MIFPA-14-04

An ω Deformation of Gauged STU Supergravity

H. Lü [1], Yi Pang [2] and C.N. Pope [2,3]

[1] *Department of Physics, Beijing Normal University, Beijing 100875, China*

[2] *George P. & Cynthia Woods Mitchell Institute for Fundamental Physics and Astronomy, Texas A&M University, College Station, TX 77843, USA*

[3] *DAMTP, Centre for Mathematical Sciences, Cambridge University, Wilberforce Road, Cambridge CB3 OWA, UK*

ABSTRACT

Four-dimensional $\mathcal{N} = 2$ gauged STU supergravity is a consistent truncation of the standard $\mathcal{N} = 8$ gauged $SO(8)$ supergravity in which just the four $U(1)$ gauge fields in the Cartan subgroup of $SO(8)$ are retained. One of these is the graviphoton in the $\mathcal{N} = 2$ supergravity multiplet and the other three lie in three vector multiplets. In this paper we carry out the analogous consistent truncation of the newly-discovered family of ω-deformed $\mathcal{N} = 8$ gauged $SO(8)$ supergravities, thereby obtaining a family of ω-deformed STU gauged supergravities. Unlike in some other truncations of the deformed $\mathcal{N} = 8$ supergravity that have been considered, here the scalar potential of the deformed STU theory is independent of the ω parameter. However, it enters in the scalar couplings in the gauge-field kinetic terms, and it is non-trivial because of the minimal couplings of the fermion fields to the gauge potentials. We discuss the supersymmetry transformation rules in the ω-deformed supergravities, and present some examples of black hole solutions.

Contents

1 Introduction

It had long been thought that the maximal $\mathcal{N}=8$ gauged $SO(8)$ supergravity theory that was constructed by de Wit and Nicolai in 1982 [1] was unique. However, based on a gauging-independent formulation of the theory [2], evidence was recently found indicating that in fact there exists a one-parameter family of inequivalent $\mathcal{N}=8$ gauged $SO(8)$ supergravity theories, characterised by an angular parameter ω [3]. When written in the purely electric frame, the new theories were given in [4].

There have subsequently been a number of studies in which truncations of the new ω-deformed maximal supergravity have been made, typically with the focus being on finding scalar-field truncations in which the scalar potential still has a non-trivial dependence on the parameter ω [5–11]. This can lead to a richer structure of anti-de Sitter (AdS) stationary points and domain-wall solutions, with the nature of the vacuum state now being dependent on ω.

It is also of interest to study truncations of the new $\mathcal{N}=8$ supergravities that give rise to families of supergravities with reduced supersymmetry. In this paper, we investigate the consistent embedding of $\mathcal{N}=2$ gauged $U(1)^4$ STU supergravity, and also the embedding of $\mathcal{N}=4$ gauged $SO(4)$ supergravity.[1] In both cases, with the embedding we consider, we find that the scalar potential of the truncated theory no longer carries any dependence on the deformation parameter of the larger $\mathcal{N}=8$ theory, even though the $\mathcal{N}=8$ potential does, of course, depend upon ω. However, this does not mean that the entire bosonic sector of the truncated theory is necessarily independent of ω. In the case of the truncation to the $U(1)^4$ gauged STU models, we find that the scalar couplings in the gauge-field kinetic terms carry non-trivial ω dependence, and so, for example, charged black hole solutions will, in general, acquire ω-dependent modifications.

If one stays purely within the bosonic sector, the ω dependence of the scalar couplings to the gauge-field kinetic terms can be absorbed by means of duality transformations, together with scalar field redefinitions. However, since the gauge potentials have minimal couplings to the fermions, this means that the necessary duality transformations are not in fact symmetries of the full supergravity theory, and so ω is a non-trivial parameter of the full ω-deformed STU supergravity theories. In particular, it can affect the supersymmetry and the fermion couplings.

We also investigate some further consistent truncations, where the resulting theories are considerably simplified. One obvious such example is where one sets the four $U(1)$ gauge fields pairwise equal. At the same time, for consistency, four of the six scalar fields are set to zero, leaving just a single dilaton/axion pair. In this truncation it turns out that the ω dependence in the scalar couplings of the kinetic terms for the remaining two gauge fields can in fact be removed merely by using a shift symmetry of the axionic scalar, with no need to perform any dualisation of the gauge fields, and so then the entire theory becomes independent of the ω parameter.

This pairwise equal truncation can in fact be viewed as the abelian $U(1)\times U(1)$ truncation of $\mathcal{N}=4$ gauged $SO(4)$ supergravity. It is of interest therefore to investigate the truncation of the deformed $\mathcal{N}=8$ supergravity to $\mathcal{N}=4$. For the embedding we adopt, we

[1] In the corresponding ungauged theory, the term STU refers to the triality of $SL(2,R)\times SL(2,R)\times SL(2,R)$ global symmetries, and was extensively discussed in [12]. In the gauged STU model, the global symmetry in the bosonic sector is reduced to $U(1)\times U(1)\times U(1)$. However, since one of the factors in the symmetry group is an electric/magnetic duality symmetry, this factor is broken completely in the full STU gauged supergravity, because of the minimal coupling of the fermions to the gauge fields. Further details of the gauged STU supergravity can be found in [14, 15].

find that the same thing happens; namely, that the scalar potential is independent of ω and furthermore that the ω-dependence in the scalar couplings to the gauge-field kinetic terms can be eliminated by means of a shift symmetry transformation of the axionic scalar. Interestingly, therefore, although one does not obtain a family of deformed $\mathcal{N}=4$ gauged $SO(4)$ supergravities via truncation from the deformed $\mathcal{N}=8$ supergravities by this method, there do in fact exist deformed $\mathcal{N}=4$ gauged $SO(4)$ supergravities, as constructed by de Roo and Wagemans long ago [13]. The deformation parameter α in these theories is very similar in nature to the deformation parameter ω in the gauged $\mathcal{N}=8$ theory, being associated with a parameterisation of the duality complexion of the gauge fields prior to gauging. It may be, however, that the deformations in the $\mathcal{N}=4$ theories are intrinsic to the $\mathcal{N}=4$ gaugings, and they do not necessarily have a larger interpretation with $\mathcal{N}=8$.

Returning to the deformed $U(1)^4$ gauged STU models, we find that different further truncations are possible for which the ω deformation parameter remains non-trivial. For example, we can perform a "$1+3$" truncation in which three of the original gauge fields are set equal, giving rise to a theory with just two remaining independent gauge fields with $A_\mu^{(1)}=A_\mu^1$ and $A_\mu^{(2)}=A_\mu^{(3)}=A_\mu^{(4)}=A_\mu^2/\sqrt{3}$. At the same time, for consistency, the three dilatonic scalars of the STU theory are equated, and the three axionic scalars are equated. The resulting supergravity theory is in fact a gauged version of the Poincaré supergravity one would obtain by reducing five-dimensional minimal supergravity on a circle. It can easily be seen that the ω parameter enters non-trivially in the scalar couplings to the gauge field kinetic terms, since the duality transformation that would remove it lies outside the symmetry group of the scalar coset. Thus since this duality transformation can no longer be performed in the gauged theory, where the gauge potentials couple minimally to the fermions, we have a non-trivial family of ω-deformed gauged supergravities in this truncation.

A further consistent truncation allows the second gauge field A_μ^2 and the axionic scalar to be set to zero. Of course, this is no longer a supersymmetric theory, but it is a consistent truncation of the STU supergravity. The resulting bosonic theory is the ω-deformed generalisation of the "Kaluza-Klein theory" obtained by the circle reduction of pure five-dimensional Einstein gravity, augmented by the addition of the scalar potential. The ω parameter is again non-trivial, in the sense that it can only be removed by performing a dualisation of the remaining gauge field, which would be disallowed because its gauge potential couples minimally to the fermions, and so viewed as a truncation of the full STU model in which fermions are retained, it will still carry the imprint of the ω parameter.

The paper is organised as follows. In section 2, we describe the truncation of the fields of $\mathcal{N}=8$ supergravity to those of $\mathcal{N}=2$ STU supergravity. Substituting into the formalism of the ω-deformed $\mathcal{N}=8$ theories, we thereby obtain the corresponding ω-deformed STU supergravity theories. Our focus is on the bosonic sector, together with finding the terms in the supersymmetry transformation rules for the fermionic fields that are needed in order to investigate the supersymmetry of bosonic solutions. We also determine the range of the ω parameter in the STU supergravities that spans the space of inequivalent models. In section 3 we examine some further truncations of the ω-deformed STU supergravity theories. These include the case where one sets the four gauge fields pairwise equal, at the same time setting four of the six scalar fields to zero, which, as we mentioned above, gives a theory where the ω parameter becomes trivial. We also discuss the non-trivial case of the $1+3$ split described above, where two gauge fields are retained. We present dyonic black hole solutions in the further truncation to a single gauge potential. In section 4 we examine some features of the family of deformed $\mathcal{N}=4$ gauged $SO(4)$ supergravities that were constructed by de Roo and Wagemans, which do not arise as truncations of the ω-deformed $\mathcal{N}=8$ theories. After conclusions in section 5, we include two appendices. Appendix A contains the detailed form of the scalar couplings to the gauge-field kinetic terms in the ω-deformed STU supergravities, and appendix B contains further detailed expressions of the various tensors that appear in the supersymmetry transformation rules.

2 The Embedding of the STU Model

2.1 The bosonic sector

The standard gauged STU supergravity theory can be obtained as a consistent truncation of standard $\mathcal{N}=8$ gauged $SO(8)$ supergravity. In this embedding, the $SO(8)$ gauge fields are truncated to retain only those in the $U(1)^4$ Cartan subalgebra of $SO(8)$. As well as the metric and the four $U(1)$ gauge fields, the bosonic sector includes three scalars from the $\mathbf{35}_v$ of scalar fields, and three axionic scalars from the $\mathbf{35}_c$ of pseudoscalar fields in the gauged $SO(8)$ theory. The fermions will be discussed in section 2.3.

The embedding was described in [14] in the case of a further truncation in which the three axionic scalar fields are set to zero.[2] The complete embedding of the bosonic sector

[2]This further truncation would be inconsistent in general, in the sense that the equations of motion for the axionic scalars would not permit setting them to zero in generic solutions. However, if one restricts attention to solutions where the wedge products of pairs of field strengths vanishes, then the axions can be

of the gauged STU model, including also the axions, was given in [15]. This was achieved by giving an explicit form for the 56-bein

$$\mathcal{V} = \begin{pmatrix} u_{ij}{}^{IJ} & v_{ijKL} \\ v^{k\ell IJ} & u^{k\ell}{}_{KL} \end{pmatrix}, \tag{2.1}$$

that enters in the construction of $\mathcal{N}=8$ supergravity given in [1]. In order to construct the corresponding ω-deformed version of the gauged STU supergravity model, we may simply take the same construction of the $\mathcal{V}$ matrix, and substitute it into the formulae presented in [4] for the ω-deformed $\mathcal{N}=8$ supergravity.

We shall use the same notation and conventions as in [14] and [15]. In [15], $\mathcal{V}$ was expressed in the symmetric gauge

$$\mathcal{V} = \exp\Big\{ -\frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & \phi_{ijk\ell} \\ \phi^{mnpq} & 0 \end{pmatrix} \Big\}, \tag{2.2}$$

where $\phi^{ijk\ell}$ is totally antisymmetric and self-dual in the sense that

$$\bar{\phi}^{ijk\ell} = \phi_{ijkl} = \frac{1}{4!}\epsilon_{ijk\ell mnpq}\, \phi^{mnpq} \,. \tag{2.3}$$

Denoting the index pairs $\{12, 34, 56, 78\}$ by $\{\alpha\}$ with $\alpha = 1, 2, 3, 4$, the self-dual tensor $\phi^{ijk\ell}$ takes the form

$$\phi^{ijk\ell} = \phi_{ijk\ell} = \sqrt{2}[(\Phi^{(1)}\epsilon^{(12)} + \bar{\Phi}^{(1)}\epsilon^{(34)}) + (\Phi^{(2)}\epsilon^{(13)} + \bar{\Phi}^{(2)}\epsilon^{(24)}) + (\Phi^{(3)}\epsilon^{(14)} + \bar{\Phi}^{(3)}\epsilon^{(23)})]\,, \tag{2.4}$$

where $\epsilon^{(12)}_{ijk\ell} = \pm 1$ whenever $\{i, j, k, \ell\}$ is an even (odd) permutation of $\{1, 2, 3, 4\}$, $\epsilon^{(13)}_{ijk\ell} = \pm 1$ whenever $\{i, j, k, \ell\}$ is an even (odd) permutation of $\{1, 2, 5, 6\}$, and so on. We write the three complex scalars as

$$\Phi^{(a)} = \phi_a\, e^{\mathrm{i}\sigma_a} \,. \tag{2.5}$$

The scalar kinetic terms are constructed as $-\frac{1}{48}\mathcal{A}^{ijk\ell}_\mu \mathcal{A}^\mu_{ijk\ell}$, where $\mathcal{A}^{ijk\ell}_\mu$ is given by [1]

$$D_\mu \mathcal{V}\,\mathcal{V}^{-1} = -\frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & \mathcal{A}^{ijk\ell}_\mu \\ \mathcal{A}_{\mu\, ijk\ell} & 0 \end{pmatrix} . \tag{2.6}$$

These kinetic terms are independent of the ω deformation of the $\mathcal{N}=8$ theory.

