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Superparticles, p-Form Coordinates and the BPS Condition

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ABSTRACT

A model for n superparticles in $(d-n,n)$ dimensions is studied. The target space supersymmetry involves a product of n momentum generators, and the action has $n(n+1)/2$ local bosonic symmetries and n local fermionic symmetries. The precise relation between the symmetries presented here and those existing in the literature is explained. A new model is proposed for superparticles in arbitrary dimensions where coordinates are associated with all the p-form charges occuring in the superalgebra. The model naturally gives rise to the BPS condition for the charges.

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1 Introduction

Consider supercharges Q_α in $(d-n,n)$ dimensions where n is the number of time-like directions. Let the index α label the minimum dimensional spinor of $SO(d-n,n)$. In the case of extended supersymmetry, the index labelling the fundamental representation of the automorphism group is to be included. We will suppress that label for simplicity in notation. Quite generally, we can contemplate the superalgebra

$$\begin{aligned} \{Q_\alpha, Q_\beta\} &= Z_{\alpha\beta} \ , \\ [Z_{\alpha\beta}, Z_{\gamma\delta}] &= 0 \ , \end{aligned} \tag{1}$$

where $Z_{\alpha\beta}$ is a symmetric matrix which can be expanded in terms of suitable p-form generators in any dimension by consulting Table 3 provided in Appendix B. The Z generators have the obvious commutator with the Lorentz generators, which are understood to be a part of the superalgebra.

Next, consider dimensions in which the n th rank γ-matrix is symmetric [2]. In $(d-n,n)$ dimensions, setting the n th rank generator equal to a product of n momentum generators [1] and all the other Z generators equal to zero, one has

$$\{Q_\alpha, Q_\beta\} = (\gamma^{\mu_1\cdots\mu_n})_{\alpha\beta}\ P^1_{\mu_1}\cdots P^n_{\mu_n}\ . \tag{2}$$

Motivations for considering these algebras have been discussed elsewhere (see [1, 2, 3, 4], for example). The fact they can be realized in terms of multi-particle systems was pointed out in [2]. Superparticle models were constructed in [3] and [5], with emphasis on the cases of $n=2,3$. In [3] a multi-time model was considered in which a particle of type $i=1,2,3$ depended on time τ_i only. As a special case, an action for a superaparticle in the background of one or two other superparticles with constant momenta was obtained. In the model of [5], where single time dependence was introduced, a similar system was described. The results agree, after one takes into account the trivial symmetry transformations that depend on the equations of motion.

These models were improved significantly in [6], where all particles are taken to have arbitrary (single) time dependence. In [6], it was observed that the theory had n first class fermionic constraints, but one fermionic symmetry was exhibited (or m of them for extended supersymmetry with $N=1,...,m$). Furthermore, the form of the transformation rules given in [6] appear to be rather different than those found earlier in [3] (albeit in the context of a restricted version of the model).

[2]For example, we can consider (n,n) and $(8+n,n)$ dimensions where one can have (pseudo) Majorana-Weyl spinors, or $(4+n,n)$ dimensions where (pseudo) symplectic Majorana-Weyl spinors are possible. In the latter case, the tensor product of the antisymmetric n'th rank γ-matrices with the antisymmetric invariant tensor of the symplectic automorphism group is symmetric. The spinors need not be Weyl, and thus we can consider other dimensions as well (see Appendix B).

The purpose of this note is three-folds: (1) to provide the full κ-symmetry transformations of the action considered in [6], (2) to show the precise relation between the single (or m) κ-symmetry parameter of [6] and the n (or $m \times n$) such parameters here, as well as the relation between the bosonic and fermionic transformations of [6] and those given here and (3) to present a new model for superparticle in which coordinates are associated with the supercharges and *all* the bosonic p-form generators occuring in their anticommutator.

The new model provides a realization of the p-form charges and it gives rise to the BPS condition. In this model, we introduce the spinorial symmetric matrix coordinates $X^{\alpha\beta}$ corresponding to the generators $Z_{\alpha\beta}$ and the associated momentum variables $P_{\alpha\beta}$ [3]. In addition, we introduce the Lagrange multiplier variables $e_{\alpha\beta}$, which are symmetric or antisymmetric, depending on the signature of spacetime. The model exhibits a reach set of local and global symmetries and the Lagrange multiplier equation of motion gives rise to a general on-shell condition for the momenta, which contains the BPS condition. An example of this model is given for $(2,2)$ dimensional target space, and it contains the coordinates X^μ and $X^{\mu\nu}$.

The new model is rather universal and it works in arbitrary dimensions. We will see that there exists a particular reduction of the model which resembles the multi-superparticle model discussed in Sec. 2. The new model is hoped to provide a general scheme for realizing supersymmetry in dimensions beyond eleven, and to be more suitable for a brany generalization, in comparison with the previously considered superparticle models.

