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A Comment on the Stability of String Monopoles †

Ramzi R. Khuri*

Center for Theoretical Physics
Texas A&M University
College Station, TX 77843

In recent work a multimonopole solution of heterotic string theory was obtained. The monopoles are noted to be stable, in contrast with analogous solutions of Einstein-Maxwell or Yang-Mills-dilaton theory. The existence of this and other classes of stable solitonic solutions in string theory thus provides a possible test for low-energy string theory as distinct from other gauge + gravity theories.

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In recent work[1], a multimonopole solution of heterotic string theory was presented. An analogous solution in Yang-Mills field theory was found to have divergent action near each source. In the string solution, however, the divergences from the Yang-Mills sector are precisely cancelled by those from the gravity sector, so that the action is finite and easily computed[1,2].

In this letter we comment briefly on the stability of this solution. We note that the string monopole solution, inasmuch as it saturates a Bogomol'nyi bound between ADM mass and charge, is stable. The stability of the resultant low-energy YM + dilaton gravity field theory contrasts with the recently demonstrated instability of analogous non-string Einstein-Maxwell or YM-dilaton solutions and thus represents a possible low-energy test for string theory. We also briefly comment on the dynamics of these solitons.

The bosonic fields for the self-dual multimonopole solution of heterotic string theory with zero background fermi fields are given by[1]

$$\begin{aligned} g_{\mu\nu} &= e^{2\phi}\delta_{\mu\nu}, \quad g_{ab} = \eta_{ab}, \\ H_{\mu\nu\lambda} &= \pm\epsilon_{\mu\nu\lambda\sigma}\partial^\sigma\phi, \\ e^{2\phi} &= e^{2\phi_0} f, \\ A_\mu &= i\overline{\Sigma}_{\mu\nu}\partial_\nu \ln f, \end{aligned} \tag{1}$$

where $\mu,\nu,\lambda,\sigma = 1,2,3,4,\ a,b = 0,5,6,7,8,9,\ \overline{\Sigma}_{\mu\nu} = \overline{\eta}^{i\mu\nu}(\sigma^i/2)$ for $i = 1,2,3$ (σ^i, $i = 1,2,3$ are the 2×2 Pauli matrices) where

$$\begin{aligned} \overline{\eta}^{i\mu\nu} = -\overline{\eta}^{i\nu\mu} &= \epsilon^{i\mu\nu}, & \mu,\nu &= 1,2,3, \\ &= -\delta^{i\mu}, & \nu &= 4 \end{aligned} \tag{2}$$

and where

$$f = 1 + \sum_{n=1}^{N} \frac{m_n}{|\vec{x} - \vec{a}_n|}, \tag{3}$$

where m_n is the charge and $\vec{a}_n$ the location in the three-space (123) of the nth monopole. The anti-self-dual solution is similar, with the δ-term in (2) changing sign. This solution was shown to have multimonopole structure[1] in the four-space (0123), each source having topological charge $Q = 1$ and magnetic charge $m = 1/g$, where g is the YM coupling constant. The four-dimensional metric line-element strongly resembles that of the Kaluza-Klein monopole [3,4]. We argued in [1] that this solution is exact to all orders in α'.

If we make the identification $\Phi \equiv A_4$, then the gauge and Higgs fields may be simply written in terms of the dilaton as

$$\begin{aligned} \Phi^a &= -\frac{2}{g}\delta^{ia}\partial_i\phi, \\ A_k^a &= -\frac{2}{g}\epsilon^{akj}\partial_j\phi \end{aligned} \tag{4}$$

for the self-dual solution. For the anti-self-dual solution, the Higgs field simply changes sign. A toroidal compactification can be adopted so that we consider the dynamics of our solution in the spacetime (0123). As usual, the existence of a static multi-soliton solution depends on the "zero force" condition.

A similar solution for Yang-Mills-dilaton theory was found in [5], with analogous dilaton behaviour and corresponding to a Dirac magnetic monopole with unit magnetic charge. It was noted that this solution has infinitely many unstable modes, but has finite action resulting from the cancellation between divergences stemming from the YM field and dilaton respectively. In fact, magnetically charged solutions to the coupled Einstein-Maxwell equations are typically found to be unstable [6,7,8], with or without the presence of a dilaton.