The scalar potential V is defined in the standard way in $\mathcal{N}=8$ gauged $SO(8)$ supergravity, except that now with the ω deformation the definition of the T tensor is modified. Thus one has [4]

$$V = g^2\Big[-\tfrac{3}{4}|A_{(1)}{}^{ij}|^2 + \tfrac{1}{24}|A_{(2)\,i}{}^{jk\ell}|^2\Big]\,, \tag{2.7}$$

set to zero.

where $T_i{}^{jk\ell}$ is decomposed into its irreducible parts

$$T_i{}^{jk\ell} = -\tfrac{3}{2} A_{(1)}{}^{j[k}\,\delta^{\ell]}{}_i - \tfrac{3}{4} A_{(2)\,i}{}^{jk\ell}\,. \tag{2.8}$$

Here $A_{(1)}{}^{ij}$ is symmetric in ij and $A_{(2)\,i}{}^{jk\ell}$ is antisymmetric in $jk\ell$ and traceless, i.e. $A_{(2)\,i}{}^{ik\ell} = 0$. The T tensor in the ω-deformed theory is defined by [4]

$$T_i{}^{jk\ell} = (e^{-\mathrm{i}\omega}\, u^{k\ell}{}_{IJ} + e^{\mathrm{i}\omega}\, v^{k\ell IJ})(u_{im}{}^{JK}\, u^{jm}{}_{KI} - v_{imJK}\, v^{jmKI})\,. \tag{2.9}$$

The four gauge fields of the deformed STU model are taken to be in the $U(1)^4$ Cartan subalgebra of the $SO(8)$ gauge fields, whose field strengths are $F^{IJ} = dA^{IJ} - \frac{1}{2} g A^{IK} \wedge A^{KJ}$. Thus, taking into account an $SO(8)$ triality rotation as discussed in [14], we write

$$\begin{aligned} A^{12} &= \tfrac{1}{2}\,[A^{(1)} + A^{(2)} + A^{(3)} + A^{(4)}]\,, \quad & A^{34} &= \tfrac{1}{2}\,[A^{(1)} + A^{(2)} - A^{(3)} - A^{(4)}]\,, \\ A^{56} &= \tfrac{1}{2}\,[A^{(1)} - A^{(2)} + A^{(3)} - A^{(4)}]\,, \quad & A^{78} &= \tfrac{1}{2}\,[A^{(1)} - A^{(2)} - A^{(3)} + A^{(4)}]\,. \end{aligned} \tag{2.10}$$

The corresponding field strengths $F^{(\alpha)}$ will simply be given by

$$F^{(\alpha)} = dA^{(\alpha)}\,, \qquad \alpha = 1, 2, 3, 4\,. \tag{2.11}$$

The kinetic terms for the gauge fields are calculated from those of the gauge fields in the ω-deformed $SO(8)$ gauged supergravity, using the expressions given in [4]. Specifically, one has $e^{-1}\,\mathcal{L}_F = -\frac{1}{8}(\mathrm{i} G^{+\,\mu\nu}_{IJ}\, F^{+\,IJ}_{\mu\nu} + \text{h.c.})$, where[3]

$$\mathrm{i}\,(u^{ij}{}_{IJ} + e^{2\mathrm{i}\omega}\, v^{ijIJ}) G^+_{\mu\nu IJ} = (u^{ij}{}_{IJ} - e^{2\mathrm{i}\omega}\, v^{ijIJ}) F^{+\,IJ}_{\mu\nu}\,, \tag{2.12}$$

and where $F^+_{\mu\nu}$ is defined for any 2-form as the complex self-dual projection[4]

$$F^+_{\mu\nu} = \tfrac{1}{2}(F_{\mu\nu} + \tfrac{1}{2}\mathrm{i}\,\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma})\,. \tag{2.13}$$

Using the expressions for the u and v tensors that we can read off from (2.1) and (2.2), we are now in a position to calculate the bosonic Lagrangian for the $U(1)^4$ truncation of the ω-deformed $SO(8)$ supergravity. We find that the scalar potential (2.7) is unmodified from

[3]Note that in (2.12), and elsewhere also, we are not taking into account higher-order fermion terms. Our object in this paper is to obtain the bosonic sector of the ω-deformed STU supergravity theories, and also to obtain those terms in the supersymmetry transformation rules that are sufficient for testing the supersymmetry of bosonic solutions.

[4]We are using conventions similar to those in [16] here, in which $G_{\mu\nu} = \delta\mathcal{L}/\delta F_{\mu\nu}$ and hence $G^+_{\mu\nu} = Z\,F^+_{\mu\nu} = \mathrm{i} M F^+_{\mu\nu}$. (Our definitions of the self-dual and anti-self-dual projections, $F^{\pm}_{\mu\nu} = \frac{1}{2}(F_{\mu\nu} \pm \mathrm{i}\,{}^*F_{\mu\nu})$, with ${}^*F_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$, are opposite to those in [16].)

its expression in the original $\omega = 0$ theory. The bosonic Lagrangian for the ω-deformed STU model is then given by

$$e^{-1}\,\mathcal{L} = R - \tfrac{1}{2}\sum_{a=1}^{3}\Big((\partial\phi_a)^2 + \sinh^2\phi_a\,(\partial\sigma_a)^2\Big) - \tfrac{1}{4}\Big(F^{+\,(\alpha)}_{\mu\nu}\,\mathcal{M}_{\alpha\beta}F^{+\,(\beta)\mu\nu} + \text{h.c.}\Big) - V\,, \tag{2.14}$$

where the scalar potential is

$$V = -2g^2(\cosh\phi_1 + \cosh\phi_2 + \cosh\phi_3)\,. \tag{2.15}$$

The complete expression for $\mathcal{M}_{\alpha\beta}$, which is rather complicated, is given in appendix A.

The important point to note is that although the scalar kinetic terms and potential are unmodified by the ω deformation, the kinetic terms for the gauge fields, encapsulated in the matrix $\mathcal{M}_{\alpha\beta}$, depend upon ω in a non-trivial way. Specifically, for no choice of the constants k_a is it possible to use the residual $U(1)^3$ global symmetry $\sigma_a \to \sigma_a + k_a$ of the scalar kinetic and potential terms to absorb the ω dependence in the matrix $\mathcal{M}_{\alpha\beta}$. Of course, in the *ungauged* theory, the ω parameter could be absorbed everywhere if one made an appropriate electric/magnetic duality transformation on the field strengths. However, this transformation would not be contained within the $SL(2,R)^3$ global symmetry of the ungauged theory.[5] (See appendix A for details.) In the purely bosonic sector of the gauged theory, the kinetic terms of the field strengths are identical to those in the ungauged theory, and hence the same duality transformation would allow one to remove the ω dependence here as well. However, in the complete gauged theory including the fermions, the minimal coupling of the spinors to the gauge potentials prevents one from making the duality transformation, and hence ω cannot be removed by means of field redefinitions in the ω-deformed STU supergravities.

2.2 Range of the ω parameter

Having obtained the embedding of the STU model into the ω-deformed $\mathcal{N} = 8$ gauged supergravity, the question arises as to what is the range of the angle ω that parameterises inequivalent STU supergravity theories. In the full $\mathcal{N} = 8$ theory the inequivalent gaugings are parameterised by ω lying in the interval $0 \le \omega \le \frac{1}{8}\pi$ [3, 4]. The arguments that show that the "naive" answer of $0 \le \omega < 2\pi$ for the parameter range of inequivalent theories is actually reduced to $0 \le \omega \le \frac{1}{8}\pi$ depend quite subtly upon the use of field redefinitions involving the $\mathcal{N} = 8$ fields. In the truncation to the $\mathcal{N} = 2$ STU theories, many of the

[5] If it *were* contained within the $SL(2,R)^3$ global symmetry, then it would have been possible to absorb the ω parameter by means of a field redefinition of the scalars.

original fields are set to zero, and so it is necessary to investigate within the restricted set of fields that are retained to see whether or not the conclusions about the non-trivial range of the ω parameter remains unchanged.

Looking at the form of the matrix of scalar fields $\mathcal{M}_{\alpha\beta}$ that appears in the gauge-field kinetic terms in (2.14), and which is given in appendix A, it is evident that there is a symmetry under which we send

$$\omega \longrightarrow \omega + \frac{\pi}{2}\,, \qquad \phi_a \longrightarrow -\phi_a\,, \tag{2.16}$$

or in other words, $\mathcal{M}_{\alpha\beta}(\phi_a, \sigma_a, \omega) = \mathcal{M}_{\alpha\beta}(-\phi_a, \sigma_a, \omega + \frac{1}{2}\pi)$. This corresponds to the first of the three discrete symmetries discussed in [4], and as can be seen from (B.4) and (B.5), we have ${u_{ij}}^{KL} \longrightarrow {u_{ij}}^{KL}$ and $v_{ijKL} \longrightarrow -v_{ijKL}$. The matrix $\mathcal{M}_{\alpha\beta}$, which is symmetric, also has the property

$$\mathcal{M}^*_{\alpha\beta}(\phi_a, \sigma_a, \omega) = \mathcal{M}_{\alpha\beta}(\phi_a, -\sigma_a, -\omega)\,, \tag{2.17}$$

where the star denotes a complex conjugation. The complex conjugation of $\mathcal{M}_{\alpha\beta}$ has the effect in the Lagrangian (2.14) of reversing the sign of the $\epsilon^{\mu\nu\rho\sigma}\, F^{(\alpha)}_{\mu\nu}\, F^{(\beta)}_{\rho\sigma}$ terms, while leaving the sign of the actual kinetic terms $F^{(\alpha)}_{\mu\nu}\, F^{(\beta)\,\mu\nu}$ unchanged. This sign change can be undone by means of a parity reversal, and this is the third of the three discrete symmetries discussed in [4]. Finally, we consider the second discrete symmetry considered in [4], namely

$$\omega \longrightarrow \omega + \frac{\pi}{4}\,. \tag{2.18}$$

This involved a discrete $SU(8)$ matrix $e^{\mathrm{i}\pi/8}\, P_8$, where P_8 is a real and orthogonal 8×8 matrix with $\det(P_8) = -1$, and satisfying $P_8^2 = 1$. An example that is convenient for our purposes is

$$P_8 = \mathrm{diag}\,(-1, 1, 1, 1, 1, 1, 1, 1)\,. \tag{2.19}$$

From the transformations in equation (2.2) of [4], this will give a symmetry of the theory if ${u_{ij}}^{KL}$ reverses sign when exactly one of the four indices is a "1," and if $v_{ijKL} \longrightarrow e^{\mathrm{i}\pi/2}\, v_{ijKL}$ for all index assignments where exactly one index is a "1," and $v_{ijKL} \longrightarrow e^{-\mathrm{i}\pi/2}\, v_{ijKL}$ otherwise. It can be seen from (B.4) and (B.5) that these can be implemented by means of the transformations

$$\sigma_a \longrightarrow \sigma'_a = \sigma_a - \frac{\pi}{2} \tag{2.20}$$

of the three axionic scalars of the truncation to the STU model. The symmetry also requires sending [4]

$$A^{12}_\mu \longrightarrow -A^{12}_\mu\,, \quad (A^{34}_\mu, A^{56}_\mu, A^{78}_\mu) \longrightarrow (A^{34}_\mu, A^{56}_\mu, A^{78}_\mu)\,. \tag{2.21}$$

The upshot of these symmetries is that the ω-deformed STU theories may be viewed as inequivalent if ω lies in the interval

$$0 \le \omega \le \frac{\pi}{8}\,, \tag{2.22}$$

just as in the case of the ω-deformed $\mathcal{N}=8$ supergravities.[6]

2.3 Supersymmetry of the ω-deformed STU supergravities

Here, we shall present the terms in the supersymmetry transformation rules for the ω-deformed STU supergravities that are relevant for investigating the supersymmetry of bosonic solutions in the theories. We obtain these by substituting the STU truncation into the transformation rules of the ω-deformed $\mathcal{N}=8$ theory, which are given in [2]. Up to the order in fermion fields to which we shall be working, they are given by

$$\begin{aligned}
\delta\psi_\mu{}^i &= 2\,\mathcal{D}_\mu\epsilon^i + \tfrac14\,\hat{\mathcal{H}}^{-}_{\rho\sigma}{}^{ij}\,\gamma^{\rho\sigma}\gamma_\mu\epsilon_j + g\,A_{(1)}{}^{ij}\,\gamma_\mu\,\epsilon_j + \cdots\,,\\
\delta\chi^{ijk} &= -\,\hat{\mathcal{A}}_\mu^{ijkl}\,\gamma^\mu\epsilon_l + \tfrac{3}{2\sqrt2}\,\hat{\mathcal{H}}^{-}_{\mu\nu}{}^{[ij}\gamma^{\mu\nu}\epsilon^{k]} - \sqrt2\,g\,A_{(2)l}{}^{ijk}\,\epsilon^l + \cdots\,,\\
\delta e_\mu{}^a &= \bar\epsilon^i\gamma^a\psi_{\mu i} \;+\; \bar\epsilon_i\gamma^a\psi_\mu{}^i\,,\\
\delta A_\mu{}^{IJ} &= -\sqrt2(e^{i\omega}u_{ij}^{\ \,IJ} + e^{-i\omega}v_{ijIJ})\Big(\bar\epsilon_k\,\gamma_\mu\,\chi^{ijk} + 2\sqrt2\,\bar\epsilon^i\,\psi^j_\mu\Big) \;+\; \text{h.c.}\,,\\
\delta u^{ij}_{\ \,IJ} &= -2\sqrt2\,v_{klIJ}\Big(\bar\epsilon^{[i}\chi^{jkl]} + \tfrac{1}{24}\varepsilon^{ijklmnpq}\,\bar\epsilon_m\chi_{npq}\Big)\,,\\
\delta v^{ijIJ} &= -2\sqrt2\,u_{kl}^{\ \,IJ}\Big(\bar\epsilon^{[i}\chi^{jkl]} + \tfrac{1}{24}\varepsilon^{ijklmnpq}\,\bar\epsilon_m\chi_{npq}\Big)\,.
\end{aligned} \tag{2.23}$$

The ellipses in the transformation rules for the fermions represent terms of higher order in fermion fields. The derivation of the complete set of transformation rules in the $\mathcal{N}=2$ truncation, including all the higher-order fermion terms, would be straightforward but rather involved. One also has to take into account the compensating transformations that would be necessary in order to maintain the symmetric gauge choice (2.2) for the parameterisation of the coset for the scalar fields [17].

In the above equations, $\mathcal{D}_\mu\epsilon^i$ is defined to be

$$\mathcal{D}_\mu\epsilon^i = \nabla_\mu\epsilon^i + \tfrac12\mathcal{B}^i_{\mu j}\epsilon^j\,, \tag{2.24}$$

where $\mathcal{B}^i_{\mu j}$ is the composite and gauge connection, and $\hat{\mathcal{H}}_{\mu\nu}{}^{ij}$ is defined through

$$F_{\mu\nu}^{\ \ IJ} = (e^{i\omega}u_{ij}^{\ \,IJ} + e^{-i\omega}v_{ijIJ})\hat{\mathcal{H}}_{\mu\nu}{}^{ij}\,. \tag{2.25}$$

[6] We are very grateful to the referee for suggesting to us that the deformed STU theories might also be equivalent under the transformation (2.18).

