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COMMENTS ON COSMOLOGICAL STRING SOLUTIONS

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ABSTRACT

We comment on some physical and technical aspects of our linear-dilaton string solutions. We explain why the corresponding cosmological epoch has finite duration due to tachyonic instabilities present even in heterotic and type-II strings, argue that the size of the universe varies at a rate depending on the choice of relevant physical units, and briefly speculate on whether such epochs replace inflation. Treating formally the screening prescription previously used to define $d > 25$ amplitudes as insertions of a tachyon background, we derive a renormalized coupling constant, which exhibits a curious zero for one particular duration of the linear dilaton epoch.

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The recent spectacular progress [1,2] in understanding non-critical strings has so far stumbled at the $d = 1$ barrier. This is bad news for the region $1 < d < 25$ ($1 < d < 9$ in the supersymmetric case) where QCD strings and fluctuating phase boundaries are a priori expected to lie. It sounds like even worse news for the far-removed, but most interesting for string unification, region $d > 25$ ($d > 9$) where the string finds itself in a cosmological time-dependent background [3,4] with time being generated dynamically [5]. Yet this latter region may turn out to be simpler to understand. On the physical side rather than, for instance, "polymerize" the surface, the tachyonic instabilities are expected to drive the theory in time to a stable supersymmetric critical string. On the more technical side, it is intriguing that $d > 25$ is the only region where one can define dual factorizable amplitudes [4] by using a screening prescription for the Liouville mode analogous to that used for minimal unitary conformal models [6].

A series of questions clearly arise: how do these non-critical strings decay to the stable vacuum? can they in the process play the role of inflation? and what is the meaning of these "screened" dual amplitudes? In this note we make a series of comments which, while reflecting our present incomplete understanding, will hopefully shed some light on these issues and in particular on the all-important tachyon dynamics. First, we discuss how the tachyon provides an exit from the linear-dilaton epoch, which may be accompanied by large entropy production. Secondly, we comment on the physical interpretation of this epoch. As was pointed out by Sanchez and Veneziano [7], although the "physical" metric describes a linearly-expanding Universe, the size of the Universe does not change when measured in "fundamental string units". Here we argue that this conclusion must be modified if one uses physical units such as the lifetimes or mean-free paths of particles or the sizes of composite states. We also comment on the fact that our cosmological string solutions have features- a possible explanation of flatness [4], possible mechanisms for entropy production and evading the horizon problem- that are normally associated with inflation. Finally, treating the screening prescription of ref. [4] as a perturbation in a tachyon background, we derive formally, after truncating all divergent terms, a renormalized string coupling constant. The corresponding scaling function has no smooth dependence on d , but it exhibits a zero for one particular duration of the linear-dilaton epoch, which does depend smoothly on d . The physical meaning, if any, of this fact is as yet unclear to us.

The time-dependent string solution [3,4] that we consider has the following σ -model background fields:

$$G_{\mu\nu} = \eta_{\mu\nu} \quad ; \quad \Phi = -2Q\tau + \Phi_0 \quad (1)$$

where $\tau = X^0$ is the conformal time and Φ the dilaton field. The constant Q is related to the dimension of space (or, more generally, the central charge of the corresponding conformal theory) :

$$Q^2 = \frac{d-25}{12} \quad (2)$$

Alternatively, one may think of $\frac{d-25}{3} \equiv V_0$ as the value of the potential at some local extremum of all scalar fields other than the dilaton.

The metric with normalized Einstein action is $g_{\mu\nu} = e^{-\Phi} G_{\mu\nu}$, and describes an expanding Universe, for $Q > 0$, with scale factor $a \sim e^{Q\tau}$. Plane-wave solutions in this background take the form $e^{ip_\mu X^\mu}$, where the zeroth component of momentum has a fixed imaginary part: $p_0 = E + iQ$, and the mass-shell condition is

$$E^2 - \vec{p}^2 = -2 - Q^2 + 2N \quad (3)$$

with N an integer labelling the level of the state. If one prefers to think of τ as the conformal factor of the two-dimensional metric, then $h = \frac{1}{2}\vec{p}^2 + N$ is the conformal weight of the "matter" part of the vertex operator, $e^{ip_0\tau}$ is its gravitational dressing [2], and condition (3) guarantees that the total conformal weight is 1.

