Shape fluctuations in a Fermi system with nonlinear dissipativity

V.M. Kolomietz[1,2], S.V. Lukyanov[1] and S. Shlomo[2]

[1] *Institute for Nuclear Research,*

Prosp. Nauki 47, 03680 Kiev,

Ukraine and

[2] *Cyclotron Institute,*

Texas A&M University, College Station,

Texas 77843

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Abstract

The contribution of thermal fluctuations to the widths of isoscalar giant multipole resonances (GMR) in heated nuclei is studied. Starting from the collisional kinetic equation, it is shown that an additional contribution to the nuclear friction and the corresponding GMR widths arises due to the nonlinear dissipativity effect. It is also shown that the magnitude of the contributions of the thermal fluctuations to the nuclear friction coefficient and the GMR widths do not exceed $\sim 20\%$.

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I. INTRODUCTION

In general, the damping of collective excitations in a cold finite Fermi system, e.g. the width of a giant multipole resonance (GMR) in cold nuclei, is determined by the two-body collisions, the particle-hole energy fragmentation (Landau damping) and the escape width. The damping in cold nuclei was intensively investigated in both the quantum (RPA like) [1, 2, 3, 4, 5] and the semiclassical (kinetic theory) approaches [6, 7, 8, 9, 10, 11, 12]. The collisional damping is due to the coupling of particle-hole excitations to more complicated states. In the kinetic theory, this type of damping is simulated by the collision integral and leads to a collisional component of the intrinsic width of the collective eigenstates. The fragmentation width is caused by the interaction of particles with the time-dependent self-consistent mean field. This contribution to the intrinsic width does not reflect a motion of the system toward the thermal equilibrium but indicates rather a redistribution of the particle-hole excitations in the vicinity of the collective state. In a hot system an additional contribution to the damping of the collective excitations arises through thermodynamic fluctuations of the corresponding collective variables because of the fluctuation-dissipation theorem. In this context, one of the most important open problem is the behavior of the GMR width in hot nuclei as a function of the temperature T. There are two essential different sources for the T-dependence of the GMR width. The first one is given by the thermal contribution to the damping width from an increasing nucleon-nucleon collision rate ($2p2h$ excitations) plus a Landau spreading due to thermally allowed ph transitions [3, 4, 13, 14, 15, 16]. In the second one a temperature increasing of the width is caused by the coupling of the GDR to the thermal shape fluctuations of the nucleus [17, 18, 19]. In the present work, we study a new effect of the influence of the thermal shape fluctuations of the nucleus on the damping of the collective motion caused by the nonlinear dissipativity appearing in the higher order variations of the collision integral. We point out that in the commonly used linear order of the variation of the collision integral, the thermal fluctuations do not lead to dissipation (viscosity) in the macroscopic equations of motion because the following ensemble smearing of the kinetic equation washes out the fluctuation terms from the final macroscopic equations of motion. This paper is organized as follows. In section II we suggest a proof of the Langevin equation for nuclear local variables (particle density, velocity field and pressure tensor), starting from the collisional kinetic equation. We perform a high

order expansion of the collisional integral and derive the non-Markovian pressure tensor to the Navier–Stokes-like equation of motion. In section III we carry out the ensemble averaging and reduce the local equations of motion to the macroscopic form and derive the macroscopic response function. In the derivations, the main features of the dynamical distortion of the Fermi surface are taken into account. Results of numerical calculations are presented in section IV. We conclude and summarize in section V. The Appendixes provide a derivation of the high order variations of the collision integral with respect to the variation of the phase space distribution function.