The various tensors $u_{ij}{}^{KL}$, v_{ijKL}, $A^{ij}_{(1)}$, $A_{(2)\,i}{}^{jk\ell}$, $\mathcal{H}_{\mu\nu}{}^{ij}$, etc., in the ω-deformed STU model can be found in appendix B.

In the truncation to the $\mathcal{N}=2$ STU models, six of the eight gravitini and eight supersymmetry parameters are set to zero. Given our truncation in the bosonic sector, where we keep the four gauge $U(1)$ potentials $(A^{12}_\mu, A^{34}_\mu, A^{56}_\mu, A^{78}_\mu)$, there are four different ways we could truncate the gravitini and supersymmetry parameters. For definiteness, we shall choose the truncation where we set all ψ^i_μ and ϵ^i to zero except

$$\psi^i_\mu \quad \text{and} \quad \epsilon^i\,, \qquad \text{for } i=1,2\,. \tag{2.26}$$

The corresponding truncation for the spin-$\frac12$ fermions involves setting all $\chi^{ijk}=0$ except for the six fields

$$(\chi^{i34}\,,\chi^{i56}\,,\chi^{i78})\,, \qquad \text{for } i=1,2. \tag{2.27}$$

One can verify from the complete supersymmetry transformation rules that the truncation to the $\mathcal{N}=2$ STU theories is indeed consistent, in the sense that the variations of the fields that are set to zero remain equal to zero. It can be seen from the expressions in (B.14) for the components of the connection $\mathcal{B}^i_{\mu j}$ that appears in the covariant derivative $\mathcal{D}_\mu$ in (2.24) that with our choice for the truncation to $\mathcal{N}=2$, it is the potential A^{12}_μ that is the graviphoton in the supergravity multiplet. We present the transformation rules for the scalar fields, after taking account of the compensating transformations mentioned above, and the transformation rules for the surviving gauge potentials, at the end of appendix B.

3 Consistent Truncations of the ω-Deformed STU Model

3.1 Pairwise equal gauge fields

There is a consistent truncation in which we set the gauge fields pairwise equal, at the same time truncating out two dilaton/axion pairs. For example, we can set

$$F^{(2)}=F^{(1)}=\frac{1}{\sqrt2}F\,,\qquad F^{(4)}=F^{(3)}=\frac{1}{\sqrt2}\widetilde F\,,\qquad \phi_2=\phi_3=\sigma_2=\sigma_3=0\,. \tag{3.1}$$

This then leads to the Lagrangian

$$\begin{aligned}e^{-1}\mathcal{L} = {}& R-\tfrac12(\partial\phi_1)^2-\tfrac12\sinh^2\phi_1\,(\partial\sigma_1)^2+2g^2(2+\cosh\phi_1)\\ &-\tfrac12\Big[\frac{1+z\,e^{2\mathrm{i}\omega}}{1-z\,e^{2\mathrm{i}\omega}}\,(F^+)^2+\frac{1-z\,e^{2\mathrm{i}\omega}}{1+z\,e^{2\mathrm{i}\omega}}\,(\widetilde F^+)^2+\text{h.c.}\Big]\,,\end{aligned} \tag{3.2}$$

where

$$z=e^{\mathrm{i}\sigma_1}\,\tanh\tfrac12\phi_1\,. \tag{3.3}$$

The electrically charged rotating black hole solutions in this theory were obtained in [18]. (The general charged rotating black holes in the ungauged STU supergravity were constructed in [19].) It is evident that in this further truncation of the ω-deformed STU supergravities, the parameter ω has now become trivial, in the sense that it can be absorbed into a redefinition of the σ_1 scalar field,

$$\sigma_1 \longrightarrow \sigma_1 - 2\omega\,. \tag{3.4}$$

We shall discuss the embedding of the nonabelian extension of this theory to $\mathcal{N}=4$ gauged $SO(4)$ supergravity, and the related topic of its deformation that was discovered long ago by de Roo and Wagemans [13], in section 4.

3.2 $1+3$ split of the gauge fields

In view of the fact that, as we just saw, the ω parameter becomes trivial within the context of the pairwise-equal truncation of the deformed STU model, it is of interest to see whether there exist different consistent truncations in which ω remains non-trivial. An example is provided by making instead the following truncation.

$$\begin{aligned} &\phi_2 = \phi_3 = \phi_1\,, \qquad \sigma_2 = \sigma_3 = \sigma_1\,,\\ &A_\mu^{(1)} = A_\mu^1\,, \qquad A_\mu^{(2)} = A_\mu^{(3)} = A_\mu^{(4)} = \frac{1}{\sqrt{3}}\,A_\mu^2\,. \end{aligned} \tag{3.5}$$

If at the same time, in the fermionic sector one sets $\chi^{i34} = \chi^{i56} = \chi^{i78}$ (with $i=1,2$ as before), the resulting truncated theory is $\mathcal{N}=2$ supersymmetric. It is in fact a gauged version of the Poincaré supergravity one obtains by dimensionally reducing five-dimensional ungauged minimal supergravity on a circle. The bosonic Lagrangian takes the form

$$e^{-1}\,\mathcal{L} = R - \tfrac{3}{2}(\partial\phi_1)^2 - \tfrac{3}{2}\sinh^2\phi_1\,(\partial\sigma_1)^2 - \tfrac{1}{4}\big(F_{\mu\nu}^{+a}\,S_{ab}\,F^{+a\,\mu\nu} + \text{h.c.}\big) - V\,, \tag{3.6}$$

where

$$V = -6g^2\cosh\phi_1 \tag{3.7}$$

and the 2×2 matrix S_{ab} has components given by

$$\begin{aligned} S_{11} &= \frac{1}{2\Delta}\Big[4\cos(2\omega+\sigma_1)\cosh 2\phi_1 - (3+\cos 2\sigma_1)\sinh 2\phi_1\\ &\qquad\qquad -4\mathrm{i}\sin(2\omega+\sigma_1)\cosh\phi_1 + 2\mathrm{i}\sin 2\sigma_1\sinh\phi_1\Big]\,,\\ S_{12} &= S_{21} = -\frac{2\sqrt{3}}{\Delta}\,(\mathrm{i}\cos\sigma_1 + \cosh\phi_1\,\sin\sigma_1)\,\sinh\phi_1\sin\sigma_1\,,\\ S_{22} &= \frac{1}{2\Delta}\Big[4\cos(2\omega+\sigma_1)\cosh 2\phi_1 + (3+\cos 2\sigma_1)\sinh 2\phi_1 \end{aligned}$$

$$-4\mathrm{i}\sin(2\omega+\sigma_1)\cosh\phi_1 - 2\mathrm{i}\sin 2\sigma_1\sinh\phi_1\Big]\,, \tag{3.8}$$

with

$$\begin{aligned}\Delta = \ & 2\cos(2\omega+\sigma_1)\cosh\phi_1 + 2\cos 2\sigma_1\sinh\phi_1 - 2\mathrm{i}\sin(2\omega+\sigma_1)\cosh 2\phi_1 \\ & -\mathrm{i}\sin 2\sigma_1\sinh 2\phi_1\,. \end{aligned} \tag{3.9}$$

It was observed in [20] that the ungauged theory (and hence also the scalar and gauge-field kinetic terms in (3.6)) has a global $SL(2,R)$ symmetry under which the field strengths F^1 and F^2 and their duals transform in a 4-dimensional irreducible representation (see [21] for details). It is evident from the form of the matrix S_{ab} that the electric/magnetic duality rotation described by the $U(1)$ subgroup of this $SL(2,R)$, under which σ_1 would shift by the angle of the duality rotation, cannot be used in order to absorb the parameter ω. In other words, the duality rotations that one would have to perform in order to eliminate the parameter ω from the gauge field kinetic terms in (3.6) lie outside the global symmetry group of the theory.

In fact, it is not difficult to work out the explicit form of the duality transformation that removes the ω dependence in the gauge-field kinetic terms. The general $Sp(4,R)$ transformations that are symmetries of the gauge field equations of motion take the form [16]

$$\begin{pmatrix} F^1 \\ F^2 \\ G_1 \\ G_2 \end{pmatrix} \longrightarrow \Lambda \begin{pmatrix} F^1 \\ F^2 \\ G_1 \\ G_2 \end{pmatrix}, \qquad \Lambda = \begin{pmatrix} A & B \\ C & D \end{pmatrix}, \tag{3.10}$$

where A, B, C and D are 2×2 matrices satisfying

$$A^T C - C^T A = 0\,, \qquad B^T D - D^T B = 0\,, \qquad A^T D - C^T B = \mathbb{1}\,. \tag{3.11}$$

The required duality transformation will be such that the scalar matrix S_{ab}, viewed as a function of ω, satisfies

$$\big(C + \mathrm{i}D\,S(\omega)\big)\big(A + \mathrm{i}B\,S(\omega)\big)^{-1} = \mathrm{i}S(0)\,. \tag{3.12}$$

Solving for the matrices A, B, C and D, we find $A = D = \mathbb{1}\cos\omega$ and $B = -C = \mathbb{1}\sin\omega$, and hence the transformation matrix Λ in (3.10) is given by

$$\Lambda = \begin{pmatrix} \mathbb{1}\cos\omega & \mathbb{1}\sin\omega \\ -\mathbb{1}\sin\omega & \mathbb{1}\cos\omega \end{pmatrix}. \tag{3.13}$$

In other words, the two field strengths F^1 and F^2 are both dualised in the same way,

$$F^a \longrightarrow F^a \cos\omega + G^a \sin\omega\,. \tag{3.14}$$

This $U(1)$ transformation manifestly lies outside the $SL(2,R)$ symmetry group of the scalar coset manifold, under which the two gauge fields and their duals transform as a four-dimensional irreducible representation. Note that we can write $S(\omega)$ in terms of the undeformed matrix $S(0)$ as

$$\mathrm{i}\,S(\omega) = (\cos\omega - \mathrm{i}\,S(0)\,\sin\omega)^{-1}\,(\sin\omega + \mathrm{i}\,S(0)\,\cos\omega)\,. \tag{3.15}$$

The upshot is that in the gauged theory where the fermions couple minimally to the gauge potential of the graviphoton, and hence one therefore cannot perform duality transformations, the parameter ω must be non-trivial.

It is straightforward to see that S_{ab} given in (3.8) has the symmetries

$$S_{ab}(\phi_1,\sigma_1,\omega) = S_{ab}(-\phi_1,\sigma_1,\omega+\tfrac12\pi)\,, \tag{3.16}$$

and

$$S^*_{ab}(\phi_1,\sigma_1,\omega) = S_{ab}(\phi_1,-\sigma_1,-\omega)\,. \tag{3.17}$$

One can also see that the truncation (3.5) is compatible with the transformations (2.20) and (2.21), and so we conclude that the range of ω parameterising inequivalent theories in this truncation is again given by (2.22). In fact one can straightforwardly verify directly that the Lagrangian (3.6) is invariant under the transformation $\sigma_1 \to \sigma_1 - \frac12\pi$, combined with a shift of ω by $\frac14\pi$, together with the transformation of the gauge fields that is implied by (2.21).

3.3 Single gauge field truncation

We may perform a further consistent truncation of the ω-deformed supergravity described in section 3.2, theory in which the gauge field F^2 and the axion σ_1 are set to zero. The function $M = S_{11}$ that forms the prefactor of the remaining gauge-field kinetic term now becomes

$$M = \frac{\cos\omega - \mathrm{i}\,e^{3\phi_1}\,\sin\omega}{-\mathrm{i}\,\sin\omega + e^{3\phi_1}\,\cos\omega}\,. \tag{3.18}$$

Within the bosonic theory it is possible to absorb the parameter ω, by making a $U(1) \in Sp(2,R)$ duality transformation of the field strength and its dual,

$$\begin{pmatrix} F \\ G \end{pmatrix} \longrightarrow \Lambda \begin{pmatrix} F \\ G \end{pmatrix}\,, \qquad \Lambda = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}\,. \tag{3.19}$$

If we implement this $U(1)$ transformation with the parameter $\alpha = \omega + \frac{1}{2}\pi$, then from $\mathrm{i}\,M \longrightarrow (c+\mathrm{i}\,dM)(a+\mathrm{i}\,bM)^{-1}$ it would have the effect of replacing M in (3.18) by

$$M' = e^{3\phi_1}\,. \tag{3.20}$$

Thus, were it not for the fact that in the full supergravity theory the bare potential A_μ appears in the covariant derivatives of the fermions, one could implement this $U(1)$ duality transformation on this particular single gauge field truncation of the deformed STU theory where in addition σ_1 is set to zero, and thereby remove the ω deformation parameter. This is not possible within the framework of the supergravity theory, and so in this sense ω remains a non-trivial parameter here, even though it is trivial as far as the bosonic solutions themselves are concerned. This is very different from the situation discussed in section 3.1 for the case of the truncation with pairwise-equal field strengths. In that case the ω-parameter was removed purely by means of a constant shift redefinition of the axionic scalar field.

The Lagrangian for this $\sigma_1 = 0$ further truncation of the single gauge field truncation can be written as

$$\begin{aligned} e^{-1}\mathcal{L} = R - \tfrac{1}{2}(\partial\phi)^2 &- \frac{1}{4(e^{-\sqrt{3}\,\phi}\,\sin^2\omega + e^{\sqrt{3}\,\phi}\,\cos^2\omega)}\,F^2 \\ &- \frac{\sin 2\omega\,\sinh\sqrt{3}\phi}{8(e^{-\sqrt{3}\,\phi}\,\sin^2\omega + e^{\sqrt{3}\,\phi}\,\cos^2\omega)}\,\epsilon^{\mu\nu\rho\sigma}\,F_{\mu\nu}F_{\rho\sigma} + 6g^2\cosh(\frac{\phi}{\sqrt{3}})\,, \end{aligned} \tag{3.21}$$

where we have now given the remaining scalar field a canonical normalisation, by defining $\phi = \sqrt{3}\,\phi_1$. Note that in the undeformed case, where $\omega = 0$, this becomes the standard Lagrangian of the gauged "Kaluza-Klein theory,"[7]

$$e^{-1}\mathcal{L} = R - \tfrac{1}{2}(\partial\phi)^2 - \tfrac{1}{4}e^{-\sqrt{3}\,\phi}\,F^2 + 6g^2\cosh(\frac{\phi}{\sqrt{3}})\,. \tag{3.22}$$

(This is the theory that comes from the Kaluza-Klein reduction of five-dimensional Einstein gravity, with the added scalar potential.)

