the superconductor and ferromagnet were studied and analyzed both experimentally and theoretically [1]-[13]. The interaction between a circular FM dot and superconducting thin film is well studied for magnetization distributions either parallel or perpendicular to the SC layer. Pinning effects and transport in superconductors interacting with periodic array of ferromagnetic dots [3] or periodic stripe domains structure [11], [13] were studied.

These studies showed that in some range of temperatures and above a threshold value of the dot magnetization, the interaction between superconductivity and ferromagnetism dominates over other interactions in the system. At such conditions for the temperature and magnetization a spontaneous vortex phase forms in the superconductors. Additionally, experimental measurements and theoretical predictions assert that the usage of a periodic pinning array of ferromagnetic textures results in higher values of the superconducting critical current than those obtained from random pinning by lattice inhomogeneities. Almost all previous studies treated the interaction between superconducting vortices and FM textures focused on textures with circular geometry. However, many studies have investigated the interaction of SC vortices with elliptical lattice inhomogeneities such as elliptic holes and elliptic columnar defects [14], [15], [16], [17]. To our knowledge the interaction between SC vortices and elliptic ferromagnetic dots (EMD) has not yet been studied.

In this article, I present a theoretical treatment of the

interaction between SC vortices and elliptic ferromagnetic textures. The study of the interaction between elliptic dots and superconductivity is interesting since its results when the dot's eccentricity  $\mathcal{E}$  is zero correspond to those known results for circular dots. Another interesting limit is when  $\mathcal{E} \rightarrow 1$  which mimic a system of long magnetic stripe domains interacting with an SC film. This article is organized such that in the first section, we calculate the magnetic fields and screening currents for a system of elliptic FM dot on the top of an SC thin film. Section two is devoted to calculate the total energy and pinning forces. In section three, we discuss the dependence of the energy on the eccentricity of the FM texture. More detailed analysis of pinning forces and qualitative discussion of the transport in these systems will be presented. Concluding remarks and a summary of this work will be given in the last section.

Since the SC and FM are electronically separated, the interaction between them is mediated via their magnetic fields. The FM dot produces a magnetic field which penetrate the SC film and alters the distribution of its screening current. In turns the SC generates a magnetic field in and out of its plane which interact with the FM dot. The problem of finding the magnetic field and screening currents must be solved self consistently. To do so, let us consider a superconducting thin film of thickness  $d_s$ , whose coherence length is  $\xi$  and its penetration depth is  $\lambda$  in the  $xy$ -plane. We place on the top of it at a distance  $D \ll \lambda$  an elliptical ferromagnetic dot of major axis  $R_1$  and minor axis  $R_2$ . Let the dot magnetization  $\mathbf{M}$  be directed along the  $z$ -axis, the magnetization distribution can be written as

$$\mathbf{M}(x, y, z) = m_0 \Theta\left(1 - \frac{x^2}{R_1^2} - \frac{y^2}{R_2^2}\right) \delta(z - D) \hat{z} \quad (1)$$

where  $m_0$  is the 2D magnetization,  $\Theta(r)$  is the step function and  $\delta(r)$  is Dirac delta function. In the presence of the superconductor the magnetic vector potential  $\mathbf{A}_{\mathbf{m}}$  of

the dot satisfy the London-Maxwell equation

$$\nabla \times \nabla \times \mathbf{A}_m + \frac{1}{\lambda} \mathbf{A}_m \delta(\mathbf{z}) = 4\pi \nabla \times \mathbf{M} \quad (2)$$

Accepting the gauge  $\nabla \cdot \mathbf{A}_m = 0$ , and using the integral Fourier representation for  $\mathbf{A}_m$ , we finds

$$\tilde{\mathbf{A}}_m(\mathbf{K}) = \frac{-8\pi^2 i m_0 R_1 R_2 J_1(G(k_x, k_y))}{G(k_x, k_y)(k_z^2 + q^2)} \times \left( e^{i k_z D} - \frac{e^{-qD}}{1 + 2\lambda q} \right) \hat{z} \times \mathbf{q} \quad (3)$$

where  $\tilde{\mathbf{A}}_m$  is the magnetic dot vector potential in Fourier representation and  $\mathbf{q} = k_x \hat{x} + k_y \hat{y}$  is Fourier wave vector in the plane of the SC. The function  $J_n(r)$  is the  $n$ -th order Bessel Function, and  $G(k_x, k_y) = \sqrt{R_1^2 k_x^2 + R_2^2 k_y^2}$ . By using  $\mathbf{B} = \nabla \times \mathbf{A}$ , the components of the dot's magnetic field can be calculated

$$B_{mz} = m_0 R_1 R_2 \int \frac{q J_1(G(k_x, k_y)) Z(k_x, k_y)}{G(k_x, k_y)} \times e^{-i(k_x x + k_y y)} d^2 q \quad (4)$$