II. LOCAL EQUATIONS OF MOTION

We will start from the collisional kinetic equation in the following form [20]

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\frac{\partial f}{\partial \vec{r}} - \frac{\partial V}{\partial \vec{r}}\frac{\partial f}{\partial \vec{p}} + \vec{F}_{\rm ext}\frac{\partial f}{\partial \vec{p}} = \delta{\rm St}[f] + y. \tag{1}$$

Here, $f \equiv f(\vec{r},\vec{p},t)$ is the Wigner distribution function, $V \equiv V(\vec{r},\vec{p},t)$ is the self-consistent mean field, $\vec{F}_{\rm ext}$ is the external driving force and $\delta{\rm St}[f]$ is the collision integral which takes into account the memory effects [21]. The random variable $y \equiv y(\vec{r},\vec{p},t)$ in Eq. (1) represents the random force. As such, its ensemble average vanishes, $\langle y\rangle = 0$. To reduce Eq. (1) to closed equations of motion for the macroscopic collective variables we will follow the nuclear fluid dynamic approach [22, 23, 24, 25, 26] and take into account the dynamic Fermi-surface distortion up to multipolarity $l = 2$. Evaluating the first three moments of Eq. (1) in the $\vec{p}$ space, we reduce Eq. (1) to the hydrodynamic-like equations of motion for particle density ρ, velocity field $\vec{u}$ and the pressure tensor $P_{\alpha\beta}$ (for details, see Ref. [27])

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial r_\nu}\rho u_\nu, \tag{2}$$

$$\frac{\partial}{\partial t}m\rho u_\alpha + \frac{\partial}{\partial r_\nu}m\rho u_\nu u_\alpha + \frac{\partial}{\partial r_\nu}P_{\nu\alpha} + \rho\frac{\partial}{\partial r_\alpha}V - \rho F_{{\rm ext},\alpha} = 0, \tag{3}$$

and

$$\frac{\partial}{\partial t}P_{\alpha\beta} + \frac{\partial}{\partial r_\nu}u_\nu P_{\alpha\beta} + P_{\nu\beta}\frac{\partial}{\partial r_\nu}u_\alpha + P_{\nu\alpha}\frac{\partial}{\partial r_\nu}u_\beta = Q_{\alpha\beta} + y_{\alpha\beta}. \tag{4}$$

Here,

$$\rho = \int \frac{gd\vec{p}}{(2\pi\hbar)^3}f, \quad \vec{u} = \frac{1}{\rho}\int \frac{gd\vec{p}}{(2\pi\hbar)^3}\frac{\vec{p}}{m}f, \quad P_{\alpha\beta} = \frac{1}{m}\int \frac{gd\vec{p}}{(2\pi\hbar)^3}(p_\alpha - mu_\alpha)(p_\beta - mu_\beta)f, \tag{5}$$

$g = 4$ is the spin-isospin degeneracy factor, $Q_{\alpha\beta}$ is associated with dissipative processes

$$Q_{\alpha\beta} = \frac{1}{m}\int \frac{gd\vec{p}}{(2\pi\hbar)^3}(p_\alpha - mu_\alpha)(p_\beta - mu_\beta)\delta\mathrm{St}[f], \tag{6}$$

and $y_{\alpha\beta}$ gives the contribution from the random force

$$y_{\alpha\beta} = \frac{1}{m}\int \frac{gd\vec{p}}{(2\pi\hbar)^3}\; p_\alpha p_\beta\; y. \tag{7}$$

In Eqs. (2) to (4) and in the following expressions, repeated Greek indices are to be understood as summed over. The pressure tensor $P_{\alpha\beta}$ can be written as

$$P_{\alpha\beta} = P_{\mathrm{eq}}\delta_{\alpha\beta} + P'_{\alpha\beta}, \tag{8}$$

where

$$P'_{\alpha\beta} = \frac{1}{m}\int \frac{gd\vec{p}}{(2\pi\hbar)^3}(p_\alpha - mu_\alpha)(p_\beta - mu_\beta)\delta f_2, \tag{9}$$

