# Continuous source of phase-controlled entangled two-photon laser

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We show that an absolute coherent phase of a laser can be used to manipulate the entanglement of photon pairs in two-photon laser. We present simple physics behind a general master equation for two-photon laser. Our focus is on the generation of a continuous source of macroscopically entangled photon pairs in the double $\Lambda$ (or Raman) scheme. We show how the steady-state photon numbers and entanglement depend on the laser parameters, especially the phase. We obtain a relationship between entanglement and two-photon correlation. We derive conditions that give steady-state entanglement for the spontaneous Raman-electromagnetic-induced transparency scheme and use it to identify the region with macroscopic entanglement. No entanglement is found for the double resonant Raman scheme.

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## I. INTRODUCTION

Entangled photon pairs is an integral asset to quantum communication technology with continuous variables [1]. A bright source of entangled photon pairs could be useful also for quantum lithography [2]. Transient entanglement of a large number of photon pairs has been shown to exist for a cascade scheme [3,4], and double Raman scheme [5]. The transient regime does not provide a continuous source of entangled photon pairs that could be as useful and practical as typical lasers in continuous wave (cw) operation. One might wonder whether the entanglement still survives in the long time limit.

In this paper, we control the coherent phase of the lasers to generate a continuous source of a large number (macroscopic) of entangled photon pairs. This provides the possibility of coherently controlling the degree of entanglement in the steady state. We focus on the double Raman configuration [Fig. 1(a)]. The *Raman-EIT* (electromagnetic induced transparency) scheme has been shown to produce nonclassically correlated photon pairs in the single atom [6] and many atoms [7] cases.

First, we discuss the physics of a two-photon emission laser using the master equation in Sec. II. The physical significance of each term in the master equation is elaborated and related to the quantities of interests (in Sec. III) such as two-photon correlation and Duan's [8] entanglement measure. In Sec. IV, we show the importance of laser phase for acquiring entanglement. In Sec. V, the steady-state solutions for the photon numbers and correlation between photon pairs are given. We show that the laser phase provides a useful knob for controlling entanglement. We then use the results to derive a condition for entanglement in the double Raman scheme. By using proper values of cavity damping and laser parameters based on analysis of the entanglement condition, we obtain a macroscopic number of entangled photon pairs in the steady state. We also analyze the double resonant Raman scheme but find no entanglement.

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## II. MASTER EQUATION AND PHYSICS OF TWO-PHOTON LASER

We consider a single atom with double Raman scheme localized in a double cavity that are resonantly tuned to the Stokes ($\hat{a}_1$) and anti-Stokes ($\hat{a}_2$) modes, as in Fig. 1(a). The atom is driven by the pump "$p$" and control "$c$" lasers with Rabi frequencies $\Omega_{p,c}$. The Stokes and anti-Stokes photons spontaneously emitted into the cavity modes are amplified by stimulated emissions into strong lasing modes, thus the Hamiltonian in the interaction picture is $\hat{V}=-\hbar(\Omega_p\hat{\sigma}_{dc}e^{-i\Delta_p t}+g_1\hat{\sigma}_{db}\hat{a}_1e^{-i\Delta_1 t}+\Omega_c\hat{\sigma}_{ab}e^{-i\Delta_c t}+g_2\hat{\sigma}_{ac}\hat{a}_2e^{-i\Delta_2 t})+\text{adj}$ where $\hat{\sigma}_{\alpha\beta}=|\alpha\rangle\langle\beta|$, $\Omega_q=|\Omega_q|e^{i\varphi_q}(q=p,c)$, $g_j=|g_j|e^{i\varphi_j}(j=1\text{-Stokes}$, 2-anti-Stokes).

The derivation of the laser master equation follows the usual approach [9], starting from $\frac{d}{dt}\hat{\rho}_{\text{tot}}=\frac{1}{i\hbar}[\hat{V},\hat{\rho}_{\text{tot}}]$

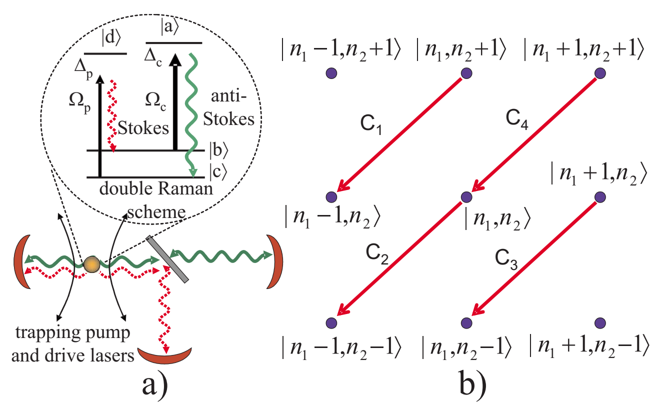


FIG. 1. (Color online) (a) Double Raman atom in a doubly resonant optical cavity. The atom is trapped by an optical dipole force and driven by a pump laser and a control laser. The *Raman-EIT* scheme $[\Omega_c,\Delta_p(=\Delta)\gg\Omega_p,\gamma$ and $\Delta_c=0]$ and *double resonant Raman* scheme $(\Omega_c=\Omega_p,\Delta_c=\Delta_p=0)$ would be the focus for analysis. (b) Photon number states for the Stokes and anti-Stokes are shown in two dimensions. The four arrows correspond to the off-diagonal density matrix elements for two-photon emission with their respective coefficients $C_k$ in Eq. (1).

 

for the atom-field density operator $\hat{\rho}_{\rm tot}$, the master equation for the radiation state $\hat{\rho}=\Sigma_{x=a,b,c}\langle x|\hat{\rho}_{\rm tot}|x\rangle$ is obtained by tracing over the internal states, which gives $\frac{d}{dt}\hat{\rho}=i(g_1\hat{a}_1\hat{\rho}_{bd}+g_2\hat{a}_2\hat{\rho}_{ca}+g_1^*\hat{a}_1^\dagger\hat{\rho}_{db}+g_2^*\hat{a}_2^\dagger\hat{\rho}_{ac})+\text{adj}$ where $\hat{\rho}_{\beta\alpha}=\Sigma_{x=a,b,c}\langle x|\hat{\sigma}_{\alpha\beta}\hat{\rho}_{\rm tot}|x\rangle$ are the atom-field coherence operators projected out by the operators $\hat{\sigma}_{\alpha\beta}$. Since the atomic dynamics vary much faster than the fields, it is possible to express $\hat{\rho}_{\beta\alpha}$ in terms of $\hat{\rho}$, $\hat{a}_j$ and $\hat{a}_j^\dagger$ from the quasi-steady-state solutions of the coupled equations for $\frac{d}{dt}\hat{\rho}_\beta(\beta=ac,ad,bc,bd)$ that contain the four channels of spontaneous emissions. By using $\hat{\rho}_{\alpha\alpha}\simeq p_{\alpha\alpha}^{st}\hat{\rho}$, $\hat{\rho}_{ab}\simeq p_{ab}^{st}\hat{\rho}$, $\hat{\rho}_{dc}\simeq p_{dc}^{st}\hat{\rho}$, where "*st*" implies the steady-state solutions of the density matrix equations (in the interaction picture) without the quantum fields $\hat{a}_j$ (the zeroth-order approximation), we obtain the master equation