$$B_{mj} = i m_0 R_1 R_2 \int \frac{k_j J_1(G(k_x, k_y)) W(k_x, k_y)}{G(k_x, k_y)} \times e^{-i(k_x x + k_y y)} d^2 q \quad (5)$$

where  $j = x, y$ , while  $Z(k_x, k_y) = e^{-q|z-D|} - \frac{e^{-q(|z|+D)}}{1+2\lambda q}$ , and  $W(k_x, k_y) = e^{-q|z-D|} \text{sign}(z-D) - \frac{e^{-q(|z|+D)}}{1+2\lambda q} \text{sign}(z)$ . The in-plane components of the EMD magnetic fields have a jump at  $z = 0$  which should be taken into account. The  $z$ -component of the dot's magnetic field is depicted in Fig.(1). The magnetic field of the dot changes strongly across the dot's circumference due to large values of  $\nabla \cdot \mathbf{M}$  there. If vortices are present in the SC film then the total magnetic field is a linear superposition of the field from the EMD and that of the vortices. The  $z$ -component of the magnetic field due to a singly quantized SV centered at the origin [18] reads

$$B_v^z(x, y, z) = \frac{\phi_0}{2\pi} \int_0^\infty \frac{q J_0(q \sqrt{x^2 + y^2}) e^{-q|z|}}{1 + 2\lambda q} dq \quad (6)$$

where  $\phi_0 = \frac{\pi \hbar c}{e}$  is the magnetic flux quantum.

Let us assume that there are an  $N$  spontaneously created vortex in the superconductor. The total energy of for a system of  $N$  vortices coupled to an FM texture is made up of four different contributions and can be written as

$$U = U_{sv} + U_{vv} + U_{mv} + U_{mm} \quad (7)$$

where  $U_{sv}$  is the energy  $N$  non-interacting singly quantized vortices,  $U_{vv}$  is the vortex-vortex interaction,  $U_{mv}$  is the interaction energy between the FM and SC, and

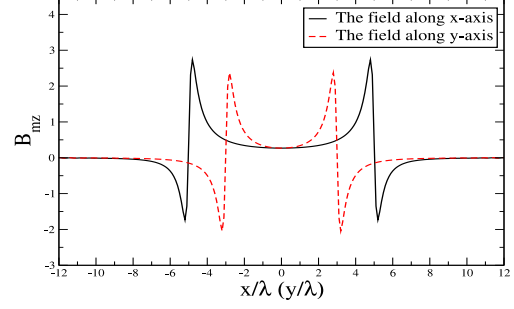


FIG. 1: The magnetic field in units of  $\frac{2m_0}{\lambda}$  measured along the semi major axis (solid line) and semi-minor axis (dashed line) for EMD with  $R_1 = 5$  and  $R_2 = 3$

$U_{mm}$  is FM dot self interaction. In [7], it was shown that the total energy of the system may be rewritten as follows:

$$E = \int \left[ \frac{n_s \hbar^2}{8m_e} (\nabla \varphi)^2 - \frac{n_s \hbar e}{4m_e c} (\nabla \varphi \cdot \mathbf{a}) - \frac{1}{2} \mathbf{m} \cdot \mathbf{b} \right] d^2 x \quad (8)$$

where  $n_s$  is the two-dimensional superconducting electrons density and  $m_e$  is their effective mass.  $\hbar$  and  $c$  are the Planck constant and the speed of light respectively. The vectorial quantities  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{m}$  are the total vector potential and magnetic field due to the  $N$  SC-vortices and the FM dot evaluated at the surface of the superconductor and the  $2D$  magnetization of the FM texture. The phase gradient of the SC order parameter in the presence of  $N$  vortices is  $\nabla \varphi = \sum_{n=1}^N \frac{(\rho - \rho_n) \times \hat{z}}{|\rho - \rho_n|^2}$ , where  $\rho_n$  is the location of the  $n$ -th vortex. In the presence of  $N > 1$  superconducting vortices the interaction of the vortices with the dot tries to lower the energy of the system due to its attractive nature while it is increased by the repulsive vortex-vortex interaction. If  $N$  vortices are coupled to the FM dot then we can recast the energy of the EMD-SC system using the identity  $\int d^2 x \rightarrow \frac{1}{4\pi^2} \int d^2 k$  as follows

$$E = N \epsilon_0 \ln \left( \frac{\lambda}{\xi} \right) + \epsilon_0 \lambda \sum_{i=1}^N \sum_{j \neq i}^N \int_0^\infty \frac{J_0(\kappa |\rho_i - \rho_j|)}{1 + 2\lambda \kappa} d\kappa - \frac{m_0 \phi_0 R_1 R_2}{\pi} \sum_{i=1}^N \Gamma(R_1, R_2, x_i, y_i) + E_{mm} \quad (9)$$

where  $\kappa$  has a dimension of inverse length,  $\epsilon_0 = \frac{\phi_0^2}{16\pi^2 \lambda}$ , and the function  $\Gamma(R_1, R_2, x_i, y_i)$  is defined as follows

$$\Gamma(R_1, R_2, x_i, y_i) = \int \frac{J_1(G(k_x, k_y)) e^{i(k_x x_i + k_y y_i)}}{G(k_x, k_y) (1 + 2\lambda q)} d^2 q \quad (10)$$

