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## Exact Embedding of $N = 1$ , $D = 7$ Gauged Supergravity in $D = 11$

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### ABSTRACT

We obtain the explicit and complete bosonic non-linear Kaluza-Klein ansatz for the consistent  $S^4$  reduction of  $D = 11$  supergravity to  $N = 1$ ,  $D = 7$  gauged supergravity. This provides a geometrical interpretation of the lower dimensional solutions from the eleven-dimensional point of view.

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Kaluza-Klein dimensional reduction has enjoyed a resurgence of attention in recent years, owing to the important role that it plays in the derivation and discussion of duality symmetries in string theories. For example, the non-perturbative U-duality symmetries of the compactified string [1, 2] are understood at the level of the effective low-energy field theory by performing Kaluza-Klein reductions on toroidal internal spaces of various dimensions. One makes contact with the conjectured non-perturbative BPS configurations of the string by showing that there exist corresponding soliton solutions of the lower-dimensional theories, which preserve some fraction of the supersymmetry. A crucial aspect of the reduction procedure is that it is *consistent*, which ensures that solutions of the lower-dimensional equations of motion will also be solutions of the original more fundamental higher-dimensional ones.

Recently, a new duality was conjectured, relating supergravities in anti-de Sitter (AdS) backgrounds to superconformal field theories on the boundary [3, 4, 5]. The relevant supergravities are *gauged* theories, which are believed to arise from the Kaluza-Klein reduction of string theory or M-theory on certain spherical internal spaces. It again becomes crucial to know whether the reduction is a consistent one, since this is important for establishing the details of the link between the supergravity theory in the bulk, and the conformal field theory on the boundary.

In the case of toroidal internal spaces, the consistency of the reduction procedure is guaranteed by simple group-theoretic arguments. The consistency of the Kaluza-Klein sphere reduction of supergravities on AdS×Sphere is more subtle to address, since there appears to be no simple group-theoretic argument that guarantees it. Indeed, there are simple arguments which demonstrate that the reduction and truncation of a generic theory on a sphere will definitely be *inconsistent*. In fact it is only because of very remarkable “conspiracies” between the various terms in the higher-dimensional supergravity theories that the consistent truncation is possible at all.

The original discussions of sphere reductions in supergravity considered only the linearised fluctuations around a fixed AdS×Sphere background [6, 7], for which no possibility of inconsistency arises. Indications of the remarkable underlying structures that lead to the consistency at the full non-linear level were seen in studies that examined leading-order non-linear contributions [8]. The exact consistent ansätze for  $N = 2$  and  $N = 3$  truncations of the  $N = 8$  supersymmetric  $S^7$  reduction of  $D = 11$  supergravity were obtained in [9, 10]. A complete demonstration of the consistency was presented in [11], although the construction was highly implicit, and did not lend itself to the explicit re-interpretation

of lower-dimensional solutions in terms of eleven-dimensional ones. Results for the full non-linear ansatz just for the metric in certain sphere reductions were also obtained in [12, 13, 14, 15].

Recently, the first examples of complete and explicit non-trivial reductions were presented, for  $S^7$  and  $S^4$  compactifications of M-theory and an  $S^5$  compactification of the type IIB string [16].<sup>1</sup> In each case, it was possible to obtain explicit results by making further truncations in which only fields associated with the Cartan subalgebras of the non-abelian Yang-Mills gauge groups were retained. Thus in the  $S^7$ ,  $S^4$  and  $S^5$  reductions only the gauge fields of  $U(1)^4$ ,  $U(1)^2$  and  $U(1)^3$  respectively were retained. These reductions were sufficient, however, for allowing the re-interpretation of certain non-trivial BPS solutions of the lower-dimensional supergravities as solutions in  $D = 11$  or  $D = 10$ . In particular, it was shown that charged AdS black-holes in the lower dimensions could be interpreted as the near-horizon limits of corresponding rotating M-branes or D3-brane in  $D = 11$  and  $D = 10$  [17, 16].

