

Bound States of Black Holes and Other P -branesM. J. Duff[†] and J. Rahmfeld*Center for Theoretical Physics
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In the process of identifying heterotic and Type *II* BPS string states with extremal dilaton black holes, it has been suggested that solutions with scalar/Maxwell parameters $a = \sqrt{3}$, 1 , $1/\sqrt{3}$ and 0 correspond to $1-$, $2-$, $3-$ and $4-$ particle bound states at threshold. (For example, the Reissner-Nordstrom black hole is just a superposition of four Kaluza-Klein black holes). Here we show that not only the masses, electric charges and magnetic charges but also the spins and supermultiplet structures of the string states are consistent with this interpretation. Their superspin L corresponds to the Kerr-type angular momentum and hence only the $L = 0$ elementary BPS states are black holes. Moreover, these results generalize to super p -branes in D -dimensions. By constructing multi-centered p -brane solitons, the new super p -branes found recently with various values of $a^2 = \Delta - 2(p+1)(D-p-3)/(D-2)$ are seen to be bound states of the fundamental ones with $\Delta = 4$.

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1 Introduction

In a previous paper [1], we argued that the spectrum of elementary BPS ($N_R = 1/2$) states of compactified heterotic strings could be identified with extremal electrically charged black holes. Further evidence for this interpretation was supplied in [2, 3, 4, 5, 6]. In particular, the $N_L = 1$ states and the $N_L > 1$ states (with vanishing left-moving internal momentum) admit a single scalar/Maxwell interpretation with parameters $a = \sqrt{3}$ or $a = 1$ respectively. In other words, by choosing appropriate combinations of dilaton and moduli fields to be the scalar field ϕ and appropriate combinations of the field strengths to be the Maxwell field F , the field equations can be consistently truncated to a form given by the Lagrangian

$$\mathcal{L} = \frac{1}{32\pi} \sqrt{-g} \left[R - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4} e^{-a\phi} F^2 + \dots \right] \quad (1.1)$$

for these two values of a , these combinations being just those corresponding to the quantum numbers of the string states¹. In the case of zero angular momentum, the ADM mass M_{black} of the extremal black hole solutions of (1.1) is given by

$$M_{black}^2 = Q^2/4(1 + a^2) \quad (1.2)$$

where $Q = \int \tilde{F}/8\pi = \int e^{-a\phi} *F/8\pi$ is the electric charge, where $*$ denotes the Hodge dual and where, for simplicity, we have set the asymptotic value of ϕ to zero. The $a = 1$ case yields the supersymmetric dilaton black hole [7]. The $a = \sqrt{3}$ case corresponds to the Kaluza-Klein black hole and the ‘‘winding’’ black hole [8] which are related to each other by T -duality. The Kaluza-Klein solution has been known for some time [7] but only recently recognized [8] as a heterotic string solution. We also argued that the corresponding solitonic magnetically charged and dyonic spectrum, predicted by S-duality [9], is also described by extremal black holes. Indeed, we were first motivated to make the black hole conjecture for the elementary states by first noting that the ‘‘winding’’ magnetic monopoles are extremal black holes [8] and then noting that string/string duality interchanges the roles of S -duality and T -duality and therefore that the solitonic monopoles play the same role for the dual string as the elementary winding states play for the fundamental string². By allowing F to describe combinations of field strengths and their duals, other dyons, preserving fewer supersymmetries and therefore not predicted by S-duality alone, can also be assigned a values. One finds $a = 1/\sqrt{3}$ and $a = 0$. The $a = 0$ case yields the Reissner-Nordstrom solution³ which, notwithstanding contrary claims in the literature, does solve the low-energy string equations [8, 1]. The $a = 1/\sqrt{3}$ black hole [12] was identified as a dyonic solution in [13]. These four values of a yield solutions

¹A *consistent* truncation is defined to be one for which all solutions of the truncated theory are solutions of the original theory. The dots in (1.1) refer to terms involving a pseudoscalar combination of axion and moduli fields which are in general required for consistency but which do not contribute to non-rotating black hole solutions. We are grateful to R. Myers for pointing out their omission in the original version of this paper.

²Of course this involves extending the classical notion of a black hole from the weak coupling to the strong coupling regime. We will therefore take the liberty of describing a state by the words *black hole* if there exists at least one string picture in which its mass exceeds the Planck mass for weak coupling.

³The Reissner-Nordstrom black hole is not a solution of dimensionally reduced pure gravity but has long been known to be a solution of M -theory [10, 11].

which are special cases of the most general solution subsequently found in [14], and shown to be exact to all orders in α' in [15]. In the $N = 2$ theory, black holes with $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$ all preserve 1 supersymmetry and therefore belong to fundamental supermultiplets with maximum spins $1/2$ [13]. In the $N = 4$ theory, black holes with $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$ preserve 2, 2, 1, 1 supersymmetries, respectively, and therefore belong to fundamental supermultiplets with maximum spins $1, 1, 3/2, 3/2$ [16, 1, 13]. This same black hole interpretation could be extended to the spectrum of BPS states of toroidally compactified Type *II* strings [17] where the supersymmetric black holes admitting a single scalar/Maxwell interpretation correspond once again to the same four values of $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$. They preserve 4, 2, 1, 1 of the $N = 8$ supersymmetries, respectively [18, 19], and therefore belong to fundamental supermultiplets with maximum spins $2, 3, 7/2, 7/2$.

On the basis of their mass and charge assignment, it was further suggested [1, 13] that we interpret these four values of a as 1-, 2-, 3- and 4-particle bound states with zero binding energy. This is reviewed in section (2). For example, the Reissner-Nordstrom ($a = 0$) black hole equals four Kaluza-Klein ($a = \sqrt{3}$) black holes! This zero-binding-energy bound-state conjecture can, in fact, be verified in the classical black hole picture by finding explicit 4-centered black hole solutions which coincide with the $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$ solutions as we bring 1, 2, 3, 4 centers together and take the remaining 3, 2, 1, 0 centers out to infinity [20]. Such a construction is possible because of the appearance of four independent harmonic functions [15]. Moreover, this provides a novel realization of the *no-force* condition in that the charge carried by each black hole corresponds to a different $U(1)$. Thus the gravitational attraction cannot be cancelled by an electromagnetic repulsion but rather by a subtle repulsion due to scalar exchange. This phenomenon was also observed in [21]. In section (3) we shall provide further evidence for the bound state interpretation in the quantum string state picture by showing that not only the masses, electric charges and magnetic charges but also the spins, and supermultiplet structures are consistent with this interpretation of the $a = 0, 1/\sqrt{3}, 1$ string states being merely bound states of the fundamental $a = \sqrt{3}$ states. This is entirely consistent with the claim of [22], using completely different reasoning, that only $a > 1$ dilaton black holes can be interpreted as *elementary* particles. Can this interpretation also apply to the recently discussed *massless* black holes [23, 25, 24, 21, 26]? Classically, the answer is yes in that there exist 2-centered solutions which coincide with the massless black hole as we bring the two centers together. In this case, however, it is necessary to assume that one of the constituents has a negative mass and it therefore seems unlikely that this bound state interpretation can survive quantum-mechanically.