$$\begin{aligned}\frac{d}{dt}\hat{\rho}=[&C_{\rm loss1}(\hat{a}_1\hat{\rho}\hat{a}_1^\dagger-\hat{\rho}\hat{a}_1^\dagger\hat{a}_1)+C_{\rm gain1}(\hat{a}_1^\dagger\hat{\rho}\hat{a}_1-\hat{a}_1\hat{a}_1^\dagger\hat{\rho})\\&+C_{\rm loss2}(\hat{a}_2\hat{\rho}\hat{a}_2^\dagger-\hat{a}_2^\dagger\hat{a}_2\hat{\rho})+C_{\rm gain2}(\hat{a}_2^\dagger\hat{\rho}\hat{a}_2-\hat{\rho}\hat{a}_2\hat{a}_2^\dagger)\\&+e^{-i\varphi_t}(C_1\hat{a}_2\hat{\rho}\hat{a}_1-C_2\hat{\rho}\hat{a}_1\hat{a}_2+C_3\hat{a}_1\hat{\rho}\hat{a}_2-C_4\hat{a}_1\hat{a}_2\hat{\rho})]\\&+\text{adj},\end{aligned}\tag{1}$$

with the effective phase $\varphi_t=\varphi_p+\varphi_c-(\varphi_1+\varphi_2)$. The phases $\varphi_\alpha(z)=k_\alpha z+\phi_\alpha$ of the lasers depend on both the position $z$ of the atom and the controllable absolute phases $\phi_\alpha$ of the lasers. So, $\phi_1=\phi_2=0$. Since $k_p+k_c-k_s-k_a=0$, the effective phase becomes $\varphi_t=\phi_p+\phi_c=\phi$. The explicit expressions for $C_{{\rm loss}j}$, $C_{{\rm gain}j}$, and $C_k$ (where $j=1,2$ and $k=1,2,3,4$) are given in Appendix A. Equation (1) already includes the cavity damping Liouvillean $L\hat{\rho}=-\Sigma_{j=1,2}\kappa_j(\hat{a}_j^\dagger\hat{a}_j\hat{\rho}+\hat{\rho}\hat{a}_j^\dagger\hat{a}_j-2\hat{a}_j\hat{\rho}\hat{a}_j^\dagger)$ since $C_{{\rm loss}j}$ depend on $\kappa_j$, the cavity damping rates for the Stokes ($j=1$) and anti-Stokes ($j=2$) modes.

The $C_{{\rm gain}j}$ are due to the emissions processes of the atom in the excited levels and Raman process via the laser fields which provide gain. On the other hand, the $C_{{\rm loss}j}$ are due to cavity dissipation $\kappa_j$ and absorption processes of the atom in the ground levels which create loss. The terms with $C_k$ coefficients correspond to the coherence between $n_j$ and $n_j\pm1$ such that the difference between the total photon number in the bra and in the ket is always 2. These terms give rise to squeezing and will be elaborated on in future presentations. Figure 1(b) illustrates the essence of each diagonal term in Eq. (1) in two-dimensional photon number space.

We find that the relation holds,

$$C_1+C_3=C_2+C_4.\tag{2}$$

The consequence of this relation for a large number of photons $n_j\gg1$ is that the coherences due to the terms $\hat{a}_2\hat{\rho}\hat{a}_1,\hat{\rho}\hat{a}_1\hat{a}_2,\hat{a}_1\hat{\rho}\hat{a}_2,\hat{a}_1\hat{a}_2\hat{\rho}$ and their adjoint are approximately equal. Hence, the contribution of the off-diagonal terms vanish and the master equation reduces to the classical rate equation. Since the off-diagonal terms give rise to entanglement (as we show below), we can understand that there will be no entanglement for very large $n_j$.

Note that Eq. (1) generalizes the master equation for the cascade scheme [10] in which $C_3=C_{\rm gain2}=0$, and $C_{{\rm loss}j}=\kappa_j$.

## III. RELATION BETWEEN ENTANGLEMENT AND TWO-PHOTON CORRELATION

Two-photon correlation for the Raman-EIT (large detuning and weak pump) case for single atom [6] and extended medium [7] show nonclassical properties such as antibunching and violation of Cauchy-Schwarz inequality. It is useful to show how nonclassical correlation relates to entanglement. The normalized two-photon correlation at zero time delay is

$$g^{(2)}(t)\doteq\frac{|\langle\hat{a}_2\hat{a}_1\rangle|^2}{\langle\hat{a}_2^\dagger\hat{a}_2\rangle\langle\hat{a}_1^\dagger\hat{a}_1\rangle}+1,\tag{3}$$

$$|\langle\hat{a}_2\hat{a}_1\rangle|=\sqrt{\bar{n}_1\bar{n}_2[g^{(2)}(t)-1]}.\tag{4}$$

Thus, the $g^{(2)}(t)$ does not provide phase $\phi_{21}$ information of the correlation $\langle\hat{a}_2\hat{a}_1\rangle$. We bring out the phase information by writing

$$\langle\hat{a}_2\hat{a}_1\rangle=|\langle\hat{a}_2\hat{a}_1\rangle|e^{i\phi_{21}}.\tag{5}$$

