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On Non-Canonical Kinetic Terms and the Tilt of the Power Spectrum

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We argue that in models of inflation with two scalar fields and non-canonical kinetic terms there is a possibility of obtaining a red tilt of the power spectrum of curvature perturbations from noncanonicality-induced interactions between the curvature and isocurvature perturbations. We describe an extremely simple model realizing this idea, study numerically its predictions for the perturbations and discuss applications in realistic scenarios of inflation. We discuss to what extent in this model the scale of the inflationary potential can be decoupled from the amplitude of the density fluctuations.

By now inflation has become a paradigm of cosmological evolution, solving the horizon problem of the hot big bang scenario and explaining the flatness of the universe. In addition, it provides a mechanism for the origin of the 10^{-5} density contrast observed in the cosmic microwave background and the growth of large structure (see, e.g., [1]). However, it is still a subject of lively discussion how inflation fits into fundamental models of particle physics, such as supergravity or, ultimately, string theory (see, e.g., [2] for classical references and [3] for a summary of recent developments). In simple single-field models of slow-roll inflation, the measured normalization of the power spectrum of the curvature perturbations $\mathcal{P}_\mathcal{R}(k)$ and its spectral index $n_s = 1 + \frac{\mathrm{d}\ln\mathcal{P}_\mathcal{R}}{\mathrm{d}\ln k}$ are given by:

$$\mathcal{P}_\mathcal{R} \simeq \frac{1}{24\pi^2 M_P^4}\frac{V_*}{\epsilon_*}\,, \qquad n_s \simeq 1 - 6\epsilon_* + 2\eta_*\,. \quad (1)$$

The slow-roll parameters are defined in terms of the Hubble expansion rate H and the scalar field potential V as $\epsilon \equiv -\frac{\dot H}{H^2} \simeq \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2$, $\eta \equiv \frac{V''}{3H^2}$. The primes are derivatives with respect to the field χ that drives inflation, and the subscript $*$ denotes the value at Hubble radius crossing. With the observed deviation of n_s from unity by few per cent [4] and with a typical relation $\epsilon \lesssim \eta$, one gets $V^{1/4} \lesssim 10^{16}\,\mathrm{GeV}$. In other words, observations require a hierarchy, $m \ll H \ll M_P$, between the mass m of the inflaton, the Hubble parameter and the Planck scale. In the slow-roll approximation, the power spectrum can be rewritten as $\mathcal{P}_\mathcal{R} \simeq \frac{1}{36\pi^2 M_P^4}\frac{V_*^2}{\dot\chi_*^2}$. Since it is natural to expect that with time the velocity of the field χ increases at the expense of the potential energy V, the later a mode with a comoving wave number k leaves the Hubble radius, the smaller power spectrum it corresponds to. Hence, one expects a red-tilted power spectrum, $n_s < 1$ [1]. Similar conclusions hold for inflation accompanied by the evolution of another scalar field, such as a modulus, driving the potential V towards smaller values.

In this letter we propose a new mechanism for generating a red tilt of the curvature power spectrum and we show that in specific realizations it can be the dominant one. To this end, we go beyond minimal models of inflation and consider scenarios with more than one scalar field active during inflation (see *e.g.* [5]), focusing in particular on the role of non-canonical kinetic terms[1]. This type of setting is well-motivated by known examples of string compactifications, which typically contain a large number of light scalar fields X^I, or moduli, whose dynamics is governed by a non-trivial moduli space metric G_{IJ}. As long as the moduli-space metric is not flat, one is generically led to non-canonical kinetic terms. Although these effects may (but do not have to) be suppressed by the high scale of the corresponding UV physics (e.g. moduli masses, string scale), they can still affect the inflationary dynamics because of the above mentioned hierarchy of the mass parameters of inflationary models.