For the purely electric elementary string states, where it makes sense to assign an oscillator number, the massive BPS states in the heterotic theory are given by the ($N_R = 1/2, N_L \geq 1$) states [1] and the massless BPS states belong to the ($N_R = 1/2, N_L = 0$) sector [24]. Curiously, however, it is possible to extend the black hole bound state interpretation to non-BPS states, for example the non-supersymmetric $a = 1$ dilaton black hole of [27] corresponds to ($N_R = 3/2, N_L = 1$).

There is now a consensus that all of string theory and its duality properties follow from an underlying eleven-dimensional theory [28, 29, 30, 31, 32, 33], now known as *M-theory*. In section (4) we turn to the black branes of *M-theory* [34, 35, 36, 37], where (1.1) is now in an arbitrary spacetime dimension $D \leq 11$ and the F is now a $(p + 2)$ -form. The parameter

a can be conveniently re-expressed as [38]

$$a^2 = \Delta - \frac{2(p+1)(D-p-3)}{D-2}, \quad (1.3)$$

since Δ is a parameter that is preserved under dimensional reduction [38]. Originally, attention was focussed on the $\Delta = 4$ solutions [39, 40, 41] but various new supersymmetric solitons with $\Delta \neq 4$ have recently been studied [38, 18, 42]. These authors proposed to classify p -branes into “rusty” or “stainless” according as they can or cannot be “oxidized” to isotropic brane solutions of a higher-dimensional supergravity. (Oxidation is the opposite of double dimensional reduction [28, 43].) Examples of new stainless solutions included a $\Delta = 2$ 5-brane in $D = 9$ and $\Delta = 2, 4/3$ strings in $D = 5$ ⁴. The authors of [38] then raised the question of whether these new $\Delta \neq 4$ branes deserve to be treated as fundamental in their own right. In this section we shall generalize the treatment of extremal black holes [20] and confirm on the level of classical solutions that these new $\Delta = 4/n$ p -branes can also be regarded as bound states with zero binding energy of n fundamental $\Delta = 4$ branes. We find new $1 \leq n \leq m$ -centered p -brane solutions which reproduce the $\Delta = 4/n$ solutions of [42] as we allow n of the centers to coincide and take the remaining $(m - n)$ out to infinity. In particular, the $\Delta = 2$ fivebrane is a bound state of two $\Delta = 4$ fivebranes and the $\Delta = 2$ and $\Delta = 4/3$ strings are respectively bound states of two and three fundamental $\Delta = 4$ strings.

In section (5), we also discuss the temperature and entropy of the extreme $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$ black holes. Here a good deal more needs to be understood since only for the $a = 0$ solution is the dilaton vanishing and so the classical entropy prediction for the other three cases (namely zero) is unreliable. Similar remarks apply to the macroscopic entropy and temperature of the other p -branes.

2 The bound state conjecture

Let us begin by recalling the bound state conjecture in the context of the four-dimensional heterotic string obtained by toroidal compactification. At a generic point in the moduli space of vacuum configurations the unbroken gauge symmetry is $U(1)^{28}$ and the low energy effective field theory is described by $N = 4$ supergravity coupled to 22 abelian vector multiplets. Using the canonical metric, the bosonic part of the Lagrangian is given by [44]

$$\mathcal{L} = \frac{1}{32\pi} \sqrt{-g} \left[R - \frac{1}{2} (\partial\eta)^2 - \frac{1}{12} e^{-2\eta} H^2 + \frac{1}{8} \text{Tr}(\partial\mathcal{M}L\partial\mathcal{M}L) - \frac{1}{4} e^{-\eta} F^T(L\mathcal{M}L)F \right], \quad (2.1)$$

where L is the metric of $O(6, 22)$. $\mathcal{M} = \mathcal{M}^T \in O(6, 22)/O(6) \times O(22)$ parametrizes the scalars in the sigma model, the 28 $F_{\mu\nu}$'s are the $U(1)$ fields strengths, η is the four dimensional dilaton and H the 3-form field strength with the usual Chern-Simons terms. The string theory has a perturbative $O(6, 22; Z)$ T -duality that transforms Kaluza-Klein states into winding states, and a non-perturbative $SL(2, Z)$ S -duality that transforms electric states into magnetic states. This is reflected in the $O(6, 22; R)$ invariance of the Lagrangian (2.1) and the $SL(2, R)$ invariance of its equations of motion.

⁴The $\Delta = 4$ 6-brane in $D = 9$ can, in fact, be oxidized to a Type *IIB* 7-brane in $D = 10$.

Let us work at a special point in the moduli space and set the asymptotic value of \mathcal{M} to I , and the asymptotic dilaton field to zero. We shall return to the general case later. Let us define $\vec{Q}_{R,L}$ and $\vec{P}_{R,L}$ by

$$\begin{aligned}\vec{Q}_{R,L} &= \frac{1}{2}(I \pm L)\vec{Q} \\ \vec{P}_{R,L} &= \frac{1}{2}(I \pm L)\vec{P},\end{aligned}\tag{2.2}$$

where \vec{Q} and \vec{P} are the 28-dimensional electric and magnetic charge vectors. Denoting by N_L and N_R the number of left and right oscillators respectively, the mass of an elementary (purely electric) string state is given by

$$M_{string}^2 = \frac{1}{8}(\vec{Q}_R^2 + 2N_R - 1) = \frac{1}{8}(\vec{Q}_L^2 + 2N_L - 2)\tag{2.3}$$

See, for example, [44]. In this $N = 4$ theory, states (whether elementary or solitonic) fall into 3 categories according as they are annihilated by $q = 2, 1, 0$ supersymmetries, in which case their masses are given by [13]

$$\begin{aligned}M_{central} &= Z_1 = Z_2 & q = 2 \\ &= Z_1 > Z_2 & q = 1 \\ &> Z_1 \geq Z_2 & q = 0\end{aligned}\tag{2.4}$$

where Z_1 and Z_2 are the moduli of the central charges in the supersymmetry algebra given by [14, 13]

$$Z_{1,2}^2 = \frac{1}{8} \left[\vec{Q}_R^2 + \vec{P}_R^2 \pm 2 \left(\vec{Q}_R \vec{P}_R^2 - (\vec{Q}_R \vec{P}_R)^2 \right)^{\frac{1}{2}} \right].\tag{2.5}$$

It follows by comparing M_{string} of (2.3) and $M_{central}$ of (2.5) that the elementary string states, being purely electric, are either BPS states with $N_R = 1/2$ or else non-supersymmetric states with $N_R > 1/2$.

The $N_R = 1/2$ states correspond to that subset of the full spectrum that is annihilated by half of the supersymmetry generators ($q = 2$), belongs to short representations of the $N = 4$ supersymmetry algebra and saturates the Bogomol'nyi bound $M_{central} = Z_1 = Z_2$. The basic superspin $L = 0$ multiplet is the 16 dimensional ($J_{max} = 1$) multiplet (1, 4, 5). This is the only multiplet appearing for $N_L = 0$. However, higher values of superspin L may appear for higher N_L . Since the left moving oscillators have spins 0 if they lie in the 22 compact dimensions or 1 if they lie in the spacetime dimensions, the superspin obeys the bound $L \leq N_L$. In particular for $N_L = 1$, we have in addition to the above $L = 0$ multiplet the 48 dimensional ($J_{max} = 2$) multiplet (1, 4, 6, 4, 1). For $N_L > 1$ we have $J_{max} = L + 1$.