Among the various measures of entanglement for continuous variables [11], we choose to employ the Duan's criteria due to its convenience for the present problem and applicability to Gaussian states [12] such as in the present case where the bosonic operators in the master equation come in pairs. Besides, it has been used in Refs. [3–5]. The sufficient condition (and necessary condition for Gaussian states) for entanglement between the two modes $\hat{a}_1$ and $\hat{a}_2$ is $D(t)=\langle(\Delta\hat{u})^2\rangle+\langle(\Delta\hat{v})^2\rangle<2$, where $\hat{u}=\hat{x}_1+\hat{x}_2$ and $\hat{v}=\hat{p}_1-\hat{p}_2$ are the EPR-type operators, with the real operators $\hat{x}_j=\frac{1}{\sqrt{2}}(\hat{a}_j+\hat{a}_j^\dagger)$, and $\hat{p}_j=\frac{1}{i\sqrt{2}}(\hat{a}_j-\hat{a}_j^\dagger)$, $g^{(2)}(t)\doteq|\langle\hat{a}_2\hat{a}_1\rangle|^2/\langle\hat{a}_2^\dagger\hat{a}_2\rangle\langle\hat{a}_1^\dagger\hat{a}_1\rangle+1$. Hence, the $D(t)$ function may be written as

$$\begin{aligned}D(t)=2\{&1+\bar{n}_1+\bar{n}_2+2\sqrt{\bar{n}_1\bar{n}_2[g^{(2)}(t)-1]}\cos\phi_{21}-|\langle\hat{a}_2\rangle|^2\\&-|\langle\hat{a}_1\rangle|^2-\langle\hat{a}_2\rangle\langle\hat{a}_1\rangle-\langle\hat{a}_2^\dagger\rangle\langle\hat{a}_1^\dagger\rangle\}.\end{aligned}\tag{6}$$

Clearly, the presence of inseparability or entanglement is entirely determined by the phase $\phi_{21}$ in Eq. (6). We now find a knob for controlling entanglement, i.e., $\cos\phi_{21}$ must be negative or $\pi/2<\phi_{21}<3\pi/2$.

If the two modes are in coherent states, the second line in Eq. (6) becomes $-|\alpha_1|^2-|\alpha_2|^2-(\alpha_1\alpha_2+\alpha_1^*\alpha_2^*)$. Here, there is no correlation, i.e., $g^{(2)}(t)=1$, and we have

$$D(t)=2[1-(\alpha_1\alpha_2+\alpha_1^*\alpha_2^*)],\tag{7}$$

which shows that if the phases of the coherent modes $\alpha_j=r_j\exp(i\theta_j)$ are locked such that $\cos(\theta_1+\theta_2)>0$, the modes can be entangled.

In the following, we consider initial vacuum state and the modes that have not evolved into the coherent state, in which the second line of Eq. (6) vanishes. Then, the condition for inseparability or entanglement $0<D(t)<2$ can be rewritten in terms of the phase and the two-photon correlation,

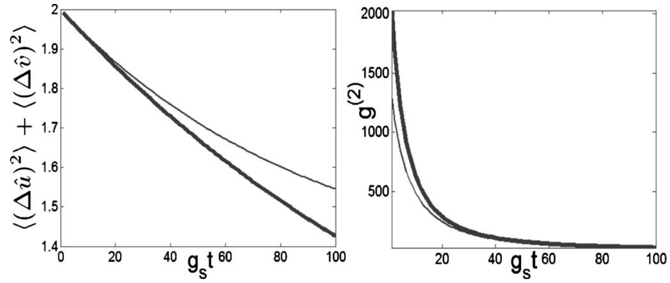


FIG. 2. The $\langle(\Delta\hat{u})^2\rangle + \langle(\Delta\hat{v})^2\rangle$ and $g^{(2)}(t)$ vary in a similar manner as a function of time for decoherence $\gamma_{bc}=0.6\gamma_{ac}$ (thin line) and without decoherence $\gamma_{bc}=0$ (thick line). Parameters used are $\kappa_1=\kappa_2=0.001g_1$, $\Omega_p=2g_1$, $\Delta_p=40g_1$, $\Omega_c=25g_1$, $\Delta_c=0$. We have assumed $g_2=g_1$, with $\gamma_{ac}=\gamma_{dc}=\gamma_{ab}=\gamma_{db}=\gamma$.

$$-\frac{\bar{n}_1+\bar{n}_2+1}{2\sqrt{\bar{n}_1\bar{n}_2(g^{(2)}-1)}} < \cos\phi_{21} < -\frac{\bar{n}_1+\bar{n}_2}{2\sqrt{\bar{n}_1\bar{n}_2(g^{(2)}-1)}}, \quad (8)$$

where the lower limit corresponds to maximum entanglement. The midpoint value $\cos\phi_{21}=-\frac{\bar{n}_1+\bar{n}_2+1/2}{2\sqrt{\bar{n}_1\bar{n}_2(g^{(2)}-1)}}$ gives $D(t)=1$. When $\bar{n}_1=\bar{n}_2$, we have $-\frac{1+1/2\bar{n}}{\sqrt{g^{(2)}-1}} < \cos\phi_{21} < -\frac{1}{\sqrt{g^{(2)}-1}}$.

Note that for a small correlation $g^{(2)}\gtrsim 1$ the entanglement window for $\phi_{21}$ can be quite large when $\bar{n}$ is small. For large two-photon correlation $g^{(2)}\gg 1$ and large photon numbers $\bar{n}_1\simeq\bar{n}_2\gg 1$, the range for entanglement becomes quite restrictive, $\cos\phi_{21}\simeq -\frac{1}{\sqrt{g^{(2)}-1}}$ becomes very small in magnitude (but negative) and from Eq. (6) we have $D(t)\lesssim 2$, i.e., the entanglement is small. This explains the results in Fig. 2 where large transient correlation is accompanied by small entanglement.

In the long time limit, Fig. 2 shows that the correlation vanishes (corresponding to photon antibunching) and the entanglement increases, $D\ll 2$. Although both the correlation and entanglement are quantum mechanical properties they do not vary in the same way. The entanglement increases with time while the correlation decreases with time. This clearly shows that correlation and entanglement are distinct terminologies and should be carefully discerned from each other. Figure 2 also shows that the decoherence $\gamma_{bc}$ tends to reduce the degree of entanglement and the magnitude of correlation, as expected.

## IV. LASER PHASE FOR ENTANGLEMENT

Here, we show by using a simple example from the resonant cascade work of Ref. [3] that the *nonzero phase* of the paired correlation $\langle\hat{a}_2\hat{a}_1\rangle$ is necessary for entanglement. Let us analyze the transient equation (written in their notations with zero laser phase)

$$\frac{d\langle\hat{a}_2\hat{a}_1\rangle}{dt} = -\langle\hat{a}_2\hat{a}_1\rangle(\beta_{22}^* - \beta_{11} + \kappa_2 + \kappa_1) - \beta_{21}^*(\langle\hat{a}_1^\dagger\hat{a}_1\rangle + 1) + \beta_{12}\langle\hat{a}_2^\dagger\hat{a}_2\rangle. \quad (9)$$

The coefficients for the resonant case are such that $\beta_{11},\beta_{22}$ are real while $\beta_{12}=i\alpha_{12}$ and $\beta_{21}=i\alpha_{21}$ are purely imaginary. Clearly we have an imaginary value for

$$\langle\hat{a}_2\hat{a}_1\rangle(t) = i\int_0^t e^{-K(t-t')}\{\alpha_{12}\langle\hat{a}_2^\dagger\hat{a}_2\rangle(t') + \alpha_{21}[\langle\hat{a}_1^\dagger\hat{a}_1\rangle(t')+1]\}dt' \simeq iX, \quad (10)$$

where $K=\beta_{22}-\beta_{11}+\kappa_2+\kappa_1$ and $X$ is real, whose expression is not important for the present discussion.

