Weighted uniform consistency of kernel density estimators with general bandwidth sequences

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Let $f_{n,h}$ be a kernel density estimator of a continuous, bounded and positive d-dimensional density f. Let $\psi(t)$ be a positive continuous function such that $\|\psi f^\beta\|_\infty < \infty$ for some $0 < \beta < 1/2$. We are interested in the rate of consistency of such estimators with respect to the weighted sup-norm determined by ψ. This problem has been considered by Giné, Koltchinskii and Zinn ([1], 2004) for a deterministic bandwidth h_n. We show that there is also a "uniform in h" version of this result allowing us to extend the results of the above paper to kernel density estimators with bandwidth sequences depending on the data and/or the location. We give precise conditions when

$$\sup_{a_n \le h \le b_n} \sqrt{\frac{nh^d}{|\log h|}} \|\psi(f_{n,h} - Ef_{n,h})\|_\infty$$

is stochastically and almost surely bounded, where $a_n = a(n)$ and $b_n = b(n)$ are regularly varying functions with negative exponent. We use techniques from the theory of empirical processes, especially a recent moment inequality of Einmahl and Mason ([2], 2005).

[1] Giné, Koltchinskii and Zinn (2004). *Weighted uniform consistency of kernel density estimators* Ann. Probab. **32**, 2570–2605.

[2] Einmahl and Mason (2005). *Uniform in bandwidth consistency of kernel function-type estimators* Ann. Stat. **33**, 1601–1624.