The $N_R > 1/2$ states, belonging to the long representations, are annihilated by no supersymmetries ($q = 0$) and satisfy $M_{central} > Z_1 \geq Z_2$. The $L = 0$ multiplet is 256 dimensional with ($J_{max} = 2$). No elementary states belong to the intermediate representation, which are annihilated by one supersymmetry ($q = 1$) and satisfy $M_{central} = Z_1 > Z_2$. These $L = 0$ multiplets are 64 dimensional with ($J_{max} = 3/2$). The situation changes when we allow for solitonic states of the string theory which carry magnetic charge. Then we can have all three

categories of supermultiplet [1, 13], and indeed we predicted the existence of new dyonic ($J_{max} = 3/2$) multiplets in the string spectrum, over and above the ($J_{max} = 1$) dyonic states related by S -duality to the elementary states and predicted by Schwarz and Sen [9].

In [1] we considered these elementary electrically charged massive $N_R = 1/2$ states, and showed that the spin zero, superspin zero, states correspond to extreme limits of non-rotating black hole solutions which preserve 1/2 of the spacetime supersymmetries. By supersymmetry, the black hole interpretation then applies to all members of the $N = 4$ supermultiplet [45, 46], which has $J_{max} = 1$. For a subset of states the low-energy string action can be truncated to (1.1). The scalar-Maxwell parameter is given by $a = \sqrt{3}$ for $N_L = 1$ and $a = 1$ for $N_L > 1$ (and vanishing left-moving internal momenta). The other superspin zero states with $N_L > 1$ are extremal non-rotating black holes too, but are not described by a single scalar truncation of the type (1.1). We also made a similar identification for the dyonic states [13]. To see this, let us recall that general black hole solutions are determined by the 56 components of the electric and magnetic charge vectors \vec{Q} and \vec{P} . We can simplify the problem by applying an $O(6) \times O(22)$ T-duality transformation which eliminates all but four electric and four magnetic components. This corresponds to a truncation involving just four field strengths: F_1, F_2, F_3 and F_4 , and just three complex scalars: $S = a + ie^{-\eta}$, $T = b + ie^{-\sigma}$, $U = c + ie^{-\rho}$.

$$\begin{aligned} \mathcal{L} = \frac{1}{32\pi} \sqrt{-g} & \left\{ R - \frac{1}{2} [(\partial\eta)^2 + (\partial\sigma)^2 + (\partial\rho)^2] - \right. \\ & - \frac{1}{2} [e^{2\sigma}(\partial b)^2 + e^{2\rho}(\partial c)^2] - \frac{1}{12} e^{-2\eta} H^2 - \\ & - \frac{e^{-\eta+\sigma+\rho}}{4} \left[|T|^2 |U|^2 F_1^2 + |T|^2 F_2^2 + F_3^2 + |U|^2 F_4^2 + 2bF_2F_3 + 2cF_3F_4 + \right. \\ & \left. \left. - 2c|T|^2 F_1F_2 - 2b|U|^2 F_1F_4 - cb(F_1F_3 - F_2F_4) \right] \right\}, \quad (2.6) \end{aligned}$$

It is noteworthy that this procedure keeps only the degrees of freedom which arise from the toroidal compactification from six to four dimensions: $N = 2$ supergravity coupled to three vector multiplets. F_1, F_2 are the Kaluza-Klein fields, F_3, F_4 are the winding fields: S is the axion/dilaton, T the Kahler-form and U the complex structure. The axion field is obtained by dualizing H , which satisfies the standard Bianchi identity. The Lagrangian (2.6) was thoroughly analyzed in [13] where, in particular the triality of the three duality groups, $SL(2; Z)_S, SL(2; Z)_T, SL(2; Z)_U$, was emphasized: (2.6) is obviously invariant under $T \leftrightarrow U$, $F_2 \leftrightarrow F_4$ exchange, but the equations of motion are also invariant under $S \leftrightarrow T$, $S \leftrightarrow U$ exchange (accompanied by an appropriate electric/magnetic transformation of the field strengths), which lead to the interpretation of a triality of the S, T and U strings. To simplify further, we shall here consider solutions with vanishing pseudoscalars so that the reduced Lagrangian is

$$\begin{aligned} \mathcal{L} = \frac{1}{32\pi} \sqrt{-g} & \left\{ R - \frac{1}{2} [(\partial\eta)^2 + (\partial\sigma)^2 + (\partial\rho)^2] - \right. \\ & \left. - \frac{e^{-\eta}}{4} [e^{-\sigma-\rho} F_1^2 + e^{-\sigma+\rho} F_2^2 + e^{\sigma+\rho} F_3^2 + e^{\sigma-\rho} F_4^2] \right\}, \quad (2.7) \end{aligned}$$

where one has to keep in mind the constraints imposed by the requirement of vanishing pseudoscalars.

The extremal black holes we have in mind to illustrate the point, namely those that allow for a description by an action of type (1.1) with $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$, can be obtained from (2.7) by making various truncations:

$$\begin{aligned}
a = \sqrt{3} : & \quad F_1 \neq 0, & \quad F_2 = F_3 = F_4 = 0, & \quad Q^2 = 1 \\
a = 1 : & \quad F_1 = F_3 \neq 0, & \quad F_2 = F_4 = 0, & \quad Q^2 = 2 \\
a = 1/\sqrt{3} : & \quad F_1 = F_3 = \tilde{F}_2 \neq 0, & \quad F_4 = 0, & \quad Q^2 = 3 \\
a = 0 : & \quad F_1 = F_3 = \tilde{F}_3 = \tilde{F}_4 \neq 0, & & \quad Q^2 = 4
\end{aligned} \tag{2.8}$$

Table 1 summarizes the charge and mass quantum numbers for those and a few more black holes and string states in the heterotic string theory. (If so desired, one may then use $S - T - U$ triality to describe them in the dual Type II string pictures [13].) Based on this Table it can be easily verified [1, 13] that the mass and charge quantum numbers of the $a = 0, 1/\sqrt{3}, 1, \sqrt{3}$ black holes admit the interpretation of 4, 3, 2, 1 particle bound state with zero binding energy. For the purposes of illustration we have chosen the special cases where all non-zero charges are equal to unity but it is easily generalized to the case of different charges Q_1, P_2, Q_3 and P_4 [14] where the interpretation is that of a $(Q_1 + P_2 + Q_3 + P_4)$ -particle bound state with zero binding energy.