The background (1) has two sources of potential instability. First, bosonic states that would be massless in flat space-time, such as the graviton, photon and moduli, obtain a "tachyonic mass-shift" $-Q^2$. This gives new one-loop divergences, but at least for $Q > 0$ does not lead to classically growing modes as seen from the form of the plane-wave solutions. In fact, all known exact solutions of the coupled dilaton-moduli equations for $d > 25$ have the asymptotic behaviour (1), the moduli approaching fixed values at large times [8]. Thus moduli and other would-be massless fields only give rise to quantum instabilities. The second source of instability, on the other hand, occurs already at the classical level and is hence more severe: it corresponds to states that would be tachyons even in flat space-time. These cannot be projected out of the spectrum for generic Q , even in the heterotic and type-II supersymmetric strings, because the GSO projection necessarily involves both left- and right-movers on the world sheet [4]. Fine-tuning these tachyons to zero is unnatural and one should allow them to participate in the dynamics.

^b In the interest of simplicity we will limit most of our discussion to the bosonic string; we will comment on the heterotic and type-II cases when necessary.

^c An exception occurs when the dimension exceeds the critical one by an integer multiple of 16.

Let us therefore try to modify the solution (1) by introducing a tachyon background that we take for simplicity constant in space: $T(\tau)$. The most general background satisfying the linearized equations of motion is an arbitrary superposition of two on-shell "plane-waves":

$$T(\tau) = \mu_+ e^{\alpha_+ \tau} + \mu_- e^{\alpha_- \tau} = \epsilon_+ e^{\alpha_+ (\tau - \tau_0 + \frac{1}{2}\delta)} + \epsilon_- e^{\alpha_- (\tau - \tau_0 - \frac{1}{2}\delta)} \quad (4)$$

where:

$$\alpha_{\pm} = -(Q \pm \sqrt{Q^2 + 2}) \quad (5)$$

In equation (4) we have absorbed the two arbitrary coefficients μ_{\pm} into the choice of origin τ_0 for the conformal time, a relative "phase-shift" δ between the two waves, and the sign-factors $\epsilon_{\pm} = \text{sign}(\mu_{\pm})$. The reason for this rewriting will soon become clear. Note that, from the two-dimensional point of view, the above tachyon-background is a linear combination of the two screening operators in the conformal theory of the time-coordinate, which is a free field with a background charge. Note also that in heterotic and type-II supersymmetric strings tachyons will generally carry extra quantum numbers, a fact that should be kept in mind if one wants to discuss specific cosmological scenarios.

Clearly, one may trust the linearized approximation only when $|T| \ll 1$. This means that the linear-dilaton epoch has a finite duration of order δ , corresponding to the time the tachyon-field spends near its local potential maximum: $V(T) = V_0 - 2T^2 + \dots$, as illustrated in figure 1*. Before and after this epoch the tachyon runs away to large positive or negative values: what happens then is anyone's guess. The tachyon could transfer part of its energy to the graviton, dilaton or other backgrounds with which it interacts, or dissipate energy through particle production. One point is nevertheless worth stressing: since the tachyon background is a generalized two-dimensional cosmological constant, periods of large positive or negative T have quite different behaviour. The first tend to expel individual strings, so that the linear-dilaton epoch in case I of fig.1 would act as a "time-trap". Negative T on the other hand favours more and larger individual strings, signaling a new instability that is probably accompanied by large entropy production. One can imagine a cosmological scenario, reminiscent of chaotic inflation, where the tachyon returns near a local maximum at $T = 0$ in different parts of the Universe, and on several occasions before

* This should be contrasted to the case $d < 1$, where both screening charges have the same sign, and the linearized approximation can be trusted asymptotically at large values of the corresponding space-coordinate [9]

the (heterotic) theory settles down to some stable supersymmetric vacuum. One would thus get several linear-dilaton epochs with different values and/or sign of Q , as shown schematically in fig.2. For large values of V_0 the tachyon would roll back to positive values while later, when V_0 and hence Q become small enough, it could pass over the hill to negative values (as in case II of figure 1) dissipating its energy through entropy production.