Vortex configurations for  $N = 1$  and  $N = 2$  are shown in Fig.(2). For  $N = 1$ , the vortex appears under the

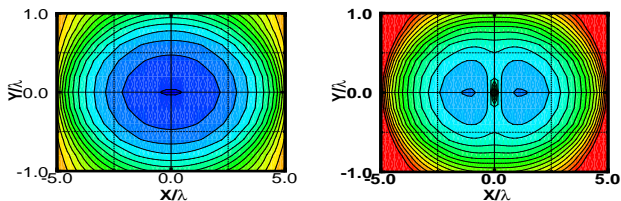


FIG. 2: (Energy profiles for  $N = 1$  and  $N = 2$  in the EMD-SC system. The EMD has major (minor) axis  $R_1 = 5\lambda$  ( $R_2 = \lambda$ ) and  $m_0\phi_0 = 10\epsilon_0$ .

center of the dot while for  $N = 2$ , vortices centers are located on the semi-major axis at equal distances from the center of the dot to minimize the total energy of the system. The degeneracy of the two vortices locations in the case of circular dot on the top of the SC film is lifted by the shape anisotropy of the dot elliptic dot. The creation of vortex configurations with  $N > 2$  requires larger values of  $\delta_m = \frac{m_0\phi_0}{\epsilon_0}$  to overcome the pearl energy and the repulsive vortex-vortex interaction. Vortex arrangements of  $N > 2$  depend on the ratio  $\frac{R_2}{R_1}$ . For  $R_2 \sim \lambda$ , vortices would line-up forming a straight chain of vortices extending under the semi-major axis of the dot. When  $R_2 \gg \lambda$ , the arrangement of vortices becomes more complex. Energy and vortex lattice structure for  $N \gg 1$  can be found numerically by minimizing the total energy of the system given by Eq.(9).

The energy of the single vortex depends on the eccentricity of the dot. To study this dependence, the energy of a single SC vortex coupled to an FM dot of fixed  $R_1$  and variable  $R_2$  must be calculated. The energy dependence on  $R_2$  is represented by the solid curve in Fig.(3). This shows that the lowest energy for  $N = 1$  configuration is reached when  $R_2 = R_1$ . However, this does not imply that spontaneous creation of superconducting vortices is more energy favorable if the dot is circular. This is because the magnetic flux supplied by the dot is maximum when  $R_1 = R_2$ . To better understand this, I compare the energy necessary to spontaneously create a single vortex by an elliptic dot with fixed  $R_1$  and varying  $R_2$  to the energy of a vortex created by a circular dot with the same per unit area magnetization  $m_0$  and radius  $R_c = \sqrt{R_1 R_2}$ . The magnetic flux due to both dots is equal since their areas are equal. The curves in Fig(3) shows that the creation of vortices by an elliptic dots is more energy favorable than those created by circular ones and has the same magnetic flux. The difference between the two curve is a reminiscence of the shape anisotropy of the FM dot.

The appearance of a vortex under the dot changes the energy of the system by an amount of  $\Delta = U_{sv} + U_{vv} + U_{mv}$  the vortex appear when  $\Delta = 0$ . This criterion produces a surface in 3D space parametrized by  $\frac{R_1}{\lambda}$ ,  $\frac{R_2}{\lambda}$ . The surface  $\Delta = 0$  separates between regimes with and without vortices. Phase transitions from  $N = 0$  regime

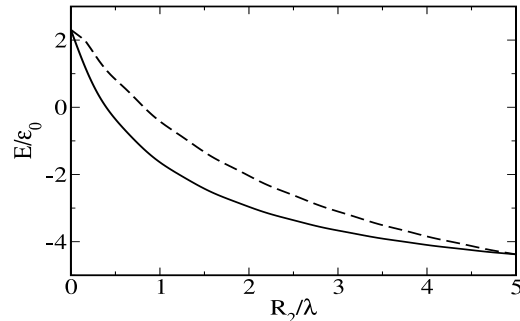


FIG. 3: The solid line is energy of a single vortex in the presence of an elliptic dot whose semi-major axis  $R_1 = 5\lambda$  as a function of  $R_2$ . The dashed line is the energy of a single vortex created by a circular dot of radius  $R_c = \sqrt{R_1 R_2}$ . In either cases  $\delta_m = 2$

to  $N = 1$  and  $N = 2$  regimes are shown in Fig.(4). Note that for strongly eccentric dot *i.e.*  $R_2 \ll R_1$  the spontaneous creation of vortices requires large values of  $\frac{m_0\phi_0}{\epsilon_0}$  due to small stray field of the dot.