Note, by the way, that even the extremal, but *non-supersymmetric*, $a = 1$ black hole with electric charges $(1, 0, -1, 0)$ fits into the string spectrum with the assignments $N_L = 1, N_R = 3/2$. (Since the masses of non-supersymmetric states are not protected from quantum corrections by any non-renormalization theorems we do usually not expect (2.3) to give the correct answer for arbitrary black holes.) This solution is related by T-duality [1] to the non-supersymmetric black hole of [27] which has just one non-vanishing charge corresponding to one of the 16 $U(1)$ s in the Yang-Mills sector. It is frequently claimed that extremal black holes and supersymmetric black holes are synonymous, but while $M_{string}^2 = \vec{Q}_L^2/8$ and $M_{string}^2 = \vec{Q}_R^2/8$ are both extremal, only $M_{string}^2 = \vec{Q}_R^2/8$ is supersymmetric, owing to the left/right asymmetry of the heterotic string. (Note, however, that this solution preserves one quarter of the supersymmetries when embedded into maximal $N = 8$ supergravity coming from the Type II string).

As discussed in [1], although the superpartners of the non-rotating black holes are themselves black holes, they are *not* rotating black holes in the sense of Kerr. On the contrary, as explained in [46], it is their fermionic hair that carries the angular momentum in contrast to conventional rotating black holes where the angular momentum is bosonic. Rather this bosonic angular momentum is supplied by the left moving oscillators, which leads us to identify the Kerr-type angular momentum with the superspin L . However, as also discussed in [1], these $N_R = 1/2$ string states cannot then be rotating *black holes* since these mass=charge solutions have event horizons only for vanishing Kerr angular momentum.

3 Black hole supermultiplets

Since the basic quantities like mass and charge are not going to change if we move to the $N = 8$ or $N = 2$ theories, we may as well extend the conjecture to these theories also. The number of preserved supersymmetries is given in Table 2. In this section we

| Quantum Numbers | | | String States | | | | Black Holes | |
|------------------------|------------------------|---|---------------|---------------|-----------------|-----------------|----------------------|-----------------|
| (Q_1, Q_2, Q_3, Q_4) | (P_1, P_2, P_3, P_4) | q | N_L | N_R | M_{string}^2 | $M_{central}^2$ | a | M_{black}^2 |
| (1,0,0,0) | (0,0,0,0) | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{16}$ | $\frac{1}{16}$ | $\sqrt{3}$ | $\frac{1}{16}$ |
| (1,0,1,0) | (0,0,0,0) | 2 | 2 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | 1 | $\frac{1}{4}$ |
| (2,1,1,1) | (0,0,0,0) | 2 | 4 | $\frac{1}{2}$ | $\frac{13}{16}$ | $\frac{13}{16}$ | x | $\frac{13}{16}$ |
| (1,0,-1,0) | (0,0,0,0) | 2 | 0 | $\frac{1}{2}$ | 0 | 0 | x | 0 |
| (1,0,-1,0) | (0,0,0,0) | 0 | 1 | $\frac{3}{2}$ | $\frac{1}{4}$ | 0 | 1 | $\frac{1}{4}$ |
| (1,0,0,0) | (0,1,0,0) | 1 | x | x | x | $\frac{1}{4}$ | 1 | $\frac{1}{4}$ |
| (1,0,1,0) | (0,1,0,0) | 1 | x | x | x | $\frac{9}{16}$ | $\frac{1}{\sqrt{3}}$ | $\frac{9}{16}$ |
| (1,0,1,0) | (0,1,0,1) | 1 | x | x | x | 1 | 0 | 1 |
| (1,0,-1,0) | (0,1,0,-1) | 0 | x | x | x | 0 | 0 | 1 |

Table 1: Masses and charges of black holes and string states.

| | $a = 0$ | $a = 1/\sqrt{3}$ | $a = 1$ | $a = \sqrt{3}$ |
|---------|---------|------------------|---------|----------------|
| $N = 2$ | 1 | 1 | 1 | 1 |
| $N = 4$ | 1 | 1 | 2 | 2 |
| $N = 8$ | 1 | 1 | 2 | 4 |

Table 2: Number of preserved supersymmetries, q , for black holes with parameters $a = 0, 1/\sqrt{3}, 1, \sqrt{3}$ in $N = 2, 4, 8$ theories.

check that the supermultiplet structure of the black holes is consistent with this bound state interpretation. The relevant group theory may be found in [48]. Let us recall that an N -extended supersymmetry algebra admits $N/2$ central charges $Z_1, \dots, Z_{N/2}$. States fall into $1 + N/2$ categories according as they are annihilated by $N/2 \geq q \geq 0$ supersymmetry generators. q also counts the number of Z s that obey the bound $M_{central} = Z_{max}$. We are primarily interested in multiplets with states with spin $J = 0$ and superspin $L = 0$, which we identify with the non-rotating black hole solutions. The rest of the $L = 0$ supermultiplet may then be filled out using the fermionic zero modes [46]. In the fundamental multiplets the spin will run from $J = 0$ up to $J = (N - q)/2$ whereas for multiplets with non-zero superspin L , the spin will run from $J = 0$ to $J = L + (N - q)/2$ for $L < (N - q)/2$ and from $J = L - (N - q)/2$ to $J = L + (N - q)/2$ for $L \geq (N - q)/2$. The multiplets are constructed in the spirit of [48] by combining massless supermultiplets and then employing the Higgs mechanism to obtain the massive multiplet. For each number q of preserved supersymmetries we give the results up to $J_{max} = 4$ (implying superspins $L = 0, \frac{1}{2}, 1, \frac{3}{2}$ and $L = 2$ for the case of four preserved supersymmetries and so on). The results are shown in Tables 3, 4 and 5 for the $N = 8$, $N = 4$ and $N = 2$ theories, respectively.

Let us begin by considering black hole solutions of the $N = 8$ supergravity theory with $Z_1 \geq Z_2 \geq Z_3 \geq Z_4$. Here we have the five categories:

$$\begin{aligned}
M_{central} &= Z_1 = Z_2 = Z_3 = Z_4 & q = 4 \\
&= Z_1 = Z_2 = Z_3 > Z_4 & q = 3 \\
&= Z_1 = Z_2 > Z_3 \geq Z_4 & q = 2 \\
&= Z_1 = Z_2 \geq Z_3 \geq Z_4 & q = 1 \\
&> Z_1 \geq Z_2 \geq Z_3 \geq Z_4 & q = 0
\end{aligned} \tag{3.1}$$

In particular, the $a = \sqrt{3}$ black holes preserve $q = 4$ supersymmetries and must belong to short supermultiplets which we assume to be the maximum spin $J = 2$, superspin $L = 0$ multiplet $(1, 8, 27, 48, 42)$ appearing in Table 3. The bound state interpretation requires that the $a = 1$ black holes preserving $q = 2$ supersymmetries appear in the product of two $a = \sqrt{3}$ representations. Ignoring the internal quantum numbers, this product will decompose into multiplets with $4 \geq J_{max} \geq 2$ as follows

$$\begin{aligned}
1 \times [4] \oplus 8 \times \left[\frac{7}{2} \right] \oplus 27 \times [3] \oplus 48 \times \left[\frac{5}{2} \right] \oplus 42 \times [2] & q = 4 \\
1 \times [4] \oplus 6 \times \left[\frac{7}{2} \right] \oplus 14 \times [3] \oplus 14 \times \left[\frac{5}{2} \right] & q = 3 \\
1 \times [4] \oplus 4 \times \left[\frac{7}{2} \right] \oplus 5 \times [3] & q = 2 \\
1 \times [4] \oplus 2 \times \left[\frac{7}{2} \right] & q = 1 \\
1 \times [4] & q = 0
\end{aligned} \tag{3.2}$$

