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A detailed comparison of LEP data with the predictions of the minimal supersymmetric $SU(5)$ GUT

John Ellis
CERN-Geneva

S. Kelley and D.V. Nanopoulos^(a)
Center for Theoretical Physics, Department of Physics
Texas A & M University, College Station, TX 77843-4242, USA
and

Astroparticle Physics Group, Houston Advanced Research Center (HARC)
The Woodlands, TX 77381, USA

ABSTRACT

We confront the precise LEP determinations of $\sin^2\theta_W$ and the strong coupling $\alpha_3(m_{Z^0})$ with the predictions of the minimal supersymmetric $SU(5)$ GUT. We incorporate $\mathcal{O}(\alpha_{em}\alpha_3)$ effects in the extraction of $\sin^2\theta_W$ from LEP data. We incorporate distinct thresholds for the supersymmetric partners of the different species of Standard Model particles, parameterized in terms of a scalar mass m_0 and a gaugino mass $m_{1/2}$ that are assumed to be universal at the GUT scale. We also allow for uncertainties in the top, Higgs and Higgsino masses. We use the full two-loop renormalization group equations including top, bottom and tau Yukawa couplings. We show that GUT threshold effects are small because proton stability prevents triplet Higgs particles from weighing much less than 10^{16} GeV. Using $1 - \sigma$ errors for the experimental inputs and plausible ranges for unknown supersymmetric model parameters, we find that either $3.0 \times 10^{12} \text{ TeV} > m_{1/2} > 21 \text{ TeV}$ or $m_{1/2} < 65 \text{ GeV}$, with the intermediate range allowed at the $2 - \sigma$ level. It is not possible at present to fix the supersymmetry breaking scale with any precision.

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1. INTRODUCTION

Experiments at LEP are providing unparalleled precision in tests of the Standard Model and measurements of its parameters [1]. Since very good agreement is found with Standard Model predictions, LEP data can also be used to constrain those of its parameters that are not yet measurable directly, such as the mass of the top quark and/or the Higgs boson [1,2]. The data can also be used to constrain or find evidence for new physics beyond the Standard Model. LEP data already seems to exclude many models in which elementary Higgs fields are replaced by composite fields [3]. If the Higgs field responsible for electroweak symmetry breaking is indeed elementary, there are strong arguments [4] that the hierarchical ratio $m_W/m_P \ll 1$ must be protected by supersymmetry against the depredations of radiative corrections. These arguments gain force in the context of Grand Unified Theories (GUTs) [5] that include a prior stage of gauge symmetry breaking at a large mass scale $m_{GUT} = 10^{14}$ to 10^{17} GeV [6,7].

To perform their protective role, supersymmetric particles should weigh less than about 1 TeV. So far, direct searches rule out most species of electroweakly-interacting sparticles weighing up to about $m_{Z^0}/2$, and strongly-interacting sparticles weighing up to about 150 GeV [8]. These searches leave open a considerable window up to the TeV scale that can so far only be probed indirectly. However, the comparisons of different electroweak measurements at LEP and elsewhere are not very sensitive to sparticles with masses satisfying the present direct experimental lower bounds [9].

To go further, it is necessary to compare electroweak measurements with strong interaction data, which is possible within the context of a Grand Unified Theory, where the 3 a priori independent Standard Model gauge couplings are reduced to 2 independent parameters, which can be taken as the GUT symmetry breaking scale m_{GUT} and the value α_{GUT} of the unified gauge coupling at that scale. This reduction in the number of parameters enables a prediction to be made, typically one for the electroweak mixing parameter $\sin^2\theta_W$ given measurements of α_{em} and $\alpha_3(m_{Z^0})$ [6]. Since m_{GUT} is very large, this prediction requires an enormous extrapolation of our present knowledge. This extrapolation can be made reliably using the two-loop renormalization group equations [10] once the light particle spectrum is specified. In minimal GUTs, these are just the particles of the Standard Model.

It was known already before LEP that the low-energy data disfavoured minimal non-supersymmetric GUTs, but were apparently consistent with minimal supersymmetric GUTs [11]. The first of these conclusions has been greatly sharpened by LEP data, and the debate is now about which variant of supersymmetric GUTs best fits the data [12]. If the minimal supersymmetric $SU(5)$ GUT is used to fit

the LEP data, one should ask what range of supersymmetry-breaking mass scales is preferred [12-14]. If this range proved to be far above 1 TeV, the radiative corrections to the electroweak mass scale would no longer be naturally small, and one could question [12] whether some other GUT model such as flipped $SU(5)$ [15] would fit the data better.

Flipped $SU(5)$ is the only known GUT derived from the string [16]. In any specific string theory, the string unification scale (at which all the extrapolated low-energy gauge couplings should appear to become equal) is calculable [17], enabling a second prediction to be made. It is time to ask whether the grand unification scale estimated by extrapolation of the experimental gauge couplings coincides with the string unification scale calculated [18] in some interesting model derived from string.

These comparisons of LEP data with GUT predictions require care in the extraction of the low-energy couplings, in the treatment of the supersymmetry threshold, in the extrapolation across the desert, and in the treatment of the GUT threshold. The purpose of this paper is to advance the state of the art in three of these areas, to give a simple formula for the inferred value of the supersymmetry-breaking scale as a function of the measured values of $\sin^2\theta_W$ and $\alpha_3(m_{Z^0})$, and to discuss the compatibility of minimal supersymmetric $SU(5)$ with LEP data.

In section 2 we discuss the extraction of $\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$ from the low-energy data, including all effects of order $\alpha_{em}\alpha_3$ in the relevant vacuum polarization diagrams. These had been included previously in extracting the value of the mass-shell definition of $\sin^2\theta_W$ from LEP and other high-energy data [19], but not in translating the mass-shell value into the \overline{MS} value relevant for GUT tests.

In section 3 we discuss the parametrization of the supersymmetry threshold, reminding the reader of the numerical factors between the nominal masses of massive particles of spin J and the scales at which the extrapolated low-energy and high-energy couplings should appear to become equal in the \overline{MS} renormalization scheme [20]. We parametrize the sparticle masses in terms of universal spin-0 and spin-1/2 masses (m_0 and $m_{1/2}$) at the GUT scale, argue that the trilinear supersymmetry breaking parameter A is numerically irrelevant to a GUT analysis, and discuss the role of the Higgs superpotential mixing parameter μ [21].

In section 4 we discuss the GUT threshold, pointing out that in the minimal supersymmetric $SU(5)$ GUT [22-24] there is no latitude for lowering the prediction for $\sin^2\theta_W$ at low energies [22,25]. This could happen only if triplet Higgs particles were much lighter than m_{GUT} , but the stability of the proton against decays induced by dimension-5 Higgs exchange operators constrains them to weigh at least 10^{16} GeV [26].

