New Rotating Non-Extremal Black Holes in $D = 5$ Maximal Gauged Supergravity

Jianwei Mei and C.N. Pope

George P. and Cynthia W. Mitchell Institute for Fundamental Physics,

Texas A&M University, College Station, TX 77843, USA

ABSTRACT

We obtain new non-extremal rotating black hole solutions in maximal five-dimensional gauged supergravity. They are characterised by five parameters, associated with the mass, the two angular momenta, and two independently-specifiable charge parameters. Two of the three charges associated with the $U(1)^3$ Cartan subgroup of the $SO(6)$ gauge group are equal, whilst the third can be independently specified. These new solutions generalise all the previously-known rotating solutions in five-dimensional gauged supergravity with independent angular momenta. They describe regular black holes, provided the parameters lie in appropriate ranges so that naked singularities and closed-timelike curves (CTCs) are avoided. We also construct the BPS limit, and show that regular supersymmetric black holes or topological solitons arise if the parameters are further restricted in an appropriate manner.

1 Introduction

The development of the AdS/CFT correspondence in string theory [1, 2, 3] has led to a growing interest in the construction of five-dimensional solutions in gauged supergravity, which can be related to four-diemensional boundary field theories. Of particular interest in this context are the solutions for five-dimensional black holes which are asymptotic to AdS spacetime at large distance. The AdS/CFT correspondence is more solidly grounded in the case of BPS configurations, which are protected by supersymmetry. Asymptotically-AdS BPS black holes must necessarily have non-zero rotation in order to be free from naked singularities or other pathologies. Thus when considering non-BPS black holes, it is appropriate to include rotation as well, so that one can take a smooth limit, free of pathologies, to reach the BPS configurations.

From the standpoint of the AdS/CFT correspondence one is most interested in finding such black-hole solutions within the maximal $SO(6)$-gauged $\mathcal{N} = 8$ five-dimensional supergravity, since this is the theory that arises from the Pauli reduction of the type IIB superstring on S^5. Black holes with Abelian gauge fields can therefore carry 3 independent charges, associated with the three $U(1)$ factors in the Cartan subgroup of $SO(6)$. Equivalently, one can think of such charged black holes as solutions of $\mathcal{N} = 2$ gauged five-dimensional supergravity coupled to two additional vector multiplets. (These two vectors, plus the graviphoton of the $\mathcal{N} = 2$ supergravity itself, carry the three charges.) The general black-hole solution should then be characterised by its mass, the two independent angular momenta associated with rotations in the two orthogonal spatial 2-planes, and the three independent charge parameters.

The currently-known charged non-extremal rotating black-hole solutions in the five-dimensional gauged supergravity are as follows. For black holes with two the independent rotation parameters, the uncharged solution (Kerr-AdS) was found in [4] and the solution with all three charges equal was found in [5]. In addition, a solution with only one charge non-zero was found in [6], and a solution where two charges are equal, with the third having a specific non-vanishing charge related to the other two was found in [7]. In the much simpler situation where the two rotation parameters are set equal, the solution with three independent charges was obtained in [8].

The general non-extremal solution with two independent rotations and three independent charges is still unknown. The purpose of the present paper is to advance one step further to this goal. Here, we construct the solution for a non-extremal black hole in five-diemensional gayged supergravity with the two independent angular momenta and with

two independent charge parameters. This corresponds to the situation where two of the three charges in the general solution are set equal, whilst the third can be independently specified. For appropriate specialisations of the charge parameters in our new solution, all the previous cases in [5], [6] and [7] mentioned above can be obtained.

Having constructed the non-extremal solution we may also consider the limit where a BPS bound is attained. In general this describes a supersymmetric configuration with singularities or closed timelike curves (CTCs) outside a Killing horizon. By making a further specialisation of the parameters, we can obtain a class of "regular" BPS black holes, with neither naked singularities nor naked CTCs. This additional specialisation also ensures that the Hawking temperature at the horizon is zero, as it must be for a regular supersymmetric black hole.