2 The Model for n Superparticles in $(d-n,n)$ Dimensions

Consider n superparticles which propagate in $(d-n,n)$ dimensional spacetime. Let the superspace coordinates of the particles be denoted by $X_i^\mu(\tau)$ and $\theta_i^\alpha(\tau)$ with $i=1,...,n$, $\mu=0,1,...,d-1$ and $\alpha=1,...,dim\,Q$, where $dim\,Q$ is the minimum real dimension of an $SO(d-n,n)$ or $SO(d-n,n)\times G$ spinor, with G being the automorphism group. Working in first order formalism, one also introduces the momentum variables $P_\mu^i(\tau)$ and the Lagrange multipliers $e_{ij}(\tau)$.

[3] The notation $X^{\alpha\beta}$ has been used in the literature for situations where it stands for $X^{\alpha\beta} \equiv X^\mu \gamma_\mu^{\alpha\beta}$ (see, for example, [7]), whereas in our model $X^{\alpha\beta}$ contains *all* the symmetric γ-matrices that can occur in the expansion. A superparticle model was proposed in [8] in which extra bosonic coordinates were introduced only for the usual central charges that are singlets of the Lorentz group. Coordinates for p-forms were used in [9] in the context of a search for string theory in eleven dimensions, and in the context of twistor superparticle action in ten dimensions [10] (We thank D. Sorokin for pointing this reference to us). Coordinates for super p-form charges in general were introduced in [11, 12], in the construction of generalized super p-brane actions. The new superparticle model presented here does not correspond to the particle limit of these brane actions. Morover, the constraints of our model differ from those arising in the twistor supercparticle model of [10]. The p-form coordinates have also arisen in the context of a free differential algebra [13] extension of the usual supertranslation group in which the p-form potentials that are required in the construction may be viewed as coordinates on an extended group manifold [14].

The superalgebra (2) can be realized in terms of the supercharge

$$Q_\alpha = \partial_\alpha + \Gamma_{\alpha\beta}\, \theta^\beta \ , \tag{3}$$

where, using the notation of [6], we have defined

$$\Gamma \equiv \frac{1}{n!}\, \varepsilon_{i_1\cdots i_n}\, \gamma^{\mu_1\cdots\mu_n}\, P^{i_1}_{\mu_1}\cdots P^{i_n}_{\mu_n}\ . \tag{4}$$

The spinorial derivative is defined as $\partial_\alpha = \partial/\partial\theta^\alpha$, acting from the right. The transformations generated by the supercharges Q_α are [6]

$$\delta_\epsilon X^\mu_i = -\bar\epsilon\, V^\mu_i\theta\ , \qquad \delta_\epsilon\theta = \epsilon\ , \qquad \delta_\epsilon P^\mu_i = 0\ , \qquad \delta_\epsilon e_{ij} = 0\ , \tag{5}$$

where, in the notation of [6], we have the definition

$$V^\mu_i \equiv \frac{1}{(n-1)!}\, \varepsilon_{ii_2\cdots i_n}\, \gamma^{\mu\mu_2\cdots\mu_n}\, P^{i_2}_{\mu_2}\cdots P^{i_n}_{\mu_n}\ . \tag{6}$$

We use a convention in which all fermionic bilinears involve γC-matrices with the charge conjugation matrix C suppressed, e.g. $\bar\theta\gamma^{\mu\nu}d\theta \equiv \theta^\alpha(\gamma^{\mu\nu}C)_{\alpha\beta}\, d\theta^\beta$. Otherwise (i.e. when there are free fermionic indices), it is understood that the matrix multiplications involve northeast-southwest contractions, e.g. $(\Gamma\,\theta)_\alpha \equiv (\Gamma)_\alpha{}^\beta\theta_\beta$. Note that

$$P^i_\mu V^\mu_i = n\Gamma\ , \qquad V^\mu_i\ dP^i_\mu = d\Gamma\ , \qquad P^i_\mu\ dV^\mu_i = (n-1)d\Gamma\ , \tag{7}$$

where $d \equiv \partial/\partial\tau$.

The action constructed in [6] is given by

$$I = \int d\tau\ \left(P^i_\mu\Pi^\mu_i - \tfrac{1}{2}e_{ij}P^i_\mu P^{j\mu}\right)\ , \tag{8}$$

where

$$\Pi^\mu_i = dX^\mu_i + \tfrac{1}{n}\bar\theta V^\mu_i d\theta\ . \tag{9}$$

It should be noted that the line element (9) is not invariant under supersymmetry, but it is defined such that $P^i_\mu\Pi^\mu_i$ transforms into a total derivative as $P^i_\mu\,(\delta_\epsilon\Pi^\mu_i) = (1-n)d(\bar\epsilon\,\Gamma\theta)$. Consequently, the action (8) is invariant under the global supersymmetry transformations (5). It also has the local bosonic symmetry

$$\delta_\Lambda e_{ij} = d\Lambda_{ij}\ , \qquad \delta_\Lambda X^\mu_i = \Lambda_{ij}P^{j\mu}\ , \qquad \delta_\Lambda P^\mu_i = 0\ , \qquad \delta_\Lambda\theta = 0\ , \tag{10}$$

where the transformation parameters have the time dependence $\Lambda_{ij}(\tau)$. These transformations are equivalent to those given in [6] by allowing gauge transformations that depend on the equations of motion, as will be shown in Appendix A.