If we stay within the framework of the gauged theory coupled to fermionic fields, so that one is not allowed to make duality transformations on the field strength $F_{\mu\nu}$, then it is easy to see that the bosonic theories described by (3.21) are inequivalent for values of the ω parameter lying in the interval $0 \le \omega \le \frac{1}{4}\pi$. This follows from the fact that (3.21) has a symmetry under sending

$$\omega \longrightarrow \omega + \frac{\pi}{2}\,, \qquad \phi \longrightarrow -\phi\,. \tag{3.23}$$

[7]David Chow has pointed out to us that the Lagrangian (3.21) in the special case that ω is set equal to $\frac{1}{4}\pi$ was encountered in [22] as a consistent truncation in an Einstein-Sasaki reduction of $D = 11$ supergravity.

Furthermore, if we send $\omega \longrightarrow -\omega$ the sign of the $\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}F_{\rho\sigma}$ term is reversed, and this can be undone by means of a parity transformation. Note that because the axionic scalar σ_1 has been set to zero in this truncation, we no longer have the further symmetry under $\omega \to \omega + \frac{1}{4}\pi$ that was present in the STU model, and thus here the parameter range for inequivalent theories is $0 \le \omega \le \frac{1}{4}\pi$ rather than $0 \le \omega \le \frac{1}{8}\pi$.

As mentioned above, in this single gauge field truncation with σ_1 also truncated out, the bosonic solutions are all equivalent to solutions in the $\omega = 0$ theory, if we ignore the minimal coupling of A_μ to the fermions and then allow duality transformations of the field $F_{\mu\nu}$. Thus we can construct solutions to the theory (3.21) by making such duality rotations on known solutions of (3.22). As an example, we may consider the static dyonic black hole solution of the theory (3.22), which was recently constructed in [23]. Implementing the duality rotation we arrive at the conclusion that the following dyonic black hole solves the equations of motion coming from (3.21):

$$
\begin{aligned}
ds^2 &= -(H_1H_2)^{-\frac{1}{2}} f\, dt^2 + (H_1H_2)^{\frac{1}{2}}\Big(\frac{dr^2}{f} + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\Big)\,,\\
\phi &= \tfrac{\sqrt{3}}{2}\log\frac{H_2}{H_1}\,, \qquad f = f_0 + g^2r^2H_1H_2\,, \qquad f_0 = 1 - \frac{2\mu}{r}\,,\\
A &= \sqrt{2}\Big[\Big(\frac{(1-\beta_1 f_0)}{\sqrt{\beta_1\gamma_2}\,H_1}\Big)\cos\omega - \Big(\frac{(1-\beta_2 f_0)}{\sqrt{\beta_2\gamma_1}\,H_2}\Big)\sin\omega\Big]\,dt\\
&\quad + 2\sqrt{2}\mu\,\Big[\frac{\sqrt{\beta_2\gamma_1}}{\gamma_2}\cos\omega + \frac{\sqrt{\beta_1\gamma_2}}{\gamma_1}\sin\omega\Big]\cos\theta d\varphi\,,\\
H_1 &= \gamma_1^{-1}(1 - 2\beta_1 f_0 + \beta_1\beta_2 f_0^2)\,, \qquad H_2 = \gamma_2^{-1}(1 - 2\beta_2 f_0 + \beta_1\beta_2 f_0^2)\,,\\
\gamma_1 &= 1 - 2\beta_1 + \beta_1\beta_2\,, \qquad \gamma_2 = 1 - 2\beta_2 + \beta_1\beta_2\,.
\end{aligned} \tag{3.24}
$$

The physical electric and magnetic charges of this black hole are given by

$$
\begin{aligned}
Q &= \frac{\mu}{\sqrt{2}}\Big(\frac{\sqrt{\beta_1\gamma_2}}{\gamma_1}\cos\omega - \frac{\sqrt{\beta_2\gamma_1}}{\gamma_2}\sin\omega\Big)\,,\\
P &= \frac{\mu}{\sqrt{2}}\Big(\frac{\sqrt{\beta_2\gamma_1}}{\gamma_2}\cos\omega + \frac{\sqrt{\beta_1\gamma_2}}{\gamma_1}\sin\omega\Big)
\end{aligned} \tag{3.25}
$$

and its mass is

$$
M = \frac{(1-\beta_1)(1-\beta_2)(1-\beta_1\beta_2)\mu}{\gamma_1\gamma_2}\,. \tag{3.26}
$$

In a similar fashion the purely electric and purely magnetic rotating black holes in the "gauged Kaluza-Klein" theory, which were constructed in [24,25], can be duality rotated into solutions of the ω-deformed theory described by (3.21). Rotating black holes with charges carried by two of the four gauge fields of STU supergravity were constructed in [26]. These can similarly be duality rotated to give solutions in the full ω-deformed STU supergravities.

It is instructive also to examine how the supersymmetry of specific solutions depends on the value of the ω deformation parameter. For example, within the class of black-hole solutions (3.24) we may consider the case where we set $\beta_2 = 0$ and where we also take the extremal limit, which is achieved by setting

$$\beta_1 = \frac{1}{2} - \frac{\mu}{q}\,, \tag{3.27}$$

and then sending μ to zero. This gives

$$M = \frac{1}{4}q\,, \qquad Q = \tfrac{1}{4}q\,\cos\omega\,, \qquad P = \tfrac{1}{4}q\,\sin\omega\,. \tag{3.28}$$

At $\omega = 0$, this solution is just a standard supersymmetric black hole in the gauged STU supergravity theory, with a single field strength carrying an electric charge. When the deformation parameter is non-vanishing, the solution is still extremal, with $M = \sqrt{Q^2 + P^2}$, but, as one can verify from the transformation rules (2.23), it is no longer supersymmetric. This can most easily be seen by looking at the gaugino transformation rule, which can be written in the general form $\delta\chi \sim \Xi\,\epsilon$. One can verify that the determinant of the matrix Ξ has a factor $(1 - e^{2\mathrm{i}\omega})$, which is non-vanishing unless $\omega = 0$. This example suffices to illustrate the point that although a family of solutions in the ω-deformed STU supergravities may be obtained from an $\omega = 0$ solution merely by means of a duality transformation, the supersymmetry of the solution is nevertheless dependent on the value of the ω parameter.

Of course, while it can be relatively easy to check the supersymmetry of a family of ω-deformed solutions that are obtained, as above, by means of duality transformations of previously-known solutions of the undeformed theory, it is likely to be more challenging to discover new supersymmetric solutions in the deformed theory that are not related to previously known such solutions in the $\omega = 0$ theory.

4 $\mathcal{N} = 4$ Gauged $SO(4)$ Supergravities

The pairwise-equal field strength truncation that we discussed in section 3.1 can also be viewed as an abelian $U(1) \times U(1)$ truncation of $\mathcal{N} = 4$ gauged $SO(4)$ supergravity. The embedding of the $\mathcal{N} = 4$ theory into the standard $\mathcal{N} = 8$ gauged $SO(8)$ supergravity was discussed in detail in [27], where the explicit form of the S^7 reduction from eleven-dimensional supergravity was also presented. One can easily verify from the expressions for the u^{ijIJ} and v_{ijIJ} tensors given in [27] that if one substitutes them into the ω-deformed $\mathcal{N} = 8$ gauged supergravity, then again the ω parameter becomes trivial, since it can be absorbed by means of a shift transformation of the axionic scalar field. In other words, one cannot by this means derive a non-trivial family of $\mathcal{N} = 4$ gauged $SO(4)$ supergravities as an embedding in the ω-deformed $\mathcal{N} = 8$ theory. It is interesting that nonetheless a family of deformed $\mathcal{N}=4$ gauged $SO(4)$ supergravities does exist; it was constructed long ago by de Roo and Wagemans in [13].

The bosonic Lagrangian of the de Roo and Wagemans gauged $SO(4)$ theory, parameterised by the extra angle α, takes the form [13]

$$e^{-1}\mathcal{L} = R - \frac{2\partial_\mu Z\,\partial^\mu \bar{Z}}{(1-|Z|^2)^2} - \tfrac{1}{4}\Big[\frac{1+Ze^{\mathrm{i}(\alpha-\beta)}}{1-Ze^{\mathrm{i}(\alpha-\beta)}}\,(F^{+i}_{(1)})^2 + \frac{1+Ze^{-\mathrm{i}(\alpha+\beta)}}{1-Ze^{-\mathrm{i}(\alpha+\beta)}}\,(F^{+i}_{(2)})^2 + \text{h.c.}\Big] - V\,, \quad (4.1)$$

where the potential is given by

$$V = -\frac{1}{(1-|Z|^2)}\Big[(g_1^2+g_2^2)(1+|Z|^2) - |e^{\mathrm{i}\alpha}g_1^2 + e^{-\mathrm{i}\alpha}g_2^2|\,(Z+\bar{Z})\Big] - 4g_1 g_2 \sin\alpha\,, \qquad (4.2)$$

and the angle β is defined in terms of α, g_1 and g_2 by

$$e^{\mathrm{i}\beta} = \frac{e^{\mathrm{i}\alpha}g_1^2 + e^{-\mathrm{i}\alpha}g_2^2}{|e^{\mathrm{i}\alpha}g_1^2 + e^{-\mathrm{i}\alpha}g_2^2|}\,. \qquad (4.3)$$

The standard $\mathcal{N}=4$ gauged $SO(4)$ supergravity corresponds to taking $\alpha=\frac{1}{2}\pi$.

If we now introduce scalar fields φ and χ, and define

$$Z = \frac{\zeta-1}{\zeta+1}\,, \qquad \zeta = -\mathrm{i}\chi + e^{-\varphi}\,, \qquad (4.4)$$

then the scalar kinetic terms in (4.1) become $-\frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}e^{2\varphi}\,(\partial\chi)^2$ and the potential becomes

$$V = -\tfrac{1}{2}(g_1^2+g_2^2)(X^2+\widetilde{X}^2) - \tfrac{1}{2}(g_1^4+g_2^4+2g_1^2g_2^2\cos 2\alpha)^{1/2}\,(X^2-\widetilde{X}^2) - 4g_1g_2\sin\alpha\,, \quad (4.5)$$

where

$$X^2 = e^{\varphi}\,, \qquad \widetilde{X}^2 = e^{-\varphi} + \chi^2 e^{\varphi}\,. \qquad (4.6)$$

One can in fact define new gauge coupling constants, in terms of which the scalar potential becomes independent of the parameter α. We do this be introducing $\tilde{g}_1$ and $\tilde{g}_2$, defined by

$$\tilde{g}_1^2 = \tfrac{1}{2}(g_1^2+g_2^2)+\tfrac{1}{2}(g_1^4+g_2^4+2g_1^2g_2^2\cos 2\alpha)^{1/2}\,, \quad \tilde{g}_2^2 = \tfrac{1}{2}(g_1^2+g_2^2)-\tfrac{1}{2}(g_1^4+g_2^4+2g_1^2g_2^2\cos 2\alpha)^{1/2}\,. \qquad (4.7)$$

The potential (4.5) then becomes simply

$$V = -\tilde{g}_1^2\,X^2 - \tilde{g}_2^2\,\widetilde{X}^2 - 4\tilde{g}_1\tilde{g}_2\,, \qquad (4.8)$$

which is the form for the standard $N=4$ gauged supergravity potential. In particular, it is now independent of the de Roo-Wagemans parameter α. Of course, there is a price to be paid, namely that the gauge field kinetic terms will have a more complicated dependence on α, since the expression (4.3) defining β must now be written in terms of $\tilde{g}_1$ and $\tilde{g}_2$

rather than g_1 and g_2. Also, the gauge coupling parameters g_1 and g_2 appearing in the definitions of the Yang-Mills field strengths, $F^i_{(1)} = dA^i_{(1)} + \frac{1}{2} g_1 \epsilon^i{}_{jk} A^j_{(1)} \wedge A^k_{(1)}$ and $F^i_{(2)} = dA^i_{(2)} + \frac{1}{2} g_2 \epsilon^i{}_{jk} A^j_{(2)} \wedge A^k_{(2)}$, and in the gauge covariant derivatives acting on the fermion fields, will now have more complicated expressions in terms of $\tilde{g}_1$ and $\tilde{g}_2$. Note that from (4.7) we have

$$g_1^2 + g_2^2 = \tilde{g}_1^2 + \tilde{g}_2^2 \,, \qquad g_1 g_2 \sin\alpha = \tilde{g}_1 \tilde{g}_2 \,. \tag{4.9}$$

An alternative way to write the bosonic sector of the de Roo-Wagemans theory is to absorb the angle $(\alpha - \beta)$ as a phase factor in a new complex scalar

$$z = Z e^{\mathrm{i}(\alpha-\beta)} \,, \tag{4.10}$$

in terms of which (4.1) becomes

$$e^{-1}\mathcal{L} = R - \frac{2\partial_\mu z\, \partial^\mu \bar{z}}{(1-|z|^2)^2} - \tfrac{1}{4}\Big[\frac{1+z}{1-z}\,(F^{+i}_{(1)})^2 + \frac{1+z e^{-2\mathrm{i}\alpha}}{1-z e^{-2\mathrm{i}\alpha}}\,(F^{+i}_{(2)})^2 + \text{h.c.}\Big] - V \,, \tag{4.11}$$

where now the scalar potential V is written as

$$V = -\frac{1}{1-|z|^2}\left(g_1^2\,\left|1+z\right|^2 + g_2^2\,\left|1+z e^{-2i\alpha}\right|^2\right) - 4 g_1 g_2 \sin\alpha \,. \tag{4.12}$$

Since the scalar potential in the de Roo-Wagemans theory is independent of α when written in terms of the redefined gauge couplings $\tilde{g}_1$ and $\tilde{g}_2$ as in (4.7), then if we restrict attention to gauge fields in the abelian $U(1) \times U(1)$ subgroup of $SO(4) \sim SU(2) \times SU(2)$, we may then remove all remaining dependence on the parameter α in the bosonic equations of motion by performing appropriate dualisations of the abelian gauge fields, say $F^3_{(1)\,\mu\nu}$ and $F^3_{(2)\,\mu\nu}$, such that the $e^{\mathrm{i}(\pm\alpha-\beta)}$ phases are removed from the gauge-field kinetic term prefactors that arise from (4.1).[8] We can then use the same kind of technique that we used in section 3.3, to generate solutions of the deformed family of theories simply by making duality rotations of already-known solutions of the original undeformed theory. For example, a rotating dyonic black hole solution of the pairwise-equal STU gauged supergravity was recently constructed in [29], generalising the purely electric rotating black holes [18]. It is then straightforward to construct dyonic solutions in the de Roo-Wagemans theory, by taking the solution in [29] with its field strengths $\bar{F}_{(1)}$ and $\bar{F}_{(2)}$, with their duals $\bar{G}_{(1)}$ and

[8]The required duality transformations lie outside those contained in the $SL(2,R)$ global symmetry of the kinetic terms in the Lagrangian, and hence the phases cannot simply be absorbed by means of scalar field redefinitions. The situation here is analogous to that for the ω-deformed STU supergravities, as we discussed in section 2.