The ansatz of (4) represents an infinite-action solution of YM + scalar field theory in 3+1 dimensions which satisfies the Bogomol'nyi bound $G_{ij}^a = \epsilon_{ijk}D_k\Phi^a$. Since the action is finite away from the singularity, the Bogomol'nyi bound guarentees stability of the solution outside arbitrary finite enclosures around each source. Since the string solution has finite action owing to the cancellation between gauge and gravitational divergences, we can use a Bogomol'nyi bound to demonstrate stability everywhere. We see this by noting that the ADM mass of the multimonopole solution written in terms of the canonical metric saturates a Bogomol'nyi bound in terms of the charges of the string monopoles[9]

$$M = \frac{2\pi}{\kappa^2}e^{\phi_0/2}\sum_{i=1}^{N} m_i. \tag{5}$$

Thus the string multimonopole solution sits at a minimum energy point and is automatically stable against perturbations independent of x_4 (presumably if we allow perturbations depending on the compactified direction the string monopoles will decay into heterotic instanton fivebranes [10–14], which possess a higher symmetry[1]). As an exercise, one can

[1] This was pointed out to me by Jianxin Lu.

explicitly show that there are no unstable modes of the string monopole away from the singularity, from either the gauge or gravitational sectors. The same divergence cancellation feature which leads to a finite action solution then allows us to circumvent the singularity in each sector when considering the stability of the string solution.

Since the divergence cancellation in the action is also possessed by the unstable solution in [5], however, this property is clearly insufficient in itself to explain the stability of the string solution, which is rooted in the (4,4) superconformal invariance of its underlying sigma-model [13,14]. It is also linked to the existence of a zero-dynamical force condition obeyed by this and other classes of heterotic multi-soliton solutions [15–19], in addition to the usual zero-static force condition. Since the string solutions all saturate Bogomol'nyi bounds and are hence stable, their existence in suitably compactified form provides a possible test for string theory as distinct from other gauge + gravity models.

A study of the low-energy dynamics of the string monopoles was done in [20], where it was found from both a test-monopole approach and a computation of the Manton metric on moduli space that the monopoles scatter trivially in the low-energy limit. The latter method relied on the construction of an $O(\beta)$ solution to the constraint equations of motion, which had the piecewise Lorentz boosted form

$$
\begin{aligned}
e^{2\phi(\vec{x},t)} &= 1 + \sum_{n=1}^{N} \frac{m_n}{|\vec{x}-\vec{a}_n(t)|}, \\
g_{00} &= -1, \qquad g^{00} = -1, \qquad g_{ij} = e^{2\phi}\delta_{ij}, \qquad g^{ij} = e^{-2\phi}\delta_{ij}, \\
g_{0i} &= -\sum_{n=1}^{N} \frac{m_n \vec{\beta}_n \cdot \hat{x}_i}{|\vec{x}-\vec{a}_n(t)|}, \qquad g^{0i} = e^{-2\phi} g_{0i}, \\
H_{ijk} &= \epsilon_{ijkm}\partial_m e^{2\phi}, \\
H_{0ij} &= \epsilon_{ijkm}\partial_m g_{0k} = \epsilon_{ijkm}\partial_k \sum_{n=1}^{N} \frac{m_n \vec{\beta}_n \cdot \hat{x}_m}{|\vec{x}-\vec{a}_n(t)|},
\end{aligned}
\tag{6}
$$

where $i,j,k,m = 1,2,3,4$ and we use a flat space ϵ-tensor. Note that g_{00}, g_{ij} and H_{ijk} are unaffected to order β. According to the conjecture of Bergshoeff and de Roo[21,22,1], from the exactness condition $A_\mu = \Omega_{\pm\mu}$[23,24] (where $\Omega_{\pm\mu}$ is the generalized connection defined in [1]), the higher order in α' terms drop out from the action. The contributions of these terms also cancel in the equations of motion, provided we assume a similar $O(\beta)$ solution for the YM field A_μ to balance the piecewise-boosted generalized connection $\Omega_{\pm\mu}$. We therefore argue that our low-energy scattering analysis in [20] is also exact, suggesting that the dynamic YM force for these string monopoles is precisely cancelled by the dynamic gravity sector force to all orders in α'. This argument can equally well be applied to the heterotic instanton fivebranes.

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