In section 5 we assemble these calculations to give a formula valid in the minimal supersymmetric $SU(5)$ GUT for the spin-1/2 supersymmetry-breaking parameter $m_{1/2}$ in terms of the measured values of $\alpha_3(m_{Z^0})$ and $\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$, taking into account uncertainties due to our ignorance of m_0 (which has a small effect), of μ , and of the Higgs mass. Inserting the best available experimental values of $\alpha_3(m_{Z^0})$ and $\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$ we find, using $1 - \sigma$ errors for the experimental inputs and plausible ranges for unknown supersymmetric model parameters, that the minimal supersymmetric $SU(5)$ GUT requires that $m_{1/2}$ be less than 65 GeV or greater than 21 TeV. It is not possible at present to fix the supersymmetry breaking scale with any precision.

2. LOW-ENERGY INPUTS

There are six low-energy inputs that play roles in the tests of minimal supersymmetric $SU(5)$ that we make in this paper. They comprise three gauge couplings, which we parameterize all in the \overline{MS} scheme by $\alpha_{em}(m_{Z^0})_{\overline{MS}}$, $\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$ and $\alpha_3(m_{Z^0})$, and m_t , m_b and m_τ . In this section we discuss the values of these quantities that we extract from LEP and other data.

$\alpha_{em}(m_{Z^0})$

This is unfortunately not as precisely determined as the value of the fine structure constant in the Thomson limit, the extra error being induced by uncertainties in the finite renormalization between scales $Q \leq m_e$ and $Q = m_{Z^0}$. These uncertainties are dominated by hadronic vacuum polarization contributions to the running of α_{em} , particularly from the energy range between $Q = 3$ GeV and $Q = 10$ GeV. Using the estimate of these hadronic contributions given in Ref. [27], it was found in Ref. [28] that

$$\alpha_{em}(m_{Z^0}) = 1/(127.9 \pm 0.2) \quad (1)$$

A part of the error in (1) is associated with the uncertainty in m_t discussed below. However, as we will see later, the error in (1) is not dominant in the unification predictions that we make later.

$\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$

This may be inferred from recent global analyses [1,2] of precision electroweak data, including those from low energies as well as LEP and $p\bar{p}$ colliders. In Ref. [2], attention is focussed on the mass-shell definition

$$\sin^2\theta_W = 1 - m_W^2/m_{Z^0}^2 \quad (2)$$

which is related to $\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$ by the following vacuum polarization correc-

tion in $\mathcal{O}(\alpha)$:

$$\sin^2\theta_W(m_{Z^0})_{\overline{MS}} = \sin^2\theta_W - X_{\overline{MS}}\cos^2\theta_W \quad (3)$$

where

$$X_{\overline{MS}} = \text{Re}\left[\frac{\Pi_{ZZ}(m_{Z^0}^2)}{m_{Z^0}^2} - \frac{\Pi_{WW}(m_W^2)}{m_W^2}\right]_{\overline{MS}} \quad (4)$$

The precision of LEP and other data now merit the inclusion of $\mathcal{O}(\alpha_{em}\alpha_3)$ terms in the vacuum polarization functions Π_{ij} [19]. The relevant $\mathcal{O}(\alpha_{em}\alpha_3)$ terms have already been included in the fitting of the mass-shell $\sin^2\theta_W$ (2) to the global data set in Ref. [2]. In the Standard Model, $\mathcal{O}(\alpha_{em}\alpha_3)$ terms relevant to the vacuum polarizations (4) appearing in $X_{\overline{MS}}$ can be extracted from Ref. [19], and take the form

$$\begin{aligned} \Pi_{ZZ}(s) &= \frac{\alpha_3\alpha_{em}}{4\pi^2\sin^2\theta_W\cos^2\theta_W} [v_t^2\Pi^v(s, m_t) + \Pi^a(s, m_t) + v_b^2\Pi^v(s, m_b) + \Pi^a(s, m_b)] \\ \Pi_{WW}(s) &= \frac{\alpha_3\alpha_{em}}{2\pi^2\sin^2\theta_W} [\Pi^v(s, m_t, m_b) + \Pi^a(s, m_t, m_b)] \end{aligned} \quad (5)$$

where we make the approximation $m_b = 0$ and

$$\begin{aligned} \Pi^v(s, m) &= \frac{m^2}{\pi^2} \left[\frac{s}{4m^2} A(m) + V_1\left(\frac{s}{4m^2}\right) \right] \\ \Pi^a(s, m) &= \frac{m^2}{\pi^2} \left[B(m) + \frac{s}{4m^2} A(m) + A_1\left(\frac{s}{4m^2}\right) \right] \\ \Pi^v(s, m, 0) = \Pi^a(s, m, 0) &= \frac{m^2}{\pi^2} \left[\frac{B(m)}{4} + \frac{s}{4m^2} A(m) + F_1\left(\frac{s}{m^2}\right) \right] \\ v_t &= 1 - \frac{8}{3}\sin^2\theta_W \quad v_b = -1 + \frac{4}{3}\sin^2\theta_W \\ A(m) &= \frac{1}{2\epsilon} - l - \frac{4}{\zeta(3)} + \frac{55}{12} \\ B(m) &= \frac{3}{2\epsilon^2} + \frac{-12l + 11}{4\epsilon} + 3l^2 - \frac{11l}{2} + 6\zeta(3) + 3\zeta(2) - \frac{11}{8} \\ n &= 4 - 2\epsilon \quad l = \ln\left(\frac{m^2}{\mu^2}\right) \end{aligned} \quad (6)$$

Note that $X_{\overline{MS}}$ is defined by dropping divergent terms and setting $\mu = m_Z$. The functions A_1 , V_1 , and F_1 are given in Ref [19].

Within the Standard Model, the extracted values of $\sin^2\theta_W$, $X_{\overline{MS}}$ and hence $\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$ depend on the unknown masses of the top quark and the Higgs boson. There are in principle additional uncertainties arising from unknown sparticle masses in the minimal supersymmetric extension of the Standard Model, but

these are negligible [9], so we neglect them here. The more important Standard Model uncertainty is that due to our ignorance of m_t . The above-mentioned global fit [2] to the electrowak data yields $m_t = 125 \pm 30$ GeV and $m_h < 400$ GeV at the 68 % confidence level. Using equations (3),(4),(5),(6), and the 68% confidence level ranges of m_t and m_h , we find

$$\sin^2\theta_W(m_{Z^0})_{\overline{MS}} = .2331 \pm 0.0013 \quad (7)$$

Although the uncertainty has been reduced relative to that for the “on-shell” definition of $\sin^2\theta_W$, it is still one of our most significant sources of error in testing unification models. The error in (7) is again largely due to our ignorance about m_t , which could be significantly reduced in the near future by improved LEP data and/or a direct measurement at the FNAL $\bar{p}p$ collider.