The action (8) is also invariant under the following κ-symmetry transformations

$$\begin{aligned}
\delta_\kappa \theta &= \gamma^\mu \kappa_i P^i_\mu \ , \\
\delta_\kappa X^\mu_i &= -\bar\theta V^\mu_i (\delta_\kappa \theta) \ , \\
\delta_\kappa P^\mu_i &= 0 \ , \\
\delta_\kappa e_{ij} &= -\tfrac{4}{(n+9)} \ \bar\kappa_{(i} \ V^\mu_{j)} \gamma_\mu \ d\theta \ .
\end{aligned} \tag{11}$$

In showing the κ-symmetry, the following lemmas are useful:

$$P^i_\mu \, (\delta_\kappa \Pi^\mu_i) = \tfrac{(1-n)}{n} \ d\left(\bar\theta_k \ \Gamma \delta_\kappa \theta_k\right) + \tfrac{2}{n} (\delta_\kappa \bar\theta_k) \ \Gamma \ d\theta_k \ , \tag{12}$$

$$\Gamma \gamma^\mu P^i_\mu = \tfrac{1}{(n+9)} \ M^{ij} V^\mu_j \gamma_\mu \ , \tag{13}$$

where

$$M^{ij} \equiv P^i_\mu P^{j\mu} \ . \tag{14}$$

A special combination of the transformations (11) was found in [6], where it was also observed that there is a total of n first class constraints in the model. The κ-symmetry transfomations given above realize the symmetries generated by these constraints.

The commutator of two κ-transformations closes on-shell onto the Λ-transformations

$$[\delta_{\kappa_{(1)}}, \delta_{\kappa_{(2)}}] = \delta_{\Lambda_{(12)}} \ , \tag{15}$$

with the composite parameter given by

$$\Lambda^{(12)}_{ij} = -4 \bar\kappa^{(2)}_{(i} \ \Gamma \ \kappa^{(1)}_{j)} \ . \tag{16}$$

It is clear that the remaining part of the algebra is $[\delta_\kappa, \delta_\Lambda] = 0$ and $[\delta_{\Lambda_1}, \delta_{\Lambda_2}] = 0$.

Finally, we note that the field equations following from the action (8) take the form

$$P^i_\mu P^{j\mu} = 0 \ , \qquad dP^i_\mu = 0 \ , \qquad \Gamma d\theta = 0 \ , \qquad dX^\mu_i + \bar\theta V^\mu_i d\theta - e_{ij} P^{j\mu} = 0 \ . \tag{17}$$

The precise relation between the bosonic symmetry transformations (10), the κ-symmetry transformations (11), and those presented in [6] is discussed in Appendix A.

In this section we have used a notation suitable to simple (i.e. $N = 1$) supersymmetry in the target superspace. One can easily account the extended supersymmetry case by introducing an extra index $A = 1, ..., m$ for the fermionic variables, thereby letting $\epsilon \to \epsilon^A$, $\theta \to \theta^A$ and $\kappa^i \to \kappa^{Ai}$, etc. All the formulae of this section still hold, since no need arises for any Fierz rearrangents that might potentially put restrictions on the dimensionality of the target superspace.

3 A New Model and the BPS Condition

3.1 A Universal Model

The model described above picks out the n th rank antisymmetric bosonic generator in the algebra (1). Furthermore, a polynomial in momenta occurs in the definition of the 'torsion' tensor, making it difficult to interpret the model in curved space. This led us to consider a different realization of the algebra (1). We take the most democratic point of view and define a generalized superspace in which coordinates are associated with Q_α and $Z_{\alpha\beta}$:

$$(Q_\alpha \ , \ Z_{\alpha\beta}) \quad \rightarrow \quad (\theta^\alpha \ , \ X^{\alpha\beta}) \ .$$

In addition, we introduce the momenta $P_{\alpha\beta}$ (symmetric) and the Lagrange multipliers $e_{\alpha\beta}$ with the same symmetry property of the charge conjugation matrix (see Appendix B):

$$e_{\alpha\beta} = \epsilon_0 \, e_{\beta\alpha} \ , \qquad C^T = \epsilon_0 \, C \ . \tag{18}$$

The enlargement of the superspace at this scale, where the bosonic coordinates do not necessarily include the familiar Lorentz vector X^μ need not alarm us, because the model may allow reductions to familiar settings in lower dimensions, and at the same time give rise to new physical situations.

Having defined the basic fields of the model, we propose the following action

$$I = \int d\tau \left(P_{\alpha\beta} \, \Pi^{\alpha\beta} + \tfrac{1}{2} e_{\alpha\beta} \, (P^2)^{\alpha\beta} \right) \ , \tag{19}$$

where

$$\Pi^{\alpha\beta} = dX^{\alpha\beta} - \theta^{(\alpha} d\theta^{\beta)} \ , \tag{20}$$

and $(P^2)^{\alpha\beta} \equiv P^{\alpha\gamma} P_\gamma{}^\beta$. The raising and lowering of spinor indices is with charge conjugation matrix C which has symmetry property (18).