Setting aside these amusing speculations, let us turn now to the physical interpretation of the linear-dilaton epoch which, even if one ignores the tachyon, raises some very subtle issues, particular to strings [7]. Indeed, even though the metric with normalized Einstein action $g_{\mu\nu}$ describes a linearly-expanding ($Q > 0$) or -contracting ($Q < 0$) Universe with scale factor $a = e^{-\frac{1}{2}\phi}$, the low-energy Lagrangian of say a scalar in this background reads [4]:

$$\mathcal{L}_{scal} = - \int \sqrt{g} [g^{\mu\nu} \partial_{\mu} \phi \partial_{\nu} \phi + h a^{-2} \phi^2] \quad (6)$$

with h the conformal weight of the corresponding string excitation. It follows that elementary string masses: $m = \sqrt{h} a^{-1}$ scale precisely as the (inverse) size of the Universe, and if one used them as measuring rods one would detect no net expansion at all. This point has been stressed particularly in ref. [7]. Here we want to make two comments. First, the size of the Universe *does vary*, when expressed in most other, physically sensible, units which invariably depend on the string coupling strength: $g^2 \sim e^{\phi} \sim e^{-2Q\tau} = a^{-2}$. Examples range from life-times of heavy unstable particles, to mean free paths, to sizes of bound states such as nucleons, atoms or crystals, and to vacuum expectation values of Higgs fields. In particular, our initial claim for linear expansion implicitly assumed that one used as measuring rod the only intrinsically gravitational scale, namely Newton's constant which measures the strength of gravitational interactions. The second comment we want to make is that, setting aside string interactions, the trajectories of individual particles get modified during the linear-dilaton epoch in a way that, if taken at face-value, could solve the horizon problem. A better understanding of the instabilities of this solution is, however, necessary before taking this statement seriously.

To be more precise, let us assume, for the sake of argument, that a linear-dilaton epoch is followed immediately (and perhaps preceded) by periods where the Universe is flat and static. Because of the "tachyonic mass-shift", eq. (3), phase velocities increase during the linear-dilaton epoch, as if one was entering a medium with anomalous refraction index; a typical world-line is shown in fig.3. Light in particular, as well as all

other excitations whose mass is less than Q , travel during this epoch outside the light-cone, and can get arbitrarily fast at sufficiently low frequency. This acausal behaviour, which would never occur in pure gravitational backgrounds, is due to the way in which the dilaton enters in the Lagrangian of radiation, playing the role of a time-varying Planck's constant:

$$\mathcal{L}_{rad} = \int e^{-\Phi} \sqrt{g} F_{\mu\nu} F^{\mu\nu} \quad (7)$$

This is a particularly stringy phenomenon, not shared by Brans-Dicke type theories. It is, as already discussed, probably a signal of quantum instability of our background, but if taken seriously it could solve the horizon problem, by allowing causally disconnected regions to communicate during the linear-dilaton epochs. In any case such behaviour is drastically different from what one would expect in a static Universe.

Concerning the precise rate of expansion or contraction, this depends as we mentioned above on the choice of relevant physical units, in which the size of the Universe is to be measured. In the Planck era, rather than masses of excited states, one may use their life-times, which vary typically as:

$$\delta t \sim m^{-1} g^{-2} \sim a^3$$

so that the size of the Universe contracts in these units as a^{-2} . In a radiation-dominated era, a more relevant unit may be an interaction cross-section:

$$\sigma \sim g^4 (E^{-2} \text{ or } T^{-2}) \quad (8)$$

or a mean free path $\lambda \sim \frac{1}{n}$, where $n \sim E^3 \text{ or } T^3$ is the number density of particles. Even later, when bound states are formed, one may use as measuring rod the size of the proton [10]:

$$\ln(\tau_p) \sim -\ln(\Lambda_{QCD}) \sim C/g^2 \quad (9)$$

with C a model-dependent constant. In this case the size of the Universe varies exponentially fast, since $g \sim a^{-1}$. The same conclusion would hold if one uses, instead of

* One can of course change contraction to expansion by flipping the sign of Q .

proton radii, electron Compton wavelengths, assuming that the vacuum expectation value of the Higgs field giving mass to the electron depends exponentially on g^2 as a result of some dynamical mass-generation mechanism [11]:

$$\ln(m_{el}) \sim \frac{C'}{g^2} + \dots \quad (10)$$

with C' another model-dependent constant. Finally, even more complicated behaviours can be obtained if one looks at such units as Bohr radii, transition temperatures in crystals etc. etc.

It has not escaped our attention that our cosmological string solutions may potentially exhibit several features normally associated with inflation: a possible solution to the flatness problem[†], and possible mechanisms for entropy production and the avoidance of the horizon problem. We hope to return to these issues in a future publication.