$\alpha_3(m_{Z^0})$

A phenomenological reanalysis of the available data from deep inelastic scattering, Υ decay and LEP jet data was recently given in Ref. [29]. There arguments were given why the central LEP value of $\alpha_3(m_{Z^0})$ might be significantly lower than those quoted in some experimental papers, as a result of taking into account the known exponentiation of the leading infrared divergences, leading to a value closer to that obtained from the extrapolation of the lower energy data. On the basis of Ref. [29], we use here

$$\alpha_3(m_{Z^0}) = 0.113 \pm 0.004 \quad (8)$$

and note that the error is essentially independent of that on $\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$ (7).

m_t

This and the other third-generation fermion masses appear in the two-loop renormalization group equations for the gauge couplings that we use in subsequent sections. As already noted, we take $m_t = 125 \pm 30$ GeV [2]. There is a small region of parameter space where $\tan\beta$ is near one, and m_t is above 140 GeV, where the top quark Yukawa coupling becomes too large for the two-loop renormalization group equations [10] to remain reliable. However, in all but this region, the top quark Yukawa coupling has a negligible effect on the results of our two-loop calculations. Near the non-perturbative region, the effect of the top quark Yukawa coupling raises the minimal SUSY $SU(5)$ prediction for $\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$ by at most $\mathcal{O}(10^{-4})$, which is small compared with the experimental error (7).

m_b and m_τ

These play unimportant roles in the two-loop renormalization group equations for the gauge couplings [10], except possibly for very large values of $\tan\beta$ (the ratio of supersymmetric Higgs vacuum expectation values), but we nonetheless include them in an exaggerated sense of fair play. More importantly, the ratio m_b/m_τ offers an important test of the renormalization of the minimal $SU(5)$ GUT prediction $m_b = m_\tau$ [30], [7] at the grand unification scale, as we discuss in Section 5. We take $m_b = 4.9 \pm 0.1$ GeV from a selection of potential models [31]. This is to be understood as the physical mass-shell value applicable at values of the three-momentum $|p| \lesssim 1$ GeV.

3. LIGHT THRESHOLD CORRECTIONS

We present our treatment of these in some detail, since the results for $\sin^2\theta_W$ and especially the supersymmetry breaking scale are very sensitive to them, and they have not always been treated correctly in the literature.

To obtain the prediction for $\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$ in a model where the $SU(3) \times SU(2) \times U(1)$ couplings unify at the same scale, we solve the renormalization group equations for α_3 , α_2 and α_Y using the boundary conditions

$$\alpha_3(m_{GUT}) = \alpha_2(m_{GUT}) = \alpha_Y(m_{GUT}) = \alpha_{GUT} \quad (9)$$

to evaluate $\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$. In a supersymmetric theory, it is natural to renormalize the couplings in the \overline{DR} scheme [32], which is also that favoured in string theory [18]. The \overline{MS} and \overline{DR} couplings are related by [32]

$$\frac{1}{\alpha_i^{\overline{DR}}} = \frac{1}{\alpha_i^{\overline{MS}}} - \frac{C_{A_i}}{12\pi} \quad (10)$$

where the C_{A_i} are the quadratic Casimir coefficients of the adjoint representations of the gauge factors: $C_{A_3} = 3$, $C_{A_2} = 2$, $C_{A_1} = 0$. We convert between the \overline{MS} and \overline{DR} schemes in the desert between the sparticle masses and m_{GUT} . Because of the scheme-dependent factors occurring in a proper treatment of thresholds, the result is independent of the scale at which the conversion is implemented. Our solution of the renormalization group equations [10] with the boundary conditions (9) can be written in the form:

$$\sin^2\theta_W(m_{Z^0})_{\overline{MS}} = .2 + \frac{7\alpha_{em}}{15\alpha_3} + 0.0029 + \delta_s(light) + \delta_s(heavy) + \delta_s(conv) \quad (11)$$

The first two terms are the one-loop result. The third term makes the two-loop correction for central values of the inputs. Threshold corrections of light mass fields

are included in $\delta_s(light)$ and similar corrections of heavy mass fields in $\delta_s(heavy)$. The term $\delta_s(conv)$ accounts for the conversion between the \overline{MS} and \overline{DR} schemes in the desert: it changes $\sin^2\theta_W(m_{Z^0})_{\overline{MS}}$ by $\mathcal{O}(10^{-5})$ and is therefore not an important effect.

In computing $\delta_s(light)$ and $\delta_s(heavy)$ the thresholds are integrated out in a step approximation at a scale equal to a spin- and scheme-dependent constant times the physical mass, $t_J m$, to match the results of properly decoupling the heavy thresholds at one loop. As we convert from the \overline{MS} scheme to the \overline{DR} scheme in the desert, the t_J used in $\delta_s(light)$ are the \overline{MS} values [20] $t_0 = t_{1/2} = 1$, and $t_1 = \exp(-1/21)$, while those used in $\delta_s(heavy)$ are the \overline{DR} values [32] $t_0 = t_{1/2} = t_1 = 1$.

The solution of (9) gives

$$\delta_s(light) = \frac{\alpha_{em}(m_{Z^0})}{20\pi} \Sigma C_s(l) \ln\left(\frac{t_J m_l}{m_Z}\right) \quad (12)$$

The logs are weighted by a simple combination of the one-loop beta function coefficients of the representation R

$$C_s(R) = \frac{10}{3} b_Y(R) - 8 b_2(R) + \frac{14}{3} b_3(R) \quad (13)$$

In our calculation, we turn off the full beta function for a doublet at the mass of the heavier member of the doublet in calculating the renormalization of the charged $SU(2)$ current. If we were calculating the renormalization of the neutral $SU(2)$ current, half the beta function of the doublet would be turned off at the mass of each member of the doublet. The calculations of the C_s for $\delta_s(light)$ are summarized in Table 1.

To parametrize the sparticle thresholds, we consider [12] the mass spectrum of the minimal supersymmetric extension of the Standard Model with universal soft supersymmetry breaking at the unification scale [21]. We neglect all but the top Yukawa, λ_t , and drop $SU(2) \times U(1)$ breaking vevs wherever they are not multiplied by λ_t in the mass matrices. These assumptions are reasonable unless $\tan\beta$ is large or the supersymmetry breaking scale near m_Z , and render all but the stop mass matrices diagonal.

Using one-loop renormalization group equations for the soft supersymmetry-breaking masses, we find that the physical sparticle masses (apart from stops) are

given by [21]

$$\begin{aligned} m_{\tilde{g}} &= \frac{\alpha_3(m_{\tilde{g}})}{\alpha_{GUT}} m_{1/2}, \quad m_{\tilde{W}} = \frac{\alpha_2(m_{\tilde{W}})}{\alpha_{GUT}} m_{1/2}, \quad m_{\tilde{h}} \approx \mu \\ m_{\tilde{t}_L} &= \sqrt{m_0^2 + c_{\tilde{t}_l} m_{1/2}^2}, \quad m_{\tilde{t}_R} = \sqrt{m_0^2 + c_{\tilde{t}_r} m_{1/2}^2}, \quad m_{\tilde{q}} = \sqrt{m_0^2 + c_{\tilde{q}} m_{1/2}^2} \end{aligned} \quad (14)$$