For initial conditions $\langle\hat{a}_j(0)\rangle=0$, the Duan's parameter becomes

$$D = 2(1 + \langle\hat{a}_1^\dagger\hat{a}_1\rangle + \langle\hat{a}_2^\dagger\hat{a}_2\rangle + \langle\hat{a}_2\hat{a}_1\rangle + \langle\hat{a}_1^\dagger\hat{a}_2^\dagger\rangle) \quad (11)$$

which clearly shows there is no entanglement ($D>2$).

For finite phase $\phi$ associated to the pump laser, the correlation $\langle\hat{a}_2\hat{a}_1\rangle$ becomes $\langle\hat{a}_2\hat{a}_1\rangle e^{i\phi}$ but the photon numbers are not affected. The parameter becomes

$$D = 2(1 + \langle\hat{a}_1^\dagger\hat{a}_1\rangle + \langle\hat{a}_2^\dagger\hat{a}_2\rangle + 2X\sin\phi) \quad (12)$$

which gives maximum entanglement when $\phi=-\pi/2$ or $3\pi/2$, and no entanglement when $\phi=0$.

## V. STEADY-STATE ENTANGLEMENT

The master equation (1) is linear and does not include saturation. One might wonder whether steady-state solutions exist. We find that there are steady-state solutions when the photon numbers $\bar{n}_j$ do not increase indefinitely but saturate at large times. Parameters that give non-steady-state solutions manifest as a negative value of $D$ and should be disregarded. The study of entanglement via nonlinear theory will be presented elsewhere.

In the case of initial vacuum, the coupled equations for $\frac{d\bar{n}_1}{dt}$, $\frac{d\bar{n}_2}{dt}$, $\frac{d\langle\hat{a}_2\hat{a}_1\rangle}{dt}$, $\frac{d\langle\hat{a}_1^\dagger\hat{a}_2^\dagger\rangle}{dt}$ are sufficient to compute the Duan's entanglement parameter, where $\bar{n}_j=\langle\hat{a}_j^\dagger\hat{a}_j\rangle$, $j=1,2$. The full expressions for the coupled equations and the corresponding steady-state solutions for $\bar{n}_1$, $\bar{n}_2$, and $\langle\hat{a}_2\hat{a}_1\rangle$ are given in Appendix B.

From Eq. (11), together with the steady-state solutions $\langle\hat{a}_2\hat{a}_1\rangle=Ee^{i\phi}$, $\bar{n}_1$ and $\bar{n}_2$ given by Eqs. (B6)–(B9), the necessary condition for entanglement is $Ee^{i\phi}+E^*e^{-i\phi}<-(\bar{n}_1+\bar{n}_2)$. If $E$ is real positive there would be no entanglement in the region $\cos\phi>0$. Entanglement is still possible even if $\phi=0$ provided $2\,\mathrm{Re}\{E\}<-(\bar{n}_1+\bar{n}_2)$. Thus, the phase $\phi$ is not necessary for entanglement, but it provides an *extra knob* for controlling entanglement.

Let us search for entanglement conditions in the limiting cases of Raman-EIT scheme which produces nonclassically correlated photon pairs, and the double resonant Raman (DRR) scheme.

### A. Raman-EIT case

For this scheme, $\Omega_c$, $\Delta_p=\Delta_1(=\Delta)\gg\Omega_p$, $\gamma_{\alpha\beta}$ ($\alpha,\beta=a,b,c,d$) and $\Delta_c=\Delta_2=0$. Thus, we have $p_{ba}=\frac{-i\Omega_c^*}{\gamma_{ab}}(p_{bb}-p_{aa})\simeq 0$ since the population is primarily in level $c$ ($p_{cc}\simeq 1, p_{bb}\simeq p_{aa}\simeq 0$) and $p_{cd}=\frac{-\Omega_p}{\Delta}=p_{dc}$. The coefficients for the Raman-EIT case are given in Appendix C. For moderate cavity damping, typically $\frac{|g_j|^2}{\Omega_c^2}\frac{\Omega_p^2}{\Delta}\ll\kappa_j$. Thus, the only

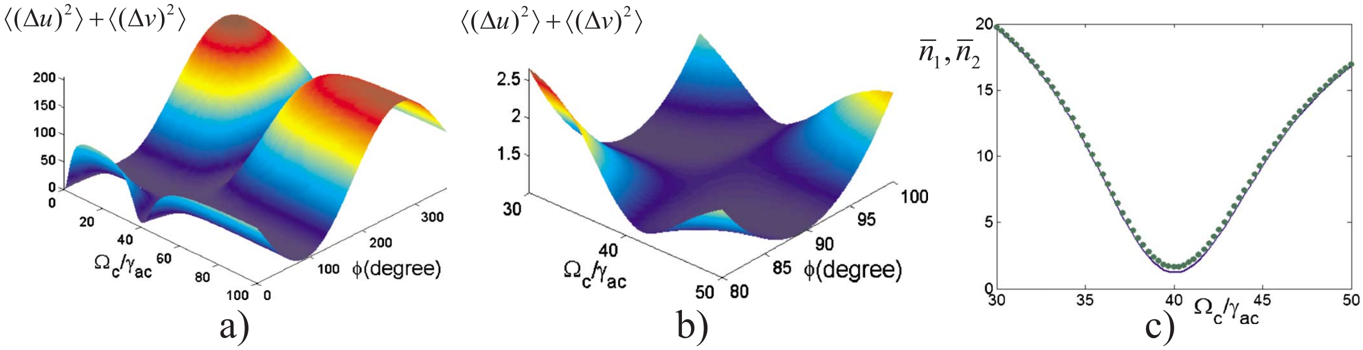


FIG. 3. (Color online) Entanglement parameter $D$ versus $\Omega_c$ and $\phi$ for Raman-EIT scheme in (a) wide view, (b) magnified view of the highly entangled region in (a), and (c) mean photon numbers $\bar{n}_1$ (solid line) and $\bar{n}_2$ (dots), for $\phi=\pi/2$. The parameters are $\Omega_p=\gamma_{ac}$, $\Delta_p=40\gamma_{ac}$, $\Delta_c=0$. Cavity damping values $\kappa_2=\kappa_1=1.01|C_2|$ ensures a minimum value of the denominator of the condition in Eq. (18). We have assumed $g_2=g_1=\gamma$, $\gamma_{bc}=0$ and $\gamma_{ac}=\gamma_{dc}=\gamma_{ab}=\gamma_{db}=\gamma$.