$\delta f_2 \equiv \delta f_2(\mathbf{r}, \mathbf{p}; t)$ is the small quadrupole deviation of the distribution function f from the one in equilibrium, $f_{\mathrm{eq}} \equiv f_{\mathrm{eq}}(\vec{r}, \vec{p})$, and P_{eq} is the pressure due to the Fermi motion of the nucleons in the ground state of the nucleus. Assuming the Thomas-Fermi approximation for f_{eq}, one has

$$P_{\mathrm{eq}} = \frac{2}{3}\int \frac{gd\vec{p}}{(2\pi\hbar)^3}\frac{p^2}{2m}f_{\mathrm{eq}} = \frac{2}{5}\frac{\hbar^2}{m}\left(\frac{6\pi^2}{g}\right)^{2/3}\rho_{\mathrm{eq}}^{5/3},$$

where $\rho_{\mathrm{eq}} \equiv \rho_{\mathrm{eq}}(\vec{r})$ is the particle density in equilibrium. Using Eq. (8) we will rewrite Eq. (4) as

$$\frac{\partial}{\partial t}P'_{\alpha\beta} + P_{\mathrm{eq}}\Lambda_{\alpha\beta} + \hat{L}P'_{\alpha\beta} = Q_{\alpha\beta} + y_{\alpha\beta}, \tag{10}$$

where

$$\Lambda_{\alpha\beta} = \frac{\partial u_\alpha}{\partial r_\beta} + \frac{\partial u_\beta}{\partial r_\alpha} - \frac{2}{3}\delta_{\alpha\beta}\frac{\partial u_\nu}{\partial r_\nu}, \tag{11}$$

and the operator $\hat{L}$ is defined by

$$\hat{L}P'_{\alpha\beta} = u_\nu\frac{\partial}{\partial r_\nu}P'_{\alpha\beta} + P'_{\alpha\beta}\frac{\partial u_\nu}{\partial r_\nu} + P'_{\alpha\nu}\frac{\partial u_\beta}{\partial r_\nu} + P'_{\beta\nu}\frac{\partial u_\alpha}{\partial r_\nu}. \tag{12}$$

To evaluate the tensor $Q_{\alpha\beta}$ in Eq. (10) we will use the collision integral in the following general form [20]

$$\delta\mathrm{St}[f] = \int \frac{g^2 d\vec{p}_2 d\vec{p}_3 d\vec{p}_4}{(2\pi\hbar)^6} w(\{\vec{p}_j\})Q(\{f_j\})\;\delta(\Delta\epsilon)\delta(\Delta\vec{p}), \tag{13}$$

where $w(\{\vec{p}_j\}) \equiv w(\vec{p}_1, \vec{p}_2; \vec{p}_3, \vec{p}_4)$ is the spin-isospin averaged probability for two-body scattering, $Q(\{f_j\}) = f_1 f_2(1 - f_3)(1 - f_4) - (1 - f_1)(1 - f_2)f_3 f_4$ is the Pauli blocking factor,

$\Delta\vec{p} = \vec{p}_1 + \vec{p}_2 - \vec{p}_3 - \vec{p}_4$ and $\Delta\epsilon = \epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4$, with $\epsilon_j = p_j^2/2m + V(r_j)$ being the single-particle energy. In performing the variation of the Pauli blocking factor $Q(\{f_j\})$ with respect to δf , we will keep all the terms, up to the third order in δf. The collision integral then takes the following form (see Appendix A)

$$\delta\mathrm{St} = \delta\mathrm{St}_1 + \delta\mathrm{St}_2 + \delta\mathrm{St}_3, \tag{14}$$

where $\delta\mathrm{St}_n$ is the variation of the collision integral $\delta\mathrm{St}$ in the n-th order of δf. Considering Eqs. (6) and (14), the tensor $Q_{\alpha\beta}$ can be written as

$$Q_{\alpha\beta} = Q^{(1)}_{\alpha\beta} + Q^{(2)}_{\alpha\beta} + Q^{(3)}_{\alpha\beta}, \tag{15}$$

where $Q^{(n)}_{\alpha\beta}$ is due to the contribution of $\delta\mathrm{St}_n$ in Eq. (6). The first order term $Q^{(1)}_{\alpha\beta}$ is simplified as [28]