$\bar{G}_{(2)}$, and then taking the field strengths in the de Roo-Wagemans theories to be given by

$$F_{(1)} = \bar{F}_{(1)} \cos\tfrac{1}{2}(\beta - \alpha) - \bar{G}_{(1)} \sin\tfrac{1}{2}(\beta - \alpha)\,,$$
$$F_{(2)} = \bar{F}_{(2)} \cos\tfrac{1}{2}(\beta + \alpha) - \bar{G}_{(2)} \sin\tfrac{1}{2}(\beta + \alpha)\,, \qquad (4.13)$$

with the metric, dilaton and axion fields left unchanged.

5 Conclusions

In this paper we have constructed a family of deformed $\mathcal{N} = 2$ gauged STU supergravities, starting from the recently discovered family of ω-deformed $\mathcal{N} = 8$ gauged $SO(8)$ supergravities. The STU theories have a field content comprised of the $\mathcal{N} = 2$ gauged supergravity multiplet coupled to three vector multiplets. The four $U(1)$ gauge fields lie in the Cartan subgroup of the original $SO(8)$ gauge fields of the $\mathcal{N} = 8$ supergravities.

Unlike some other truncations of the ω-deformed $\mathcal{N} = 8$ supergravities that have been studied recently, in the case of the STU model truncation the scalar potential is unchanged in the presence of the ω parameter. However, the scalar functions that multiply the kinetic terms of the $U(1)^4$ gauge fields do depend upon ω in a non-trivial way, in the sense that it cannot be absorbed merely by means of redefinitions of the scalar fields. If one were free to make duality transformations of the gauge field strengths also, then the ω parameter could be absorbed. However, the fermions have minimal couplings to the gauge potentials, and thus in the full ω-deformed STU supergravity one cannot make duality transformations in order to absorb the ω parameter, and so it is in this sense non-trivial. Thus although purely bosonic solutions in the ω-deformed STU supergravities can be rotated into solutions of the usual STU model, their supersymmetry properties and their couplings to the fermion fields are different.

We then studied two different supersymmetric consistent truncations of the ω-deformed STU supergravities. In the first, where the four gauge fields are set pairwise equal, we arrived at a theory where the ω parameter becomes trivial, since it can now be absorbed by means of a shift symmetry transformation of the axion that remains in the truncation. By contrast, in a different consistent truncation in which again two gauge fields remain, but this time achieved by equating three of the original gauge fields, we showed that the ω deformation parameter remains non-trivial. The crucial difference between the two cases is that in the first, the duality transformation that eliminates the ω parameter lies within the global symmetry group of the scalar coset manifold, and so the same effect can be achieved by performing a symmetry transformation on the scalar fields, thereby absorbing the ω parameter. In the second truncation, the required duality transformation that would eliminate ω lies *outside* the global symmetry group of the scalar manifold, and so ω cannot be removed by means of scalar symmetry transformations in this case. Since the duality transformations are disallowed in the gauged theory because of the minimal couplings of the gauge potentials to the fermions, the ω parameter remains non-trivial in this case.

We presented a class of static dyonic black holes that generalise some solutions that were recently constructed in the usual gauged STU model in [23]. These black holes are embedded within a truncation of the STU theory in which all except one of the gauge fields are set to zero, at the same time setting the three dilatonic scalars ϕ_a equal, and the three axionic scalars σ_a equal. Viewed purely as bosonic configurations, the solutions we obtained in this paper would not be genuinely "new" in the sense that we obtained them by making a duality rotation on the gauge field strength of the previously obtained solutions. This duality transformation would be a symmetry relating one member of the family of ω-deformed theories to another, with different ω, were it not for the minimal coupling of the gauge potential to the gravitini. Thus as solutions of the full deformed STU supergravities, the dyonic black holes we constructed can be viewed as being new.

The pairwise-equal truncation of the ω-deformed supergravities, where, as mentioned above, ω becomes trivial, is in fact itself a $U(1)\times U(1)$ truncation of $\mathcal{N}=4$ gauged $SO(4)$ supergravity. This raises the question as to whether the ω parameter again becomes trivial if one embeds the full $\mathcal{N}=4$ theory into the ω-deformed $\mathcal{N}=8$ supergravities. The embedding for the $\mathcal{N}=4$ truncation was given in detail in [27], and using this, we were able to demonstrate that here too, the ω parameter can be absorbed once the truncation is performed.

It is intriguing that nevertheless, there does exist a one-parameter family of deformed $\mathcal{N}=4$ gauged supergravities with $SO(4)$ gauge group, constructed in [13]. We studied some properties of these theories, and presented some examples of bosonic solutions that could be obtained by making dualisations of the field strengths in previously-obtained solutions.

The discovery of the ω-deformed $\mathcal{N}=8$ gauged supergravities has raised many intriguing questions, and opens the way for the investigation of the solutions and the truncations to smaller theories. There is also the important question as to whether the deformed supergravities have a higher-dimensional origin.

Acknowledgements

We are grateful to Andrea Borghese, David Chow, Hadi Godazgar, Mahdi Godazgar, Henning Samtleben and Oscar Varela for useful discussions. The research of H.L. is supported in part by the NSFC grants 11175269 and 11235003. The work of Y.P. and C.N.P. is supported in part by DOE grant DE-FG02-13ER42020.

A Gauge Field Terms in ω-Deformed STU Supergravity

Here we present the complex matrix $\mathcal{M}_{\alpha\beta}$ that gives the kinetic terms for the gauge fields in the ω-deformed STU supergravity model that we constructed in section 2. The matrix can be conveniently written in the form

$$\mathcal{M}_{\alpha\beta} = \frac{1}{D}\mathcal{N}_{\alpha\beta}\,, \tag{A.1}$$

where the denominator D is given by

$$D = \tilde{\alpha}_4 e^{-4\mathrm{i}\omega} + \tilde{\alpha}_2 e^{-2\mathrm{i}\omega} + \alpha_0 + \alpha_2 e^{2\mathrm{i}\omega} + \alpha_4 e^{4\mathrm{i}\omega}\,, \tag{A.2}$$

with

$$\begin{aligned}
\tilde{\alpha}_4 &= \tfrac{1}{2}(-1+\hat{c}_1^2+\hat{c}_2^2+\hat{c}_3^2+2\hat{c}_1\hat{c}_2\hat{c}_3)\,, \qquad \alpha_4 = \tfrac{1}{2}(-1+\hat{c}_1^2+\hat{c}_2^2+\hat{c}_3^2-2\hat{c}_1\hat{c}_2\hat{c}_3)\,,\\
\tilde{\alpha}_2 &= \tfrac{1}{2}\hat{s}_1\hat{s}_2\hat{s}_3\big[e^{\mathrm{i}(\sigma_1+\sigma_2+\sigma_3)}+e^{\mathrm{i}(\sigma_1-\sigma_2-\sigma_3)}+e^{\mathrm{i}(-\sigma_1+\sigma_2-\sigma_3)}+e^{\mathrm{i}(-\sigma_1-\sigma_2+\sigma_3)}\big]\,,\\
\alpha_2 &= -\tfrac{1}{2}\hat{s}_1\hat{s}_2\hat{s}_3\big[e^{-\mathrm{i}(\sigma_1+\sigma_2+\sigma_3)}+e^{-\mathrm{i}(\sigma_1-\sigma_2-\sigma_3)}+e^{-\mathrm{i}(-\sigma_1+\sigma_2-\sigma_3)}+e^{-\mathrm{i}(-\sigma_1-\sigma_2+\sigma_3)}\big]\,,\\
\alpha_0 &= -\hat{s}_1^2\cos 2\sigma_1-\hat{s}_2^2\cos 2\sigma_2-\hat{s}_3^2\cos 2\sigma_3\,.
\end{aligned} \tag{A.3}$$

Here, we have defined

$$\hat{s}_a = \sinh\phi_a\,, \qquad \hat{c}_a = \cosh\phi_a\,. \tag{A.4}$$

The matrix $\mathcal{M}_{\alpha\beta}$ is symmetric in its indices α and β.

The diagonal components of the numerator matrix $\mathcal{N}_{\alpha\beta}$ are given by

$$\begin{aligned}
\mathcal{N}_{11} &= \tilde{\alpha}_4 e^{-4\mathrm{i}\omega} - \alpha_4 e^{4\mathrm{i}\omega} + (\beta_{01}+\beta_{02}+\beta_{03})\\
&\quad +(\tilde{\beta}_{20}+\tilde{\beta}_{21}+\tilde{\beta}_{22}+\tilde{\beta}_{23})e^{-2\mathrm{i}\omega}+(\beta_{20}+\beta_{21}+\beta_{22}+\beta_{23})e^{2\mathrm{i}\omega}\,,\\
\mathcal{N}_{22} &= \tilde{\alpha}_4 e^{-4\mathrm{i}\omega} - \alpha_4 e^{4\mathrm{i}\omega} + (\beta_{01}-\beta_{02}-\beta_{03})\\
&\quad +(\tilde{\beta}_{20}+\tilde{\beta}_{21}-\tilde{\beta}_{22}-\tilde{\beta}_{23})e^{-2\mathrm{i}\omega}+(\beta_{20}+\beta_{21}-\beta_{22}-\beta_{23})e^{2\mathrm{i}\omega}\,,\\
\mathcal{N}_{33} &= \tilde{\alpha}_4 e^{-4\mathrm{i}\omega} - \alpha_4 e^{4\mathrm{i}\omega} + (-\beta_{01}+\beta_{02}-\beta_{03})\\
&\quad +(\tilde{\beta}_{20}-\tilde{\beta}_{21}+\tilde{\beta}_{22}-\tilde{\beta}_{23})e^{-2\mathrm{i}\omega}+(\beta_{20}-\beta_{21}+\beta_{22}-\beta_{23})e^{2\mathrm{i}\omega}\,,
\end{aligned}$$

$$\begin{aligned}\mathcal{N}_{44} &= \tilde{\alpha}_4 e^{-4\mathrm{i}\omega} - \alpha_4 e^{4\mathrm{i}\omega} + (-\beta_{01} - \beta_{02} + \beta_{03}) \\ &\quad + (\tilde{\beta}_{20} - \tilde{\beta}_{21} - \tilde{\beta}_{22} + \tilde{\beta}_{23}) e^{-2\mathrm{i}\omega} + (\beta_{20} - \beta_{21} - \beta_{22} + \beta_{23}) e^{2\mathrm{i}\omega}\,, \end{aligned} \tag{A.5}$$

where

$$\begin{aligned} \beta_{01} &= 2\hat{c}_1\hat{s}_2\hat{s}_3\cos\sigma_2\cos\sigma_3\,, \quad \beta_{02} = 2\hat{c}_2\hat{s}_1\hat{s}_3\cos\sigma_1\cos\sigma_3\,, \quad \beta_{03} = 2\hat{c}_3\hat{s}_1\hat{s}_2\cos\sigma_1\cos\sigma_2\,, \\ \tilde{\beta}_{20} &= \tilde{\alpha}_2\,, \qquad \beta_{20} = -\alpha_2\,, \\ \tilde{\beta}_{21} &= \hat{s}_1(\hat{c}_2\hat{c}_3 + \hat{c}_1)\cos\sigma_1\,, \quad \tilde{\beta}_{22} = \hat{s}_2(\hat{c}_1\hat{c}_3 + \hat{c}_2)\cos\sigma_2\,, \quad \tilde{\beta}_{23} = \hat{s}_3(\hat{c}_1\hat{c}_2 + \hat{c}_3)\cos\sigma_3\,, \\ \beta_{21} &= \hat{s}_1(\hat{c}_2\hat{c}_3 - \hat{c}_1)\cos\sigma_1\,, \quad \beta_{22} = \hat{s}_2(\hat{c}_1\hat{c}_3 - \hat{c}_2)\cos\sigma_2\,, \quad \beta_{23} = \hat{s}_3(\hat{c}_1\hat{c}_2 - \hat{c}_3)\cos\sigma_3\,. \end{aligned} \tag{A.6}$$

The off-diagonal components of $\mathcal{N}_{\alpha\beta}$ are as follows.