significant terms are $C_{\text{loss1}}\simeq\kappa_1$ and $C_{\text{loss2}}\simeq\kappa_2$. Since $\Delta \gg \gamma_{bc}$ we have $C_1 \ll C_2 \simeq -C_4 \simeq i\Xi$, where

$$\Xi = \frac{-g_2 g_1 \Omega_c \Omega_p[\Delta^2 - (\Omega_c^2 - \Omega_p^2)]/\Delta}{(\Omega_c^2 - \Delta^2)\gamma_{ac}\gamma_{bc} - \Delta^2\Omega_c^2 + (\Omega_c^2 - \Omega_p^2)^2}, \tag{13}$$

which becomes $g_2 g_1 \frac{\Omega_p\Omega_c}{\Delta(\gamma_{ac}\gamma_{bc}+\Omega_c^2)}$ when $\Omega_c \gg \Omega_p$. It follows that

$$K_1 = -2\kappa_1, \quad K_2 = -2\kappa_2, \quad K_{12} = -(\kappa_1+\kappa_2). \tag{14}$$

Hence, we have the steady-state solutions

$$n_1 = \Xi^2 \frac{\kappa_2}{(\kappa_1+\kappa_2)(\kappa_2\kappa_1 - \Xi^2)}, \tag{15}$$

$$n_2 = \Xi^2 \frac{\kappa_1}{(\kappa_1+\kappa_2)(\kappa_2\kappa_1 - \Xi^2)}, \tag{16}$$

$$\langle \hat{a}_1 \hat{a}_2 \rangle = e^{i\phi} i\Xi \frac{\kappa_2\kappa_1}{(\kappa_1+\kappa_2)(\kappa_2\kappa_1 - \Xi^2)}. \tag{17}$$

The entanglement criteria can be rewritten as

$$\Xi \frac{\Xi - \dfrac{\kappa_2\kappa_1}{(\kappa_1+\kappa_2)} 2\sin\phi}{\kappa_2\kappa_1 - \Xi^2} < 0. \tag{18}$$

Note that the sign of the detuning $\Delta$ in Eq. (13) is important for entanglement generation. There are many ways for arranging the laser parameters and the cavity rates $\kappa_j$ in Eq. (18) to obtain entanglement.

For negative detuning $\Xi<0$, there are two possibilities:

(i) if $\kappa_2\kappa_1<\Xi^2$ entanglement occurs in the region $\frac{\kappa_2\kappa_1}{(\kappa_1+\kappa_2)}2\sin\phi>\Xi$,

(ii) if $\kappa_2\kappa_1>\Xi^2$ we have entanglement in $\frac{\kappa_2\kappa_1}{(\kappa_1+\kappa_2)}2\sin\phi<\Xi$.

Similarly, for positive detuning $\Xi>0$:

(i) if $\kappa_2\kappa_1<\Xi^2$ we need $\Xi>\frac{\kappa_2\kappa_1}{(\kappa_1+\kappa_2)}2\sin\phi$,

(ii) if $\kappa_2\kappa_1>\Xi^2$ then we need $\Xi<\frac{\kappa_2\kappa_1}{(\kappa_1+\kappa_2)}2\sin\phi$.

To obtain large entanglement, we tune the cavity damping such that the denominator $\kappa_2\kappa_1-\Xi^2$ in Eq. (18) is small and $\sin\phi\sim 1$. Figure 3 is plotted using $\kappa_2=\kappa_1=1.01|C_2|$ and $\Delta=40\gamma_{ac}$ for $\phi\sim 90°$. The region $\Omega_c\sim\Delta$ gives a large entanglement where $\Xi\to\frac{\Omega_c\Omega_p}{(2\Omega_c^2-\Omega_p^2)\Delta}$, but the photon numbers are minimum. This seems to prevent the generation of steady-state macroscopic entanglement. The region of maximum entanglement occurs around $\phi=90°$. We verify that if we change to a negative detuning $\Delta=-40\gamma_{ac}$ there is no entanglement. Although entanglement can occur over a wide range of large $\Omega_c$, the photon numbers $\bar{n}_j$ decrease as $\Omega_c$ increases.

Figure 4 shows that it is possible to obtain a continuous bright source of entangled photons. We realize that the number of nonclassical photon pairs in the Raman-EIT case is limited by the weak pump field. Thus, by increasing the pump field we can generate more Stokes photons [Fig. 4(a)]. At the same time, the detuning is increased as well to ensure that the scheme remains in the Raman-EIT regime ($\Delta_p \gg \Omega_p$). By further applying the condition Eq. (18) we obtained a larger (macroscopic) number of entangled photon pairs [Fig. 4(b)].

### B. Double resonant Raman

Numerical results seem to show that steady-state entanglement in the double resonant Raman case ($\Omega_c=\Omega_p, \Delta_c=\Delta_p=0$) is hardly possible. In the following, we verify this analytically. Here, the coefficients (given in Appendix D) $C_{ac,ac}$, $C_{ac,bd}$, $C_{bd,ac}$, $C_{bd,bd}$ are real and positive while $C_{ac,ad}$, $C_{ac,bc}$, $C_{bd,ad}$, $C_{bd,bc}$ are purely imaginary (positive or negative). Since $p_{cd}=-i|p_{cd}|$, $p_{ba}=-i|p_{ba}|$, all $C_j$, $C_{\text{loss}j}$, and $C_{\text{gain}j}$ are real but could be negative. Thus, we have $K_j=2C_{\text{gain}j}-2C_{\text{loss}j}$ and $K_{12}=\frac{1}{2}(K_1+K_2)$.