Let us finally come to the technical comments, concerning the calculation of amplitudes in the linear-dilaton epoch. In the absence of a tachyon background, there are infrared divergences at $\tau \rightarrow \pm\infty$ (for $Q < \text{or} > 0$), where both the string coupling and plane waves blow up. These divergences are regulated by a positive tachyon background, like the background I of fig.1, which effectively restricts the τ integration to the finite epoch of size δ , centered around τ_0 . Of course, we do not know how to treat the string theory exactly in such combined dilaton-tachyon backgrounds, nor do we anyway know the precise form of these backgrounds outside the above finite epoch. The next few steps we take are therefore only formal. Nevertheless, the result is, we believe, intriguing enough to merit further reflection.

First, let us imagine treating the tachyon field $T(\tau)$ of eq. (4) as a perturbation, and consider for concreteness a 3-point tree amplitude. It reads:

$$\mathcal{A}^{(3)} = \mathcal{N}^{-1} \sum \frac{\mu_{\pm}^N \mu_{\mp}^M}{N! M!} \langle V_{+}^N V_{-}^M \prod_{i=1}^3 V(i) \rangle \quad (11)$$

where \mathcal{N} is a constant proportional to the infinite volume of $SL(2, C)$, $V_{\pm} = \int d^2z : e^{\alpha_{\pm} \tau(z)} :$ are the screening operators,

[†] As discussed in detail in the first ref. [4], string theory gives new quantization conditions which may force the three-space curvature to vanish, or else be of order M_{Planck}

and $V_{(j)} = Z^{-\frac{1}{2}} \int d^2z : \mathcal{P}_{(j)}(\partial\bar{X}) e^{i\eta_{(j)} X(z)} :$ are vertex operators creating normalized external states. The factor Z ensures the correct normalization, and will turn out to be independent of the precise vertex operator considered. The external states are physical, i.e. transverse, so that the only dependence on the time coordinate $X_0 = \tau$ is in the exponential. Since the space part of the theory will play no role in the sequel, we may as well limit our discussion to external tachyons for which $\mathcal{P}_{(j)} = 1$. The perturbative expansion (11) has had infrared-divergences when one integrates over the time zero-mode. Since as we just argued these divergences are cut off in the full theory, at least for positive μ_{\pm} , let us *decree that we only keep finite terms in the above expansion*. Since each external momentum has a fixed imaginary part iQ in its zeroth component, and there is a background charge $-2iQ$ at infinity, finite terms are those for which:

$$N\alpha_+ + M\alpha_- - Q = 0 \quad (12)$$

These turn out, furthermore, to be the only $SL(2, C)$ -invariant terms, all other terms being also ultraviolet-divergent on the world-sheet.

We can rewrite condition (12) as:

$$Q^2 = \frac{2(N-M)^2}{(2N+1)(2M+1)} \quad (13)$$

so that, if n, m are the smallest positive integers satisfying it, the general solution is:

$$N = n + (2n+1)k ; \quad M = m + (2m+1)k \quad (14)$$

with k an arbitrary non-negative integer. Note incidentally that, as (13) shows, we can only define our amplitudes for certain rational values of Q . These are, however, dense in the real line, and we should expect that any physically sensible result can be continued analytically to all values of Q .

The sum in (11) reduces, with our prescription, to a sum over k . Furthermore, the energy-dependence of the amplitude does not depend on k , i.e. on the precise manner of "screening", and can thus be factored outside the sum. Hence the result takes the form:

$$\mathcal{A}^{(3)} = g_{str}(\mu_+, \mu_-, Q) \prod_{j=1}^3 F(E_{(j)}, Q) \delta(E_{(1)} + E_{(2)} + E_{(3)}) \delta^{(d)}(\vec{p}_{(1)} + \vec{p}_{(2)} + \vec{p}_{(3)}) \quad (15)$$

where the function F , calculated in [4], reads:

$$F(E, Q) = \frac{f(E; N, M)}{f(0; N, M)} \quad (16)$$

with

$$f(E; N, M) = \prod_{i=1}^N \frac{\Gamma(-M + i\alpha_+ E - i\frac{\alpha_+^2}{2})}{\Gamma(1 + M - i\alpha_- E + i\frac{\alpha_-^2}{2})} \prod_{i=1}^M \frac{\Gamma(i\alpha_+ E + i\frac{\alpha_+^2}{2})}{\Gamma(1 - i\alpha_- E - i\frac{\alpha_-^2}{2})} \quad (17)$$

and to see that it does not depend on k one must use the Gauss-Legendre multiplication formula:

$$\prod_{r=0}^{m-1} \Gamma(z + \frac{r}{m}) = (2\pi)^{\frac{1}{2}(m-1)} m^{\frac{1}{2} - mz} \Gamma(mz) \quad (18)$$

Note also that at least in the high-energy limit the Q -dependence of $F \sim |E|^{-\frac{d}{2}}$ is indeed smooth as it should be [4].