Renormalization group calculations of the various coefficients give $\frac{\alpha_3(m_{\tilde{g}})}{\alpha_{GUT}} \approx 2.7$, $\frac{\alpha_2(m_{\tilde{g}})}{\alpha_{GUT}} \approx 0.79$, $c_{\tilde{t}_l} \approx 0.5$, $c_{\tilde{t}_r} \approx 0.15$ and $c_{\tilde{q}} \approx 6$. General stop mass eigenstates may be written as

$$\tilde{t}_1 = \cos \phi \tilde{t}_L + \sin \phi \tilde{t}_R, \quad \tilde{t}_2 = -\sin \phi \tilde{t}_L + \cos \phi \tilde{t}_R \quad (15)$$

with the beta functions

$$\begin{aligned} \tilde{t}_1 : \quad b_Y &= \frac{1}{15} \left(\frac{1}{2} \cos \phi + 2 \sin \phi \right)^2, \quad b_2 = \frac{1}{2} \cos^2 \phi, \quad b_3 = \frac{1}{6} \\ \tilde{t}_2 : \quad b_Y &= \frac{1}{15} \left(-\frac{1}{2} \sin \phi + 2 \cos \phi \right)^2, \quad b_2 = \frac{1}{2} \sin^2 \phi, \quad b_3 = \frac{1}{6} \end{aligned} \quad (16)$$

which give

$$C_s(\tilde{t}_1) = \frac{2}{9} \sin 2\phi + \frac{29}{6} \sin^2 \phi - \frac{19}{6}, \quad C_s(\tilde{t}_2) = -\frac{2}{9} \sin 2\phi - \frac{29}{6} \sin^2 \phi + \frac{5}{3} \quad (17)$$

Note that $C_s(\tilde{t}_1) + C_s(\tilde{t}_2) = -\frac{3}{2}$ is independent of the mixing angle ϕ . We approximate the 2×2 stop mass-squared matrix by common values $m_{\tilde{q}}^2$ along the diagonal (we assume $m_{\tilde{q}}^2 \gg m_t^2$) and a parameter \bar{m}^2 in the off-diagonal elements. This form of matrix is diagonalized by $\phi = \frac{\pi}{4}$ giving

$$\begin{aligned} m_{\tilde{t}_1}^2 &= m_{\tilde{q}}^2 + \bar{m}^2 & C_s(\tilde{t}_1) &= -\frac{19}{36} \\ m_{\tilde{t}_2}^2 &= m_{\tilde{q}}^2 - \bar{m}^2 & C_s(\tilde{t}_2) &= -\frac{35}{36} \end{aligned} \quad (18)$$

Note that this reduces to the unmixed case in the limit $\bar{m}^2 \rightarrow 0$.

Among the 8 real degrees of freedom in the 2 Higgs doublets, 3 are Goldstone bosons ‘eaten’ by the W^{+-} and Z^0 , another neutral scalar is expected to weigh $\mathcal{O}(m_{Z^0})$ and two neutral plus one charged scalar can be heavy. Therefore we take half the degrees of freedom below m_{Z^0} where they do not contribute to $\delta_s(\text{light})$, and the other half degenerate at some scale m_h .

Putting this spectrum into (12) with the C_s from Table 1 we obtain:

$$\begin{aligned} \delta_s(light) = & \frac{\alpha_{em}(m_{Z^0})}{20\pi} \left[-3\ln\left(\frac{m_t}{m_Z}\right) + \frac{28}{3}\ln\left(\frac{2.7m_{1/2}}{m_Z}\right) - \frac{32}{3}\ln\left(\frac{.79m_{1/2}}{m_Z}\right) - \ln\left(\frac{m_h}{m_Z}\right) \right. \\ & - 4\ln\left(\frac{\mu}{m_Z}\right) + \frac{5}{2}\ln\left(\frac{m_{1/2}\sqrt{6+y}}{m_Z}\right) - 3\ln\left(\frac{m_{1/2}\sqrt{.5+y}}{m_Z}\right) \\ & + 2\ln\left(\frac{m_{1/2}\sqrt{.15+y}}{m_Z}\right) - \frac{19}{36}\ln\left(\frac{m_{1/2}\sqrt{6+y+w}}{m_Z}\right) \\ & \left. - \frac{35}{36}\ln\left(\frac{m_{1/2}\sqrt{6+y-w}}{m_Z}\right) \right] \end{aligned} \quad (19)$$

where $y = m_0^2/m_{1/2}^2$ and $w = \tilde{m}^2/m_{1/2}^2$. Note that this depends on the five parameters $m_{1/2}$, y , w , μ , m_h . A simple parametrization [14] in terms of a universal supersymmetry-breaking mass-scale m_{SUSY} is inadequate.

4. HEAVY THRESHOLD CORRECTIONS

These have not been treated in the literature, and are potentially very important, but highly model-dependent. To give a specific result for $\delta_s(heavy)$, we choose the minimal supersymmetric $SU(5)$ model [22,23] with Higgs fields contained in matter superfields of the $\underline{24}$, $\underline{5}$ and $\bar{\underline{5}}$ representations of $SU(5)$. The most general form of renormalizable superpotential for the $\underline{24}$ representation Σ of $SU(5)$ is

$$W = \frac{\beta_3}{3} \text{Tr}(\Sigma^3) + \frac{M}{2} \text{Tr}(\Sigma^2). \quad (20)$$

which we now use to estimate $\delta_s(heavy)$.

We note, however, that this model has several theoretical and phenomenological difficulties [33], and that other models could give very different contributions to $\delta_s(heavy)$.

There are several degenerate minima of the scalar potential resulting from (20). Phenomenologically, we must choose the vacuum which breaks $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ with a vev for the $\underline{24}$ of Higgs of the form:

$$\langle \Sigma \rangle = \frac{M}{\beta_3} \text{diagonal } (2, 2, 2, -3, -3) \quad (21)$$

Since the GUT scale is much larger than the soft SUSY breaking terms, the spectrum is nearly supersymmetric with the X and Y gauge and matter superfields having a mass $m_X = \frac{5}{\sqrt{2}} \frac{gM}{\beta_3}$. The left-over color octet and the $SU(2)$ triplet from the $\underline{24}$ matter superfield have mass $m_{24} = 5M$ which can be greater or less than the m_X , depending on the value of β_3 .

The mass of the D^c matter superfields contained in the $\underline{5}$ and $\bar{\underline{5}}$ cannot be much less than 10^{16} GeV if dimension-five proton decay operators are not to violate experimental limits [26]. To get this color triplet mass large while keeping the doublet Higgs masses light as needed for electroweak breaking requires extra terms in the superpotential involving the $\underline{5}$ and $\bar{\underline{5}}$ with absurdly fine-tuned coefficients [33]. We sidestep this weakness of the model and set $m_{D^c} = \lambda m$ with λ arbitrary.

Since we are integrating out the thresholds with a step function, the unification scale is where the three beta functions become equal. This is when the full $SU(5)$ -symmetric spectrum is functioning in the renormalization group equations, which corresponds to the largest mass.