The construction in [16] showed that the reduction must be performed at the level of the higher-dimensional equations of motion, rather than the Lagrangian (even when it exists). Furthermore, it also demonstrated how the consistency of the reduction depends crucially on “conspiracies” involving an interplay between the various fields in the higher-dimensional theory, and emphasised the fact that the Kaluza-Klein reduction of a “generic” theory on a sphere would, by contrast, be inconsistent.

In this letter, we shall consider the Kaluza-Klein reduction of eleven-dimensional supergravity on  $S^4$ . A complete Kaluza-Klein reduction ansatz for the  $S^4$  compactification to  $N = 2$  gauged supergravity in  $D = 7$  was recently presented [18]. Although much simpler than the corresponding ansatz for the  $S^7$  reduction in [11], owing to the smaller dimension of the sphere, it is still quite a complicated construction. Here, we shall consider a somewhat simpler situation, where we still reduce  $D = 11$  supergravity on  $S^4$ , but where we truncate to the bosonic fields of  $N = 1$  gauged supergravity. These comprise the metric, a dilatonic scalar field, a 3-form potential and the gauge fields of  $SU(2)$  Yang-Mills. This is

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<sup>1</sup>We define “non-trivial” reductions to mean ones that involve scalar fields that parameterise inhomogeneous deformations of the compactifying sphere, implying that the consistency of the reduction will not be explicable purely by a simple group-invariance argument. Furthermore, we emphasise the completeness of the reduction ansatz because it is only when one has the full set of non-linear expressions, including in particular the ansätze for the higher-dimensional field strengths as well as the metric, that one is able to obtain a consistent reduction. The metric sector by itself gives rise to inconsistencies that are resolved only by virtue of “miraculous” conspiracies between the higher-dimensional metric and antisymmetric tensor fields.

still a non-trivial reduction, in that the scalar field parameterises inhomogeneous deformations of the 4-sphere, and the gauge fields of a non-abelian Yang-Mills group are retained. In addition, the emergence of the topological mass term for the 3-form potential can be seen. Nonetheless, the construction is completely explicit, and any solution of the seven-dimensional equations of motion can be straightforwardly re-interpreted back in  $D = 11$ . Note that the embedding that we shall discuss in this letter is qualitatively different from the one presented in [18], in that we give our ansatz on the 3-form potential and metric of the eleven-dimensional theory in its original second-order formulation, rather than in the new first-order formalism presented in [18].

Our starting point is the bosonic sector of eleven-dimensional supergravity, which is described by the Lagrangian [19]

$$\mathcal{L}_{11} = \hat{R} \hat{*} \mathbf{1} - \frac{1}{2} \hat{*} \hat{F}_{(4)} \wedge \hat{F}_{(4)} - \frac{1}{6} \hat{F}_{(4)} \wedge \hat{F}_{(4)} \wedge \hat{A}_{(3)} , \quad (1)$$

where  $\hat{F}_{(4)} = d\hat{A}_{(3)}$ , and we use hats to denote eleven-dimensional fields and the eleven-dimensional Hodge dual  $\hat{*}$ . The equations of motion following from this Lagrangian are

$$\begin{aligned} \hat{R}_{MN} &= \frac{1}{12} (\hat{F}_{MN}^2 - \frac{1}{12} \hat{F}_{(4)}^2 \hat{g}_{MN}) , \\ d\hat{*} \hat{F}_{(4)} &= \frac{1}{2} \hat{F}_{(4)} \wedge \hat{F}_{(4)} . \end{aligned} \quad (2)$$

We obtained the ansatz for the reduction to the bosonic sector of  $N = 1$  gauged supergravity in  $D = 7$  by first taking the results for the  $U(1)^2$  abelian truncation given in [16], setting the two scalars  $X_1$  and  $X_2$  of that theory equal, and also the two  $U(1)$  gauge fields. Having done this, the metric on the internal 4-sphere takes the form

$$ds_4^2 = X^3 \Delta d\xi^2 + \frac{1}{4} X^{-1} \cos^2 \xi \left( d\theta^2 + \sin^2 \theta d\varphi^2 + (d\psi + \cos \theta d\varphi - g A_{(1)})^2 \right) , \quad (3)$$