Which of the above possibilities is actually realized, however, will depend on the charge assignments of the two constituents. Suppose each is singly charged under just one of the Kaluza-Klein $U(1)$ s, then the bound state will again belong to a $q = 4$ multiplet if the $U(1)$ s are the same. On the other hand, if one carries a Kaluza-Klein charge and the other a winding charge in the same dimension, then we get $q = 2$. Since these are precisely the quantum numbers of the $a = 1$ black hole, which has twice the mass of the $a = \sqrt{3}$ black

| q | J | $L = 0$ | $L = \frac{1}{2}$ | $L = 1$ | $L = \frac{3}{2}$ | $L = 2$ |
|-----|---------------|---------|-------------------|---------|-------------------|---------|
| 4 | 4 | | | | | 1 |
| | $\frac{7}{2}$ | | | | 1 | 8 |
| | $\frac{3}{2}$ | | | 1 | 8 | 28 |
| | $\frac{5}{2}$ | | 1 | 8 | 28 | 56 |
| | 2 | 1 | 8 | 28 | 56 | 70 |
| | $\frac{3}{2}$ | 8 | 28 | 56 | 70 | 56 |
| | 1 | 27 | 56 | 70 | 56 | 28 |
| | $\frac{1}{2}$ | 48 | 69 | 56 | 28 | 8 |
| | 0 | 42 | 48 | 27 | 8 | 1 |
| 3 | 4 | | | | 1 | |
| | $\frac{7}{2}$ | | | 1 | 10 | |
| | $\frac{3}{2}$ | | 1 | 10 | 45 | |
| | $\frac{5}{2}$ | 1 | 10 | 45 | 120 | |
| | 2 | 10 | 45 | 120 | 210 | |
| | $\frac{3}{2}$ | 44 | 120 | 210 | 252 | |
| | 1 | 110 | 209 | 252 | 210 | |
| | $\frac{1}{2}$ | 165 | 242 | 209 | 120 | |
| | 0 | 132 | 165 | 110 | 44 | |
| 2 | 4 | | | 1 | | |
| | $\frac{7}{2}$ | | 1 | 12 | | |
| | $\frac{3}{2}$ | 1 | 12 | 66 | | |
| | $\frac{5}{2}$ | 12 | 66 | 220 | | |
| | 2 | 65 | 220 | 495 | | |
| | $\frac{3}{2}$ | 208 | 494 | 792 | | |
| | 1 | 429 | 780 | 923 | | |
| | $\frac{1}{2}$ | 527 | 858 | 780 | | |
| | 0 | 429 | 572 | 429 | | |
| 1 | 4 | | 1 | | | |
| | $\frac{7}{2}$ | 1 | 14 | | | |
| | $\frac{3}{2}$ | 14 | 91 | | | |
| | $\frac{5}{2}$ | 90 | 664 | | | |
| | 2 | 350 | 1000 | | | |
| | $\frac{3}{2}$ | 910 | 1988 | | | |
| | 1 | 1638 | 2912 | | | |
| | $\frac{1}{2}$ | 2002 | 3068 | | | |
| | 0 | 1430 | 2002 | | | |
| 0 | 4 | 1 | | | | |
| | $\frac{7}{2}$ | 16 | | | | |
| | $\frac{3}{2}$ | 119 | | | | |
| | $\frac{5}{2}$ | 544 | | | | |
| | 2 | 1700 | | | | |
| | $\frac{3}{2}$ | 3808 | | | | |
| | 1 | 6188 | | | | |
| | $\frac{1}{2}$ | 7072 | | | | |
| | 0 | 4862 | | | | |

Table 3: Massive supersymmetry representations of $N = 8$

| q | J | $L = 0$ | $L = \frac{1}{2}$ | $L = 1$ |
|-----|---------------|---------|-------------------|---------|
| 2 | 2 | | | 1 |
| | $\frac{3}{2}$ | | 1 | 4 |
| | 1 | 1 | 4 | 6 |
| | $\frac{1}{2}$ | 4 | 6 | 4 |
| | 0 | 5 | 4 | 1 |
| 1 | 2 | | | 1 |
| | $\frac{3}{2}$ | 1 | 6 | |
| | 1 | 6 | 15 | |
| | $\frac{1}{2}$ | 14 | 20 | |
| | 0 | 14 | 14 | |
| 0 | 2 | 1 | | |
| | $\frac{3}{2}$ | 8 | | |
| | 1 | 27 | | |
| | $\frac{1}{2}$ | 48 | | |
| | 0 | 42 | | |

Table 4: Massive supersymmetry representations for $N = 4$

| q | J | $L = 0$ | $L = \frac{1}{2}$ |
|-----|---------------|---------|-------------------|
| 1 | 1 | | 1 |
| | $\frac{1}{2}$ | 1 | 2 |
| | 0 | 2 | 1 |
| 0 | 1 | 1 | |
| | $\frac{1}{2}$ | 4 | |
| | 0 | 5 | |

Table 5: Massive supersymmetry representations for $N = 2$

hole, this is entirely consistent with our hypothesis. Note, however, that although one might also expect to obtain $q = 3$ multiplets, a closer look at the internal quantum numbers shows that these do not in fact arise, since the $M_{central} = Z_1 = Z_2 = Z_3 > Z_4$ configuration can never be achieved with 2-particle states.

The same arguments go through for an embedding of the black holes into $N = 4$ or $N = 2$ supergravity with an appropriate number of matter multiplets. For the $N = 4$ case the compositions for the product of two short multiplets are given by ⁵

$$\begin{aligned}
1 \times [2] \oplus 4 \times \left[\frac{3}{2} \right] \oplus 5 \times [1] & \quad q = 2 \\
1 \times [2] \oplus 2 \times \left[\frac{3}{2} \right] & \quad q = 1 \\
1 \times [2] & \quad q = 0
\end{aligned} \tag{3.3}$$

For example, the appearance of the $q = 0$ multiplet is consistent with the interpretation of the non-supersymmetric $a = 1$ black hole as a bound state of a supersymmetric $a = \sqrt{3}$ positively charged Kaluza=Klein black hole and a supersymmetric $a = \sqrt{3}$ negatively charged winding black hole. Similarly, for $N = 2$ we find

$$\begin{aligned}
1 \times [1] \oplus 2 \times \left[\frac{1}{2} \right] & \quad q = 1 \\
1 \times [1] & \quad q = 0
\end{aligned} \tag{3.4}$$

Therefore, we have shown that the supermultiplet structures are consistent with our hypothesis in the case of 2-particle bound states. It is straightforward to show by taking further tensor products that things go through in a similar way for the 3- and 4-particle states.