In the same way as we obtained $\delta_s(light)$, we find

$$\delta_s(heavy) = -\frac{\alpha_{em}(m_{Z^0})}{20\pi} \Sigma C_s(h) \ln\left(\frac{m_{GUT}}{m_h}\right) \quad (22)$$

where the differences between the expressions $\delta_s(light)$ and $\delta_s(heavy)$ are because in $\delta_s(light)$ we want to remove the influence of the light field in the scaling between m_Z and $t_j m_l$ and in $\delta_s(heavy)$ we want to include the influence of the heavy field in the scaling between m_h and m_{GUT} in the \overline{DR} scheme where all the t_j are one, where m_{GUT} is defined to be the scale at which the \overline{DR} couplings become equal. The beta functions and coefficients for the heavy fields are given in Table 2.

Putting this all together we obtain

$$\delta_s(heavy) = \frac{\alpha_{em}(m_{Z^0})}{20\pi} \left[-6\ln\left(\frac{x}{\lambda}\right) + 4\ln\left(\frac{x}{\hat{g}}\right) + 2\ln\left(\frac{x}{5}\right) \right] \quad (23)$$

where $x = \max[5, \lambda, \hat{g} = \frac{5g}{\sqrt{2}\beta_3} \approx 2.57/\beta_3]$. Since the unification scale is the mass of the heaviest field where the beta functions become equal, all the logs are positive and the only chance of a negative contribution to $\delta_s(heavy)$ is if m_{D^c} is below m_{GUT} . Previous calculations show that even for $m_{D^c} = m_{GUT}$, the minimal model has troubles with dimension-5 proton decay unless tight constraints are obeyed by the various parameters determining the light spectrum [26]. Because these constraints raise the minimum contribution from $\delta_s(light)$ as m_{D^c} decreases more quickly than they lower the contribution from $\delta_s(heavy)$, we take a lower bound on $\delta_s(heavy)$ of zero and leave the parameters determining the light spectrum unconstrained. This is a conservative estimate, as even with $m_{D^c} \sim m_{GUT} \sim 10^{16}$ GeV there are significant constraints necessary in the light particle spectrum to avoid proton decay [26].

5. CONFRONTATION WITH EXPERIMENT

We now check the minimal GUT predictions for $\sin^2 \theta_W$ and m_b/m_τ against experiment, and study whether a precise prediction for the supersymmetry breaking scale $m_{1/2}$ can be obtained. We show in Fig. 1 the one-sigma error ellipse for $(\alpha_3, \sin^2 \theta_W(m_{Z^0})|_{\overline{MS}})$ confronted with the minimal supersymmetric GUT prediction if low- and high-energy thresholds are neglected, which would be valid if all sparticles weighed m_Z and all heavy particles m_{GUT} . The double line of the theoretical prediction corresponds to the range (1) of $\alpha_{em}(m_{Z^0})|_{\overline{MS}}$, and just misses the top of the ellipse, indicating that sparticles probably weigh more than m_Z . We have also calculated the physical b quark mass, for different values of m_t , $\tan \beta$ and α_3 , as seen in Fig. 2, again neglecting high and heavy threshold corrections. Figures 2a and 2b show contours of constant $m_b(m_b)$ in the $(\tan \beta, m_t)$ plane for the two extreme values of α_3 [29]. This calculation uses the full two-loop renormalization group equations for the minimal supersymmetric extension of the Standard Model above m_Z . Below m_Z we use one-loop gauge corrections to the masses for α_3 and α_{em} , where α_3 is calculated to two-loop order and α_{em} is calculated to one-loop order. We take $m_\tau = 1.784$ GeV, drop the bottom and top quark from the renormalization group equations at their physical masses, and take all other light thresholds at m_Z . In almost the whole $(\tan \beta, m_t)$ plane, m_b agrees with the value from potential models for some α_3 in the range $.113 \pm .004$. The dotted regions are areas where the Yukawa couplings are non-perturbative.

The agreement of both $\sin^2 \theta_W(m_{Z^0})|_{\overline{MS}}$ and m_b with the values determined by experiment (0.2331 ± 0.0013 and 4.9 ± 0.1 GeV, respectively) is striking.

Writing

$$\begin{aligned} \sin^2 \theta_W(m_{Z^0})|_{\overline{MS}} &= .2029 + \frac{7\alpha_{em}(m_{Z^0})}{15\alpha_3} + \delta_s \\ \delta_s &= \delta_s(light) + \delta_s(heavy) + \delta_s(conv) \end{aligned} \quad (24)$$

the range of δ_s compatible with the $1 - \sigma$ error ellipse defined by the values given in Section 2 can be determined by solving for the values of δ_s for which the curve (24) just touches the top and bottom of the ellipse:

$$-3.9 \times 10^{-3} < \delta_s < -3 \times 10^{-4} \quad (25)$$

In section 3 we showed that $\delta_s(conv)$ is negligible and in section 4 we showed that $\delta_s(heavy) > 0$. If we assume that $\delta_s(heavy) = 0$, which gives a conservative upper

estimate of the range of possible values of $\delta_s(light)$, we have

$$-3.9 \times 10^{-3} < \delta_s(light) < -3 \times 10^{-4} \quad (26)$$

and a central value for $\sin^2\theta_W(m_{Z^0})_{\overline{MS}} = .2331$ and $\alpha_3(m_{Z^0}) = .113$ of $\delta_s(light) = -2.1 \times 10^{-3}$.

If one were to make the naive assumption [14] that all the light fields in $\delta_s(light)$ except the top could be integrated out at a common scale m_{SUSY} , i.e., ignoring the predicted spectrum relations (14) and the other theoretical parameters (μ, m_h, y, w) , one would find

$$\delta_s(light) = \frac{\alpha_{em}}{20\pi} [-3\ln(\frac{m_t}{m_Z}) - \frac{19}{3}\ln(\frac{m_{SUSY}}{m_Z})]. \quad (27)$$

This would correspond to

$$\ln(\frac{m_{SUSY}}{m_Z}) = 2.8 \pm 2.4 \quad (28)$$

whose central value is similar to that quoted in Ref. [14]. However, we re-emphasize that such a simple parametrization of the diverse sparticle thresholds is inadequate.