The bosonic symmetry of the action takes the form

$$\delta_\Lambda e_{\alpha\beta} = d\Lambda_{\alpha\beta} \ , \qquad \delta_\Lambda X^{\alpha\beta} = -\Lambda^{(\alpha}{}_\gamma \, P^{\beta)\gamma} \ , \qquad \delta_\Lambda P_{\alpha\beta} = 0 \ , \qquad \delta_\Lambda \theta = 0 \ . \tag{21}$$

In addition, there is a trivial bosonic Σ-symmetry given by

$$\delta_\Sigma e_{\alpha\beta} = \tfrac{1}{2} \left(\Sigma_{\gamma\alpha} \ P_\beta{}^\gamma + \epsilon_0 \, \Sigma_{\gamma\beta} \ P_\alpha{}^\gamma \right) \ , \qquad \delta_\Sigma P_{\alpha\beta} = \delta_\Sigma X^{\alpha\beta} = \delta_\Sigma \theta^\alpha = 0 \ , \tag{22}$$

where the parameter is antisymmetric: $\Sigma_{\alpha\beta} = -\Sigma_{\beta\alpha}$ and ϵ_0 is defined in (18). The action (19) has also fermionic symmetries. Firstly, it is invariant under the global supersymmetry transformations

$$\delta_\epsilon \theta^\alpha = \epsilon^\alpha \ , \qquad \delta_\epsilon X^{\alpha\beta} = \epsilon^{(\alpha} \theta^{\beta)} \ , \qquad \delta_\epsilon P_{\alpha\beta} = 0 \ , \qquad \delta_\epsilon e_{\alpha\beta} = 0 \ , \tag{23}$$

which clearly realize the algebra (1). The action (19) also has local κ-symmetry given by

$$\begin{aligned} \delta_\kappa \theta^\alpha &= P^{\alpha\beta}\,\kappa_\beta\ , \\ \delta_\kappa X^{\alpha\beta} &= \theta^{(\alpha}\,\delta_\kappa\theta^{\beta)}\ , \\ \delta_\kappa e_{\alpha\beta} &= 2(\kappa_\beta\, d\theta_\alpha + \epsilon_0\,\kappa_\alpha\, d\theta_\beta)\ , \\ \delta_\kappa P_{\alpha\beta} &= 0\ . \end{aligned} \tag{24}$$

Recall that $C^T = \epsilon_0\, C$. These transformations close on-shell on the bosonic Λ-transformations and Σ-transformations:

$$[\delta_{\kappa_{(1)}}, \delta_{\kappa_{(2)}}] = \delta_\Lambda + \delta_\Sigma\ , \tag{25}$$

where the composite gauge transformation parameters are

$$\begin{aligned} \Lambda_{\alpha\beta} &= \left(\kappa^{(2)}_\beta\ P_\alpha{}^\gamma\ \kappa^{(1)}_\gamma + \epsilon_0\,\kappa^{(2)}_\alpha\ P_\beta{}^\gamma\ \kappa^{(1)}_\gamma\right) - (1 \leftrightarrow 2)\ , \\ \Sigma_{\alpha\beta} &= 4\epsilon_0\,\kappa^{(1)}_{[\alpha}\ d\kappa^{(2)}_{\beta]} - (1 \leftrightarrow 2)\ . \end{aligned} \tag{26}$$

We need the $e_{\alpha\beta}$ equation of motion in showing the closure on $X^{\alpha\beta}$, and vice versa.

The field equations that follow from the action (19) are

$$P_{\alpha\beta}\, P^{\beta\gamma} = 0\ , \tag{27}$$

$$dP_{\alpha\beta} = 0\ , \qquad P_{\alpha\beta}\ d\theta^\beta = 0\ , \tag{28}$$

$$dX^{\alpha\beta} - \theta^{(\alpha} d\theta^{\beta)} + e^{\gamma(\alpha}\, P^{\beta)}{}_\gamma = 0\ . \tag{29}$$

In particular (27) implies

$$\det\ P = 0\ , \tag{30}$$

which is the familiar BPS condition. We will come back to this point below.

The last equation can be solved for P yielding the result

$$P = -2\epsilon_0 e^{-1}\,(E\ \wedge\ \Pi)\ , \tag{31}$$

where

$$E \equiv \left(\frac{1}{1+e^{-1}}\right)\ , \tag{32}$$

and the definition

$$e^n \wedge \Pi\ \equiv\ e^n\, \Pi\, e^{-n}\ ,$$

is to be applied to every term that results from the expansion of E in e. Substituting (31) into the action (19), we find

$$I = -2\int d\tau\ \mathrm{tr}\ \left[e^{-1}\,(E \wedge \Pi)\right]^2\ . \tag{33}$$

Going back to the first order form of the action (19), it is possible to introduce a constant mass parameter m by shifting everywhere the momenta $P_{\alpha\beta}$ occur (including the κ-symmetry transformations) as

$$Massive\ Model: \qquad P_{\alpha\beta} \ \rightarrow \ P_{\alpha\beta} + m\, C_{\alpha\beta}.$$

The charge conjugation matrix $C_{\alpha\beta}$ has to be symmetric, i.e. $\epsilon_0 = 1$, for this to make sense.