2 The Black Hole Solution

As with earlier work on non-extremal asymptotically-AdS rotating black holes, the complexity of the equations of motion and the absence of any solution-generating techniques means that the only practicable method for finding solutions involves a large measure of guesswork and trial and error, followed by explicit verification of the field equations. In this task we were aided greatly by knowledge of the solutions in the previously-obtained special cases, especially those in [5], [6] and [7].

The five-dimensional Lagrangian for the bosonic sector of $\mathcal{N} = 2$ gauged supergravity coupled to two vector multiplets can be written as

$$L = \sqrt{-g}\left[R - \tfrac{1}{2}\sum_{\alpha=1}^{2}(\partial\varphi_\alpha)^2 + \sum_{i=1}^{3}\left(4g^2 X_i^{-1} - \tfrac{1}{4}\mathcal{F}^i_{\mu\nu}\mathcal{F}^{i\mu\nu}\right)\right] + \frac{1}{24}|\varepsilon_{ijk}|\varepsilon^{uv\rho\sigma\lambda}\mathcal{F}^i_{uv}\mathcal{F}^j_{\rho\sigma}\mathcal{A}^k_\lambda, \; (1)$$

where g is the gauge-coupling constant, and the quantities X_i are formed from the two scalar fields φ_1 and φ_2 in the vector multiplets:

$$X_1 = e^{-\frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2}, \qquad X_2 = e^{-\frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2}, \qquad X_3 = e^{\frac{2}{\sqrt{6}}\varphi_1}. \qquad (2)$$

We find that the following is a solution of the resulting equations of motion:

$$ds^2 = H_1^{2/3}H_3^{1/3}\left\{(x^2-y^2)\left(\frac{dx^2}{X} - \frac{dy^2}{Y}\right) - \frac{x^2 X(dt+y^2 d\sigma)^2}{(x^2-y^2)fH_1^2} + \frac{y^2 Y\left[dt+(x^2+2ms_1^2)d\sigma\right]^2}{(x^2-y^2)(\gamma+y^2)H_1^2}\right.$$
$$\left. -U\left(dt+y^2 d\sigma + \frac{(x^2-y^2)fH_1\left[abd\sigma+(\gamma+y^2)d\chi\right]}{ab(x^2-y^2)H_3 - 2ms_3 c_3(\gamma+y^2)}\right)^2\right\}, \qquad (3)$$

$$\mathcal{A}^1 = \mathcal{A}^2 = \frac{2ms_1c_1(dt+y^2d\sigma)}{(x^2-y^2)H_1}\,,$$

$$\mathcal{A}^3 = \frac{2m\left\{s_3c_3(dt+y^2d\sigma)-(s_1^2-s_3^2)\left[abd\sigma+(\gamma+y^2)d\chi\right]\right\}}{(x^2-y^2)H_3}\,, \tag{4}$$

$$X_1 = X_2 = \left(\frac{H_3}{H_1}\right)^{1/3}, \qquad X_3 = \left(\frac{H_1}{H_3}\right)^{2/3}, \tag{5}$$

where

$$f = x^2 + \gamma + 2ms_3^2\,, \qquad \gamma = 2abs_3c_3 + (a^2+b^2)s_3^2\,,$$

$$U = \frac{\left[ab(x^2-y^2)H_3 - 2ms_3c_3(\gamma+y^2)\right]^2}{(x^2-y^2)^2(\gamma+y^2)fH_1^2H_3}\,,$$

$$H_1 = 1 + \frac{2ms_1^2}{x^2-y^2}\,, \qquad H_3 = 1 + \frac{2ms_3^2}{x^2-y^2}\,, \tag{6}$$

and $s_i \equiv \sinh\delta_i$, $c_i \equiv \cosh\delta_i$. The functions X and Y are given by

$$X = \frac{-2mx^2+(\tilde{a}^2+x^2)(\tilde{b}^2+x^2)+g^2(\tilde{a}^2+2ms_1^2+x^2)(\tilde{b}^2+2ms_1^2+x^2)(2ms_3^2+\gamma+x^2)}{x^2}\,,$$

$$Y = \frac{(\tilde{a}^2+y^2)(\tilde{b}^2+y^2)\left[1+g^2(\gamma+y^2)\right]}{y^2}\,, \quad \text{with} \ \ \tilde{a} \equiv ac_3+bs_3\,, \ \ \tilde{b} \equiv bc_3+as_3\,. \tag{7}$$