A more realistic parametrization of $\delta_s(light)$ consistent with the general philosophy of grand unification involves $m_0, m_{1/2}, \mu$ and m_h as described in Section 3. Assembling the calculations of the previous sections and assuming values of the variables so that all thresholds are above m_Z , we can solve for $\ln(\frac{m_{1/2}}{m_Z})$:

$$\begin{aligned} \ln(\frac{m_{1/2}}{m_Z}) = & \frac{15\pi}{\alpha_{em}(m_{Z^0})} [0.2029 + \frac{7\alpha_{em}(m_{Z^0})}{15\alpha_3(m_{Z^0})} - \sin^2\theta_W(m_{Z^0})_{\overline{MS}} + \delta_s(heavy)] \\ & + f(\frac{m_0^2}{m_{1/2}^2}, \frac{\bar{m}^2}{m_{1/2}^2}) - \frac{9}{4}\ln(\frac{m_t}{m_Z}) - \frac{3}{4}\ln(\frac{m_h}{m_Z}) - 3\ln(\frac{\mu}{m_Z}) + 8.839 \end{aligned} \quad (29)$$

where

$$\begin{aligned} f(y, w) = & \frac{15}{16}\ln(6+y) - \frac{19}{96}\ln(6+y+w) - \frac{35}{96}\ln(6+y-w) \\ & - \frac{9}{8}\ln(.5+y) + \frac{3}{4}\ln(.15+y) \end{aligned} \quad (30)$$

This result holds in general for the $SU(3) \times SU(2) \times U(1)$ minimal supersymmetric Standard Model with soft supersymmetry breaking that is universal at a scale where all three couplings unify. The effects of an arbitrary spectrum of extra heavy

representations included in $\delta_s(\text{heavy})$. To get any mileage out of this result, we must eliminate some of this arbitrariness by choosing a specific model. Our choice in this work has been minimal supersymmetric $SU(5)$ model [22,23] with $\underline{24}$, $\underline{5}$ and $\underline{\bar{5}}$ matter superfields containing the Higgs fields which break the gauge symmetry. In this case $\delta_s(\text{heavy}) > 0$. The function $f(y, w)$ increases with physically relevant values of w and $f(y, 0)$ has a maximum of .386 at $y = .703$ and approaches zero as y becomes very large. We will henceforth use $\delta_s(\text{heavy}) = 0$ and $w = 0$, noting that possible departures from these assumptions can only increase the value of $\ln(\frac{m_{1/2}}{m_Z})$, and take $f = 0.193 \pm 0.19$.

Within this parameterization, using the experimental central values and errors on the input parameters α_{em} , α_3 , $\sin^2\theta_W$ and m_t quoted in Section 2, allowing $m_0/m_{1/2}$ to vary between 0 and infinity, taking theoretical ranges $\mu = 100 \times 3^{0\pm1}$ GeV and $m_h = 300 \times 3^{0\pm1}$ GeV, we find

$$\ln(\frac{m_{1/2}}{m_Z}) = 19.8 \mp 7.3 \mp 6.9 \mp .3 \begin{smallmatrix} -3.3 \\ +0.3 \end{smallmatrix} \mp 0.2 \mp 0.8 \quad (31)$$

The first three errors are the experimental errors, the first due to the highly-correlated values of $\sin^2\theta_W$ and m_t , the second due to α_3 and the third due to α_{em} . The last three errors are due to the theoretical uncertainty in m_h , $y = m_0^2/m_{1/2}^2$, and μ . The lower bounds on m_h and μ are phenomenological, while the upper bounds are from naturalness [34] and may be relaxed depending on just what amount of fine tuning one wants to tolerate.

Adding the first three experimental errors in quadrature and taking naïvely the extreme values of the last three theoretical uncertainties in (31) would give 3.0×10^{12} TeV $> m_{1/2} > 21$ TeV, which is **uncomfortably high**. However, Eq. (31) does not apply when some of the thresholds are below m_Z . We find that in this case, the minimum contribution of the squarks and sleptons to $\delta_s(\text{light})$ is still 0 for very large y . Choosing the maximum values of $m_t = 155$ GeV, $\mu = 300$ GeV, and $m_h = 900$ GeV which minimize $\delta_s(\text{light})$, eq. (19) and the bound (26) used in the region where some thresholds are below m_Z , imply $m_{1/2} < 65$ GeV. This value should be taken only as approximate as several approximations made in the supersymmetric mass matrices break down for low values of $m_{1/2}$. For such low values of $m_{1/2}$, supersymmetric radiative corrections should also be included [9] in the global analysis of LEP and other low-energy precision data. The range $65 \text{ GeV} < m_{1/2} < 21 \text{ TeV}$ is allowed at the $2 - \sigma$ level.

We conclude that the minimal supersymmetric $SU(5)$ model is consistent with LEP data also for a small supersymmetry breaking scale. However, it is not possible at present to estimate $m_{1/2}$ precisely, and it is clearly possible that the minimum supersymmetry breaking scale is uncomfortably large [35], with a re-emergence of

the hierarchy problem (admittedly at a scale smaller than m_{GUT}). Our results provide a framework to monitor the minimum supersymmetry breaking scale in this model (and others), and possibly eliminate specific models on the basis of too large a supersymmetry breaking scale as experimental data sharpen.

In particular, our favorite model is supersymmetric $SU(5) \times U(1)$, which economically solves all of the problems of the minimal SUSY $SU(5)$ model [15,16]. In addition, the supersymmetric $SU(5)$ predictions for $\sin^2\theta_W$ become an upper bound in the flipped model [36,12,18], which gives a simple explanation why the prediction with all light thresholds at m_Z lies to the top of the LEP oval. Of course, there are many other models which could conceivably produce predictions of $\sin^2\theta_W$ in the bullseye of the LEP oval. Their predictions may be extracted using the methods of this paper.

Even if future $2 - \sigma$ ovals [35] happen to require large supersymmetry breaking scales in the minimal supersymmetric $SU(5)$ model [22-24], there are several ways out. Extra representations are often added to the model to remedy its manifest defects [33], and could easily give large threshold corrections. Relations between various sparticle masses could conceivably allow a lower value of m_{D^c} . For $\tan\beta$ of around 1 and $m_t > 140$ GeV, the top quark Yukawa coupling is too large to use perturbation theory. Non-universal SUSY breaking scenarios could alter the contributions of the light thresholds.

There will always be such ‘what ifs’, but in the context of GUT and string models which are becoming very specific, it is clear that LEP data provide a very stringent test of unified models using coupling constant unification, and may eventually provide useful constraints on the supersymmetry breaking scale.

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R	$b_Y(R)$	$b_2(R)$	$b_3(R)$	$C_s(R)$
\tilde{g}	0	0	2	$\frac{28}{3}$
\tilde{l}_l	$\frac{3}{10}$	$\frac{1}{2}$	0	-3
\tilde{l}_r	$\frac{3}{5}$	0	0	2
\tilde{w}	0	$\frac{4}{3}$	0	$-\frac{32}{3}$
$\tilde{q} - \tilde{t}$	$\frac{49}{60}$	1	$\frac{5}{3}$	$\frac{5}{2}$
\tilde{t}_l	$\frac{1}{60}$	$\frac{1}{2}$	$\frac{1}{6}$	$-\frac{19}{6}$
\tilde{t}_r	$\frac{4}{15}$	0	$\frac{1}{6}$	$\frac{5}{3}$
\tilde{h}	$\frac{2}{5}$	$\frac{2}{3}$	0	-4
heavy higgs	$\frac{1}{10}$	$\frac{1}{6}$	0	-1
SUSY total	$\frac{5}{2}$	$\frac{25}{6}$	4	$-\frac{19}{3}$
Standard Model	$\frac{41}{10}$	$-\frac{19}{6}$	-7	$\frac{19}{3}$
Standard SUSY Model	$\frac{33}{5}$	1	-3	0
t	$\frac{17}{30}$	1	$\frac{2}{3}$	-3

Table 1 - Beta function coefficients for the light representations, R . Note that the whole $SU(2)$ charge of a doublet is given to its heaviest member, $C_s(R) = \frac{10}{3}b_Y(R) - 8b_2(R) + \frac{14}{3}b_3(R)$, and SUSY total + Standard Model = Standard SUSY model.