$$\begin{aligned} \mathcal{N}_{12} &= \Big\{ -\tfrac{1}{4}\hat{s}_1\hat{s}_2\hat{s}_3[e^{\mathrm{i}(\sigma_1+\sigma_2+\sigma_3)} + e^{\mathrm{i}(\sigma_1-\sigma_2-\sigma_3)} - e^{\mathrm{i}(-\sigma_1+\sigma_2-\sigma_3)} - e^{\mathrm{i}(-\sigma_1-\sigma_2+\sigma_3)}] \\ &\quad + \mathrm{i}\,\hat{s}_1(\hat{c}_2\hat{c}_3 + \hat{c}_1)\,\sin\sigma_1 \Big\} e^{-2\mathrm{i}\omega} \\ &\quad + \Big\{ -\tfrac{1}{4}\hat{s}_1\hat{s}_2\hat{s}_3[e^{-\mathrm{i}(\sigma_1+\sigma_2+\sigma_3)} + e^{-\mathrm{i}(\sigma_1-\sigma_2-\sigma_3)} - e^{-\mathrm{i}(-\sigma_1+\sigma_2-\sigma_3)} - e^{-\mathrm{i}(-\sigma_1-\sigma_2+\sigma_3)}] \\ &\quad + \mathrm{i}\,\hat{s}_1(\hat{c}_2\hat{c}_3 - \hat{c}_1)\,\sin\sigma_1 \Big\} e^{2\mathrm{i}\omega} \\ &\quad + \big[\mathrm{i}\,\hat{s}_1^2\sin 2\sigma_1 + 2\hat{c}_1\hat{s}_2\hat{s}_3\sin\sigma_2\sin\sigma_3\big] \end{aligned} \tag{A.7}$$

and

$$\begin{aligned} \mathcal{N}_{34} &= \Big\{ -\tfrac{1}{4}\hat{s}_1\hat{s}_2\hat{s}_3[e^{\mathrm{i}(\sigma_1+\sigma_2+\sigma_3)} + e^{\mathrm{i}(\sigma_1-\sigma_2-\sigma_3)} - e^{\mathrm{i}(-\sigma_1+\sigma_2-\sigma_3)} - e^{\mathrm{i}(-\sigma_1-\sigma_2+\sigma_3)}] \\ &\quad - \mathrm{i}\,\hat{s}_1(\hat{c}_2\hat{c}_3 + \hat{c}_1)\,\sin\sigma_1 \Big\} e^{-2\mathrm{i}\omega} \\ &\quad + \Big\{ -\tfrac{1}{4}\hat{s}_1\hat{s}_2\hat{s}_3[e^{-\mathrm{i}(\sigma_1+\sigma_2+\sigma_3)} + e^{-\mathrm{i}(\sigma_1-\sigma_2-\sigma_3)} - e^{-\mathrm{i}(-\sigma_1+\sigma_2-\sigma_3)} - e^{-\mathrm{i}(-\sigma_1-\sigma_2+\sigma_3)}] \\ &\quad - \mathrm{i}\,\hat{s}_1(\hat{c}_2\hat{c}_3 - \hat{c}_1)\,\sin\sigma_1 \Big\} e^{2\mathrm{i}\omega} \\ &\quad + \big[\mathrm{i}\,\hat{s}_1^2\sin 2\sigma_1 - 2\hat{c}_1\hat{s}_2\hat{s}_3\sin\sigma_2\sin\sigma_3\big]\,, \end{aligned} \tag{A.8}$$

with $\{\mathcal{N}_{13}, \mathcal{N}_{24}\}$ being given by making the cyclic replacements

$$(\hat{s}_1, \hat{s}_2, \hat{s}_3) \longrightarrow (\hat{s}_2, \hat{s}_3, \hat{s}_1)\,, \quad (\hat{c}_1, \hat{c}_2, \hat{c}_3) \longrightarrow (\hat{c}_2, \hat{c}_3, \hat{c}_1)\,, \quad (\sigma_1, \sigma_2, \sigma_3) \longrightarrow (\sigma_2, \sigma_3, \sigma_1) \tag{A.9}$$

in $\{\mathcal{N}_{12}, \mathcal{N}_{34}\}$ respectively. Finally, $\{\mathcal{N}_{14}, \mathcal{N}_{23}\}$ are obtained by again applying the cyclic replacements (A.9) to $\{\mathcal{N}_{13}, \mathcal{N}_{24}\}$ respectively.

It can easily be verified that the deformation parameter ω cannot be absorbed by making shift transformations of the axial scalar fields σ_1, σ_2 and σ_3, and hence ω is a non-trivial parameter in the deformed STU model that we have constructed. In fact, it can be seen that the required duality transformation of the gauge fields that removes the ω parameter from the matrix $\mathcal{M}_{\alpha\beta}$ is

$$\begin{pmatrix} F^{(\alpha)} \\ G_{(\alpha)} \end{pmatrix} \longrightarrow \begin{pmatrix} \mathbb{1}\cos\omega & \mathbb{1}\sin\omega \\ -\mathbb{1}\sin\omega & \mathbb{1}\cos\omega \end{pmatrix} \begin{pmatrix} F^{(\alpha)} \\ G_{(\alpha)} \end{pmatrix}, \tag{A.10}$$

that is to say, a simultaneous duality rotation of all four fields through the same angle ω. This element of the $Sp(8, R)$ symmetry of the gauge-field kinetic terms lies outside the $SL(2, R)^3$ symmetry of the scalar coset, and hence the ω parameter is non-trivial here. It then follows that we can write the matrix $\mathcal{M}_{\alpha\beta}$ of the ω-deformed STU theory in terms of $\mathcal{M}_{0\,\alpha\beta}$, the corresponding matrix in the undeformed ($\omega = 0$) theory, as

$$\mathrm{i}\,\mathcal{M} = (\cos\omega + \mathrm{i}\,\mathcal{M}_0\,\sin\omega)^{-1}\,(\mathrm{i}\,\mathcal{M}_0\,\cos\omega - \sin\omega)\,. \tag{A.11}$$

B Expressions in ω-deformed STU Supergravity

Here, we collect some expressions for the various quantities that appear in the Lagrangian and the supersymmetry transformation rules in the ω-deformed STU supergravity. The starting point for obtaining these is the expression for the 56×56 matrix $\mathcal{V}$, defined by (2.2) and (2.4). It is useful to note that the matrices defined by the three terms in (2.4), proportional to $\Phi^{(1)}$, $\Phi^{(2)}$ and $\Phi^{(3)}$, all commute, and thus we may write $\mathcal{V}$ is the product of three commuting terms,

$$\mathcal{V} = \mathcal{V}_1\,\mathcal{V}_2\,\mathcal{V}_3\,. \tag{B.1}$$

The individual factors are given by evaluating (2.2) with $\phi_{ijk\ell}$ replaced by

$$\phi^{(1)}_{ijk\ell} = \sqrt{2}\,(\Phi^{(1)}\,\epsilon^{(12)} + \bar\Phi^{(1)}\,\epsilon^{(34)})\,, \tag{B.2}$$

to obtain $\mathcal{V}_1$, and analogously for $\mathcal{V}_2$ and $\mathcal{V}_3$. From these, we can read off the tensors $u(a)_{ij}{}^{KL}$ and $v(a)_{ijKL}$, defined from $\mathcal{V}_a$ for $a = 1, 2, 3$ in the obvious way using (2.1). This gives[9]

$$\begin{aligned} u(a)_{AB}{}^{CD} &= \cosh\tfrac12\phi_a\,\delta^{CD}_{AB}\,, \quad u(a)_{\bar A\bar B}{}^{\bar C\bar D} = \cosh\tfrac12\phi_a\,\delta^{\bar C\bar D}_{\bar A\bar B}\,, \quad u(a)_{A\bar B}{}^{C\bar D} = \tfrac12\delta^C_A\,\delta^{\bar D}_{\bar B}\,, \\ v(a)_{ABCD} &= \tfrac12\sinh\tfrac12\phi_a\,e^{-\mathrm{i}\,\sigma_a}\,\epsilon_{ABCD}\,, \qquad v(a)_{\bar A\bar B\bar C\bar D} = \tfrac12\sinh\tfrac12\phi_a\,e^{\mathrm{i}\,\sigma_a}\,\epsilon_{\bar A\bar B\bar C\bar D}\,, \end{aligned} \tag{B.3}$$

[9]The $u(a)_{ij}{}^{KL}$ and $v(a)_{ijKL}$ tensors for any specific choice of a can be seen to coincide with the $u_{ij}{}^{KL}$ and v_{ijKL} given in [27] (modulo conventions and correcting a typographical error in [27]) for the embedding of $\mathcal{N} = 4$ gauged $SO(4)$ supergravity into the maximal gauged theory. This should be no surprise, since the scalar sectors then coincide. What is perhaps less obvious is that the scalar embedding of the complete STU supergravity should just be given by a product of the three commuting factors $\mathcal{V} = \mathcal{V}_1\,\mathcal{V}_2\,\mathcal{V}_3$.

where we divide the index range $(1,\ldots,8)$ into $(A=\{1,2,3,4\}\,,\bar{A}=\{5,6,7,8\})$ when $a=1$; $(A=\{1,2,5,6\}\,,\bar{A}=\{3,4,7,8\})$ when $a=2$; and $(A=\{1,2,7,8\}\,,\bar{A}=\{3,4,5,6\})$ when $a=3$.

The complete $u_{ij}{}^{KL}$ and v_{ijKL} tensors for the matrix $\mathcal{V}$ can then be seen to be as follows. The tensor $u_{ij}{}^{KL}$ is given by

$$
\begin{aligned}
u_{12}{}^{12} &= u_{34}{}^{34} = u_{56}{}^{56} = u_{78}{}^{78} = \tfrac{1}{2}c_1c_2c_3\,,\\
u_{12}{}^{34} &= \tfrac{1}{2}c_1s_2s_3/(t_1t_2)\,, \qquad u_{34}{}^{12} = \tfrac{1}{2}c_1s_2s_3t_1t_2\,,\\
u_{12}{}^{56} &= \tfrac{1}{2}c_2s_1s_3/(t_1t_3)\,, \qquad u_{56}{}^{12} = \tfrac{1}{2}c_2s_1s_3t_1t_3\,,\\
u_{12}{}^{78} &= \tfrac{1}{2}c_3s_1s_1/(t_1t_2)\,, \qquad u_{78}{}^{12} = \tfrac{1}{2}c_3s_1s_2t_1t_2\,,\\
u_{34}{}^{56} &= \tfrac{1}{2}c_3s_1s_2t_2/t_1 \qquad u_{56}{}^{34} = \tfrac{1}{2}c_3s_1s_2t_1/t_2\,,\\
u_{34}{}^{78} &= \tfrac{1}{2}c_2s_1s_3t_3/t_1\,, \qquad u_{78}{}^{34} = \tfrac{1}{2}c_2s_1s_3t_1/t_3\,,\\
u_{56}{}^{78} &= \tfrac{1}{2}c_1s_2s_3t_3/t_2\,, \quad u_{78}{}^{56} = \tfrac{1}{2}c_1s_2s_3t_2/t_3\,,\\
u_{13}{}^{13} &= u_{14}{}^{14} = u_{23}{}^{23} = u_{24}{}^{24} = u_{57}{}^{57} = u_{58}{}^{58} = u_{67}{}^{67} = u_{68}{}^{68} = \tfrac{1}{2}c_1\,,\\
u_{15}{}^{15} &= u_{16}{}^{16} = u_{25}{}^{25} = u_{26}{}^{26} = u_{37}{}^{37} = u_{38}{}^{38} = u_{47}{}^{47} = u_{48}{}^{48} = \tfrac{1}{2}c_2\,,\\
u_{17}{}^{17} &= u_{18}{}^{18} = u_{27}{}^{27} = u_{28}{}^{28} = u_{35}{}^{35} = u_{36}{}^{36} = u_{45}{}^{45} = u_{46}{}^{46} = \tfrac{1}{2}c_3\,,
\end{aligned}
\tag{B.4}
$$

and the tensor v_{ijKL} is given by

$$
\begin{aligned}
v_{1212} &= \tfrac{1}{2}s_1s_2s_3/(t_1t_2t_3)\,, \qquad v_{3434} = \tfrac{1}{2}s_1s_2s_3t_2t_3/t_1\,,\\
v_{5656} &= \tfrac{1}{2}s_1s_2s_3t_1t_3/t_2\,, \qquad v_{7878} = \tfrac{1}{2}s_1s_2s_3t_1t_2/t_3\,,\\
v_{1234} &= v_{3412} = \tfrac{1}{2}c_2c_3s_1/t_1\,, \quad v_{1256} = v_{5612} = \tfrac{1}{2}c_1c_3s_2/t_2\,, \quad v_{1278} = v_{7812} = \tfrac{1}{2}c_1c_2s_3/t_3\,,\\
v_{3456} &= v_{5634} = \tfrac{1}{2}c_1c_2s_3t_3\,, \quad v_{3478} = v_{7834} = \tfrac{1}{2}c_1c_3s_2t_2\,, \quad v_{5678} = v_{7856} = \tfrac{1}{2}c_2c_3s_1t_1\,,\\
v_{1324} &= v_{2413} = -v_{1423} = -v_{2314} = -\tfrac{1}{2}s_1/t_1\,,\\
v_{1526} &= v_{2615} = -v_{1625} = -v_{2516} = -\tfrac{1}{2}s_2/t_2\,,\\
v_{1728} &= v_{2817} = -v_{1827} = -v_{2718} = -\tfrac{1}{2}s_3/t_3\,,\\
v_{5768} &= v_{6857} = -v_{5867} = -v_{6758} = -\tfrac{1}{2}s_1t_1\,,\\
v_{3748} &= v_{4837} = -v_{3847} = -v_{4738} = -\tfrac{1}{2}s_2t_2\,,\\
v_{3546} &= v_{4635} = -v_{3645} = -v_{4636} = -\tfrac{1}{2}s_3t_3\,,
\end{aligned}
\tag{B.5}
$$

where we use the notation

$$
c_a = \cosh\tfrac{1}{2}\phi_a\,, \qquad s_a = \sinh\tfrac{1}{2}\phi_a\,, \qquad t_a = e^{\mathrm{i}\sigma_a}\,. \tag{B.6}
$$