$$Q^{(1)}_{\alpha\beta} = -\frac{1}{\tau_2} P'_{\alpha\beta}, \tag{16}$$

where τ_2 is the two-body relaxation time in the case of quadrupole deformation of the Fermi surface. The higher order terms $Q^{(2)}_{\alpha\beta}$ and $Q^{(3)}_{\alpha\beta}$ can be reduced to the following forms (see Appendix A, Eqs. (A18) and (A30))

$$Q^{(2)}_{\alpha\beta} = \frac{mP'_0}{\zeta}\, P'_{\alpha\beta}, \quad Q^{(3)}_{\alpha\beta} = \frac{m^2 P'^2_0}{\xi} P'_{\alpha\beta}, \tag{17}$$

where the quantities ζ and ξ are deduced from the collision integral (see Appendix A, Eqs. (A19) and (A31)) and

$$P'_0 = \frac{1}{2}(P'_{xx} + P'_{yy} - P'_{zz}). \tag{18}$$

To simplify the derivations, we will introduce the operator $\hat{N}$ as

$$\hat{N} P'_{\alpha\beta} = Q^{(2)}_{\alpha\beta} + Q^{(3)}_{\alpha\beta}. \tag{19}$$

Equation (10) is then rewritten as

$$\frac{\partial}{\partial t} P'_{\alpha\beta} + P_{\mathrm{eq}}\Lambda_{\alpha\beta} + \frac{1}{\tau_2} P'_{\alpha\beta} + \hat{L} P'_{\alpha\beta} = \hat{N} P'_{\alpha\beta} + y_{\alpha\beta}. \tag{20}$$

We will look for a solution of Eq. (20) in the following form

$$P'_{\alpha\beta} = P'^{(0)}_{\alpha\beta} + P'^{(NL)}_{\alpha\beta}. \tag{21}$$

Here, the tensor $P'^{(0)}_{\alpha\beta}$ is obtained as a solution to the following linear differential equation [29]

$$\frac{\partial}{\partial t}P'^{(0)}_{\alpha\beta} + P_{\rm eq}\Lambda_{\alpha\beta} + \frac{1}{\tau_2}P'^{(0)}_{\alpha\beta} = y_{\alpha\beta}, \tag{22}$$

and it is given by the following non-Markovian form

$$P'^{(0)}_{\alpha\beta}(t) = -\int_{-\infty}^{t} dt' \ \exp\left(\frac{t'-t}{\tau_2}\right)\left[P_{\rm eq}\ \Lambda_{\alpha\beta}(t') - y_{\alpha\beta}(t')\right]. \tag{23}$$

The tensor $P'^{(NL)}_{\alpha\beta}$ in Eq. (21) satisfies the nonlinear differential equation

$$\frac{\partial}{\partial t}P'^{(NL)}_{\alpha\beta} + \frac{1}{\tau_2}P'^{(NL)}_{\alpha\beta} + \hat{L}P'_{\alpha\beta} - \hat{N}P'_{\alpha\beta} = 0. \tag{24}$$

To solve Eq. (24), we will use the iteration procedure. The first order iteration $P'^{(NL)}_{\alpha\beta,1}(t')$ to Eq. (24) reads

$$P'^{(NL)}_{\alpha\beta,1}(t) = \int_{-\infty}^{t} dt' \ \exp\left(\frac{t'-t}{\tau_2}\right)\ \left[\hat{N}P'^{(0)}_{\alpha\beta}(t') - \hat{L}P'^{(0)}_{\alpha\beta}(t')\right], \tag{25}$$

and the second iteration is given by

$$P'^{(NL)}_{\alpha\beta,2}(t) = \int_{-\infty}^{t} dt' \ \exp\left(\frac{t'-t}{\tau_2}\right)$$
$$\times\left[\hat{N}\left(P'^{(0)}_{\alpha\beta}(t') + P'^{(NL)}_{\alpha\beta,1}(t')\right) - \hat{L}\left(P'^{(0)}_{\alpha\beta}(t') + P'^{(NL)}_{\alpha\beta,1}(t')\right)\right]. \tag{26}$$