R	$b_Y(R)$	$b_2(R)$	$b_3(R)$	$C_s(R)$
D^c matter superfield	$\frac{1}{5}$	0	$\frac{1}{2}$	3
X,Y gauge superfields	-15	-9	-6	-6
X,Y matter superfields	5	3	2	2
W matter superfield	0	2	0	-16
g matter superfield	0	0	3	14

Table 2 - Beta function coefficients for the heavy representations, R , and $C_s(R) \equiv \frac{10}{3}b_Y(R) - 8b_2(R) + \frac{14}{3}b_3(R)$. Note that the minimal SU(5) model considered in this paper has two D^c 's, and the X,Y,W and g matter superfields are contained in the 24 matter superfield used to break the GUT symmetry.

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Figure captions

Fig. 1 - One-sigma error ellipses in the $(\alpha_3, \sin^2 \theta_W(m_{Z^0})|_{\overline{MS}})$ plane compared with the band predicted by the minimal supersymmetric $SU(5)$ GUT, assuming that all spartners of Standard Model particles weigh m_Z , and neglecting GUT threshold corrections. The width of the band corresponds to the error (1) in $\alpha_{em}(m_{Z^0})$. The $1 - \sigma$ errors quoted in this paper are shown by the solid ellipse and those of previous works [12-14] by dotted ellipses. Also shown is the line in the $(\alpha_3, \sin^2 \theta_W(m_{Z^0})|_{\overline{MS}})$ plane predicted by a minimal non-supersymmetric $SU(5)$ GUT.

Fig. 2 - Contours of the physical on-shell value of the b quark mass $m_b(m_b)$, obtained in the minimal supersymmetric $SU(5)$ GUT, assuming that all spartners of Standard Model particles weigh m_Z , and neglecting GUT threshold corrections. The predictions are given as contours in the $(\tan \beta, m_t)$ plane for (a) $\alpha_3 = 0.109$, (b) $\alpha_3 = 0.117$. In the dotted region our two-loop perturbative calculation fails.