where  $X = e^{-\phi/\sqrt{10}}$  parameterises the scalar field in terms of a canonically-normalised dilaton  $\phi$ , and

$$\Delta = X^{-4} \sin^2 \xi + X \cos^2 \xi . \quad (4)$$

At  $X = 1$ , in the absence of the  $U(1)$  gauge field  $A_{(1)}$ , this metric describes a unit 4-sphere as a foliation of 3-spheres that are parameterised by Euler angles  $(\theta, \varphi, \psi)$ , with “latitude” coordinate  $\xi$ . The 3-sphere itself is a  $U(1)$  bundle over the 2-sphere with metric  $d\theta^2 + \sin^2 \theta d\varphi^2$ , with  $\psi$  as the coordinate on the  $U(1)$  fibres. When the 7-dimensional scalar  $X$  is excited, it describes inhomogeneous deformations of the 4-sphere, leaving the 3-sphere foliations intact. Excitations of the  $U(1)$  gauge field  $A_{(1)}$  in seven dimensions describe deformations of the 3-sphere’s  $U(1)$  bundle.

It is now natural to consider a non-abelian generalisation of the deformations of the 4-sphere metric, by introducing the three left-invariant 1-forms  $\sigma^i$  on  $S^3$ , which satisfy  $d\sigma^i = -\frac{1}{2}\epsilon_{ijk}\sigma^j \wedge \sigma^k$ . These can be written in terms of the Euler angles as  $\sigma_1 + i\sigma_2 = e^{-i\psi}(d\theta + i\sin\theta d\varphi)$ ,  $\sigma_3 = d\psi + \cos\theta d\varphi$ . We are naturally led to generalise (3) to

$$ds_4^2 = X^3 \Delta d\xi^2 + \frac{1}{4}X^{-1} \cos^2 \xi \sum_{i=1}^3 (\sigma^i - g A_{(1)}^i)^2 . \quad (5)$$

Note that this reduces to (3) in the abelian limit where the  $i = 1$  and  $i = 2$  components of the  $SU(2)$  gauge potentials  $A_{(1)}^i$  are set to zero.

Having established our notation, we can now present our ansätze for the  $D = 11$  metric and 3-form potential:

$$\begin{aligned} ds_{11}^2 &= \Delta^{1/3} ds_7^2 + 2g^{-2} X^3 \Delta^{1/3} d\xi^2 + \frac{1}{2}g^{-2} \Delta^{-2/3} X^{-1} \cos^2 \xi \sum_i (\sigma^i - g A_{(1)}^i)^2 , \quad (6) \\ \hat{A}_{(3)} &= \sin \xi A_{(3)} + \frac{1}{2\sqrt{2}}g^{-3}(2\sin \xi + \sin \xi \cos^2 \xi \Delta^{-1} X^{-4}) \epsilon_{(3)} \\ &\quad - \frac{1}{\sqrt{2}}g^{-2} \sin \xi F_{(2)}^i \wedge h^i - \frac{1}{\sqrt{2}}g^{-1} \sin \xi \omega_{(3)} . \quad (7) \end{aligned}$$

where  $h^i \equiv \sigma^i - g A_{(1)}^i$  and  $\epsilon_{(3)} \equiv h^1 \wedge h^2 \wedge h^3$ . The  $SU(2)$  Yang-Mills field strengths  $F_{(2)}^i$  are given by  $F_{(2)}^i = dA_{(1)}^i + \frac{1}{2}g \epsilon_{ijk} A_{(1)}^j \wedge A_{(1)}^k$ , and we have defined  $\omega_{(3)} \equiv A_{(1)}^i \wedge F_{(2)}^i - \frac{1}{6}g \epsilon_{ijk} A_{(1)}^i \wedge A_{(1)}^j \wedge A_{(1)}^k$ , so that  $d\omega_{(3)} = F_{(2)}^i \wedge F_{(2)}^i$ . Note that all the fields  $X$ ,  $A_{(3)}$  and  $A_{(1)}^i$ , and the metric  $ds_7^2$ , appearing on the right-hand sides of (6) and (7), are taken to depend only on the coordinates of the seven-dimensional spacetime.