Below, we will apply Eqs. (2) to (4) to the small-amplitude vibrations of the particle density $\delta\rho$ near the equilibrium. We point out that we do not assume the velocity field $\vec{u}$ to be small. Finally, taking into account the above mentioned derivations we will rewrite Eq. (3) as

$$m\rho_{\rm eq}\frac{\partial u_\alpha}{\partial t} + m\rho_{\rm eq}u_\nu\frac{\partial u_\alpha}{\partial r_\nu} + \rho_{\rm eq}\frac{\partial}{\partial r_\alpha}\left[\left.\frac{\delta^2\varepsilon}{\delta\rho^2}\right|_{\rm eq}\delta\rho\right] + \frac{\partial P'^{(0)}_{\alpha\beta}}{\partial r_\alpha} + \frac{\partial P'^{(NL)}_{\alpha\beta}}{\partial r_\alpha} - \rho_{\rm eq}F_{{\rm ext},\alpha} = 0 \tag{27}$$

where ε is the particle energy density.

III. ENSEMBLE AVERAGING AND MACROSCOPIC RESPONSE

Let us introduce the displacement field $\vec{\chi}$ related to the velocity field $\vec{u}$ by $\vec{u}(\vec{r},t) = \dot{\vec{\chi}}(\vec{r},t)$, where the dot denotes a time derivative. For the displacement field we will assume the following separable form $\vec{\chi}(\vec{r},t) = \beta(t)\vec{v}(\vec{r})$. Using this separable form of $\vec{\chi}(\vec{r},t)$, we reduce Eq. (27) to the equation of motion for the collective variable $\beta(t)$ in the presence of the

external field $\mathcal{F}_{\rm ext}(t)$ and the random force $\widetilde{y}(t)$ (see Appendix C, Eq. (C1)). Below, we will look for the response of a nucleus to the periodic external field $\mathcal{F}_{\rm ext}(t) = \mathcal{F}_\omega \exp(i\omega t)$. Because of the random force $\widetilde{y}(t)$ in Eq. (C1), we will separate the description of the collective motion into two parts with $\beta(t) = \widetilde{\beta}(t) + \delta\beta(t)$. The first motion is related to the driving force $\mathcal{F}_{\rm ext}(t)$ and it is associated with the velocity $\dot{\widetilde{\beta}}$. The second one is due to the random force $\widetilde{y}(t)$ with the velocity $\delta\dot{\beta}$. We will assume that $|\delta\dot{\beta}| \gg |\dot{\widetilde{\beta}}|$. Performing the ensemble averaging, one can write

$$\left\langle \dot{\beta}(t_1)\dot{\beta}(t_2)\dot{\beta}(t_3)\right\rangle \approx \dot{\widetilde{\beta}}(t_1)\left\langle \delta\dot{\beta}(t_2)\ \delta\dot{\beta}(t_3)\right\rangle + \dot{\widetilde{\beta}}(t_2)\left\langle \delta\dot{\beta}(t_1)\ \delta\dot{\beta}(t_3)\right\rangle + \dot{\widetilde{\beta}}(t_3)\left\langle \delta\dot{\beta}(t_2)\ \delta\dot{\beta}(t_1)\right\rangle . \tag{28}$$

We will also assume the following ergodic property for the correlation function $\left\langle \delta\dot{\beta}(t)\ \delta\dot{\beta}(t')\right\rangle$, see Ref. [30], Ch.12,

$$\left\langle \delta\dot{\beta}(t)\ \delta\dot{\beta}(t')\right\rangle = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\left(\delta\dot{\beta}^2\right)_\omega e^{-i\omega(t-t')}. \tag{29}$$