It should be stressed that we are not claiming that all bound states can be identified with oscillator states. In the $N = 4$ table, for example, $L = 1/2, J_{max} = 3/2$ states appear in the tensor product, whereas there are no $L = 1/2$ supermultiplets in the oscillator spectrum of the $N = 4$ heterotic string because the left-moving sector is purely bosonic. Similarly, the $L = 1, J_{max} = 2$ states appearing in the tensor product will be black holes whose angular momentum is fermionic in origin [46], whereas the $L = 1$ oscillator states have a bosonic Kerr-type angular momentum and cannot therefore be black holes.

So far we have put ourselves at the special point in moduli space $S = T = U = i$. At generic points, the mass formula becomes [47, 13]

$$M_{central} = \frac{1}{\text{Im } S \text{ Im } T \text{ Im } U} \max |(\alpha_1 + U\alpha_2 + T\alpha_4 - TU\alpha_3) \pm (S\beta_1 - SU\beta_2 - ST\beta_4 - STU\beta_3)| \tag{3.5}$$

where α and β are the quantized electric and magnetic charge vectors. Thus, although the bound state interpretation continues to apply, the zero binding energy phenomenon does not. The triangle inequality ensures that at generic points the bound states have non-zero binding energy (except of course when the charges are not relatively prime).

⁵Since the same group theory applies, this suggests that higher superspin multiplets also appear in the spectrum of global $N = 4$ Yang-Mills theories, where traditionally attention is focussed only on maximum spin 1.

4 Multi-black brane solutions of M -theory

| | 4-form | 3-form | 2-form | 1-form |
|------|--------|--------|-------------------------|--|
| D=11 | 4 | | | |
| D=10 | 4 | 4 | 4 | |
| D=9 | 4 | 4 | 4, 2 | 4 |
| D=8 | 4 | 4 | 4, 2 | 4, 2 |
| D=7 | | 4 | 4, 2 | 4, 2 |
| D=6 | | 4, 2 | 4, 2 | 4, 2, $\frac{4}{3}$, 1 |
| D=5 | | | 4, 2, $\frac{4}{3}$ | 4, 2, $\frac{4}{3}$, 1 |
| D=4 | | | 4, 2, $\frac{4}{3}$, 1 | 4, 2, $\frac{4}{3}$, 1, $\frac{4}{5}$, $\frac{2}{3}$, $\frac{4}{7}$ |

Table 6: Δ values for supersymmetric p -branes

Let us now turn to the black branes of M -theory [34, 35]. Starting from eleven dimension, toroidal compactification gives rise to a variety of $(d+1)$ -form field strengths and hence fundamental $(d-1)$ -brane and solitonic $(\tilde{d}-1)$ -brane solutions in the lower dimension $D = d + \tilde{d} + 2$. The compactified eleven-dimensional supergravity theory admits a consistent truncation to the following set of fields: the metric tensor g_{MN} , a set of n scalars $\vec{\phi} = (\phi_1, \dots, \phi_n)$, and n field strengths F_α of rank $(\tilde{d}+1)$. The D -dimensional action describing the p -branes under consideration is then given by [42]

$$\mathcal{L} = \sqrt{-g} \left[R - \frac{1}{2}(\partial\vec{\phi})^2 - \frac{1}{2 \times (d+1)!} \sum_{\alpha=1}^n e^{\vec{a}_\alpha \vec{\phi}} F_{d+1}^{\alpha 2} \right], \quad (4.1)$$

where n is the number of participating field strengths. If all active charges are equal, this can be further truncated to the Lagrangian (1.1) involving a single scalar and single field strength where a , ϕ and F are given by

$$\begin{aligned} a^{-2} &= \sum_{\alpha\beta} (M^{-1})_{\alpha\beta} \\ \phi &= a \sum_{\alpha\beta} (M^{-1})_{\alpha\beta} \vec{a}_\alpha \vec{\phi} \\ (F_\alpha)^2 &= a^2 \sum_{\alpha\beta} (M^{-1})_{\alpha\beta} F^2, \end{aligned} \quad (4.2)$$

where the matrix $M_{\alpha\beta}$ is given by

$$M_{\alpha\beta} = \vec{a}_\alpha \vec{a}_\beta. \quad (4.3)$$

The parameter a can conveniently be expressed as

$$a^2 = \Delta - \frac{2d\tilde{d}}{d+\tilde{d}} \quad (4.4)$$

As discussed in [42], supersymmetric p -branes solutions can arise only when the value of Δ is given by $\Delta = 4/n$. This occurs when

$$M_{\alpha\beta} = 4\delta_{\alpha\beta} - 2\frac{d\tilde{d}}{d+\tilde{d}}. \quad (4.5)$$

Originally, attention was focussed on the $\Delta = 4$ solutions [39, 40, 41] but various new supersymmetric solitons with $\Delta \neq 4$ have recently been studied [38, 18, 42]. In this section we shall generalize the treatment of extremal black holes [20] and confirm on the level of classical solutions that these new p -branes can also be regarded as bound states with zero binding energy of fundamental $\Delta = 4$ branes. We find new $1 \leq n \leq m$ -centered p -brane solutions which reproduce the $\Delta = 4/n$ solutions of [42] as we allow n of the centers to coincide and take the remaining $(m - n)$ out to infinity. Table 6 summarizes in which dimensions the various solitons in maximal supergravities arise. Below eight (six) dimensions the four(three)-form field strengths are dualized.

For the purposes of exhibiting the multi-centered solutions, we shall work with (4.1). The equations of motion are

$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}e^{\vec{a}_\alpha\vec{\phi}}F_{d+1}^{\alpha MN}) = 0 \quad (4.6)$$

$$\frac{1}{\sqrt{-g}}\partial_M(\sqrt{-g}\partial^M\phi_i) = \sum_n \frac{a_{i\alpha}}{2 \times (d+1)!} F_{d+1}^{\alpha 2} \quad (4.7)$$

$$R_{MN} = \frac{1}{2}\partial_M\vec{\phi}\partial_N\vec{\phi} + \frac{1}{2d!}\sum_n e^{\vec{a}_\alpha\vec{\phi}}\left(F_{MN}^{\alpha 2} - \frac{d}{n(d+\tilde{d})}F^{\alpha 2}g_{MN}\right) \quad (4.8)$$

The one-center solutions of these equations have been intensively studied. Here we find the *multi-center* solution which reads for the solitonic case:

$$f_\alpha = \left(1 + \frac{\lambda_\alpha}{d|\vec{y} - \vec{y}_{0,\alpha}|^d}\right) \quad (4.9)$$

$$ds^2 = \prod_{\alpha=1}^n f_\alpha^{-\frac{d}{d+\tilde{d}}} dx^\mu dx_\mu + \prod_{\alpha=1}^n f_\alpha^{\frac{\tilde{d}}{d+\tilde{d}}} dy^m dy_m \quad (4.10)$$

$$e^{-\vec{a}_\alpha\vec{\phi}} = f_\alpha^2 \prod_{\beta=1}^n f_\beta^{-\frac{d\tilde{d}}{d+\tilde{d}}} \quad (4.11)$$

$$F_{m_1\dots m_{d+1}}^\alpha = \lambda_\alpha \epsilon_{m_1\dots m_{d+1}p} \frac{y^p}{|\vec{y} - \vec{y}_{0,\alpha}|^{d+2}} \quad (4.12)$$