In this paper we will consider inflationary models described by an effective Lagrangian of the form [7]:

$$\mathcal{L} = \frac{R}{16\pi G} + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi + \frac{e^{2\,b(\phi)}}{2}\,\partial_\mu\chi\,\partial^\mu\chi - V(\phi,\chi)\,. \quad (2)$$

In many string motivated examples $M_P\,\frac{\mathrm{d}b}{\mathrm{d}\phi}$ can be rather large, *i.e.* of order 1. The non-canonical kinetic terms can enhance the coupling between the curvature perturbations (measured in the CMB) and the isocurvature perturbations, affecting the predictions of such models. In this letter we show that such an *enhanced coupling can result in a negative contribution to the spectral index* of the curvature perturbations. In a very simple model, we show that this can be the dominant effect responsible for the deviation of the spectral index from unity. We defer a more complete and general treatment of the evolution of the curvature and isocurvature perturbations to [9], focusing here on a particularly simple case where the coupling is moderate. Motivated by the plethora of scalar

[1] For canonical kinetic terms, an enhanced red tilt can be arranged, *e.g.* as in [6]

fields with non-canonical kinetic terms in effective field theories originating from string theory, we construct an analytically tractable toy model of inflation, in which the noncanonicality leads to interactions between curvature and isocurvature perturbations.

We begin with a brief review of the dynamics of cosmological perturbations during inflation. The background equations of motion for (2) are:

$$0 = \ddot{\phi} + 3H\dot{\phi} + V_\phi - b_\phi e^{2b(\phi)}\dot{\chi}^2 \,, \quad (3)$$
$$0 = \ddot{\chi} + 3H\dot{\chi} + 2b_\phi\dot{\phi}\,\dot{\chi} + e^{-2b(\phi)}V_\chi \quad (4)$$

where $b_\phi \equiv \frac{\mathrm{d}b}{\mathrm{d}\phi}$. As usual, the Hubble parameter is given by $H = (T+V)/(3M_P^2)$, where $T = (\dot{\phi}^2 + e^{2b}\dot{\chi}^2)/2$. Following [10], we utilize the so-called adiabatic-entropy decomposition, i.e. we introduce 'polar coordinates' in the space of field velocities, i.e. $\dot{\sigma} = (\dot{\phi}^2 + e^{2b}\dot{\chi}^2)^{1/2}$, $\cos\theta = \dot{\phi}/\dot{\sigma}$ and $\sin\theta = e^b\dot{\chi}/\dot{\sigma}$. The slow-roll parameters are then given by $\epsilon \equiv -\dot{H}/H^2$ and $\eta_{ij} \equiv \frac{V_{ij}}{3H^2}$, where $V_{i_1,\ldots,i_n}$ denotes derivatives of the potential along directions parallel (σ) or orthogonal (s) to the inflationary trajectory in the field space. The instantaneous *adiabatic* (curvature) and *entropy* (isocurvature) components read: $Q_\sigma = \cos\theta Q_\phi + \sin\theta e^b Q_\chi$ and $\delta s = -\sin\theta Q_\phi + \cos\theta e^b Q_\chi$, where Q_ϕ and Q_χ are the Mukhanov-Sasaki variables related to the linear perturbations of the fields ϕ and χ. In the so-called comoving gauge, the perturbation Q_σ is directly related to the three-dimensional curvature $\mathcal{R}$ of the constant time slices by $\mathcal{R} = H(\dot{\phi}^2 + e^{2b}\dot{\chi}^2)^{-1/2}Q_\sigma$.

The equations of motion for Q_σ and δs have been given in the closed form in [11]. On scales larger than the Hubble radius and in the slow-roll case, $|\dot{H}| \ll H^2$, these rather complicated formulae simplify to [8]:

$$\frac{1}{H}\dot{Q}_\sigma = A\,Q_\sigma + B\,\delta s \,, \qquad \frac{1}{H}\dot{\delta s} = D\,\delta s \,, \quad (5)$$

where

$$A = -\eta_{\sigma\sigma} + 2\epsilon - \xi\cos\theta\sin 2\theta \,, \quad (6)$$
$$B = -2\eta_{\sigma s} + 2\xi\sin 3\theta \simeq \frac{2}{H}\dot{\theta} + 2\xi\sin\theta \,, \quad (7)$$
$$D = -\eta_{ss} + \xi\cos\theta(1+\sin 2\theta) \quad (8)$$

with $\xi = \sqrt{2\epsilon}\, b_\phi M_P$. From (5) it is apparent that the quantity B parametrizes the *coupling between the curvature and the isocurvature perturbations*. Thanks to the presence of the noncanonicality, encoded by ξ, this coupling does not vanish on super-Hubble scales even if $\dot{\theta} = 0$.