The solution is characterised by the mass parameter m, the two rotation parameters a and b, and the two charge parameters δ_1 and δ_3. It is evident from (4) that the charges carried by the gauge fields $\mathcal{A}^1$ and $\mathcal{A}^2$ are equal, whilst that carried by $\mathcal{A}^3$ is independently specificiable.

The solution can be rewritten in an asymptotically non-rotating frame, in terms of a canonically-normalised time coordinate τ and azimuthal coordinates ϕ and ψ having independent periodicities 2π by means of the transformation

$$t = \frac{(1+g^2\gamma)\tau}{\Xi_a\Xi_b} - \frac{a(a^2+\gamma)\phi}{(a^2-b^2)\Xi_a} + \frac{b(b^2+\gamma)\psi}{(a^2-b^2)\Xi_b}\,,$$

$$\sigma = \frac{g^2\tau}{\Xi_a\Xi_b} - \frac{a\phi}{(a^2-b^2)\Xi_a} + \frac{b\psi}{(a^2-b^2)\Xi_b}\,,$$

$$\chi = \frac{g^4ab\tau}{\Xi_a\Xi_b} - \frac{b\phi}{(a^2-b^2)\Xi_a} + \frac{a\psi}{(a^2-b^2)\Xi_b}\,, \tag{8}$$

where

$$\Xi_a \equiv 1 - g^2a^2\,, \qquad \Xi_b \equiv 1 - g^2b^2\,. \tag{9}$$

It is also useful to defined new coordinates r and θ to replace x and y,

$$x^2 = r^2 - \gamma - \tfrac{2}{3}m(2s_1^2+s_3^2)\,,$$

$$y^2 = -\tilde{a}^2\cos^2\theta - \tilde{b}^2\sin^2\theta = -\gamma - a^2\cos^2\theta - b^2\sin^2\theta\,. \tag{10}$$

For later convenience, we also define a new radial metric function $\Delta_r(r)$ by

$$\Delta_r(r) = \frac{x^2 X(x)}{r^2} \quad \implies \quad \frac{dx^2}{X(x)} = \frac{dr^2}{\Delta_r(r)}, \tag{11}$$

where x is given in (10). After rewriting the full metric (3) in terms of these new coordinates as defined in (8) and (10), it can be seen that it describes a rotating black hole with an horizon of S^3 topology located at the largest root $r = r_0$ of the function $\Delta_r(r)$. At large distance, $r \to \infty$, the metric approaches anti-de Sitter spacetime $(R_{\mu\nu} \to -4g^2 g_{\mu\nu})$,

$$ds^2 \sim -\frac{(1+g^2 r^2)\Delta_\theta}{\Xi_a \Xi_b} d\tau^2 + \frac{dr^2}{g^2 r^2} + \frac{\rho^2 d\theta^2}{\Delta_\theta} + \frac{r^2 + a^2}{\Xi_a} \sin^2\theta d\phi^2 + \frac{r^2 + b^2}{\Xi_b} \cos^2\theta d\psi^2, \tag{12}$$

where

$$\rho^2 \equiv r^2 + a^2 \cos^2\theta + b^2 \sin^2\theta, \qquad \Delta_\theta \equiv 1 - g^2 a^2 \cos^2\theta - g^2 b^2 \sin^2\theta. \tag{13}$$