where μ refers to the \tilde{d} world-volume coordinates of the solitonic $(\tilde{d} - 1)$ -brane and m to the $D - \tilde{d} = d + 2$ transverse coordinates. In the special case $d = 0$ we choose

$$f_\alpha = \left(1 + \lambda_\alpha \ln \frac{|\vec{y} - \vec{y}_{0,\alpha}|}{r_{0,\alpha}}\right) \quad (4.13)$$

instead of (4.9). The solutions for elementary multi- p -branes are easily obtained by generalizing the single-centered solutions of [42] along the lines above. Since (4.6) and (4.7) are

essentially linear in the contributions of each field strength they are obviously satisfied for the multi-center solutions. The only slightly non-trivial check comes from the Einstein equations, since here the scalar fields and R_{mn} involve non-linearities in the individual soliton contributions. If we plug the ansatz into the field equations we find the following condition for vanishing non-linearities:

$$\begin{aligned} & \frac{d\tilde{d}^2}{2(d+\tilde{d})} + \tilde{d}M_d^{-1} \left[(n-2)\left(\frac{d\tilde{d}}{d+\tilde{d}}\right)^2 - 2\left(2 - \frac{d\tilde{d}}{d+\tilde{d}}\right)\frac{d\tilde{d}}{d+\tilde{d}} \right] + \\ & + \tilde{d}M_n^{-1} \left[\left(2 - \frac{d\tilde{d}}{d+\tilde{d}}\right)^2 + \left(\frac{d\tilde{d}}{d+\tilde{d}}\right)^2 - 2(n-2)\left(2 - \frac{d\tilde{d}}{d+\tilde{d}}\right)\frac{d\tilde{d}}{d+\tilde{d}} + \right. \\ & \left. + 2(n-2)\left(\frac{d\tilde{d}}{d+\tilde{d}}\right)^2 + (n-2)(n-3)\left(\frac{d\tilde{d}}{d+\tilde{d}}\right)^2 \right] = 0, \quad (4.14) \end{aligned}$$

where $M_{d(n)}^{-1}$ denote the (off)-diagonal elements of the matrix $M_{\alpha\beta}$ defined in (4.3). Indeed, for all the new supersymmetric p -branes the above condition holds, allowing us to generalize the single-center solutions to multi-center ones. However, there is one subtlety, for certain values of d, n, D, M is singular and cannot be inverted. The problem arises for $(D=4, n=4, d=1)$ and $(D=5, n=3, d=1)$. The first case was considered in [20] and shown to be a valid solution. The second one can also be shown to work by an independent calculation. If so desired, one may now consider the special case of coincident centers and equal charges to obtain the $\Delta = 4/n$ solutions, thus confirming that these admit the interpretation of bound states with zero binding energy of n fundamental $\Delta = 4$ branes.

Most of our results are not based on the fact that we consider maximal supergravities. So we can ask ourselves what kind of bound states survive in the heterotic theory. It appears that all 1-form and 2-form types do. In four dimensions we also have the four classic black hole types and also strings with up to seven participating field strengths. In the heterotic theory the number seven finds a very natural explanation in the presence of one S -field 3 T -fields and 3 U -fields of section (2) [13, 49]. The multi-string solution found in [49] with non-vanishing S and T fields is given by

$$\begin{aligned} ds^2 &= -dt^2 + dz^2 + S_2 T_2^{(1)} T_2^{(2)} T_2^{(3)} (dx^2 + dy^2) \\ S &= S_1 + iS_2 = \sum_{i=1}^n s_i \ln \frac{x + iy - w_i}{r_i} \end{aligned} \quad (4.15)$$

$$T^{(a)} = T_1^{(a)} + iT_2^{(a)} = \sum_{i=1}^n t_i^{(a)} \ln \frac{x + iy - w_i^{(a)}}{r_i^{(a)}} \quad (4.16)$$

where $w_i = x_i + iy_i$ and r_i denote the positions and sizes of the sources. The solution preserves 1/2, 1/4 and 1/8 of the supersymmetries for one, two and three T -fields but without an S charge. In the presence of the S field the supersymmetries get halved with the exception of the configuration with 3 T and one S field which breaks either seven eighths or all supersymmetries, depending on the chirality choice. It is straightforward to generalize these results to include the U fields.

The presence of only one 3-form in all dimensions (above five) forbids the $D=6, d=2, n=2$ solution. Nevertheless, we have a very interesting solution in $D=6, d=2$, which

also can be viewed as a bound state: the dyonic string of [49, 50]:

$$\begin{aligned}
\Phi &= \Phi_E + \Phi_M, \\
ds^2 &= e^{\Phi_E}(-dt^2 + dz^2) + e^{\Phi_M} dx^i dx_i \\
e^{-\Phi_E} &= 1 + \frac{Q}{y^2} \quad e^{-\Phi_M} = 1 + \frac{P}{y^2} \\
H_3 &= 2Q\epsilon_3 + 2Pe^\Phi * \epsilon_3
\end{aligned} \tag{4.17}$$

with $y^2 = x^i x_i$, $i = 1, 2, 3, 4$ and (in general)

$$e^{\Phi_E} \partial^2 e^{-\Phi_E} = e^{\Phi_M} \partial^2 e^{-\Phi_M} = 0 \tag{4.18}$$

Since the electric as well as the magnetic part are determined by two independent harmonic functions, we can easily generalize (4.17) to a multi-string solution. The same holds for the ten dimensional dyonic string also found in [49]:

$$\begin{aligned}
ds^2 &= e^{\Phi_{E_1} + \Phi_{E_2}}(-dt^2 + dz^2) + e^{\Phi_{M_1}} \delta^{ij} dy_i dy_j + e^{\Phi_{M_2}} \delta^{ab} dy_a dy_b \\
e^{\Phi_{E_\alpha}} \partial_1^2 e^{-\Phi_{E_\alpha}} &= 0 \quad e^{\Phi_{M_\alpha}} \partial_2^2 e^{-\Phi_{M_\alpha}} = 0
\end{aligned} \tag{4.19}$$

with $i, j = 2, 3, 4, 5$; $a, b = 6, 7, 8, 9$ and $\alpha = 1, 2$. $\partial_{1,2}^2$ denote the d'Alemberts in the (2, 3, 4, 5) and (6, 7, 8, 9) subspaces respectively. The antisymmetric tensor field is given as

$$B_{01} = e^{\Phi_{E_1 + E_2}}, \quad H_{ijk} = \epsilon_{ijkl} \partial^l \Phi_{M_1}, \quad H_{abc} = \epsilon_{abcd} \partial^d \Phi_{M_2}. \tag{4.20}$$

(4.19) is written in ten dimensional string coordinates.

Another interesting aspects of the multi- p -brane solutions is that the charge parameters of the individual constituents are independent. In particular, if we have only two p -branes, we can choose their charges to be of equal magnitude but different sign, which leads to massless solutions [25, 24, 21, 26, 18, 51, 50]. The interpretation is that two supersymmetric p -branes, one with positive and one with negative mass, can combine to form a supersymmetric massless p -brane. Certainly, an isolated brane with negative mass does not make sense quantum-mechanically but there may be some quantum confinement mechanism that allows it to exist only as a bound state.