Now we write down a potential which allows for a large coupling between the curvature and isocurvature perturbations. As follows from (5)-(8), the choice $\cos\theta = 0$ makes the form of these equations identical to the case of canonical kinetic terms – with the exception of introducing a *large coupling* $\sim \sqrt{\epsilon}$ between the curvature and the isocurvature modes. Hence, such choice will make the impact of the noncanonicality on the inflationary spectra particularly clear. Moreover, $\cos\theta = 0$ corresponds to $\phi = \text{const}$, which means that the normalization of the inflaton field χ, given by the prefactor $e^{2b(\phi)}$ in the kinetic term, remains unchanged during inflation. Yet a nontrivial curvature of the field space metric enables the isocurvature perturbations to affect the curvature perturbations even in the absence of a direct interaction term in the potential, unlike in models in which the field space metric is flat.

From (3) the requirement $\cos\theta = 0$ corresponds to $V_\phi = b_\phi e^{2b}\dot{\chi}^2$. Although a trajectory $\cos\theta = 0$ is a slow-roll ($\epsilon \ll 1$) solution to the full background equations of motion (3)-(4), it is *not* an approximate solution to the simplified form of these equations with neglected double time derivatives. What we have results from balancing the last two terms in (3), which are formally of different order in the slow-roll parameters. Nevertheless, as we shall show in a numerical example, a moderate hierarchy between the parameters of the potential is sufficient for this purpose. Since the usual slow roll solution $\dot{\chi} \approx M_P e^{-2b}V_\chi/(\sqrt{3V})$ is applicable here, we find the following condition for a trajectory with $\cos\theta = 0$:

$$3VV_\phi = M_P^2 b_\phi e^{-2b} V_\chi^2 \,. \quad (9)$$

Note that the relation (9) should be satisfied for a range of values of χ relevant for inflation, but only for a single value of ϕ. Therefore, in order to look for appropriate evolution of the background fields we only need to expand the potential V to the linear order in ϕ. If the potential can be written as $V = U(\phi)\,\tilde{U}(\chi)$, we obtain from (9) that $\tilde{U}'/\tilde{U}$ is a constant, which is solved by any function $\tilde{U}$ proportional to $e^{\beta\,\chi/M_P}$ with an appropriate value of β. In practice, it is enough to satisfy (9) approximately, e.g. by a function $\tilde{U}(\chi)$ admitting an expansion $\tilde{U}_0(1+\beta(\chi-\chi_0)/M_P)$ over a sufficiently large range of the arguments.

Following these observations, we shall from now on assume that the scalar potential is very flat in *both* directions ϕ and χ, so that it suffices to expand V as

$$V(\phi,\chi) = V_0\left(1 + \alpha\cdot\frac{\phi-\phi_0}{M_P} + \beta\cdot\frac{\chi-\chi_0}{M_P}\right) \,, \quad (10)$$

and that the relations (9) and $\dot{\phi} = 0$ are satisfied as an initial condition. As long as the potential can be reliably expanded as (10), one can estimate how much the field χ can shift during N e-folds of inflation without spoiling the condition $\dot{\phi} = 0$. It follows from (3) that this requires the field velocity $\dot{\chi}$ to be approximately constant, which is guaranteed by the form of (10).

Rewriting the equations (5) in terms of $\mathcal{R} = (H/\dot{\sigma})Q_\sigma$ and $\mathcal{S} = (H/\dot{\sigma})\delta s$ and noting that in our simple example the parameters $\tilde{D} = D - A$ and B are constant, one finds [8] that $\mathcal{R}(N) = \mathcal{R}_* + (B/\tilde{D})(e^{\tilde{D}N} - 1)\mathcal{S}_*$ and $\mathcal{S}(N) = e^{\tilde{D}N}\mathcal{S}_*$, where $N = \ln(a/a_*)$ is the number of efolds after the Hubble radius exit of the mode with the smallest k. Barring initial correlations and assuming that $|\tilde{D}|\, N \ll 1$, the power spectra at the end of inflation are given by:

$$\mathcal{P}_{\mathcal{R}} = \mathcal{P}^0_{\mathcal{R}} \left(1 + B_*^2 N_{\rm e}^2\right) \,,\; \mathcal{P}_{\mathcal{S}} = \mathcal{P}^0_{\mathcal{R}} \left(1 + 2\tilde{D}_* N_{\rm e}\right) \,, \quad (11)$$

where $N_{\rm e}$ denotes the number of e-folds between horizon crossing and the end of inflation. The initial condition $\mathcal{P}^0_{\mathcal{R}} = \frac{V_*}{24\pi^2\epsilon_*}$ for *both* the curvature and isocurvature perturbations corresponds to the value that the power spectrum of each perturbation would have in the absence of the coupling B. Noting that $\mathrm{d}\ln k = \mathrm{d}\ln(a/a_{\rm e}) = -\mathrm{d}N_{\rm e}$, we arrive at the following expression for the running of the spectral index of the curvature perturbations:

$$n_s - 1 = -\frac{2B_*}{\frac{1}{B_* N_{\rm e}} + B_* N_{\rm e}} + \mathcal{O}(\epsilon_*) \qquad (12)$$