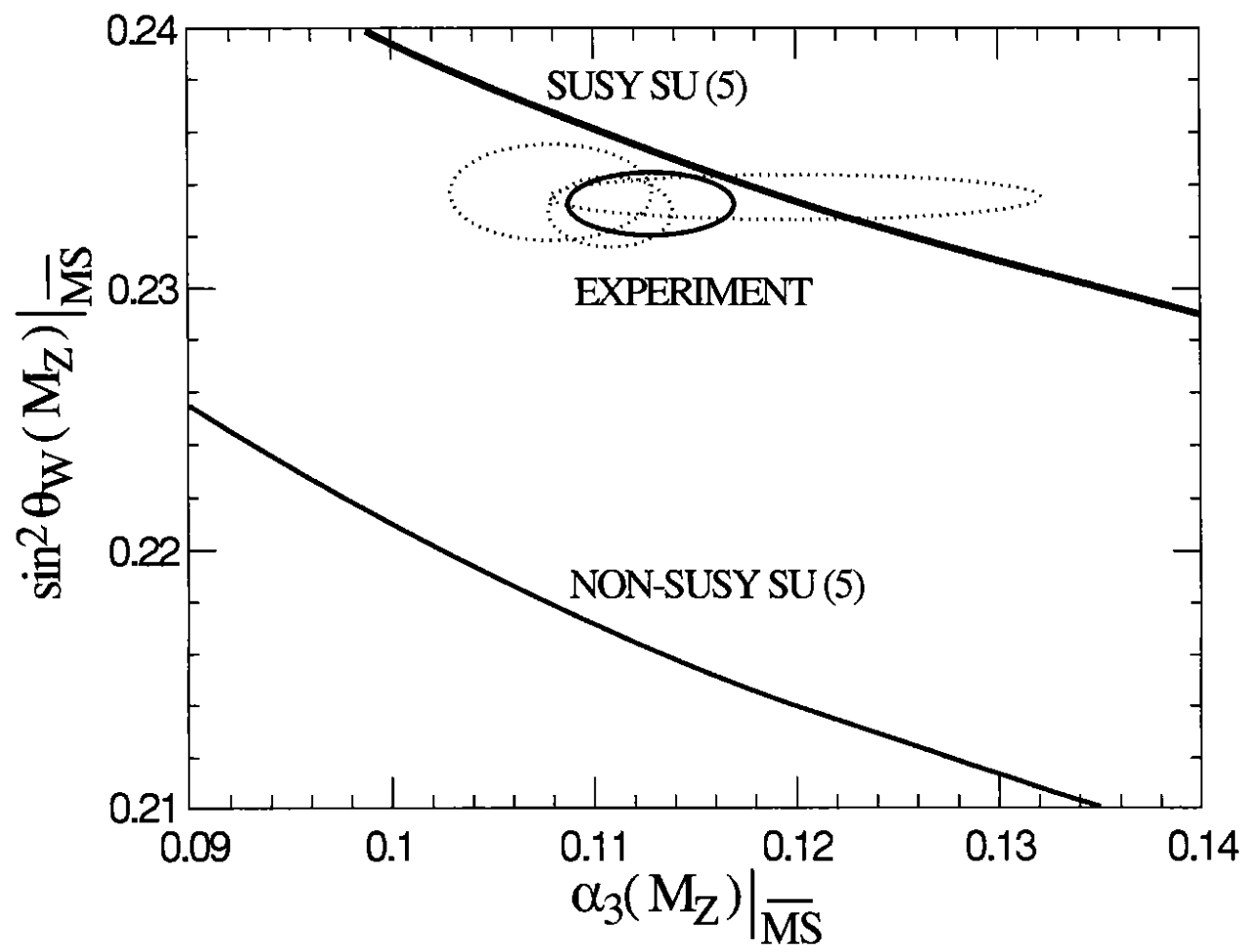


Figure 1

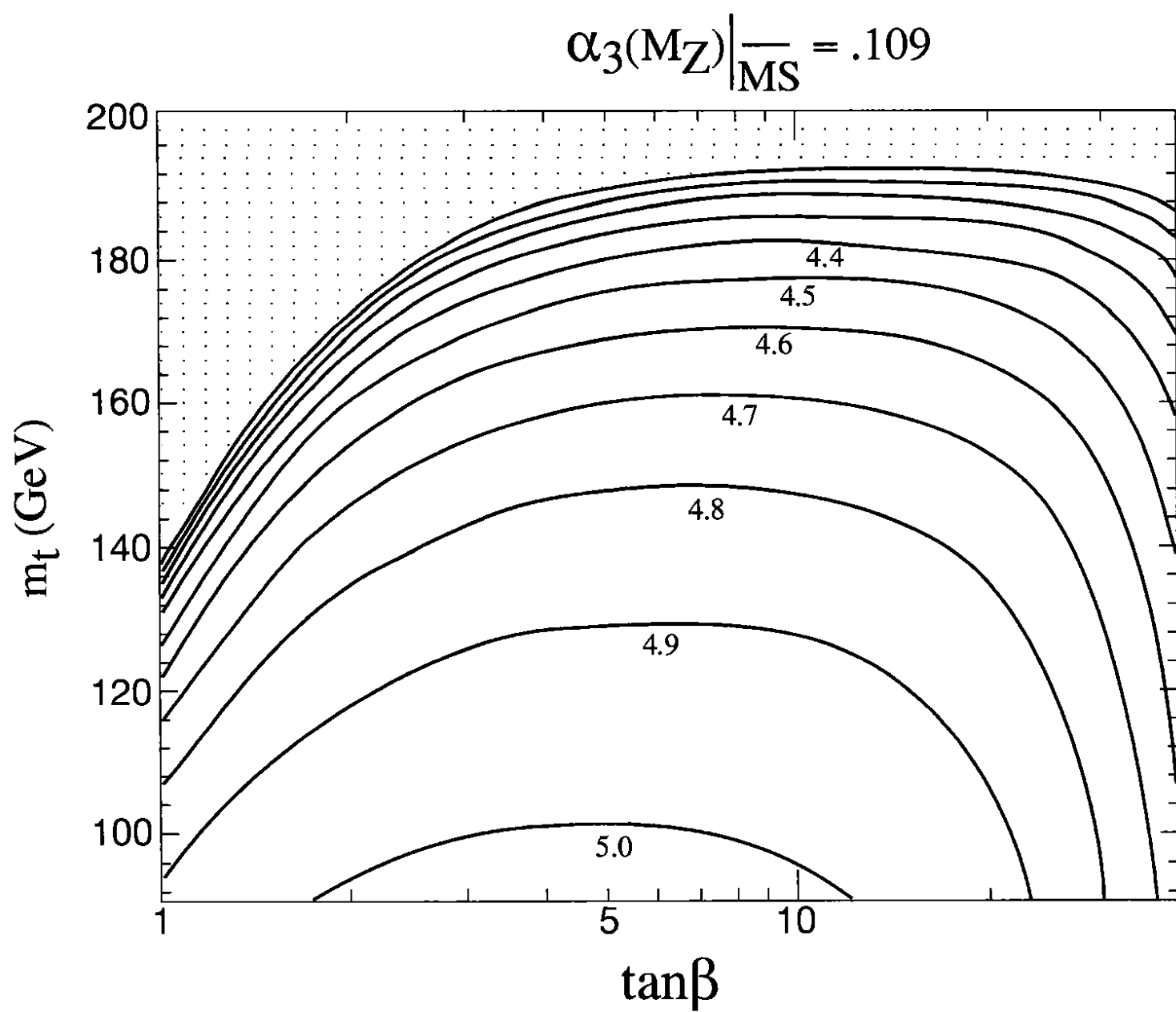


Figure 2a

$$\alpha_3(M_Z)|_{\overline{MS}} = .117$$

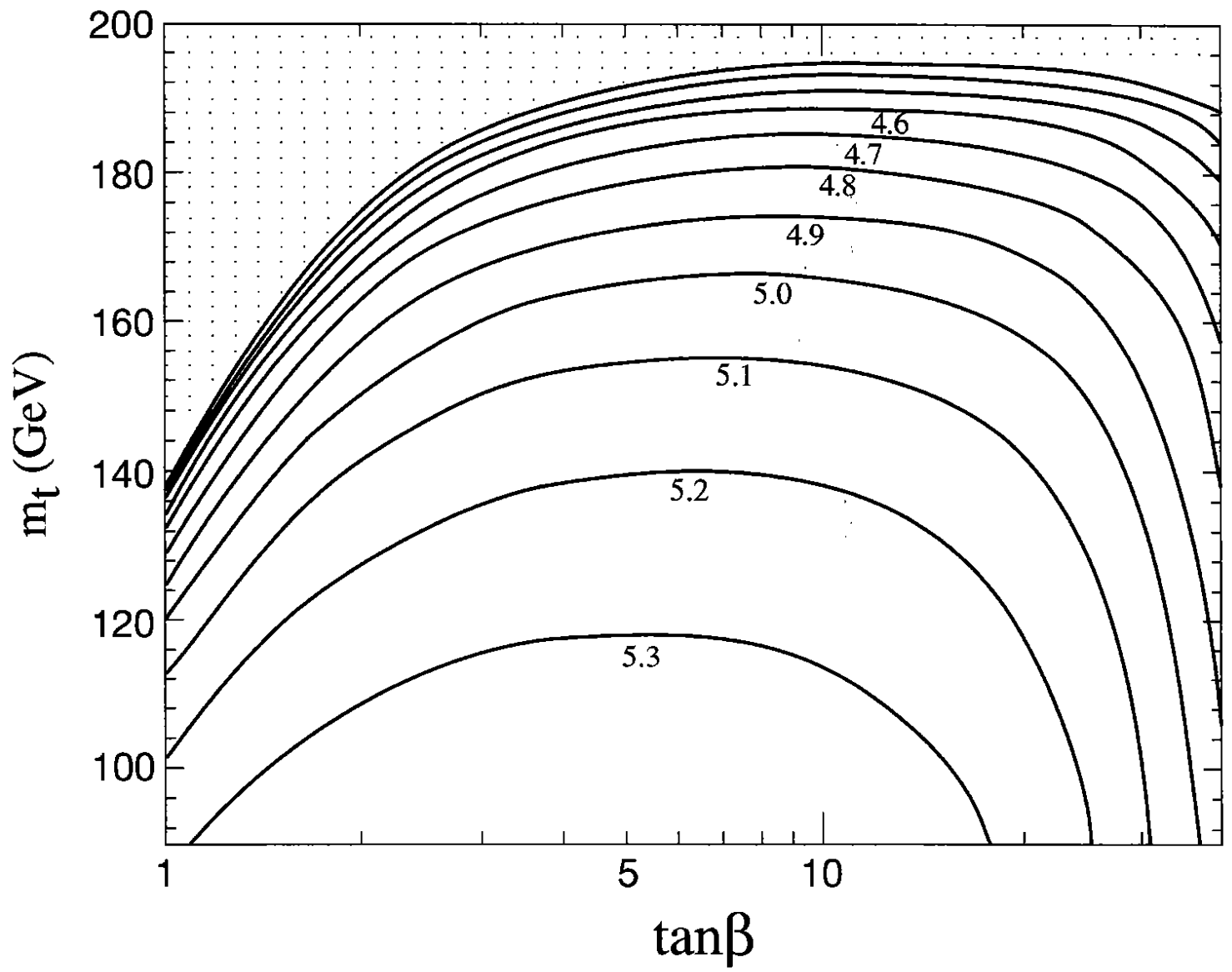


Figure 2b