For *symmetric* system $\Omega_p\simeq\Omega_c$, we find $p_{ab}=-p_{ba}=p_{dc}=-p_{cd}$, $p_{cc}\simeq p_{bb}$, and $p_{aa}\simeq p_{dd}$ [13]. Then, $C_{ac,ac}=C_{bd,bd}$ and $C_{ac,bd}=C_{bd,ac}$. If we take $T_{ac}=T_{dc}=T_{ab}=T_{db}=\gamma$ (spontaneous decay rate) with $T_{bc}=\gamma_{bc}$ and $T_{ad}=2\gamma$ we further have $C_{bd,ad}=-C_{ac,ad}$, $C_{bd,bc}=-C_{ac,bc}$. The resulting steady-state solutions for the DRR scheme can be written as

$$\bar{n}_1 = \bar{n}_2 = \frac{C_{\text{gain}}(C_{\text{gain}} - C_{\text{loss}}) + \frac{1}{2}C_2C_{12}}{C_{12}^2 - (C_{\text{gain}} - C_{\text{loss}})^2}, \tag{19}$$

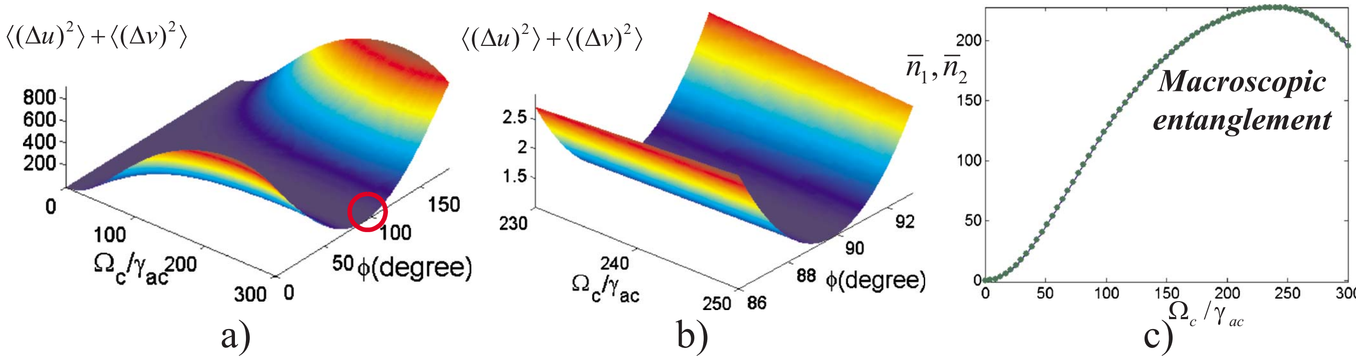


FIG. 4. (Color online) Macroscopic entanglement with larger pump field and detuning $\Omega_p = 10\gamma_{ac}$, $\Delta_p = 400\gamma_{ac}$, and cavity damping $\kappa_2 = \kappa_1 = 1.001|C_2|$. Other parameters are the same as Fig. 3. This gives larger mean photon numbers, i.e., macroscopically entangled photon pairs. (a) A wide view of $D$ versus $\Omega_c$ and $\phi$ with an entangled region (red circle). (b) Magnified view of the entangled region in (a). (c) Photon numbers $\bar{n}_1 \simeq \bar{n}_2$ up to 200 can be generated for $\phi = \pi/2$.

$$\langle \hat{a}_1 \hat{a}_2 \rangle = -e^{i\phi}\left(\frac{C_1 C_{\text{gain}} - \frac{1}{2}C_2(C_{\text{gain}} + C_{\text{loss}})}{C_{12}^2 - (C_{\text{gain}} - C_{\text{loss}})^2}\right), \tag{20}$$

and the corresponding entanglement condition becomes

$$\bar{n}_1 + \bar{n}_2 < 2\xi \cos\phi, \tag{21}$$

where $\xi$ is the term in the bracket $(\cdots)$ of Eq. (20).

In order to determine whether Eq. (21) can be met, we consider a simpler case where $\gamma_{bc} = 0$. From the coefficients in Appendix D, we have $C_{\text{loss}} - C_{\text{gain}} = \kappa$ and $C_{12} = \eta(p_{cc} - p_{aa})$ with $\eta = g^2/\gamma$, $C_1 = \eta p_{aa}$, $C_2 = \eta(2p_{aa} - p_{cc})$, $C_{\text{loss}} = \frac{1}{2}\eta p_{bb} + \kappa$, and $C_{\text{gain}} = \frac{1}{2}\eta p_{cc}$. These results are used to rewrite Eqs. (19) and (20) as

$$\bar{n}_j = \frac{1}{2}\eta \frac{\eta(2p_{aa} - p_{cc})(p_{cc} - p_{aa}) - p_{cc}\kappa}{[\eta(p_{cc} - p_{aa})]^2 - \kappa^2}, \tag{22}$$

$$\langle \hat{a}_1 \hat{a}_2 \rangle = \frac{e^{i\phi}}{2}\eta \frac{(2p_{aa} - p_{cc})\kappa - \eta(p_{cc} - p_{aa})p_{cc}}{[\eta(p_{cc} - p_{aa})]^2 - \kappa^2}. \tag{23}$$

For *strong fields*, the populations in the upper and lower levels are equally distributed, i.e., $p_{cc} \simeq p_{aa} = 0.25$. The steady solutions become $\bar{n}_1 = \bar{n}_2 = g^2/8\gamma\kappa$, $\langle \hat{a}_1 \hat{a}_2 \rangle = -(g^2/8\gamma\kappa)e^{i\phi}$, and $D = 2\left[1 + (g^2/2\gamma\kappa)\sin^2\frac{1}{2}\phi\right]$, i.e., no entanglement.

For *weak fields*, one-half of the population is in level $b$ and one-half in level $c$, so $p_{cc} \simeq p_{bb} \simeq 0.5, p_{aa} \simeq p_{dd} \simeq 0$. The steady solutions are $\bar{n}_1 = \bar{n}_2 = \frac{g^2}{4\gamma}\frac{1}{\kappa - g^2/2\gamma}$, $\langle \hat{a}_1 \hat{a}_2 \rangle = \frac{g^2}{4\gamma}\frac{e^{i\phi}}{\kappa - g^2/2\gamma}$ with $\kappa > g^2/2\gamma$ and hence $D = 2\left(1 + \frac{2g^2}{2\gamma\kappa - g^2}\sin^2\frac{1}{2}\phi\right)$, again no entanglement. Here, the cavity damping must be sufficiently large $(\kappa > g^2/2\gamma)$ to ensure $\bar{n}_1$ and $\bar{n}_2$ are positive, i.e., the existence of steady-state solutions. If the cavity damping is small $\kappa < g^2/2\gamma$, negative values of $D$ and $\bar{n}_j$ would appear, corresponding to the non-steady-state regime.

Thus, we have shown that there is *no* steady-state entanglement for the DRR scheme in both weak field and strong field regimes, in contrast to the Raman-EIT photon pairs, which are entangled in the steady state. This is compatible with the corresponding $G^{(2)}$, which shows classical two-photon correlation [14].