(since the potential is linear, we have $\eta_{i,j} = 0$ for $i, j = \sigma, s$ and we consider a very flat potential). The fraction in (12) is maximized for $BN_{\rm e} \sim 1$, in which case it assumes a value $\sim 1/N_{\rm e}$. Note, however, that during $N_{\rm e}$ efolds the field variation is $\Delta\chi \approx \beta N_{\rm end} M_P$, while $B \approx 2\beta b_\phi M_P$, so the maximal effect is obtained for a field variation close to the Planck scale. The possibility of obtaining such a large field variation in string models is a subject of intensive research (see *e.g.* [12]).

We validate the discussion of our simple model by a numerical analysis of a specific example of two-field inflation with noncanonical kinetic terms, realizing the ideas presented above. We use a potential of the form (10) and assume that the noncanonicality has a form $b(\phi) = -\gamma\phi/M_P$, with γ a constant, typical of many supergravity constructions. According to (9), there must be a relation between the parameters of the model (α, β and γ) and the initial condition ϕ_0 for the field ϕ, to ensure a straight trajectory in the field space. We chose $\beta = 1/30$, $\gamma = 1$, $\phi_0 = \chi_0 = 0$, $\dot{\phi} = 0$, $\dot{\chi} = -e^{-2b} V_\chi/(3H)$ and we adjusted $\alpha = -3.7 \cdot 10^{-4}$ so that the condition (9) was satisfied. The equations of motion for the background values of the fields ϕ and χ, as well as for the gauge invariant curvature and isocurvature perturbations, can be solved using the techniques described in [11]. The initial conditions are specified at 8 efolds before the largest modes relevant for the CMB temperature fluctuations leave the Hubble radius. Since we expand the potential up to linear order in the fields, we cannot trace the model-dependent exit from inflation, but we presume that rolling of a standard waterfall field triggered by the inflaton field χ ends the slow-roll period. Therefore, we trace the evolution up to $N_{\rm e} = 51$ efolds after the observable modes with the smallest k leave the Hubble radius. The parameters A, B and D are practically constant during inflation – their values at Hubble radius exit are $A_* = 1.1 \cdot 10^{-3}$, $B_* = 0.067$ and $D_* = 6 \cdot 10^{-6}$. We find that $n_s - 1 = -0.038$ and that (12) gives precisely

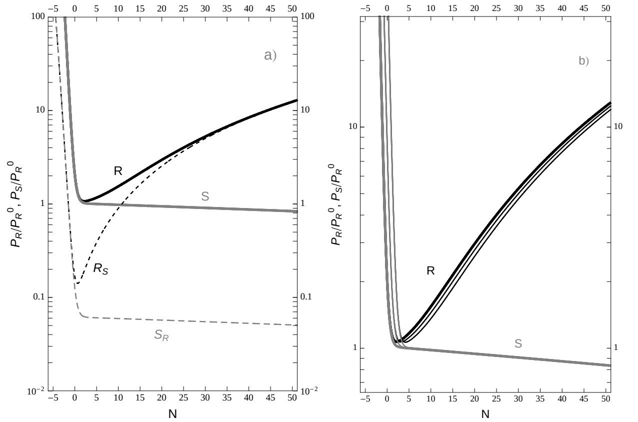

FIG. 1: *(a) Evolution of the instantaneous curvature and isocurvature perturbations shown in terms of the power spectra. Initial conditions are imposed 8 efolds before the Hubble radius exit at $N = 0$. Solid lines $\mathcal{R}$ and $\mathcal{S}$ show the total curvature and isocurvature perturbations. Dashed lines $\mathcal{R}_\mathcal{S}$ ($\mathcal{S}_\mathcal{R}$) correspond to the components of the curvature (isocurvature) perturbations generated from initial pure isocurvature (curvature) perturbations. (b) Comparison of the curvature ($\mathcal{R}$) and isocurvature ($\mathcal{S}$) modes with different wave numbers, leaving the Hubble radius at $N = 0, 1, 2$.*