3   Conserved Charges and Thermodynamics

The angular momenta can be determined from the Komar integrals $J = 1/(16\pi) \int_{S^3} *dK$, where $K = K_\mu dx^\mu$ and $K^\mu \partial/\partial x^\mu = \partial/\partial\phi$ or $\partial/\partial\psi$. These give, respectively,

$$\begin{aligned}
J_\phi &= \frac{\pi m \left[a \left(c_3^2 + s_3^2 + \Xi_b(s_1^2 - s_3^2) \right) + bc_3 s_3(1 + g^2 a^2) \right]}{2\Xi_a^2 \, \Xi_b}, \\
J_\psi &= \frac{\pi m \left[b \left(c_3^2 + s_3^2 + \Xi_a(s_1^2 - s_3^2) \right) + ac_3 s_3(1 + g^2 b^2) \right]}{2\Xi_a \Xi_b^2},
\end{aligned} \tag{14}$$

The conserved electric charges are given by $Q_i = 1/(16\pi) \int_{S^3} (X_i^{-2} *\mathcal{F}^i + \frac{1}{2}|\varepsilon_{ijk}| \mathcal{A}^j \wedge \mathcal{F}^k)$, evaluated over the sphere at infinity. We find

$$Q_1 = Q_2 = \frac{\pi m s_1 c_1}{2\Xi_a \, \Xi_b}, \qquad Q_3 = \frac{\pi m[s_3 c_3 - g^2 ab(s_1^2 - s_3^2)]}{2\Xi_a \, \Xi_b}. \tag{15}$$

The conserved mass E could in principle be calculated using the conformal technique of Ashtekar, Magnon and Das [9, 10], but in practice it is easier to evaluate it by integrating the first law of thermodynamics,

$$dE = TdS + \Omega_\phi dJ_\phi + \Omega_\psi dJ_\psi + \sum_i \Phi_i dQ_i, \tag{16}$$

where T is the Hawking temperature, S is the entropy, Ω_ϕ and Ω_ψ are the angular velocities of the horizon and Φ_i are the potential differences between the horizon and infinity. To do this, we first construct the Killing vector ℓ that becomes null on the horizon at $r = r_0$, given by

$$\ell = \frac{\partial}{\partial\tau} + \Omega_\phi \frac{\partial}{\partial\phi} + \Omega_\psi \frac{\partial}{\partial\psi}. \tag{17}$$

We find that the anglar velocities are given by

$$
\begin{aligned}
\Omega_\phi &= \frac{b(ab + 2ms_3c_3) + a[1 + g^2(b^2 + r_0^2 + \frac{2}{3}m(s_1^2 - s_3^2))][r_0^2 - \frac{4}{3}m(s_1^2 - s_3^2)]}{ab(ab + 2ms_3c_3) + [a^2 + b^2 + r_0^2 + \frac{2}{3}m(s_1^2 - s_3^2)][r_0^2 - \frac{4}{3}m(s_1^2 - s_3^2)]}, \\
\Omega_\psi &= \frac{a(ab + 2ms_3c_3) + b[1 + g^2(a^2 + r_0^2 + \frac{2}{3}m(s_1^2 - s_3^2))][r_0^2 - \frac{4}{3}m(s_1^2 - s_3^2)]}{ab(ab + 2ms_3c_3) + [a^2 + b^2 + r_0^2 + \frac{2}{3}m(s_1^2 - s_3^2)][r_0^2 - \frac{4}{3}m(s_1^2 - s_3^2)]},
\end{aligned} \tag{18}
$$

The surface gravity κ is given by

$$
\kappa^2 = \lim_{r \to r_0} \frac{g^{\mu\nu}(\partial_\mu \ell^2)(\partial_\nu \ell^2)}{(-4\ell^2)}, \tag{19}
$$

From this, we find that the Hawking temperature $T = \kappa/(2\pi)$ is given by

$$
T = \frac{r_0 \Delta_r'(r_0)\sqrt{r_0^2 - \frac{4}{3}m(s_1^2 - s_3^2)}}{4\pi\{ab(ab + 2ms_3c_3) + [a^2 + b^2 + r_0^2 + \frac{2}{3}m(s_1^2 - s_3^2)][r_0^2 - \frac{4}{3}m(s_1^2 - s_3^2)]\}}, \tag{20}
$$

where $\Delta_r'(r_0)$ means the derivative of $\Delta_r(r)$ evaluated at $r = r_0$.