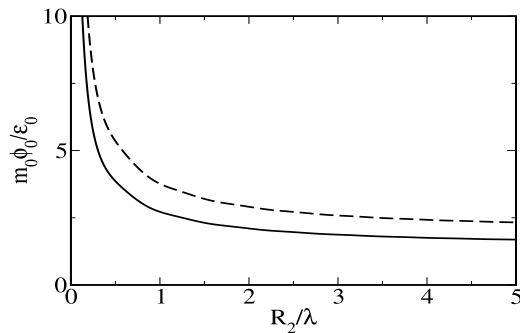


FIG. 4: The solid (dashed) curve separate the regime without vortices from the regimes with  $N = 1$  ( $N = 2$ ) vortices in the SC for  $R_1 = 5\lambda$ .

Now, let us consider a square array of identical elliptic FM dots on the top of a superconducting thin film. Let all dots have their semi-major axis aligned along the  $x$ -axis, and they are well separated so that the dipolar interaction between them could be ignored. If  $\delta_m$  is larger than a critical value then vortices appear under the dots. Due to the conservation of topological charge, equal number of antivortices will appear in the regions between the dots. In the presence of the antivortices the total energy of the system must include their interaction with the dot array and vortices and other antivortices in the system. For large enough array and a filling of one vortex per dot, vortices appear under the centers of the dots while antivortices will appear at the centers of the unit cells. This is so only if finite size effects are ignored. Since these ef-

fects violate the symmetry of the vortex lattice causing a shift in the locations of vortices and antivortices. Pinning forces acting on vortices are due to their interaction with the FM dots array and the vortex-antivortex interaction. Since the dots are well separated, the  $i$ -th vortex feels mostly the pinning potential created by the dot above it

$$U_{mv} = -\frac{m_0\phi_0 R_1 R_2}{\pi} \Gamma(R_1, R_2, x_i, y_i) \quad (11)$$

The pinning by antivortices is isotropic and regular and can be represented by a two-dimensional washboard potential. The pinning force exerted by the FM dot on a single vortex in the SC is derivable from  $U_{mv}$  and its components are

$$F_j(x_i, y_i) = -\frac{m_0\phi_0 R_1 R_2}{\pi} \Xi_j(R_1, R_2, x_i, y_i) \quad (12)$$

where  $j = x, y$ . Here the function  $\Xi_j(R_1, R_2, x, y)$  is defined as follows:

$$\Xi_j(R_1, R_2, x, y) = \int \frac{k_j J_1(G(k_x, k_y)) e^{i(k_x x + k_y y)}}{G(k_x, k_y) (1 + 2\lambda q)} d^2 q \quad (13)$$

The shape anisotropy of the dot manifests itself in the pinning potential  $U_{mv}$  and the pinning forces. Anisotropic pinning forces implies anisotropic transport properties such as anisotropic critical current. In other words the critical current  $J_c$  for this system may depend on the angle  $\theta$  between the driving current and the semi-major axis of the dots. It also must depend on  $\delta_m$  and the eccentricity of the dots. For fixed value of  $\delta_m$  and  $\frac{R_2}{R_1}$ , the strength of the transport anisotropy can be measured through the ratio  $K_1 = \frac{J_c(\theta=\frac{\pi}{2})}{J_c(\theta=0)}$ . To detect the effect of the dot's shape anisotropy on the transport properties of the underlying superconductor, one can perform resistivity measurements while changing  $\theta$ . For a dots array whose dots are very eccentric, the measurements must reflect a decrease in the resistance of the sample as  $\theta$  is increased down from 0 up to  $\frac{\pi}{2}$ . The full understanding of transport properties and the effect of the dot's shape anisotropy on vortex dynamics in this system is beyond the scope of this article.

In conclusion the system of a single elliptic dot on the top of a superconducting thin film is studied. The magnetic fields and screening currents for the FM-SC system are calculated self consistently using London-Maxwell electrodynamics. I showed that above some a threshold value of the dot's magnetization, the spontaneous creation of superconducting vortices becomes energy favorable. The appearance of the spontaneous vortex phase in a superconductor covered by an elliptic dot is more energy favorable compared to that created by a circular one that has the same area and magnetization per unit area. The phase transitions from regime without vortices

to regimes with one and more vortices are studied. We also studied the pinning forces and transport properties for a superconducting thin film covered by an array of elliptic dots. The critical current for this system is affected by the shape anisotropy of the dots and it depend on the angle between the direction of the driving current and the semi-major axis of the dot.

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