It is useful to present the field strength  $\hat{F}_{(4)} = d\hat{A}_{(3)}$  and its eleven-dimensional Hodge dual:

$$\begin{aligned} \hat{F}_{(4)} &= -\frac{1}{2\sqrt{2}}g^{-3}(X^{-8} \sin^2 \xi - 2X^2 \cos^2 \xi + 3X^{-3} \cos^2 \xi - 4X^{-3}) \Delta^{-2} \cos^3 \xi d\xi \wedge \epsilon_{(3)} \\ &\quad - \frac{5}{2\sqrt{2}}g^{-3} \Delta^{-2} X^{-4} \sin \xi \cos^4 \xi dX \wedge \epsilon_{(3)} + \sin \xi F_{(4)} \\ &\quad + \sqrt{2}g^{-1} \cos \xi X^4 *F_4 \wedge d\xi - \frac{1}{\sqrt{2}}g^{-2} \cos \xi F_{(2)}^i \wedge d\xi \wedge h^i \\ &\quad - \frac{1}{4\sqrt{2}}g^{-2} X^{-4} \Delta^{-1} \sin \xi \cos^2 \xi F_{(2)}^i \wedge h^j \wedge h^k \epsilon_{ijk} , \quad (8) \\ * \hat{F}_{(4)} &= -\frac{1}{\sqrt{2}}g(X^{-8} \sin^2 \xi - 2X^2 \cos^2 \xi + 3X^{-3} \cos^2 \xi - 4X^{-3}) \epsilon_{(7)} \\ &\quad + 5\sqrt{2}g^{-1} \sin \xi \cos \xi X^{-1} *dX \wedge d\xi + 8g^{-4} \sin \xi \cos^3 \xi \Delta^{-1} *F_{(4)} \wedge d\xi \wedge \epsilon_{(3)} \\ &\quad - \frac{1}{2\sqrt{2}}g^{-3} \cos^4 \xi \Delta^{-1} X F_{(4)} \wedge \epsilon_{(3)} - \frac{1}{4\sqrt{2}}g^{-2} \cos^2 \xi X^{-2} *F_{(2)}^i \wedge h^j \wedge h^k \epsilon_{ijk} \\ &\quad - \frac{1}{\sqrt{2}}g^{-2} \sin \xi \cos \xi X^{-2} *F_{(2)}^i \wedge d\xi \wedge h^i , \quad (9) \end{aligned}$$

where  $F_{(4)} = dA_{(3)}$ ,  $*$  denotes the seven-dimensional Hodge dual calculated in the metric  $ds_7^2$ , and  $\epsilon_{(7)}$  is the volume-form of  $ds_7^2$ .

A remark about the truncation that we are performing is in order here. What is required is the truncation to the pure  $N = 1$  supergravity multiplet in  $D = 7$ . As discussed in [20], the 3-form  $A_{(3)}$ , which is massive, would have 20 on-shell degrees of freedom if it satisfied an ordinary second-order field equation. However, the 3-form field in the supergravity multiplet should have only 10 on-shell degrees of freedom. This is achieved by requiring that it satisfy a *first order* field equation; the so-called “odd-dimensional self-duality” equation [21]. In the presence of the  $SU(2)$  Yang-Mills fields, we find that this equation will be

$$X^4 *F_{(4)} = -\frac{1}{\sqrt{2}}g A_{(3)} + \frac{1}{2}\omega_{(3)} . \quad (10)$$

Since the imposition of this equation is part of our truncation procedure, we are free to impose it, if we wish, when writing down the expressions for the eleven-dimensional fields. This we have done in writing  $\hat{F}_{(4)}$  and  $\hat{*F}_{(4)}$  above.