## VI. CONCLUSION

We have shown that two-photon laser can produce a continuous source of entangled photon pairs based on the steady-state solutions and an entanglement criteria. We obtained a relationship between entanglement and two-photon correlation, and found that both do not vary with time in the same manner. We have derived a condition for steady-state entanglement in the Raman-EIT scheme and found macroscopic steady-state entanglement. Thus, we have bypassed the constraint that a large steady-state entanglement is at the expense of a small number of photons. We reinforce the significance of the Raman-EIT scheme, by showing that the double resonant Raman scheme does not generate steady-state entangled photon pairs for any laser parameters. Finally, we foresee that a continuous source of entangled two-photon laser could be a practical tool in optics that would spawn new applications.

## ACKNOWLEDGMENT

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## APPENDIX A: COEFFICIENTS FOR DOUBLE RAMAN SCHEME

The coefficients in Eq. (1) are

$$C_{\text{loss}1} = |g_1|^2(C_{bd,ad}p_{ab} + C_{bd,bd}p_{bb}) + \kappa_1, \tag{A1}$$

$$C_{\text{gain}1} = |g_1|^2(C_{bd,bd}p_{dd} + C_{bd,bc}p_{dc}), \tag{A2}$$

$$C_{\text{loss}2} = |g_2|^2(C_{ac,ac}p_{cc} + C_{ac,ad}p_{cd}) + \kappa_2, \tag{A3}$$

$$C_{\text{gain}2} = |g_2|^2(C_{ac,ac}p_{aa} + C_{ac,bc}p_{ba}), \tag{A4}$$

$$\frac{C_1}{g_2 g_1} = C_{bd,ac}p_{cc} + C^*_{ac,bd}p_{dd} + (C_{bd,ad} + C^*_{ac,bc})p_{cd}, \tag{A5}$$

$$\frac{C_2}{g_2 g_1} = C_{bd,ac}p_{aa} + C^*_{ac,bd}p_{dd} + C_{bd,bc}p_{ba} + C^*_{ac,bc}p_{cd}, \tag{A6}$$

$$\frac{C_3}{g_2 g_1} = C_{bd,ac}p_{aa} + C^*_{ac,bd}p_{bb} + (C_{bd,bc} + C^*_{ac,ad})p_{ba}, \tag{A7}$$

$$\frac{C_4}{g_2 g_1} = C_{bd,ac}p_{cc} + C^*_{ac,bd}p_{bb} + C_{bd,ad}p_{cd} + C^*_{ac,ad}p_{ba}, \tag{A8}$$

where $J_k = \frac{C_k}{g_2 g_1}$, $g_2$, $g_1$ are atom-field coupling strengths, $C_{\alpha\beta,\gamma\delta}$ $(\alpha,\beta,\gamma,\delta=a,b,c,d)$ are complex coefficients that depend on decoherence rates $\gamma_{\alpha\beta}$, laser detunings $\Delta_p$, $\Delta_c$ and Rabi frequencies $\Omega_p$, $\Omega_c$. The $p_{\alpha\alpha}$, $p_{ab}$, $p_{cd}$ $(\alpha=a,b,c,d)$ are steady-state populations and coherences. The $C_{\alpha\beta}$ coefficients are

$$C_{ac,ac} = \frac{T^*_{ad}T^*_{bc}T_{db} + I_p T^*_{ad} + I_c T^*_{bc}}{Z}, \tag{A9}$$

$$C_{ac,ad} = -i\Omega_p \frac{T^*_{bc}T_{db} + I_p - I_c}{Z}, \tag{A10}$$

$$C_{ac,bc} = -i\Omega_c \frac{-T^*_{ad}T_{db} + I_p - I_c}{Z}, \tag{A11}$$

$$C_{ac,bd} = \Omega_c\Omega_p \frac{T^*_{bc} + T^*_{ad}}{Z}, \tag{A12}$$

$$C_{bd,ac} = \Omega_p\Omega_c \frac{T^*_{bc} + T^*_{ad}}{Z}, \tag{A13}$$

$$C_{bd,ad} = -i\Omega_c \frac{-T^*_{ac}T^*_{bc} + I_p - I_c}{Z}, \tag{A14}$$

$$C_{bd,bc} = -i\Omega_p \frac{T^*_{ac}T^*_{ad} + I_p - I_c}{Z}, \tag{A15}$$

$$C_{bd,bd} = \frac{T^*_{ac}T^*_{ad}T^*_{bc} + I_p T^*_{bc} + I_c T^*_{ad}}{Z}, \tag{A16}$$

$$Z = T^*_{ac}T^*_{ad}T^*_{bc}T_{db} + I_p T^*_{ac}T^*_{ad} + I_p T^*_{bc}T_{db} + I_c T^*_{ac}T^*_{bc} + I_c T^*_{ad}T_{db} + (I_p - I_c)^2, \tag{A17}$$

where $I_p=\Omega_p^2$, $I_c=\Omega_c^2$, the complex decay rates $T_{ac} = i\Delta_2 + \gamma_{ac}$, $T_{ad} = i(\Delta_c - \Delta_1) + \gamma_{ad}$, $T_{bc} = i(\Delta_p - \Delta_1) + \gamma_{bc}$, $T_{db} = i\Delta_1 + \gamma_{db}$, and the detunings $\Delta_p = \nu_p - \omega_{dc}$, $\Delta_c = \nu_c - \omega_{ab}$, $\Delta_1 = \nu_s - \omega_{db}$, $\Delta_2 = \nu_a - \omega_{ac}$.

## APPENDIX B: COUPLED EQUATIONS AND SOLUTIONS

From the master equation (1), we obtain

$$\frac{d\bar{n}_1}{dt} = \bar{n}_1 K_1 + e^{-i\phi}(C_1 - C_2)\langle \hat{a}_2\hat{a}_1\rangle + e^{i\phi}(C_1^* - C_2^*)\langle \hat{a}_1^\dagger \hat{a}_2^\dagger\rangle + 2\,\mathrm{Re}\,C_{\mathrm{gain}1}, \tag{B1}$$

$$\frac{d\bar{n}_2}{dt} = \bar{n}_2 K_2 + e^{-i\phi}(C_3 - C_2)\langle \hat{a}_2\hat{a}_1\rangle + e^{i\phi}(C_3^* - C_2^*)\langle \hat{a}_1^\dagger \hat{a}_2^\dagger\rangle + 2\,\mathrm{Re}\,C_{\mathrm{gain}2}, \tag{B2}$$

$$\left(\frac{d}{dt} - K_{12}\right)\langle \hat{a}_2\hat{a}_1\rangle = e^{i\phi}[\bar{n}_1(C_3^* - C_2^*) + \bar{n}_2(C_1^* - C_2^*) - C_2^*], \tag{B3}$$

where $2\,\mathrm{Re}\,C_{\mathrm{gain}j} = C_{\mathrm{gain}j} + C^*_{\mathrm{gain}j}$ and the gain and/or loss coefficients are

$$K_j = 2\,\mathrm{Re}(C_{\mathrm{gain}j} - C_{\mathrm{loss}j}), \tag{B4}$$

$$K_{12} = C_{\mathrm{gain}2} + C^*_{\mathrm{gain}1} - (C_{\mathrm{loss}2} + C^*_{\mathrm{loss}1}). \tag{B5}$$