The tensors $A_{(1)}^{ij}$ and $A_{(2)\,i}{}^{jk\ell}$ take the forms

$$
A_{(1)}^{ij} = \mathrm{diag}\,(a_1,a_1,a_2,a_2,a_3,a_3,a_4,a_4)\,, \tag{B.7}
$$

$$
\begin{aligned}
a_1 &= e^{-\mathrm{i}\omega}\,c_1c_2c_3 + e^{\mathrm{i}\omega}\,s_1s_2s_3t_1t_2t_3\,, \quad a_2 = e^{-\mathrm{i}\omega}\,c_1c_2c_3 + e^{\mathrm{i}\omega}\,s_1s_2s_3t_1/(t_2t_3)\,,\\
a_3 &= e^{-\mathrm{i}\omega}\,c_1c_2c_3 + e^{\mathrm{i}\omega}\,s_1s_2s_3t_2/(t_1t_3)\,, \quad a_4 = e^{-\mathrm{i}\omega}\,c_1c_2c_3 + e^{\mathrm{i}\omega}\,s_1s_2s_3t_3/(t_1t_2)\,,
\end{aligned}
$$

and

$$
\begin{aligned}
A_{(2)\,1}{}^{234} &= -A_{(2)\,2}{}^{134} = -(e^{-\mathrm{i}\omega}\, c_1 s_2 s_3/(t_2 t_3) + e^{\mathrm{i}\omega}\, c_2 c_3 s_1 t_1)\,,\\
A_{(2)\,1}{}^{256} &= -A_{(2)\,2}{}^{156} = -(e^{-\mathrm{i}\omega}\, c_2 s_1 s_3/(t_1 t_3) + e^{\mathrm{i}\omega}\, c_1 c_3 s_2 t_2)\,,\\
A_{(2)\,1}{}^{278} &= -A_{(2)\,2}{}^{178} = -(e^{-\mathrm{i}\omega}\, c_3 s_1 s_2/(t_1 t_2) + e^{\mathrm{i}\omega}\, c_1 c_2 s_3 t_3)\,,\\
A_{(2)\,3}{}^{456} &= -A_{(2)\,4}{}^{356} = -(e^{-\mathrm{i}\omega}\, c_3 s_1 s_2 t_2/t_1 + e^{\mathrm{i}\omega}\, c_1 c_2 s_3/t_3)\,,\\
A_{(2)\,3}{}^{478} &= -A_{(2)\,4}{}^{378} = -(e^{-\mathrm{i}\omega}\, c_2 s_1 s_3 t_3/t_1 + e^{\mathrm{i}\omega}\, c_1 c_3 s_2/t_2)\,,\\
A_{(2)\,5}{}^{678} &= -A_{(2)\,6}{}^{578} = -(e^{-\mathrm{i}\omega}\, c_1 s_2 s_3 t_3/t_2 + e^{\mathrm{i}\omega}\, c_2 c_3 s_1/t_1)\,,\\
A_{(2)\,3}{}^{124} &= -A_{(2)\,4}{}^{123} = -(e^{-\mathrm{i}\omega}\, c_1 s_2 s_3 t_2 t_3 + e^{\mathrm{i}\omega}\, c_2 c_3 s_1 t_1)\,,\\
A_{(2)\,5}{}^{126} &= -A_{(2)\,6}{}^{125} = -(e^{-\mathrm{i}\omega}\, c_2 s_1 s_3 t_1 t_3 + e^{\mathrm{i}\omega}\, c_1 c_3 s_2 t_2)\,,\\
A_{(2)\,5}{}^{346} &= -A_{(2)\,6}{}^{345} = -(e^{-\mathrm{i}\omega}\, c_3 s_1 s_2 t_1/t_2 + e^{\mathrm{i}\omega}\, c_1 c_2 s_3/t_3)\,,\\
A_{(2)\,7}{}^{348} &= -A_{(2)\,8}{}^{347} = -(e^{-\mathrm{i}\omega}\, c_2 s_1 s_3 t_1/t_3 + e^{\mathrm{i}\omega}\, c_1 c_3 s_2/t_2)\,,\\
A_{(2)\,7}{}^{568} &= -A_{(2)\,8}{}^{567} = -(e^{-\mathrm{i}\omega}\, c_1 s_2 s_3 t_2/t_3 + e^{\mathrm{i}\omega}\, c_2 c_3 s_1/t_1)\,,\\
A_{(2)\,7}{}^{128} &= -A_{(2)\,8}{}^{127} = -(e^{-\mathrm{i}\omega}\, c_3 s_1 s_2 t_1 t_2 + e^{\mathrm{i}\omega}\, c_1 c_2 s_3 t_3)\,.
\end{aligned}
\tag{B.8}
$$

Solving (2.25) gives the non-zero components $\mathbf{H}_{\mu\nu} = (\hat{\mathcal{H}}_{\mu\nu}{}^{12}, \hat{\mathcal{H}}_{\mu\nu}{}^{34}, \hat{\mathcal{H}}_{\mu\nu}{}^{56}, \hat{\mathcal{H}}_{\mu\nu}{}^{78})^T$ in terms of the four non-zero components of $F_{\mu\nu}{}^{ij}$, namely $\mathbf{F}_{\mu\nu} = (F_{\mu\nu}{}^{12}, F_{\mu\nu}{}^{34}, F_{\mu\nu}{}^{56}, F_{\mu\nu}{}^{78})^T$, as $\mathbf{H}_{\mu\nu} = \mathbf{K}\,\mathbf{F}_{\mu\nu}$, where K is the 4×4 matrix

$$
\mathbf{K} = \frac{1}{D^*}\,\mathbf{J}\,,
\tag{B.9}
$$

and D is defined in (A.2), with the star denoting complex conjugation. The components of $\mathbf{J}$ can all be given in terms of

$$
\begin{aligned}
\mathbf{J}_{11} &= e^{3\mathrm{i}\omega}\,(1+s_1^2+s_2^2+s_3^2)c_1c_2c_3 - e^{\mathrm{i}\omega}\,(c_1^2 t_2 t_3/t_1 + c_2^2 t_1 t_3/t_2 + c_3^2 t_1 t_2/t_3) s_1 s_2 s_3\\
&\quad - e^{-\mathrm{i}\omega}\,(s_1^2 t_1^2 + s_2^2 t_2^2 + s_3^2 t_3^2) c_1 c_2 c_3 + e^{-3\mathrm{i}\omega}\,(c_1^2+c_2^2+c_3^2-1) s_1 s_2 s_3 t_1 t_2 t_3\,,\\
\mathbf{J}_{12} &= e^{3\mathrm{i}\omega}\,(c_1^2-c_2^2-c_3^2) c_1 s_2 s_3 t_2 t_3 + e^{\mathrm{i}\omega}\,(s_2^2 t_2^2 t_1 + s_3^2 t_3^2 t_1 - c_1^2/t_1) s_1 c_2 c_3\\
&\quad + e^{-\mathrm{i}\omega}\,(c_2^2 t_3/t_2 + c_3^2 t_2/t_3 - s_1^2 t_1^2 t_2 t_3) c_1 s_2 s_3 + e^{-3\mathrm{i}\omega}\,(s_1^2 - s_2^2 - s_3^2) s_1 c_2 c_3 t_1\,,
\end{aligned}
\tag{B.10}
$$

by defining the cyclic operator $\mathcal{C}$ and the parity operators $\mathcal{P}_{12}$, $\mathcal{P}_{23}$ and $\mathcal{P}_{13}$ that act on the scalar fields by

$$
\begin{aligned}
\mathcal{C}:&\quad (\phi_1,\phi_2,\phi_3,\sigma_1,\sigma_2,\sigma_3) \longrightarrow (\phi_2,\phi_3,\phi_1,\sigma_2,\sigma_3,\sigma_1)\,,\\
\mathcal{P}_{12}:&\quad (\sigma_1,\sigma_2) \longrightarrow (-\sigma_1,-\sigma_2)\,,\\
\mathcal{P}_{23}:&\quad (\sigma_2,\sigma_3) \longrightarrow (-\sigma_2,-\sigma_3)\,,\\
\mathcal{P}_{13}:&\quad (\sigma_1,\sigma_3) \longrightarrow (-\sigma_1,-\sigma_3)\,.
\end{aligned}
\tag{B.11}
$$

We then have

$$
\begin{aligned}
\mathbf{J}_{22} &= \mathcal{P}_{23}(\mathbf{J}_{11})\,, &\quad \mathbf{J}_{33} &= \mathcal{P}_{13}(\mathbf{J}_{11})\,, &\quad \mathbf{J}_{44} &= \mathcal{P}_{12}(\mathbf{J}_{11})\,,\\
\mathbf{J}_{13} &= \mathcal{C}(\mathbf{J}_{12})\,, & \mathbf{J}_{14} &= \mathcal{C}(\mathbf{J}_{13})\,, & &\\
\mathbf{J}_{21} &= \mathcal{P}_{23}(\mathbf{J}_{12})\,, & \mathbf{J}_{23} &= \mathcal{P}_{23}(\mathbf{J}_{14})\,, & \mathbf{J}_{24} &= \mathcal{P}_{23}(\mathbf{J}_{13})\,,\\
\mathbf{J}_{31} &= \mathcal{P}_{13}(\mathbf{J}_{13})\,, & \mathbf{J}_{32} &= \mathcal{P}_{13}(\mathbf{J}_{14})\,, & \mathbf{J}_{34} &= \mathcal{P}_{13}(\mathbf{J}_{12})\,,\\
\mathbf{J}_{41} &= \mathcal{P}_{12}(\mathbf{J}_{14})\,, & \mathbf{J}_{42} &= \mathcal{P}_{12}(\mathbf{J}_{13})\,, & \mathbf{J}_{43} &= \mathcal{P}_{12}(\mathbf{J}_{12})\,.
\end{aligned}
\qquad \text{(B.12)}
$$

The scalar field kinetic terms in the Lagrangian (2.14) come from the quantities $\mathcal{A}_\mu^{ijk\ell}$ that are defined in (2.6). We find that they are given by

$$
\begin{aligned}
\mathcal{A}_\mu^{1234} &= \tfrac{1}{\sqrt{2}}(\partial_\mu\phi_1 - \mathrm{i}\sinh\phi_1\,\partial_\mu\sigma_1)\,e^{-\mathrm{i}\sigma_1}\,, & \mathcal{A}_\mu^{5678} &= \tfrac{1}{\sqrt{2}}(\partial_\mu\phi_1 + \mathrm{i}\sinh\phi_1\,\partial_\mu\sigma_1)\,e^{\mathrm{i}\sigma_1}\,,\\
\mathcal{A}_\mu^{1256} &= \tfrac{1}{\sqrt{2}}(\partial_\mu\phi_2 - \mathrm{i}\sinh\phi_2\,\partial_\mu\sigma_2)\,e^{-\mathrm{i}\sigma_2}\,, & \mathcal{A}_\mu^{3478} &= \tfrac{1}{\sqrt{2}}(\partial_\mu\phi_2 + \mathrm{i}\sinh\phi_2\,\partial_\mu\sigma_2)\,e^{\mathrm{i}\sigma_2}\,,\\
\mathcal{A}_\mu^{1278} &= \tfrac{1}{\sqrt{2}}(\partial_\mu\phi_3 - \mathrm{i}\sinh\phi_3\,\partial_\mu\sigma_3)\,e^{-\mathrm{i}\sigma_3}\,, & \mathcal{A}_\mu^{3456} &= \tfrac{1}{\sqrt{2}}(\partial_\mu\phi_3 + \mathrm{i}\sinh\phi_3\,\partial_\mu\sigma_3)\,e^{\mathrm{i}\sigma_3}\,.
\end{aligned}
\qquad \text{(B.13)}
$$

The non-vanishing components of the connection $\mathcal{B}_{\mu j}^i$ appearing in the covariant derivative (2.24) are given by

$$
\begin{aligned}
\mathcal{B}_{\mu 1}^1 &= \mathcal{B}_{\mu 2}^2 = -\mathrm{i}\,(\partial_\mu\sigma_1\sinh^2\tfrac12\phi_1 + \partial_\mu\sigma_2\sinh^2\tfrac12\phi_2 + \partial_\mu\sigma_3\sinh^2\tfrac12\phi_3)\,,\\
\mathcal{B}_{\mu 3}^3 &= \mathcal{B}_{\mu 4}^4 = -\mathrm{i}\,(\partial_\mu\sigma_1\sinh^2\tfrac12\phi_1 - \partial_\mu\sigma_2\sinh^2\tfrac12\phi_2 - \partial_\mu\sigma_3\sinh^2\tfrac12\phi_3)\,,\\
\mathcal{B}_{\mu 5}^5 &= \mathcal{B}_{\mu 6}^6 = -\mathrm{i}\,(-\partial_\mu\sigma_1\sinh^2\tfrac12\phi_1 + \partial_\mu\sigma_2\sinh^2\tfrac12\phi_2 - \partial_\mu\sigma_3\sinh^2\tfrac12\phi_3)\,,\\
\mathcal{B}_{\mu 7}^7 &= \mathcal{B}_{\mu 8}^8 = -\mathrm{i}\,(-\partial_\mu\sigma_1\sinh^2\tfrac12\phi_1 - \partial_\mu\sigma_2\sinh^2\tfrac12\phi_2 + \partial_\mu\sigma_3\sinh^2\tfrac12\phi_3)\,,\\
\mathcal{B}_{\mu 2}^1 &= -\mathcal{B}_{\mu 1}^2 = -gA_\mu^{12}\,, \quad \mathcal{B}_{\mu 4}^3 = -\mathcal{B}_{\mu 3}^4 = -gA_\mu^{34}\,,\\
\mathcal{B}_{\mu 6}^5 &= -\mathcal{B}_{\mu 5}^6 = -gA_\mu^{56}\,, \quad \mathcal{B}_{\mu 8}^7 = -\mathcal{B}_{\mu 7}^8 = -gA_\mu^{78}\,.
\end{aligned}
\qquad \text{(B.14)}
$$