We find that the equation of motion for  $\hat{F}_{(4)}$ , given in (2), implies the following seven-dimensional equations:

$$\begin{aligned} d(X^{-1} *dX) &= \frac{1}{5}X^4 *F_{(4)} \wedge F_{(4)} - \frac{1}{10}X^{-2} *F_{(2)}^i \wedge F_{(2)}^i - \frac{1}{5}g^2 (X^{-8} + 2X^2 - 3X^{-3}) \epsilon_{(7)} , \\ d(X^4 *F_{(4)}) &= -\frac{1}{\sqrt{2}}g F_{(4)} + \frac{1}{2}F_{(2)}^i \wedge F_{(2)}^i , \\ D(X^{-2} *F_{(2)}^i) &= F_{(4)} \wedge F_{(2)}^i , \end{aligned} \quad (11)$$

where  $D$  denotes the gauge-covariant exterior derivative,  $D\omega^i = d\omega^i + g \epsilon_{ijk} A_{(1)}^j \wedge \omega^k$ . Note that the second-order equation for  $A_{(3)}$  here is nothing but the exterior derivative of the first-order equation (10).

Substituting the ansätze (6) and (8) into the eleven-dimensional Einstein equation in (2), we again obtain the equations of motion for the scalar  $X$  and the Yang-Mills fields  $A_{(1)}^i$ , as in (11), together with the seven-dimensional Einstein equation

$$\begin{aligned} R_{\mu\nu} &= 5X^{-2} \partial_\mu X \partial_\nu X + \frac{1}{5}V g_{\mu\nu} + \frac{1}{12}X^4 (F_{(4)\mu\nu}^2 - \frac{3}{20}F_{(4)}^2 g_{\mu\nu}) \\ &\quad + \frac{1}{2}X^{-2} ((F_{(2)}^i)_{\mu\nu}^2 - \frac{1}{10}(F_{(2)}^i)^2 g_{\mu\nu}) , \end{aligned} \quad (12)$$

where  $V$  is the scalar potential, given by

$$V = g^2 (\frac{1}{4}X^{-8} - 2X^{-3} - 2X^2) . \quad (13)$$

The above seven-dimensional equations of motion can be derived from the Lagrangian

$$\begin{aligned} \mathcal{L}_7 &= R * \mathbb{1} - \frac{1}{2} *d\phi \wedge d\phi - g^2 (\frac{1}{4}e^{\frac{8}{\sqrt{10}}\phi} - 2e^{\frac{3}{\sqrt{10}}\phi} - 2e^{-\frac{2}{\sqrt{10}}\phi}) * \mathbb{1} - \frac{1}{2}e^{-\frac{4}{\sqrt{10}}\phi} *F_{(4)} \wedge F_{(4)} \\ &\quad - \frac{1}{2}e^{\frac{2}{\sqrt{10}}\phi} *F_{(2)}^i \wedge F_{(2)}^i + \frac{1}{2}F_{(2)}^i \wedge F_{(2)}^i \wedge A_{(3)} - \frac{1}{2\sqrt{2}}g F_{(4)} \wedge A_{(3)} , \end{aligned} \quad (14)$$

where we have replaced  $X$  by  $X = e^{-\phi/\sqrt{10}}$ , together with the self-duality condition (10), which is to be imposed after having obtained the equations of motion from (14). This set of equations of motion precisely describe the bosonic sector of  $N = 1$  gauged supergravity in seven dimensions. The Lagrangian (14) was also given in [22].

The fact that we have been able to extract seven-dimensional equations of motion by substituting (6) and (8) into the eleven-dimensional equations of motion demonstrates that our reduction ansätze are consistent. In other words, the eleven-dimensional equations of motion are satisfied by all the field configurations given in (6) and (8), provided that the seven-dimensional fields satisfy the equations of motion of  $N = 1$  gauged supergravity. It is worth emphasising that the ability to extract seven-dimensional equations of motion in a consistent manner depends crucially on the interplay between the contributions of the three terms in the eleven-dimensional Lagrangian (1), which leads to a precise matching, and hence factoring-out, of the dependence on the coordinates of the internal 4-sphere in the eleven-dimensional equations of motion. In particular, the specific value of the coefficient of the  $\hat{F}_{(4)}\hat{F}_{(4)}\hat{A}_{(3)}$  term in (1) is crucial. Since the magnitude of this term is also uniquely determined by supersymmetry, the calculations that we have presented here again support the notion that the consistency of non-trivial sphere reductions is closely related to supersymmetry [23].