3.2 A Multi-Superparticle Reduction of the Model

There exists an interesting reduction of (19) yielding an action analogous to that of Sec. 2. Consider the ansatze

$$P = \Gamma\ , \qquad X_i^\mu = -\mathrm{tr}\ X V_i^\mu\ . \tag{34}$$

where the spinor indices are suppressed. Recall the definitions (4) and (6) of Γ and V_i^μ, respectively. The definitions (34) lead to the action

$$I = \int d\tau \left(P_\mu^i dX_i^\mu - \bar\theta \Gamma d\theta + \tfrac{1}{2} e\,(\det\ M) \right)\ , \tag{35}$$

where we have defined $e \equiv (-1)^{n(n-1)/2}\, C^{\alpha\beta}\, e_{\alpha\beta}$, dropped a total derivative term $(n-1)d(\mathrm{tr}\, X\Gamma)$ and used the lemma $\Gamma^2 = (-1)^{n(n-1)/2} \det M$. The Λ-symmetry transformations (21) reduce to

$$\delta_\Lambda e = d\Lambda\ , \qquad \delta_\Lambda X_i^\mu = -\Lambda\ Cof\ (M)_{ij}\ P^{j\mu}\ , \qquad \delta_\Lambda P_\mu^i = 0\ , \qquad \delta_\Lambda \theta = 0\ . \tag{36}$$

The Σ-symmetry (22) becomes trivial, while the κ-symmetry transformations (24) take the form

$$\delta_\kappa \theta = \Gamma\,\kappa\ , \qquad \delta_\kappa X_i^\mu = \bar\theta\, V_i^\mu\, (\delta_\kappa \theta)\ , \qquad \delta_\kappa e = -4(-1)^{n(n-1)/2}\, d\bar\theta\,\kappa\ , \qquad \delta_\kappa P_\mu^i = 0\ . \tag{37}$$

The action (35) and its symmetries are similar to those of the model discussed in the previous section, but they are not quite the same. The last term in the new action is different and it gives the on-shell condition

$$\det\ M = 0\ , \tag{38}$$

as opposed to $M = 0$ of the previous section. Recall that $M^{ij} \equiv P_\mu^i P^{j\mu}$.

For two particles, for example, the condition (38) means that either $\vec{P}_1 = \lambda \vec{P}_2$, where λ is an arbitrary constant, or $\vec{P}_i \cdot \vec{P}_j = 0$ $(i, j = 1, 2)$. In the first case M has rank one and $P^{\mu\nu} = 0$, while in the second case M has rank zero and $P^{\mu\nu} \neq 0$. To satisfy (38) in general, either two or more momenta should be parallel, in which case $P^{\mu_1 \cdots \mu_n} = 0$, or they should all be perpendicular to each other in which case $M = 0$ and $P^{\mu_1 \cdots \mu_n} \neq 0$.

3.3 An Example in $(2,2)$ Dimensions

It is useful to consider a simple example to see what the action (19) and the condition (27) mean. Therefore, let us consider a $(2,2)$ dimensional spacetime with pseudo Majorana spinors. Then, we have the expansion

$$P = \gamma^\mu P_\mu + \gamma^{\mu\nu} P_{\mu\nu} \ . \tag{39}$$

Curiously, there are ten coordinates in total. Substituting this into the action (19), and denoting fields that occur in the γ-matrix expansion of the antisymmetric $e_{\alpha\beta}$ by (e, ϕ, ϕ_μ), we obtain

$$I = \int d\tau \left[P_\mu \Pi^\mu + P_{\mu\nu}\Pi^{\mu\nu} + e\left(P^\mu P_\mu - 2P^{\mu\nu}P_{\mu\nu}\right) + \epsilon^{\mu\nu\rho\sigma}\left(\phi P_{\mu\nu} + \phi_\mu P_\nu\right) P_{\rho\sigma}\right] \ , \tag{40}$$

where

$$\Pi^\mu = dX^\mu - \bar\theta\gamma^\mu d\theta \ , \qquad \Pi^{\mu\nu} = dX^{\mu\nu} - \bar\theta\gamma^{\mu\nu} d\theta \ . \tag{41}$$

Note, in particular, the κ-symmetry transformation of θ:

$$\delta_\kappa \theta = \left(\gamma^\mu P_\mu + \gamma^{\mu\nu} P_{\mu\nu}\right)\kappa \ . \tag{42}$$

The constraints on momenta resulting from the (e, ϕ, ϕ_μ) equations of motion, equivalent to the single equation $(P^2)^{\alpha\beta} = 0$, are

$$P^\mu P_\mu - 2P^{\mu\nu}P_{\mu\nu} = 0 \ , \qquad P_{[\mu}\, P_{\rho\sigma]} = 0 \ , \qquad P_{[\mu\nu}\, P_{\rho\sigma]} = 0 \ . \tag{43}$$