It is useful also to note that although the gauge potentials that we actually use in the STU model are $A_\mu^{(\alpha)}$, which are defined in terms of $\mathbf{A}_\mu \equiv (A_\mu^{12}, A_\mu^{34}, A_\mu^{56}, A_\mu^{78})^T$ by (2.10), the expression for the gauge-field kinetic terms can be written more simply in terms of the original fields. Thus if we also define $\mathbf{G}_{\mu\nu} \equiv (G_\mu^{12}, G_\mu^{34}, G_\mu^{56}, G_\mu^{78})^T$, then the solution to (2.12) can be written as

$$
\mathbf{G}_{\mu\nu} = \mathbf{Q}\mathbf{F}_{\mu\nu}\,, \qquad \mathbf{Q} = \frac{1}{D}\,\mathbf{R}\,, \qquad \text{(B.15)}
$$

where the 4×4 matrix $\mathbf{R}$ has components given by

$$
\begin{aligned}
\mathbf{R}_{11} &= \tfrac12 e^{4\mathrm{i}\omega}\,(2\hat c_1\hat c_2\hat c_3 + 1 - \hat c_1^2 - \hat c_2^2 - \hat c_3^2) + \tfrac12 e^{-4\mathrm{i}\omega}\,(2\hat c_1\hat c_2\hat c_3 - 1 + \hat c_1^2 + \hat c_2^2 + \hat c_3^2)\\
&\quad + \tfrac12 e^{2\mathrm{i}\omega}\,[1/(t_1t_2t_30 - t_1t_2/t_3 - t_1t_3/t_2 - t_2t_3/t_1]\hat s_1\hat s_2\hat s_3\\
&\quad + \tfrac12 e^{-2\mathrm{i}\omega}\,[t_1t_2t_3 - t_1/(t_2t_3) - t_2/(t_1t_3) - t_3/(t_1t_2)]\hat s_1\hat s_2\hat s_3\\
&\quad + \mathrm{i}\,(\hat s_1^2\sin 2\sigma_1 + \hat s_2^2\sin 2\sigma_2 + \hat s_3^2\sin 2\sigma_3)\,, \qquad\qquad (\mathrm{B}.16)\\
\mathbf{R}_{12} &= e^{2\mathrm{i}\omega}\,(\hat c_1 - \hat c_2 - \hat c_3)\hat s_1/t_1 - e^{-2\mathrm{i}\omega}\,(\hat c_1 + \hat c_2 + \hat c_3)\hat s_1 t_1 + \hat c_1\hat s_2\hat s_3\,\cos(\sigma_2-\sigma_3)\,,
\end{aligned}
$$

with the remaining components being given, as with (B.12), by

$$
\begin{aligned}
&\mathbf{R}_{22} = \mathcal{P}_{23}(\mathbf{R}_{11})\,, && \mathbf{R}_{33} = \mathcal{P}_{13}(\mathbf{R}_{11})\,, && \mathbf{R}_{44} = \mathcal{P}_{12}(\mathbf{R}_{11})\,,\\
&\mathbf{R}_{13} = \mathcal{C}(\mathbf{R}_{12})\,, && \mathbf{R}_{14} = \mathcal{C}(\mathbf{R}_{13})\,, && \\
&\mathbf{R}_{21} = \mathcal{P}_{23}(\mathbf{R}_{12})\,, && \mathbf{R}_{23} = \mathcal{P}_{23}(\mathbf{R}_{14})\,, && \mathbf{R}_{24} = \mathcal{P}_{23}(\mathbf{R}_{13})\,,\\
&\mathbf{R}_{31} = \mathcal{P}_{13}(\mathbf{R}_{13})\,, && \mathbf{R}_{32} = \mathcal{P}_{13}(\mathbf{R}_{14})\,, && \mathbf{R}_{34} = \mathcal{P}_{13}(\mathbf{R}_{12})\,,\\
&\mathbf{R}_{41} = \mathcal{P}_{12}(\mathbf{R}_{14})\,, && \mathbf{R}_{42} = \mathcal{P}_{12}(\mathbf{R}_{13})\,, && \mathbf{R}_{43} = \mathcal{P}_{12}(\mathbf{R}_{12})\,.
\end{aligned}
\qquad (\mathrm{B}.17)
$$

(Recall that the hatted quantities $\hat c_a$ and $\hat s_a$ are defined in (A.4).) The matrix $\mathcal{M}_{\alpha\beta}$ that appears in the Lagrangian terms for the gauge fields in (2.14), and that is presented in appendix A, is related to $\mathbf{Q}$ by

$$
\mathcal{M}_{\alpha\beta} = (\Omega\,\mathbf{Q}\,\Omega)_{\alpha\beta}\,, \qquad (\mathrm{B}.18)
$$

where

$$
\Omega = \tfrac12 \begin{pmatrix} 1 & 1 & 1 & 1\\ 1 & 1 & -1 & -1\\ 1 & -1 & 1 & -1\\ 1 & -1 & -1 & 1 \end{pmatrix} = \Omega^{-1} = \Omega^T \qquad (\mathrm{B}.19)
$$

is the matrix that implements the change of field variables, $A_\mu^{(\alpha)} = (\Omega\mathbf{A}_\mu)^\alpha$, as in (2.10).

We also present the supersymmetry transformation rules for the gauge potentials and the scalar fields of the ω-deformed STU supergravities, which we discussed in section 2.3.

We find for the gauge potentials

$$\begin{aligned}
\delta A_\mu^{12} &= -\sqrt{2}(e^{\mathrm{i}\omega} c_1 s_2 s_3 t_1 t_2 + e^{-\mathrm{i}\omega} c_2 c_3 s_1/t_1)\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(1)} \\
&\quad -\sqrt{2}(e^{\mathrm{i}\omega} c_2 s_1 s_3 t_1 t_3 + e^{-\mathrm{i}\omega} c_1 c_3 s_2/t_2)\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(2)} \\
&\quad -\sqrt{2}(e^{\mathrm{i}\omega} c_3 s_1 s_2 t_1 t_2 + e^{-\mathrm{i}\omega} c_1 c_2 s_3/t_3))\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(3)} \\
&\quad -2(e^{\mathrm{i}\omega} c_1 c_2 c_3 + e^{-\mathrm{i}\omega} s_1 s_2 s_3/(t_1 t_2 t_3))\bar{\epsilon}^i\,\psi_\mu^j \epsilon_{ij} \ + \ \text{h.c.}\,, \\
\delta A_\mu^{34} &= -\sqrt{2}(e^{\mathrm{i}\omega} c_1 c_2 c_3 + e^{-\mathrm{i}\omega} s_1 s_2 s_3 t_2 t_3/t_1)\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(1)} \\
&\quad -\sqrt{2}(e^{\mathrm{i}\omega} c_3 s_1 s_2 t_1/t_2 + e^{-\mathrm{i}\omega} c_1 c_2 s_3 t_3)\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(2)} \\
&\quad -\sqrt{2}(e^{\mathrm{i}\omega} c_2 s_1 s_3 t_1/t_3 + e^{-\mathrm{i}\omega} c_1 c_3 s_2 t_2))\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(3)} \ + \ \text{h.c.}\,, \\
\delta A_\mu^{56} &= -\sqrt{2}(e^{\mathrm{i}\omega} c_3 s_1 s_2 t_2/t_1 + e^{-\mathrm{i}\omega} c_1 c_2 s_3 t_3)\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(1)} \\
&\quad -\sqrt{2}(e^{\mathrm{i}\omega} c_1 c_2 c_3 + e^{-\mathrm{i}\omega} s_1 s_2 s_3 t_1 t_3/t_2)\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(2)} \\
&\quad -\sqrt{2}(e^{\mathrm{i}\omega} c_1 s_2 s_3 t_2/t_3 + e^{-\mathrm{i}\omega} c_2 c_3 s_1 t_1))\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(3)} \ + \ \text{h.c.}\,, \\
\delta A_\mu^{78} &= -\sqrt{2}(e^{\mathrm{i}\omega} c_2 s_1 s_3 t_3/t_1 + e^{-\mathrm{i}\omega} c_1 c_3 s_2 t_2)\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(1)} \\
&\quad -\sqrt{2}(e^{\mathrm{i}\omega} c_1 s_2 s_3 t_3/t_2 + e^{-\mathrm{i}\omega} c_2 c_3 s_1 t_1)\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(2)} \\
&\quad -\sqrt{2}(e^{\mathrm{i}\omega} c_1 c_2 c_3 + e^{-\mathrm{i}\omega} s_1 s_2 s_3 t_1 t_2/t_3)\bar{\epsilon}_i\,\gamma_\mu\,\chi^{i(3)} \ + \ \text{h.c.}\,,
\end{aligned} \tag{B.20}$$

where $\chi^{i(1)}$, $\chi^{i(2)}$ and $\chi^{i(3)}$ mean χ^{i34}, χ^{i56} and χ^{i78} respectively.

After making the necessary compensating transformation to restore the symmetric gauge choice for the scalar fields, we find that the scalar supersymmetry transformations become

$$\begin{aligned}
(\delta\phi_1 - \mathrm{i}\sinh\phi_1 \delta\sigma_1) &= 2\sqrt{2} e^{\mathrm{i}\sigma_1} \bar{\epsilon}^i \chi^{j(1)}\,\epsilon_{ij} \\
(\delta\phi_2 - \mathrm{i}\sinh\phi_2 \delta\sigma_2) &= 2\sqrt{2} e^{\mathrm{i}\sigma_2} \bar{\epsilon}^i \chi^{j(2)}\,\epsilon_{ij} \\
(\delta\phi_3 - \mathrm{i}\sinh\phi_3 \delta\sigma_3) &= 2\sqrt{2} e^{\mathrm{i}\sigma_3} \bar{\epsilon}^i \chi^{j(3)}\,\epsilon_{ij}\,.
\end{aligned} \tag{B.21}$$

References

[1] B. de Wit and H. Nicolai, $N = 8$ *supergravity*, Nucl. Phys. B **208**, 323 (1982).

[2] B. de Wit, H. Samtleben and M. Trigiante, *The maximal* $D = 4$ *supergravities,* JHEP **0706**, 049 (2007), arXiv:0705.2101 [hep-th].

[3] G. Dall'Agata, G. Inverso and M. Trigiante, *Evidence for a family of* $SO(8)$ *gauged supergravity theories*, Phys. Rev. Lett. **109**, 201301 (2012), arXiv:1209.0760 [hep-th].

[4] B. de Wit and H. Nicolai, *Deformations of gauged* $SO(8)$ *supergravity and supergravity in eleven dimensions*, JHEP **1305**, 077 (2013), arXiv:1302.6219 [hep-th].

[5] A. Borghese, A. Guarino and D. Roest, *All G_2 invariant critical points of maximal supergravity,* JHEP **1212**, 108 (2012), arXiv:1209.3003 [hep-th].

[6] A. Borghese, G. Dibitetto, A. Guarino, D. Roest and O. Varela, *The SU(3)-invariant sector of new maximal supergravity,* JHEP **1303**, 082 (2013), arXiv:1211.5335 [hep-th].

[7] A. Borghese, A. Guarino and D. Roest, *Triality, periodicity and stability of $SO(8)$ gauged supergravity,* JHEP **1305**, 107 (2013), arXiv:1302.6057 [hep-th].

[8] J. Blaback, A. Borghese and S.S. Haque, *Power-law cosmologies in minimal and maximal gauged supergravity,* JHEP **1306**, 107 (2013), arXiv:1303.3258 [hep-th].

[9] A. Guarino, *On new maximal supergravity and its BPS domain-walls,* arXiv:1311.0785 [hep-th].

[10] J. Tarrio and O. Varela, *Electric/magnetic duality and RG flows in AdS_4/CFT_3,* JHEP **1401**, 071 (2014), arXiv:1311.2933 [hep-th].

[11] A. Anabalon and D. Astefanesei, *Black holes in ω-defomed gauged $N = 8$ supergravity,* arXiv:1311.7459 [hep-th].

[12] M.J. Duff, J.T. Liu and J. Rahmfeld, *Four-dimensional string-string-string triality,* Nucl. Phys. B **459**, 125 (1996), hep-th/9508094.

[13] M. de Roo and P. Wagemans, *Gauge matter coupling in $N = 4$ supergravity*, Nucl. Phys. B **262**, 644 (1985).

[14] M.J. Duff and J.T. Liu, *Anti-de Sitter black holes in gauged $N = 8$ supergravity*, Nucl. Phys. B **554**, 237 (1999), hep-th/9901149.

[15] M. Cvetič, M.J. Duff, P. Hoxha, J.T. Liu, H. Lü, J.X. Lu, R. Martinez-Acosta, C.N. Pope, H. Sati and T.A. Tran, *Embedding AdS black holes in ten-dimensions and eleven-dimensions*, Nucl. Phys. B **558**, 96 (1999), hep-th/9903214.

[16] D.Z. Freedman and A. Van Proeyen, *Supergravity*, Cambridge University Press 2012.

[17] E. Cremmer and B. Julia, *The $SO(8)$ supergravity*, Nucl. Phys. B **159**, 141 (1979).

[18] Z.-W. Chong, M. Cvetič, H. Lü and C.N. Pope, *Charged rotating black holes in four-dimensional gauged and ungauged supergravities,* Nucl. Phys. B **717**, 246 (2005), hep-th/0411045.

[19] M. Cvetič and D. Youm, *Entropy of nonextreme charged rotating black holes in string theory*, Phys. Rev. D **54**, 2612 (1996), hep-th/9603147.

[20] E. Cremmer, B. Julia, H. Lü and C.N. Pope, *Higher dimensional origin of $D = 3$ coset symmetries,* hep-th/9909099.

[21] H. Lü, J. Mei and C.N. Pope, *New charged black holes in five dimensions,* Class. Quant. Grav. **27**, 075013 (2010), arXiv:0806.2204 [hep-th].

[22] A. Donos, J.P. Gauntlett, J. Sonner and B. Withers, *Competing orders in M-theory: superfluids, stripes and metamagnetism*, JHEP **1303**, 108 (2013), arXiv:1212.0871 [hep-th].

[23] H. Lü, Y. Pang and C.N. Pope, *AdS dyonic black hole and its thermodynamics*, JHEP **1311**, 033 (2013), arXiv:1307.6243 [hep-th].

[24] D.D.K. Chow, *Single-charge rotating black holes in four-dimensional gauged supergravity*, Class. Quant. Grav. **28**, 032001 (2011), arXiv:1011.2202 [hep-th].

[25] S.-Q. Wu, *General rotating charged Kaluza-Klein AdS black holes in higher dimensions*, Phys. Rev. D **83**, 121502 (2011), arXiv:1108.4157 [hep-th].

[26] D.D.K. Chow, *Two-charge rotating black holes in four-dimensional gauged supergravity*, Class. Quant. Grav. **28**, 175004 (2011), arXiv:1012.1851 [hep-th].

[27] M. Cvetič, H. Lü and C.N. Pope, *Four-dimensional $N = 4$, $SO(4)$ gauged supergravity from $D = 11$*, Nucl. Phys. B **574**, 761 (2000), hep-th/9910252.

[28] D.B. Westra, *Symmetries in $\mathcal{N} = 4$ supergravities*, PhD thesis (University Library, U. Groningen, 2006; ISBN 9064640270, 9789064640278).

[29] D.D.K. Chow and G. Compère, *Dyonic AdS black holes in maximal gauged supergravity*, arXiv:1311.1204 [hep-th].