Our approach to finding the correct ansatz, and verifying that it gives a consistent reduction, has been to substitute it into the eleven-dimensional equations of motion, and then to verify that these are satisfied provided the lower-dimensional equations of motion are satisfied. This, by definition, is what one means by consistency. An alternative approach that is sometimes discussed is to substitute the ansatz into the higher-dimensional Lagrangian, and integrate over the coordinates of the internal space. Of course before doing this, one would first have to find an independent argument for why the ansatz was a consistent one. (As is well known, the mere fact that one obtains a sensible-looking lower-dimensional action by substituting an ansatz into the higher-dimensional one does not, of itself, guarantee that the ansatz is consistent.) Since we have constructed an explicit consistent reduction ansatz in this letter, it is of interest to see what would happen if one were to substitute it into the eleven-dimensional action.

First, let us simplify the discussion by restricting just to the gravity plus scalar sector of the seven-dimensional theory. Substituting the ansätze (6) and (7) into the eleven-dimensional Lagrangian (1), we find  $\mathcal{L}_{11} = \frac{1}{2}g^{-4} \cos^3 \xi (R - \frac{1}{2}(\partial\phi)^2 + W) \sqrt{g_3} \sqrt{-g_7}$ , where

$$W = \frac{1}{12}g^2 \Delta^{-2} X^{-11} \left( -2 \sin^2 \xi (1 - 21 \sin^2 \xi) - (7 - 66 \sin^2 \xi + 51 \sin^4 \xi) X^5 \right)$$

$$+4 \cos^2 \xi (7 + 3 \sin^2 \xi) X^{10} + 24 \cos^4 \xi X^{15} \Big). \quad (15)$$

Integrating over  $\xi$ , one indeed obtains the correct gravity plus scalar sector of the seven-dimensional Lagrangian (14).<sup>2</sup> However, once the gauge fields are included in the reduction ansatz, the substitution of (6) and (7) into the eleven-dimensional action will not give rise to the seven-dimensional action (14). All these defects are avoided by sticking with the equations of motion.

In this letter, we have presented an explicit and consistent fully non-linear reduction ansatz for the compactification of eleven-dimensional supergravity on  $S^4$ , with a truncation to  $N = 1$  gauged supergravity in  $D = 7$ . In particular, we have shown how the dilaton  $\phi$  parameterises inhomogeneous deformations of the 4-sphere that leave the foliating 3-spheres undistorted. On the other hand, the  $SU(2)$  gauge fields that form the surviving subgroup of the  $SO(5)$  Yang-Mills fields of the maximal gauged theory are associated with deformations that correspond to right-translations under the  $SU(2)$  that acts on the 3-spheres. Using this geometrical embedding of the seven-dimensional theory in the eleven-dimensional one, it is now possible to re-express any solution of the seven-dimensional  $N = 1$  gauged theory as a solution in the low-energy limit of M-theory.

## References

- [1] C.M. Hull and P.K. Townsend, *Unity of superstring dualities*, Nucl. Phys. **B294** (1995) 196, hep-th/9410167.
- [2] E. Witten, *Some comments on string dynamics*, in STRINGS 95: Future Perspectives in String Theory, Los Angeles, CA, 13-18 Mar 1995, hep-th/9507121.
- [3] J. Maldacena, *The large  $N$  limit of superconformal field theories and supergravity*, Adv. Theor. Math. Phys. **2** (1998) 231, hep-th/9711200.
- [4] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, Phys. Lett. **B428** (1998) 105, hep-th/9802109.
- [5] E. Witten, *Anti-de Sitter space and holography*, Adv. Theor. Math. Phys. **2** (1998) 253, hep-th/9802150.

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<sup>2</sup>By contrast, in an analogous calculation for the gravity plus scalar sector of the  $S^4$  reduction of the massive type IIA theory, the corresponding function  $W$  becomes singular at the limit of the  $\xi$  integration, and the procedure of substituting the ansatz into the action is problematical even in the scalar sector [24].