There are a number of ways to satisy these conditions. For example, introducing a vector Q_μ, we have the solution

$$P_{\mu\nu} = P_{[\mu}\, Q_{\nu]} \ , \qquad (P\cdot Q)^2 = P^2(Q^2 - 1) \ . \tag{44}$$

Observe that the vectors P and Q do not have to be null or orthogonal, though they can be so as a special case. Another solution is obtained by using two mutually orthogonal null vectors $P^i_\mu\, (i = 1, 2)$ as follows

$$P_\mu = 0 \ , \qquad P_{\mu\nu} = \epsilon_{ij}\ P^i_\mu P^j_\nu \ , \qquad P^i_\mu\, P^{j\mu} = 0 \ , \tag{45}$$

corresponding to the two-particle model of Sec. 2.

4 Conclusions

We have presented a simplified form of the gauge and κ-symmetry transformations of an n superparticle system in $(d-n, n)$ dimensions. One can consider a version of the model in which there are n fermionic variables and all the variables have multi-time dependence. In that case, one can replace θ by $(\theta_1 + \cdots + \theta_n)/n$ and ∂ by $(\partial_1 + \cdots + \partial_n)/n$. If one then takes the variables X_i and θ_i to depend only on τ_i, for example $\theta_1(\tau_1)$, $X^2(\tau_2)$, etc, then one otains the model constructed in [3].

In Sec. 3, we have generalized the model discussed in Sec. 2 to a rather universal one which realizes all the brane charges of the superalgebra and gives rise to an on-shell condition that includes the BPS condition. In this model, one need not set the n th rank brane charge into a product of n-momenta, though this is a partricular solution to the on-shell condition.

The meaning of physical degrees of freedom and the nature of target space wave equations in presence of the p-form coordinates requires a better understanding of the representation theory behind the general superalgebra (1). Since the model is supersymmetric, whatever the degrees of freedom are, they must form a representation of the underlying superalgebra that survives the BPS condition. We refer the reader to [1] for a discussion of how these kind of superalgebras may be used in unifying the perturbative and nonperturbative states of M-theory.

The string and higher brane generalization of the new model should be of great interest. In fact, an attempt was made sometime ago to introduce p-form coordinates in trying to build a new type of string action (in eleven dimensions) [9], but the problem of how to achieve κ-symmetry was never resolved [16]. If one takes the superparticle limit [15] of $D = 11$ supermembrane, then one obtains the equations for a massless superparticle, which differ from the new model presented in Sec. 3.1. The $D = 10$ twistor superparticle model of [10] is more similar to our model. In either case, the open problem is how to construct a superbrane action in the "maximally p-form extended" superspace, in such a way that it will give the modfel presented here in the particle limit.

The new model presented here should also give some clues for the realization of the M-algebra super p-form charges [17, 11, 12]. The role of the new coordinates in the realization of duality symmetries, and as a separate development, in a covariant matrix formulation of M-theory, are some of the other aspects of the new model that merit further study.

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Appendix A

Reduction of the κ-Symmetry and Trivial Gauge Transformations

In order to facilitate comparison of the symmetries presented in Sec. 2, and those of [6], we take a particular form of the κ_i symmetry given by

$$\kappa_i = \tfrac{1}{n+9}\gamma_\mu V_i^\mu \kappa \ , \tag{46}$$

which defines the single κ-symmetry parameter κ. The transformations (11) now take the form

$$\begin{aligned} \delta_\kappa \theta &= n\Gamma\ \kappa\ , \\ \delta_\kappa X_i^\mu &= -\bar\theta V_i^\mu (\delta_\kappa \theta)\ , \\ \delta_\kappa P_i^\mu &= 0\ , \\ \delta_\kappa e_{ij} &= -4\ Cof\ (M)_{ij}\ \bar\kappa\ d\theta\ , \end{aligned} \tag{47}$$

where we have used the lemmas

$$\begin{aligned} &(\gamma^\mu P_\mu^i)\,(\gamma_\nu V_i^\nu) = n(n+9)\ \Gamma\ , \\ &\gamma_\mu V_{(i}^\mu V_{j)}{}^\nu \gamma_\nu = -(-1)^{n(n+1)/2}\ (n+9)^2\ Cof\ (M)_{ij}\ , \\ &Cof\ (M)_{ij} \equiv \tfrac{1}{(n-1)!}\,\epsilon_{ii_2\cdots i_n}\epsilon_{jj_2\cdots j_n}\ M^{i_2j_2}\cdots M^{i_nj_n}\ . \end{aligned} \tag{48}$$

We can use the field equations in the last transformation rule so that $\delta_\kappa e_{ij} = 0$, but we shall not do so in order to compare our results with those of [6]. We will show that the bosonic gauge symmetries (10) and the κ-symmetry transformations (47) are equivalent to those given in [6]. To do this, we shall make use of trivial gauge transformations that are proportional to equations of motion.