The new result we want to present here is the calculation of the energy-independent function g_{str} . The calculation is tedious but straightforward, and makes repeated use of (18), as well as of the integration formula (B10) of Dotsenko and Fateev [6]. One finds that the summation over k corresponds to a geometric series with the result:

$$g_{str}(\mu_+, \mu_-, Q) = \mu_+ \mu_- (1-A)^{-1} Z^{-\frac{1}{2}} \quad (19)$$

where

$$A = \left[\frac{\Gamma(-\varrho_+)}{\Gamma(1+\varrho_+)} \right]^{2n+1} \left[\frac{\Gamma(-\varrho_-)}{\Gamma(1+\varrho_-)} \right]^{2m+1} \quad (20)$$

Here:

$$\varrho_+ = \frac{1}{2}\alpha_+^2 = \frac{2m+1}{2n+1} ; \quad \varrho_- = \frac{1}{2}\alpha_-^2 = \frac{1}{\varrho_+} \quad (21)$$

and in (19) we have dropped a tachyon-independent constant. A similar calculation of the 2-point amplitude gives zero*, indicating that the tachyon-background does not, with our prescription, modify the mass-shell conditions. On the other hand, it does modify the norm of the two-dimensional states created by the vertex operators

* This is not trivial, since in all but the first term of the μ_{\pm} -expansion the infinite $SL(2, C)$ -volume can be eliminated by removing the integration over one screening operator.

: $e^{\mu\nu} X^\mu$: This "wave-function renormalization" can be calculated with the same methods as above. It is energy-independent and reads:

$$Z^{-1} = 1 - A \quad (22)$$

We thus finally obtain the tachyon-dependent renormalization of the string-coupling constant:

$$g_{str} = \mu_+^2 \mu_-^m (1 - A)^{\frac{1}{2}} \quad (23)$$

which can be thought of as the $d > 25$ analog of the scaling functions of $d \leq 1$ strings [1].

Substituting $\mu_\pm = \epsilon_\pm e^{\alpha_\pm(-\tau_0 \pm \frac{1}{2}\delta)}$ in eq. (23), and using eqs. (12) and (21), we see that the dependence on the conformal-time zero mode factorizes:

$$g_{str} = e^{-Q\tau_0} \tilde{g}(\delta, \epsilon_+ \epsilon_-, n, m) \quad (24)$$

This agrees with the naive expectation that the string coupling varies with the dilaton as $e^{\frac{1}{2}\phi}$. The novel feature is the function \tilde{g} , which depends on the size δ of the linear-dilaton epoch, in addition to the background charge Q . Unfortunately, this latter dependence is not smooth and cannot be continued analytically to the whole real line. In other words, \tilde{g} depends explicitly on the integers n, m , and not only through the combination (13), which makes its physical significance at best dubious. What is, however, intriguing is that g_{str} exhibits a zero at some particular value δ which does depend smoothly on Q :

$$e^{2\delta} = \epsilon_+ \epsilon_- \left[\pi \frac{\Gamma(-\varrho_+)}{\Gamma(1+\varrho_+)} \right]^{\alpha_+} \left[\pi \frac{\Gamma(-\varrho_-)}{\Gamma(1+\varrho_-)} \right]^{-\alpha_+} \quad (25)$$

This value diverges at the space-dimensions: $d = 25 + \frac{1}{2}(1-1)^2 = 25, 28, 33, \dots$. The meaning, if any, of formula (25) is unclear to us, and we may only hope that it will emerge in the future. Let us, however, repeat that the existence of dual factorizable amplitudes in these "cosmological" non-critical strings is interesting enough to merit further study.

Finally, let us point out that the linear dilaton solution is classical, and hence to trust its relevance one must assume that the string coupling was, during the corresponding epoch, small. This can of course be arranged by choosing appropriately the constant piece of the dilaton. Alternatively, assuming that the above screening prescription is physical, the theory is asymptotically free at large Q (and d) and for sufficiently high energies, and completely free for phase shifts δ obeying condition (25).

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FIGURE CAPTIONS

Fig.1 . Two possible tachyon backgrounds, corresponding to the field rolling back before reaching the local maximum at $T=0$ (case I, $\epsilon_{\pm} = +1$), or making it over the hill (case II, $\epsilon_{+} = -\epsilon_{-} = +1$). The first traps individual strings in the linear-dilaton epoch: $-\frac{\delta}{2} < \tau < \frac{\delta}{2}$, while the latter may lead to big entropy production.