The entropy S is equal to a quarter of the area of the 3-sphere horizon at $r = r_0$, and is given by

$$
S = \frac{\pi^2[ab(ab + 2ms_3c_3) + (a^2 + b^2 + r_0^2 + \frac{2}{3}m(s_1^2 - s_3^2))(r_0^2 - \frac{4}{3}m(s_1^2 - s_3^2))]}{2\Xi_a\,\Xi_b\,\sqrt{r_0^2 - \frac{4}{3}m(s_1^2 - s_3^2)}}. \tag{21}
$$

Finally, the electrostatic potentials Φ_i on the horizon are given by evaluating $\ell^\mu \mathcal{A}_\mu^i$ at $r = r_0$. The potentials at infinity vanish in the gauge we are using. Thus we find

$$
\begin{aligned}
\Phi_1 &= \Phi_2 = \frac{2ms_1c_1[r_0^2 - \frac{4}{3}m(s_1^2 - s_3^2)]}{ab(ab + 2ms_3c_3) + (a^2 + b^2 + r_0^2 + \frac{2}{3}m(s_1^2 - s_3^2))(r_0^2 - \frac{4}{3}m(s_1^2 - s_3^2))}, \\
\Phi_3 &= \frac{2m[s_3c_3(r_0^2 + \frac{2}{3}m(s_1^2 - s_3^2)) + ab(s_1^2 - s_3^2)]}{ab(ab + 2ms_3c_3) + (a^2 + b^2 + r_0^2 + \frac{2}{3}m(s_1^2 - s_3^2))(r_0^2 - \frac{4}{3}m(s_1^2 - s_3^2))}.
\end{aligned} \tag{22}
$$

Using all the above results, we are in a position to evaluate the right-hand side of the first law (16), and to integrate it up to obtain the conserved mass E. It is highly non-trivial that the right-hand side turns out to be an exact differential, and this provides a useful check on the algebra. We find that the conserved mass is given by

$$
\begin{aligned}
E &= \frac{m\pi\left[(c_3^2 + s_3^2)(2\Xi_a + 2\Xi_b - \Xi_a\,\Xi_b) + 4g^2abs_3c_3(\Xi_a + \Xi_b)\right]}{4\Xi_a^2\,\Xi_b^2} \\
&\quad + \frac{m\pi(s_1^2 - s_3^2)\left[2\left(\Xi_a + \Xi_b + g^4(a^4 + b^4)\right) + g^2(a^2 + b^2)(\Xi_a\,\Xi_b - 2)\right]}{4\Xi_a^2\,\Xi_b^2}.
\end{aligned} \tag{23}
$$

It is straightforward to check that the angular momenta, electric charges and conserved mass, along with the other thermodynamic quantities calculated here, agree in the appropriate limits with previous results. Thus setting $s_1 = s_3$ yields the previous results in [5] for the case of 3 equal charges; setting $s_1 = 0$ yields the results in [6] and [7] for the case of a single non-zero charge; and setting instead $s_3 = 0$ yields the results in [7] for the case with 2 charges equal and the third non-vanishing but related to the other two.

6

4   BPS Limit and Supersymmetric Black Holes

A BPS limit of the non-extremal solutions will arise if the conserved charges satisfy the condition[1]

$$E = gJ_\phi + gJ_\psi + \sum_{i=1}^{3} Q_i.$$ (24)

The solution then admits a Killing spinor, implying that it is a supersymmetric supergravity background. Substituting our results from the previous section, we find (24) implies that

$$e^{2\delta_1 + 2\delta_3} = 1 + \frac{2}{g(a+b)}.$$ (25)

(Recall that δ_1 and δ_3 are the charge parameters in the original metric, with $s_i = \sinh \delta_i$, etc.)