In any field theory involving a collection of fields Φ^A, there always exist a trivial gauge symmetry that is proportional to the equations of motion:

$$\delta_\omega \Phi^A = \omega^{AB}\frac{\delta\mathcal{L}}{\delta\Phi_B}\ , \tag{49}$$

where the arbitrary and possibly field dependedent transformations parameters are only required to be graded antisymmetric: $\omega^{AB} = -(-1)^{AB}\,\omega^{BA}$. In our case, we have the set of fields

$$\Phi^A \equiv (e_{ij}, X_i^\mu, P_\mu^i, \theta^\alpha)\ . \tag{50}$$

The field equatios we will need for the purposes of this section are

$$\begin{aligned} &\delta\mathcal{L}/\delta X_i^\mu = -dP_\mu^i \equiv R_\mu^i\ , \\ &\delta\mathcal{L}/\delta P_i^\mu = dX_i^\mu + \bar\theta V_i^\mu d\theta - e_{ij}P^{j\mu} \equiv S_i^\mu\ , \\ &\delta\mathcal{L}/\delta\theta = 2\Gamma\ d\theta + (d\Gamma)\ \theta \equiv \psi\ . \end{aligned} \tag{51}$$

Now, consider a special subset of these transformations, involving the (X, P) equations of motion, given by

$$\delta_\omega X_i^\mu = \omega_i{}^j \ S_i^\mu \ , \qquad \delta_\omega P_\mu^i = -R_\mu^i \ \omega_i{}^j \ . \tag{52}$$

Choosing the transformation parameter $\omega_i{}^j$ as

$$\omega_i{}^j = \Lambda_{ik} e^{kj} \ , \tag{53}$$

where $e^{ij} \equiv (e^{-1})^{ij}$, we find that the sum of the Λ-transormations (10) and the ω-transformation (52) with parameter (53) give

$$\delta e_{ij} = d\Lambda_{ij} \ , \qquad \delta X_i^\mu = \Lambda_{ik} \ e^{kj} \ (dX_j^\mu + \bar{\theta} V_j^\mu \theta) \ , \qquad \delta P_\mu^i = e^{ik} \Lambda_{kj} \ dP_\mu^j \ , \qquad \delta\theta = 0 \ . \tag{54}$$

These are precisely the bosonic gauge transformations of [6]. Note that while the Λ-transformations (10) close off-shell, the combined $(\delta_\Lambda + \delta_\omega)$ transformations (54) close on-shell.

Next we turn to the κ-transformation rules (11). It will be sufficient to consider the ω-transformations that involve the (X, P, θ) equations of motion

$$\delta_\omega \theta = \omega^{i\mu} \ S_{i\mu} \ , \qquad \delta_\omega X_{i\mu} = \omega_{i\mu}{}^{j\nu} \ S_{j\nu} \ , \qquad \delta_\omega P^{i\mu} = \bar{\psi} \ \omega^{i\mu} - R^{j\nu} \ \omega_{j\nu}{}^{i\mu} \ , \tag{55}$$

where we have suppressed the spinor indices on $\omega^{i\mu}$. Calculating the sum of the κ-transformations (47) and the ω-transformations (55) with parameters

$$\omega^{i\mu} = e^{ij} V_j^\mu \ \kappa \ , \qquad \omega_{i\mu}{}^{j\nu} = -\bar{\theta} \ V_{i\mu} e^{jk} V_k^\nu \ \kappa \ , \tag{56}$$

we find that the combined $\delta_\kappa + \delta_\omega$ transformations yield

$$\begin{aligned}
\delta\theta &= e^{ij} (dX_i^\mu + \bar{\theta} V_i^\mu \theta) V_{j\mu} \kappa \ , \\
\delta X_i^\mu &= -\bar{\theta} V_i^\mu (\delta_\kappa \theta) \ , \\
\delta P_i^\mu &= 2 \ d\bar{\theta} \ \Gamma e^{ij} V_{j\mu} \kappa \ , \\
\delta_\kappa e_{ij} &= -4 \ Cof \ (M)_{ij} \ \bar{\kappa} \ d\theta \ .
\end{aligned} \tag{57}$$

These are precisely the κ-transformations of [6], apart from a sign factor in certain dimensions in the last equation (see Appendix B for the relevant symmetry properties of the γ-matrices).

Appendix B

Properties of Spinors and γ-Matrices in Arbitrary Dimensions

Here we collect the properties of spinors and Dirac γ-matrices in (s, n) dimensions where $s(n)$ are the number of space(time) coordinates. The Clifford algebra is $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$, where $\eta_{\mu\nu}$ has the signature in which the time-like directions are negative and the spacelike directions positive. The possible reality conditions on spinors are listed in Table 1, where M, PM, SM, PSM stand for Majorana, pseudo Majorana, symplectic majorana and pseudo symplectic Majorana, respectively [18, 19] [4] (see below). An additional chirality condition can be imposed for $s - n = 0$ *mod* 4.