Fig.2 . Schematic drawing of the possibility of several linear-dilaton epochs: the tachyon approaches the crest at $T=0$ on different occasions and for different values of the other matter fields. The broken line indicates our ignorance of the large-tachyon dynamics.

Fig.3 . Trajectory of a particle or light-ray, when a linear-dilaton epoch is sandwiched between periods of flat static universes. The broken line extrapolates the trajectory as if no linear-dilaton epoch has occurred. Light rays, in particular, travel during this epoch outside the light cone. Taken at face value this could solve the horizon problem.

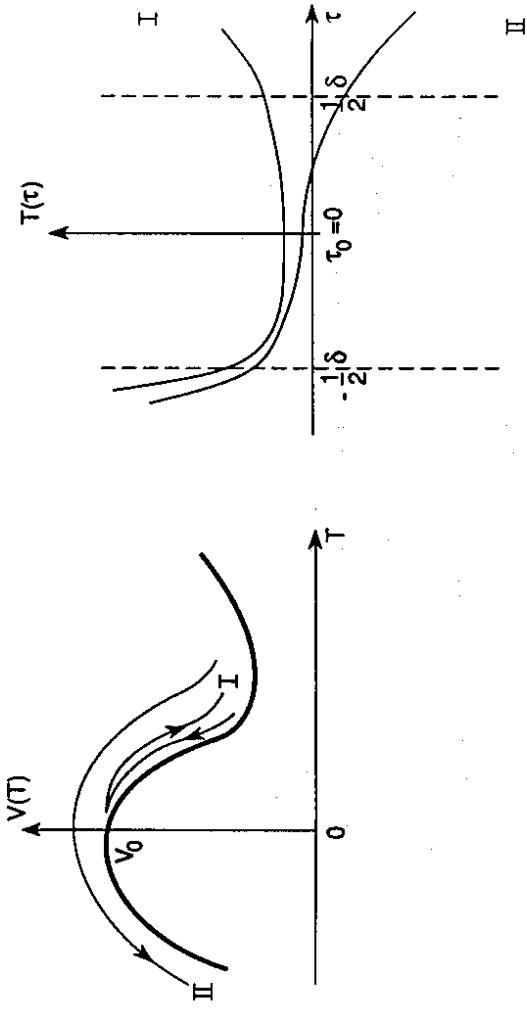


FIGURE 1

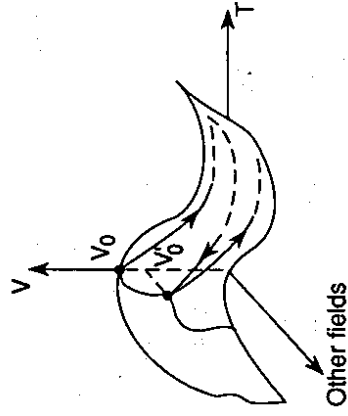


FIGURE 2

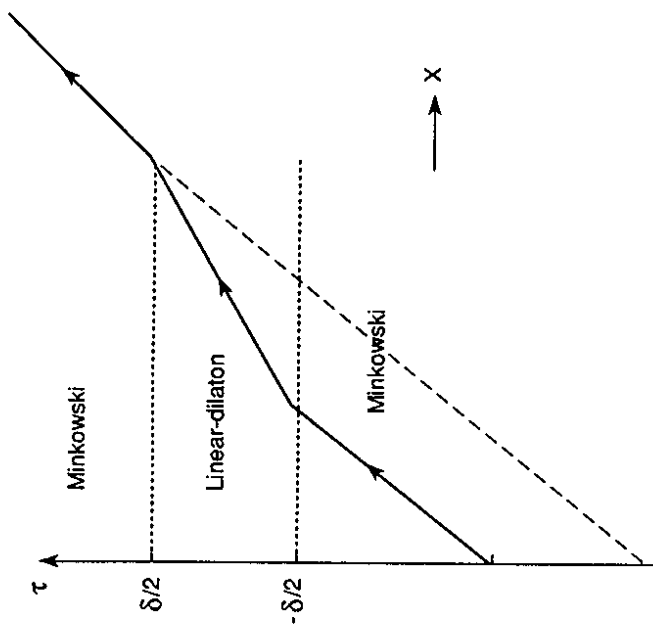


FIGURE 3