The existence of a Killing spinor η allows one to write down an everywhere-timelike Killing vector $K^\mu = \bar{\eta}\Gamma^\mu\eta$. This will take the form

$$K = \frac{\partial}{\partial\tau} + g\frac{\partial}{\partial\phi} + g\frac{\partial}{\partial\psi}.$$ (26)

Because its admits a spinorial square root, the Killing vector K has a manifestly negative norm (see, for example, [11], and also [5]), and in fact one can show that when (25) is satisfied

$$K^2 = -h_1^{-4/3}h_3^{-2/3}\left[\rho^2 + \frac{m\left[(2+ga+gb)^2 - g^2(a+b)^2 e^{4\delta_3}\right](3\Delta_\theta - (1+ga)(1+gb))}{6g^2(a+b)^2(1+ga)(1+gb)(2+ga+gb)e^{2\delta_3}}\right.$$
$$\left. -\frac{2m}{3g^2(a+b)^2 e^{2\delta_3}}\right]^2,$$ (27)

where

$$h_1 = \rho^2 + \frac{2}{3}m(s_1^2 - s_3^2) \quad , \quad h_3 = \rho^2 - \frac{4}{3}m(s_1^2 - s_3^2).$$ (28)

This result is useful for studying the occurrence of closed timelike curves (CTCs) in the BPS metric. First, we note that the metric can be cast in the form

$$ds^2 = -\frac{r^2\Delta_r(r)\Delta_\theta \sin^2\theta \cos^2\theta dt^2}{\Xi_a^2\,\Xi_b^2\,B_\phi B_\psi} + h_1^{2/3}h_3^{1/3}\left[\frac{d\theta^2}{\Delta_\theta} + \frac{dr^2}{\Delta_r(r)}\right]$$
$$+B_\psi(d\psi + v_1 d\phi + v_2 dt)^2 + B_\phi(d\phi + v_3 dt)^2,$$ (29)

where the functions B_ϕ, B_ψ and v_i can be read off by comparing (29) with the original form of the metric. In order not to have CTCs, it must be that B_ϕ and B_ψ are non-negative outside the horizon. After imposing (25), we can write

$$K^2 = -\frac{r^2\Delta_r(r)\Delta_\theta \sin^2\theta \cos^2\theta}{\Xi_a^2\,\Xi_b^2\,B_\phi B_\psi} + B_\psi(g + v_1 g + v_2)^2 + B_\phi(g + v_3)^2,$$ (30)

---

[1]Equivalent BPS conditions arise for all other choices of signs in this equation.

and so on the horizon, where $\Delta_r(r) = 0$, the negativity of K^2 implies that B_ϕ or B_ψ must be negative, and hence except for special cases there will be CTCs on and outside the horizon in the BPS solutions.

4.1 Supersymmetric black holes

One way to avoid the occurrence of CTCs outside the horizon in the BPS solutions is to arrange by means of a further condition on the parameters that K^2, given by (27), actually vanishes on the horizon. As in cases studied previously, such as that of three equal charges in [5], this condition is precisely equivalent to the condition that the derivative of the metric function $\Delta_r(r)$ vanishes on the horizon at $r = r_0$. In other words, it has a double root there:

$$\Delta_r(r_0) = 0 = \Delta_r'(r_0). \tag{31}$$

As can be seen from (20), this means that the Hawking temperature vanishes. This is indeed a necessary condition for having a regular supersymmetric black hole, since the inequivalent energy distribution functions for bosons and fermions in a thermal state at non-zero temperature are manifestly incompatible with supersymmetry.

A convenient way to solve the zero-temperature condition (31) in addition to the BPS condition (25) is to regard (25) as placing a constraint on the value of the gauge-coupling constant g as a function of the rotation and charge parameters. (This has the advantage of allowing not only the two angular momenta, but also the two charge parameters, to be adjusted freely, and this makes it easier to compare results with previously-known cases such as $\delta_1 = \delta_3$, $\delta_1 = 0$ or $\delta_3 = 0$.) The zero-temperature condition (31) can then be solved for the mass parameter m, implying that

$$M = \frac{e^{\delta_1 + \delta_3} \left[(a^2 + b^2) \sinh(2\delta_1 + 2\delta_3) + 2ab \cosh(2\delta_1 + 2\delta_3) \right]}{2 \sinh(\delta_1 + \delta_3) \sinh 2\delta_1}. \tag{32}$$

If the solution is chosen so that both (25) and (32) are satisfied, then it can describe a regular supersymmetric black hole. It is still necessary to restrict the remaining 3 parameters to lie within appropriate regions, in order that the metric be free of any CTCs outside the horizon, but these remaining conditions take the form of inequalities rather than further functional relations between the parameters. They are generalisations of the restrictions found in [5] for the case when the three charges were equal. One can, for example, see that if ga and gb are sufficiently small and positive, and the charge parameter δ_3 is sufficiently large, then there will be no CTCs outside the horizon.