The symmetry properties of the charge conjugation matrix C and $(\gamma^\mu C)_{\alpha\beta}$ are listed in Table 2. The sign factors ϵ_0 and ϵ_1 arise in the relations

$$C^T = \epsilon_0 C \ , \qquad (\gamma^\mu C)^T = \epsilon_1 (\gamma^\mu C) \ . \tag{58}$$

This information is sufficient to deduce the symmetry of $(\gamma^{\mu_1 \cdots \mu_p} C)_{\alpha\beta}$ for any p, since the symmetry property alternates for p mod 2. In any dimension with n times, one finds

$$(\gamma^{\mu_1 \cdots \mu_p} C)^T = \epsilon_p \, (\gamma^{\mu_1 \cdots \mu_p} C) \ , \qquad \epsilon_p \equiv \epsilon \, \eta^{p+n} \, (-1)^{(p-n)(p-n-1)/2} \ , \tag{59}$$

where η is a sign factor. Note that $\epsilon_n = \epsilon$ and $\epsilon_{n-1} = -\epsilon\eta$. All possible values of (n, s, p) in which $\epsilon_p = +1$, i.e. the values of p for which $\gamma^{\mu_1 \cdots \mu_p} C$ is symmetric (the antisymmetric ones occur for *mod* 4 complements p) are listed in Table 3. Other useful formulae are:

$$\gamma_\mu^T = (-1)^n \, \eta \; C^{-1} \gamma_\mu C \ , \qquad \gamma_\mu^* = \eta B \gamma_\mu B^{-1} \ , \qquad \gamma_\mu^\dagger = (-1)^n A C_\mu A^{-1} \ . \tag{60}$$

We can choose $A = \gamma_0 \gamma_1 \cdots \gamma_{n-1}$. Note that $A = BC$. The chirality matrix in even dimensions d can be defined as

$$\gamma_{d+1} \equiv \pm(-1)^{(s-n)(s-n-1)/4} \, \gamma_0 \gamma_1 \cdots \gamma_{d-1} \ , \qquad (\gamma_{d+1})^2 = 1 \ . \tag{61}$$

We can choose the sign in the first equation such that the overall factor in front is $+1$ or $+i$. Using the matrix B, we can express the reality conditions on a (pseudo) Majorana spinor $\psi^* = B\psi$ and a (pseudo)symplectic Majorona spinor as $(\psi_i)^* = \Omega^{ij} B \psi_j$, where Ω^{ij} is a constant antisymmetric matrix satisfying $\Omega_{ij}\Omega^{jk} = -\delta_i^k$ and the index i labels a pseudo-real representation of a given Lie algebra which admits such a representation (e.g. the fundamental representations of $Sp(n)$ and E_7).

[4] Corrections to formulae in p. 5 & 6 of [19]: (a) Change eq. (2) to: $(\Gamma^{d+1})^2 = (-1)^{(s-t)(s-t-1)/2}$ (our n here is denoted by t in [19]), (b) eq. (3) should read $\Gamma_\mu^\dagger = (-1)^t A \Gamma_\mu A^{-1}$, (c) change the last formula in eq. (5) to $B = CA$, (d) multiply eq. (6) with ϵ on the right hand side, (e) the prefactor in eq. (8) should be $2^{-[d/2]}$, (f) interchange the indices ν_1 and ν_2 in the last term of eq. (9). Note that here we have let $C \to C^{-1}$ relative to [19].

ϵ	η	$(s-n)$ *mod* 8	*Spinor Type*
$+$	$+$	0,1,2	M
$+$	$-$	6,7,8	PM
$-$	$+$	4,5,6	SM
$-$	$-$	2,3,4	PSM

Table 1: *Spinor types in* (s,n) *Dimensions*

n *mod* 4	ϵ_0	ϵ_1
0	$+\epsilon$	$+\epsilon\eta$
1	$-\epsilon\eta$	$+\epsilon$
2	$-\epsilon$	$-\epsilon\eta$
3	$+\epsilon\eta$	$-\epsilon$

Table 2: *Symmetries of* C *and* $(\gamma^\mu C)$ *in* (s,n) *dimensions.*

n *mod* 4	s *mod* 8	p *mod* 4	n *mod* 4	s *mod* 8	p *mod* 4
1	**1**, 2, 3	**1**, 2	**3**	**3**, 4, 5	**3**, 4
	1, 7, 8	**1**, 4		**3**, 1, 2	**3**, 2
	5, 6, 7	**3**, 4		**7**, 8, 1	**1**, 2
	5, 3, 4	**3**, 2		**7**, 5, 6	**1**, 4
2	**2**, 3, 4	**2**, 3	**4**	**4**, 5, 6	**4**, 1
	2, 8, 1	**2**, 1		**4**, 2, 3	**4**, 3
	6, 7, 8	**4**, 1		**8**, 1, 2	**2**, 3
	6, 4, 5	**4**, 3		**8**, 6, 7	**2**, 1

Table 3: *Symmetric* $(\gamma^{\mu_1\cdots\mu_p}C)$ *in* (s,n) *dimensions.*
In bold cases, an extra Weyl condition is possible.

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