this value (we arranged the parameters so that other contributions are negligible). The evolution of the curvature and isocurvature perturbations for a few selected modes is shown in Figure 1, where we represent the perturbations by their 'instantaneous power spectra' defined by $\mathcal{P}_\mathcal{R}(k)\, \delta(\mathbf{k} - \mathbf{k}') \equiv \frac{k^3}{2\pi^2} \langle \mathcal{R}^*_{\mathbf{k}'} \mathcal{R}_{\mathbf{k}} \rangle$ and analogously for $\mathcal{S}$, where the linear perturbations are treated as Gaussian random variables.

The particular toy model we chose to analyze is, of course, very simplified. The potential is linear in the fields and very flat, and the initial conditions for the field evolution are chosen so that the classical trajectory in the field space is a straight line. Although string-theory-based models of inflation with a linear potential have been put forth [12], such a negative contribution to the spectral index can also arise in more generic settings. For example, [11] studies the double quadratic potential $V(\phi, \chi) = \frac{m^2}{2}(\phi^2 + \chi^2)$ with $b(\phi) = -\phi/M_P$ and finds trajectories which are almost straight lines in the field space. Although the variations of the potential and the inflaton velocity during inflation are sufficient to drive n_s below 1, in that example the interactions between the curvature and isocurvature perturbations are still responsible for roughly half of the deviation from unity. An important feature of that example is that the perturbations orthogonal to the trajectory in the field space are light compared to the Hubble parameter, which allows one to achieve the balance condition (9) approximately.

The potential (10) employed here bears some resemblance to models of Brans-Dicke inflation with $V = e^{-\alpha\phi} U(\chi)$ [13], which also predict $n_s < 1$. In these models the evolution of the modulus ϕ suppresses the scale of the potential as inflation proceeds, hence the amplitude of the curvature perturbations leaving the Hubble radius decreases with time. In our example, the scale of the potential remains practically constant throughout inflation and the red tilt arises solely from the interactions between the perturbations.

The result shown in (12) may be viewed as particular limit of a general expression for the spectral index, obtained in [8] under assumptions of constant slow-roll parameters and uncorrelated curvature and isocurvature perturbations at the Hubble radius crossing. However, our work goes beyond this analysis in several ways. Firstly, we prove the existence of a trajectory for which these assumptions are satisfied and which also allows for a relatively large couplings between the curvature and the isocurvature perturbations. This is only possible in the presence of noncanonicality: introducing such a coupling from interactions in the potential, described by $\eta_{\sigma s}$, would produce a fast turn in the trajectory and an instantaneous sourcing of the curvature perturbations by the isocurvature ones and the $N_{\rm e}$ dependence would be lost. Secondly, the balance condition (9) for the inflationary trajectory ensures that the spectrum of the curvature perturbations is red-tilded, contrarily to what the general expression in [8] may suggest.

A recent analysis [14] studied a setup similar to ours. The crucial difference is that the authors of [14] assumed *both* scalar fields slowly rolling, *i.e.* neglected the term proportional to $\dot{\chi}^2$ in (3) as a higher order correction. This leads to different inflationary trajectories and thus to different predictions. Our class of trajectories, given by (9), has been previously studied in the literature only in the 'gelaton' scenario [15]. In [15] it was, however, assumed that the parameter B is very large and that the field ϕ is much heavier than the Hubble scale. The expansion in the coupling between the perturbations used here is insufficient in analyzing such models and we defer a detailed comparison between differences in predictions of our example and those of the 'gelaton' scenario to future work [9].

Finally, we note that in models in which the curvature perturbations are generated from isocurvature perturbations *during* inflation, the power spectrum of the curvature perturbations is larger than the single field result (1). In our simple example, this enhancement is by a factor $B^2N_{\rm e}^2$, which may serve for decoupling the scale of the inflationary potential from the amplitude of the density perturbations by means other than lowering the slow-roll parameter ϵ.

In conclusion, in this note we have studied the inflationary dynamics of a model with two scalar fields, one with a non-canonical kinetic term, and a very flat potential. We found, analytically and numerically, that the interactions between the curvature and isocurvature perturbations can give a negative contributions to n_s of the order of a few percent, accounting for practically *all* the redness of the spectrum. This offers a new principle for constructing phenomenologically viable models of inflations employing very flat potentials.

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