The Fourier component, $\left(\delta\dot{\beta}^2\right)_\omega$, of the correlation function is governed by the correlation properties of the random force $\widetilde{y}(t)$, see below. The macroscopic equation of motion (C1) is significantly simplified in the case of a Fermi distribution for the equilibrium distribution function $f_{\rm eq}$

$$f_{\rm eq} = \left[1 + \exp\left(\frac{\epsilon - \epsilon_F}{T}\right)\right]^{-1}, \tag{30}$$

where ϵ_F is the Fermi energy. In this case, one obtains from Eqs. (A19) that $1/\zeta \ll 1$ (see also Figs. 1 and 2) and the contribution of the terms with A_2, A_3 and A_5 in Eq. (C1) is negligible. Performing the ensemble averaging of Eq. (C1), using Eq. (28) and $\langle y(t)\rangle = 0$ and assuming $\langle\beta(t)\rangle = \widetilde{\beta}(t) = \widetilde{\beta}_\omega \exp(i\omega t)$, we reduce Eq. (C1) to the following form

$$-B\omega^2\widetilde{\beta}_\omega + C_{LDM}\widetilde{\beta}_\omega + \frac{i\omega\tau_2}{1+i\omega\tau_2}\left(A_0 + \tau_2^2 A_4\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\left(\delta\dot{\beta}^2\right)_{\omega'}}{1+i\omega'\tau_2} + \frac{4\tau_2^3 A_1}{1+i\omega\tau_2}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\left(\delta\dot{\beta}^2\right)_{\omega'}}{1+(\omega'\tau_2)^2}\right.$$
$$\left. + \tau_2^2 A_4\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\left(\delta\dot{\beta}^2\right)_{\omega'}}{1+i\omega'\tau_2}\frac{1}{1+i(\omega'+\omega)\tau_2} + \frac{\tau_2^2 A_4}{1+i\omega\tau_2}\int_{-\infty}^{\infty}\frac{d\omega'}{2\pi}\frac{\left(\delta\dot{\beta}^2\right)_{\omega'}}{1+i(\omega'+\omega)\tau_2}\right)\widetilde{\beta}_\omega = B\mathcal{F}_\omega. \tag{31}$$

Considering the nuclear isoscalar quadrupole mode, we will assume an irrotational motion with the displacement field $\vec{v}(\vec{r})$ given by [31]

$$\vec{v}(\vec{r}) = \vec{\nabla}\ (r^2 Y_{20}(\hat{r}))/2, \tag{32}$$

and the time dependent radius of the nucleus given by

$$R(t) = R_0 \left[1 + \widetilde{\beta}(t) Y_{20}(\hat{r})\right]. \tag{33}$$

In this particular case the calculation of the coefficients A_1 and A_4 from Eqs. (C4) and (C7) gives: $A_1 = 16A_0(mP_{\rm eq})^2/\xi$ and $A_4 = 12A_0$. The mass coefficient B of Eq. (C2) for the displacement field of Eq. (32) is given by

$$B = \frac{3}{8\pi} A m R_0^2, \tag{34}$$

where A is the nuclear mass number. Let us introduce the collective response function $\chi(\omega)$ as

$$\widetilde{\beta}_\omega = \chi(\omega)\ \mathcal{F}_\omega. \tag{35}$$

Using Eqs. (31) and (33), we obtain from Eq. (35)

$$\chi^{-1}(\omega) = -\omega^2 + \omega_0^2 + i\omega\ \gamma_0 + 12\tau_2^2 \frac{A_0}{B} \frac{i\omega\tau_2}{1 + i\omega\tau_2} K(\omega). \tag{36}$$

Here,

$$\omega_0 = \sqrt{\frac{C_{LDM} + C'(\omega)}{B}}, \qquad \gamma_0 = \frac{A_0}{B} \frac{\tau_2}{1 + (\omega\tau_2)^2} \tag{37}$$

and

$$K(\omega) = \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \frac{\left(\delta\dot{\beta}^2\right)_{\omega'}}{1 + i\omega'\tau_2} \left(1 + \frac{1}{1 + i(\omega' + \omega)\tau_2} + \frac{4}{1 + i\omega\tau_2} \frac{\tau_2/\tau''}{1 - i\omega'\tau_2} + \frac{1}{1 + i\omega\tau_2} \frac{1 + i\omega'\tau_2}{1 + i(\omega' + \omega)\tau_2}\right). \tag{38}$$