The supersymmetric black holes that we have obtained here will correspond to the $Q_1 = Q_2$ specialisation of the supersymmetric 3-charge black holes coostructed in [12].

4.2 Topological solitons

A second way of eliminating CTCs in the BPS solutions is if the product $B_\phi B_\psi$ is proportional to $\Delta_r(r)$, and hence one or other of B_ϕ or B_ψ vanishes on the horizon. In this case, the BPS condition (25) is supplemented by the further condition

$$m = \frac{2k_3 k_4 (a+b)(1+ga)(1+gb)(2+ga+gb)e^{2\delta_3}}{k_1^2 k_2}, \tag{33}$$

with

$$
\begin{aligned}
k_1 &= (2+ga+gb)^2 - g^2(a+b)^2 e^{4\delta_3} ; \tag{34}\\
k_2 &= (2+ga+gb)(a+b+2gab) - (a+b)(2+ga+gb+2g^2ab)e^{4\delta_3} ; \\
k_3 &= (2+ga+gb)\left[2a-gb^2+gab(1-ga-gb)\right] + g(a+b)^2(2+gb+g^2ab)e^{4\delta_3} ; \\
k_4 &= (2+ga+gb)\left[2b-ga^2+gab(1-ga-gb)\right] + g(a+b)^2(2+ga+g^2ab)e^{4\delta_3} .
\end{aligned}
$$

(In this case, we have chosen to use (25) to eliminate δ_1.) The metric now describes a smooth topological soliton, with $r = r_0$ being a regular origin of polar coordinates at which $B_\phi \to 0$, and free of conical singularities, provided that the quantisation condition

$$\frac{ak_4 - bk_3}{g(a-b)b(1-ga)}\left[\frac{2g^2 b}{k_1} - \frac{1+gb}{k_2} - \frac{g(a-b)(1-gb)}{k_4}\right] = 1 \tag{35}$$

is satisfied. These topological solitons generalise examples found in [5] in the case that the three charges were equal.

5 Conclusions

The most general non-extremal black holes with an S^3 horizon topology in maximal $SO(6)$-gauged five-dimensional supergravity would be characterised by a total of six parameters, comprising the mass, the two independent angular momenta, and three independent electric charges supported by the three abelian gauge fields in the $U(1)^3$ Cartan subgroup of $SO(6)$. They could equivalently be regarded as solutions in $\mathcal{N} = 2$ gauged supergravity coupled to two vector multiplets.

In this paper, we have constructed the most general such non-extremal rotating black holes found to date. They are characterised by five parameters, namely the mass, the two angular momenta, and two independently-specifiable charge parameters. They correspond to the situation where two of the three charges in the most general solution are set equal, but with no restrictions otherwise. These solutions encompass and extend all previously-obtained results for black holes with independent rotation parameters in five-dimensional gauged supergravity.

We calculated the conserved angular momenta and charges for the new solutions; the entropy and Hawking temperature; and the angular velocities and electric potentials on the horizon. From this, we showed that the first law of thermodynamics is integrable, and we obtained the expression for the mass of the black holes.

We then studied the BPS limit of the solutions, and showed how further restrictions on the remaining parameters would give rise to regular supersymmetric black holes and to smooth topological solitons.

The results we have obtained in this paper should have applications in the sudy of the AdS/CFT correspondence. It would be of considerable interest to find the more general 6-parameter black-hole solutions in five-dimensional maximal gauged supergravity, in which the three electric charges, as well as the mass and the two angular momenta, are independently specificable. These can be expected to be considerably more complicated than the solutions constructed until now.

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