5 Entropy and temperature

In this section, we ask whether the entropy and temperature of these black p -branes [34, 35, 36] is consistent with the bound state interpretation. The mass per unit volume and the charges of the multi black p -branes may be written in terms of parameters k and μ_α as [34]

$$\begin{aligned}
M_{black} &= k(\tilde{d} \sum_{\alpha=1}^n \sinh^2 \mu_\alpha + \tilde{d} + 1) \\
\lambda_\alpha &= \frac{1}{2} \tilde{d} k \sinh 2\mu_\alpha
\end{aligned} \tag{5.1}$$

The Hawking temperature T and entropy S of these multi black p -branes in the case where all centers are coincident are given by [34]

$$T = \frac{\tilde{d}}{4\pi y_+} \prod_{\alpha=1}^n (\cosh \mu_\alpha)^{-1}$$

$$S = \frac{1}{4} y_+^{\tilde{d}+1} \omega_{\tilde{d}+1} \prod_{\alpha=1}^n (\cosh \mu_\alpha) \quad (5.2)$$

where the event horizon is located at $y = y_+ = k^{1/\tilde{d}}$ and where $\omega_{\tilde{d}+1}$ is the volume of the unit $(\tilde{d} + 1)$ -sphere.

The form of the total entropy as a product of the individual entropies is puzzling from the bound state interpretation. It remains a puzzle when we take the extremal limit. To illustrate this, let us consider the special case where each of the n field strengths is equal. Then we have [34]

$$T = \frac{\tilde{d}}{4\pi y_+} (\cosh \mu)^{-n}$$

$$S = \frac{1}{4} y_+^{\tilde{d}+1} \omega_{\tilde{d}+1} (\cosh \mu)^n \quad (5.3)$$

The extremal limit corresponds to $k \rightarrow 0$, $\mu \rightarrow \infty$, holding $\lambda = \tilde{d}\sqrt{n/2}ke^\mu$ constant. Thus the entropy vanishes unless the constant a is zero and $d = 1$. This happens only for $(D = 4, n = 4, d = 1)$, which is just the Reissner Nordstrom black hole, and for the five-dimensional black hole $(D = 5, n = 3, d = 1)$. Remarkably, these were precisely the two cases where the matrix M of (4.3) was singular. At first sight this result seems strange: In $D = 4$, for example, it seems very unnatural to combine three black holes and still the entropy is zero; but adding a fourth one suddenly forces it to be finite. To see in more detail how this comes about, let us invoke [52], where a new recipe for the calculation of the horizon area was given: the scalar fields on the horizon (in our case η , σ and ρ) are determined by the requirement that the central charge becomes extremal. The entropy is then given by the value of the central charge evaluated with the scalar fields at the horizon. For the standard black holes with charges Q_1, Q_3, P_2 and P_4 the scalar fields on the horizon are fixed to [52]

$$e^{-2\eta} \rightarrow \left| \frac{P_2 P_4}{Q_1 Q_3} \right|, \quad e^{-2\sigma} \rightarrow \left| \frac{P_2 Q_3}{Q_1 P_4} \right|, \quad e^{-2\rho} \rightarrow \left| \frac{Q_3 P_4}{Q_1 P_2} \right|. \quad (5.4)$$

For each ‘‘incomplete’’ black hole, i.e. a state with not all four charges non-zero, at least one of the scalars blow up, either to plus or minus infinity. This ties in very nicely with our bound state hypothesis. For an $a = 1/\sqrt{3}$ with (for example) $Q_1 = Q_3 = P_2 = 1$ all three scalars diverge:

$$\eta \rightarrow \infty, \quad \sigma \rightarrow -\infty, \quad \rho \rightarrow \infty. \quad (5.5)$$

The elementary magnetic black hole with $P_4 = 1$ on the other hand has diverging scalars with the opposite sign:

$$\eta \rightarrow -\infty, \quad \sigma \rightarrow \infty, \quad \rho \rightarrow -\infty! \quad (5.6)$$

Since we know from [20, 15] that there is a multi-black hole solution and the scalars are additive we can safely conclude that the $a = 1/\sqrt{3}$ and $a = \sqrt{3}$ state conspire to force the scalars and therefore the entropy to be finite.

All this suggests that it is only the $a = 0$ non-dilatonic p -brane entropies that can be trusted since when the dilaton is non-trivial there will be strong coupling effects that we do not yet know how to handle. However, it was shown in [36] that if quantum corrections smooth out singularities, even black holes with $a \neq 0$ may have non-vanishing entropy.

6 Conclusions

We have reexamined the suggestion in [1] that, in the extremal limit, the non-rotating black hole solutions of string theory may be identified with both elementary and solitonic BPS string states, and have confirmed that the interpretation of certain multiply charged black holes as bound states at threshold of singly charged black holes is consistent with the masses, charges, spins and supermultiplet structure of the string states. We also confirm that the bosonic Kerr-type angular momentum, arising from the left-moving sector of the heterotic string, corresponds to the superspin L of the oscillator states and hence that only $L = 0$ BPS oscillator states can be black holes. One is tempted to conclude that the $L > 0$ oscillator states must therefore be described by naked singularities, but there also exist solutions that are asymptotically identical to such solutions but near the core have a much milder singularity and whose angular momentum is naturally Regge-bounded [53]. Moreover, this bound state interpretation generalizes to super p -branes in D dimensions. In doing so, of course we have to put ourselves at special points in moduli space. In section (2), for example, we set the asymptotic value of \mathcal{M} to I and the asymptotic value of the dilaton to zero. For generic points in moduli space, the bound state interpretation would continue to apply but we would no longer have the zero binding energy phenomenon (except of course if the charges are not relatively prime). We might add that these results are also consistent with the recent recognition that some p -branes carrying Ramond-Ramond charges admit an interpretation as Dirichlet-branes, or D -branes, and are therefore amenable to the calculational power of conformal field theory [54]. Bound states of p -branes have been discussed from the somewhat different perspective of Dirichlet branes in [55, 56, 57, 58, 59, 60]. Apart from their intrinsic importance, therefore, these black holes and black p -branes have recently come to the fore as away of providing a microscopic explanation of the Hawking entropy and temperature formulae which have long been something of an enigma. See [61] for a recent review. For reasons discussed in section (5), however, only the $a = 0$ branes are amenable to these calculations, given our current technology.

An interesting special case is provided by the $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$ black holes which admit the interpretation as 1, 2, 3, 4-particle bound states at threshold [1, 13]. One feature which appeared mysterious to us at the time was: Why the unit charge solutions singled out *four* values of a and hence why only 1, 2, 3, 4- and not 5, 6, ...-particle bound states? An explanation of this has recently been given [62, 63, 64, 65, 66] in terms of intersecting membranes [67] and fivebranes [68] in $D = 11$. This opens up a new direction for the study of black hole and black p -brane bound states.

Finally, another crucial consistency check on the black hole, bound state, string state

picture is supplied by comparing gyromagnetic and gyroelectric ratios. This turns out to be quite subtle [69] and will be treated in a separate publication [70].

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