The steady-state solution for the correlation is

$$\begin{aligned}\langle \hat{a}_1\hat{a}_2\rangle = [&-C^*_{32}(K_2K^*_{12} + C^*_{12}C_{32} - C_{12}C^*_{32})2\,\mathrm{Re}\,C_{\mathrm{gain}1} \\ &- C^*_{12}(K_1K^*_{12} + C_{12}C^*_{32} - C^*_{12}C_{32})2\,\mathrm{Re}\,C_{\mathrm{gain}2} \\ &+ C^*_2(K_1C_{12}C^*_{32} + K_2C_{32}C^*_{12}) - C^*_2K_1K_2K^*_{12} \\ &- C_2C^*_{32}C^*_{12}(K_1 + K_2)]\frac{e^{i\phi}}{M},\end{aligned} \tag{B6}$$

where

$$M = (K_1K^*_{12} + K_2K_{12})(C^*_{12}C_{32}) + \mathrm{c.c.} - (C_{12}C^*_{32} - C^*_{12}C_{32})^2 - K_1K_2K_{12}K^*_{12}. \tag{B7}$$

The steady-state solutions for the photon numbers are

$$\begin{aligned}\bar{n}_1 = {}& 2\,\mathrm{Re}\,C_{\mathrm{gain}1}\frac{K_2K_{12}K^*_{12} - (C^*_{12}C_{32}K^*_{12} + \mathrm{c.c.})}{M} \\ &+ 2\,\mathrm{Re}\,C_{\mathrm{gain}2}C^*_{12}C_{12}\frac{K^*_{12} + K_{12}}{M} \\ &+ C^*_2C_{12}\frac{K_2K^*_{12} + (C^*_{12}C_{32} - \mathrm{c.c.})}{M} \\ &+ C_2C^*_{12}\frac{K_2K_{12} + (C_{12}C^*_{32} - \mathrm{c.c.})}{M},\end{aligned} \tag{B8}$$

$$\begin{aligned}\bar{n}_2 = {}& 2\,\mathrm{Re}\,C_{\mathrm{gain}2}\frac{K_1K_{12}K^*_{12} - (C^*_{32}C_{12}K^*_{12} + \mathrm{c.c.})}{M} \\ &+ 2\,\mathrm{Re}\,C_{\mathrm{gain}1}C^*_{32}C_{32}\frac{K^*_{12} + K_{12}}{M} \\ &+ C^*_2C_{32}\frac{K_1K^*_{12} + (C_{12}C^*_{32} - \mathrm{c.c.})}{M} \\ &+ C_2C^*_{32}\frac{K_1K_{12} + (C^*_{12}C_{32} - \mathrm{c.c.})}{M}.\end{aligned} \tag{B9}$$

## APPENDIX C: COEFFICIENTS FOR RAMAN-EIT SCHEME

By noting that most parameters would be zero from $p_{cc} \simeq 1, p_{bb} \simeq p_{aa} \simeq p_{bd} \simeq 0$, and $p_{cd} = -\Omega_p/\Delta = p_{dc}$ for the REIT scheme, the coefficients reduce to

$$C_{\text{loss1}} \simeq \kappa_1, \tag{C1}$$

$$C_{\text{gain1}} \simeq |g_1|^2 i \frac{I_p}{\Delta} \frac{\gamma_{ac} i\Delta + I_p - I_c}{Z}, \tag{C2}$$

$$C_{\text{loss2}} \simeq |g_2|^2 \frac{\left(i\frac{I_p}{\Delta} - \gamma_{bc}\right)[\Delta^2 - (I_c - I_p)]}{Z} + \kappa_2, \tag{C3}$$

$$J_1 \simeq \frac{\Omega_p \Omega_c}{Z} \gamma_{bc} \left(1 - \frac{i\gamma_{ac}}{\Delta}\right), \tag{C4}$$

$$J_2 \simeq \frac{-i\Omega_c\Omega_p}{Z^*} \frac{\Delta^2 - (I_c - I_p)}{\Delta}, \tag{C5}$$

$$J_4 \simeq \frac{\Omega_p\Omega_c}{Z}\left[\gamma_{bc}\left(1 - \frac{i\gamma_{ac}}{\Delta}\right) + \frac{i[\Delta^2 - (I_c - I_p)]}{\Delta}\right], \tag{C6}$$

where

$$Z \simeq (I_c - \Delta^2)\gamma_{ac}\gamma_{bc} - \Delta^2 I_c + (I_c - I_p)^2 + iI_p\Delta(\gamma_{ac} + \gamma_{bc}). \tag{C7}$$

## APPENDIX D: COEFFICIENTS FOR DRR SCHEME

From Appendix A, we obtain the coefficients

$$C_1 = g_2 g_1 \frac{I}{Z} 2(T_{bc} p_{cc} + T_{ad} p_{aa}), \tag{D1}$$

$$C_2 = g_2 g_1 \frac{I}{Z} 2[T_{bc} p_{aa} + T_{ad}(2p_{aa} - p_{cc})], \tag{D2}$$

$$C_{12} = C_1 - C_2 = C_3 - C_2 = g_2 g_1 \Omega^2 \frac{T_{bc} + T_{ad}}{Z} 2(p_{cc} - p_{aa}), \tag{D3}$$

where $Z = \gamma[T_{ad}T_{bc}\gamma + 2I(T_{ad} + T_{bc})]$ and $I = \Omega^2$. Taking $\kappa_1 = \kappa_2 = \kappa$ we also have $C_{\text{loss1}} = C_{\text{loss2}}$, $C_{\text{gain1}} = C_{\text{gain2}}$ and $K_2 = K_1 = K_{12} = 2(C_{\text{gain}} - C_{\text{loss}})$, and hence

$$C_{\text{loss}} = |g_1|^2 \left(\frac{IT_{bc}}{Z} p_{aa} + T_{ad} \frac{TT_{bc} + I}{Z} p_{bb}\right) + \kappa, \tag{D4}$$

$$C_{\text{gain}} = |g_1|^2 \left(T_{bc} \frac{TT_{ad} + I}{Z} p_{dd} + \frac{IT_{ad}}{Z} p_{cc}\right). \tag{D5}$$


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