- [6] M.J. Duff and C.N. Pope, *Kaluza-Klein supergravity and the seven sphere*, in: Super-symmetry and supergravity 82, eds. S. Ferrara, J.G. Taylor and P. van Nieuwenhuizen (World Scientific, Singapore, 1983).
- [7] B. Biran, F. Englert, B. de Wit and H. Nicolai. *Gauged  $N = 8$  supergravity and its breaking from spontaneous compactification*, Phys. Lett. **B124** (1983) 45, Erratum-ibid. **B128** (1983) 461.
- [8] M.J. Duff, B.E.W. Nilsson, C.N. Pope and N.P. Warner, *On the consistency of the Kaluza-Klein ansatz*. Phys. Lett. **B149** (1984) 90.
- [9] C.N. Pope, *Consistency of truncations in Kaluza-Klein*, published in the Proceedings of the 1984 Santa Fe meeting.
- [10] C.N. Pope, *The embedding of the Einstein-Yang-Mills equations in  $D = 11$  supergravity*, Class. Quantum Grav. **2** (1985) L77.
- [11] B. de Wit and H. Nicolai, *The consistency of the  $S^7$  truncation in  $D = 11$  supergravity*, Nucl. Phys. B281 (1987) 211.
- [12] B. de Wit and H. Nicolai, *The parallelizing  $S^7$  torsion in gauged  $N = 8$  supergravity*, Nucl. Phys. **B231** (1984) 506.
- [13] B. de Wit, H. Nicolai and N.P. Warner, *The embedding of gauged  $N = 8$  supergravity into  $D = 11$  supergravity*, Nucl. Phys. **B255** (1985) 29.
- [14] B. de Wit and H. Nicolai, *A new  $SO(7)$  invariant solution of  $D = 11$  supergravity*, Phys. Lett. **B148** (1984) 60.
- [15] B.E.W. Nilsson, *On the embedding of  $D = 4$   $N = 8$  gauged supergravity in  $D = 11$   $N = 1$  supergravity*, Phys. Lett. **B155** (1985) 54.
- [16] M. Cvetič, M.J. Duff, P. Hoxha, J.T. Liu, H. Lü, J.X. Lu, R. Martinez-Acosta, C.N. Pope, H. Sati and T.A. Tran, *Embedding AdS black holes in ten and eleven dimensions*, hep-th/9903214, to appear in Nucl. Phys. **B**.
- [17] M. Cvetič and S.S. Gubser, *Phases of  $R$  charged black holes, spinning branes and strongly coupled gauge theories*, hep-th/9902195.
- [18] H. Nastase, D. Vaman and P. van Nieuwenhuizen, *Consistent nonlinear KK reduction of 11d supergravity on  $AdS_7 \times S_4$  and self-duality in odd dimensions*, hep-th/9905075.

- [19] E. Cremmer, B. Julia and J. Scherk, *Supergravity theory in eleven dimensions*, Phys. Lett. **B76** (1978) 409.
- [20] K. Pilch, P. van Nieuwenhuizen and P.K. Townsend, *Compactification of  $d = 11$  supergravity on  $S^4$  (or  $11 = 7 + 4$ , too)*, Nucl. Phys. **B242** (1984) 377.
- [21] P.K. Townsend, K. Pilch and P. van Nieuwenhuizen, *Selfduality in odd dimensions*, Phys. Lett. **B136** (1984) 38, Addendum-ibid. **B137** (1984) 443.
- [22] P.K. Townsend and P. van Nieuwenhuizen, *Gauged seven-dimensional supergravity*, Phys. Lett. **B125** (1983) 41.
- [23] C.N. Pope and K.S. Stelle, *Zilch currents, supersymmetry and Kaluza-Klein consistency*, Phys. Lett. **B198** (1987) 151.
- [24] M. Cvetič, H. Lü and C.N. Pope, *Gauged six-dimensional supergravity from massive type IIA*, hep-th/9906221.