We have also used the following notations

$$C'(\omega) = A_0 \frac{(\omega\tau_2)^2}{1 + (\omega\tau_2)^2} \tag{39}$$

and

$$\tau'' = \xi/(mP_{\rm eq})^2.$$

We point out that the additional contribution to the stiffness coefficient $C'(\omega)$ in Eq. (37) is caused by the distortion of the Fermi surface [29]. The expression (36) can be rewritten as

$$\chi^{-1}(\omega) = (\omega_0^2 + \Delta\omega_0^2 - \omega^2) + i\omega\ (\gamma_0 + \Delta\gamma), \tag{40}$$

where we have introduced the following notations for the additional components of the relaxation coefficient and the squared frequency ω

$$\frac{\Delta\gamma}{\gamma_0} = 12\tau_2^2 \left\{ \mathrm{Re}K(\omega) + \omega\tau_2 \mathrm{Im}K(\omega) \right\}, \tag{41}$$

$$\frac{\Delta\omega_0^2}{\omega_0^2} = 12\tau_2^2 \left\{ \mathrm{Re}K(\omega) - \frac{1}{\omega\tau_2}\mathrm{Im}K(\omega) \right\} \left(1 + \frac{C_{LDM}}{C'(\omega)} \right)^{-1}. \tag{42}$$

Note that above, ω is real. Finally, the macroscopic strength function $S(\omega) = -\mathrm{Im}\chi(\omega)$ is given by

$$S(\omega) = \frac{(\gamma_0 + \Delta\gamma)\omega}{(\omega_0^2 + \Delta\omega_0^2 - \omega^2)^2 + (\gamma_0 + \Delta\gamma)^2\omega^2}. \tag{43}$$

Both the additional spreading $\Delta\gamma$ and the resonance shift $\Delta\omega_0$ appear in the strength function (43) due to the nonlinear dissipativity effect.

IV. NUMERICAL RESULTS AND DISCUSSION

We have performed the numerical calculations assuming a Fermi distribution for the equilibrium distribution function of Eq. (30) and adopting the Fermi energy $\epsilon_F = 39$ MeV and the nuclear radius $R_0 = r_0 A^{1/3}$ with $r_0 = 1.12$ fm. The higher order relaxation parameters $\tau' = \zeta/mP_{\mathrm{eq}}$ and $\tau'' = \xi/(mP_{\mathrm{eq}})^2$ are related to the collision integral and can be evaluated using Eqs. (A19) and (A31) from Appendix A. We point out that in the limit of a cold nucleus, $T \to 0$ and $f_{\mathrm{eq}} = \Theta(\epsilon_F - \epsilon)$, the corrections τ' and τ'' take the following simple form

$$\frac{1}{\tau'} = 0, \quad \frac{1}{\tau''} \approx 1.5\ \frac{m^3 P_{\mathrm{eq}}^2 w_0}{p_F^6}, \tag{44}$$

where p_F is the nucleon Fermi momentum and the scattering probability $w_0 = 15\pi^2\hbar^5/m^3 g\alpha$ is related to the in-medium cross section σ_{in} of nucleon-nucleon scattering. We use $\alpha = 9.2$ MeV from [14], which corresponds to $\sigma_{\mathrm{in}} \approx \sigma_{\mathrm{free}}/2$, where $\sigma_{\mathrm{free}} \approx 40$ mb is the cross section for the nucleon-nucleon scattering in free space. Note that both relaxation parameters τ' and τ'' can not be directly interpreted as the corrections to the observable relaxation time. In particular, the value of $1/\tau''$ does not equal to zero in the ground state of the nucleus. The relaxation parameters τ' and τ'' determine the contribution of the viscous tensors $Q^{(2)}_{\alpha\beta}$ and $Q^{(3)}_{\alpha\beta}$ (see Eq. (17)) to the local equations of motion and both tensors $Q^{(2)}_{\alpha\beta}$ and $Q^{(3)}_{\alpha\beta}$ disappear in the ground state. We also point out that the relaxation parameters τ' and τ'' as well as

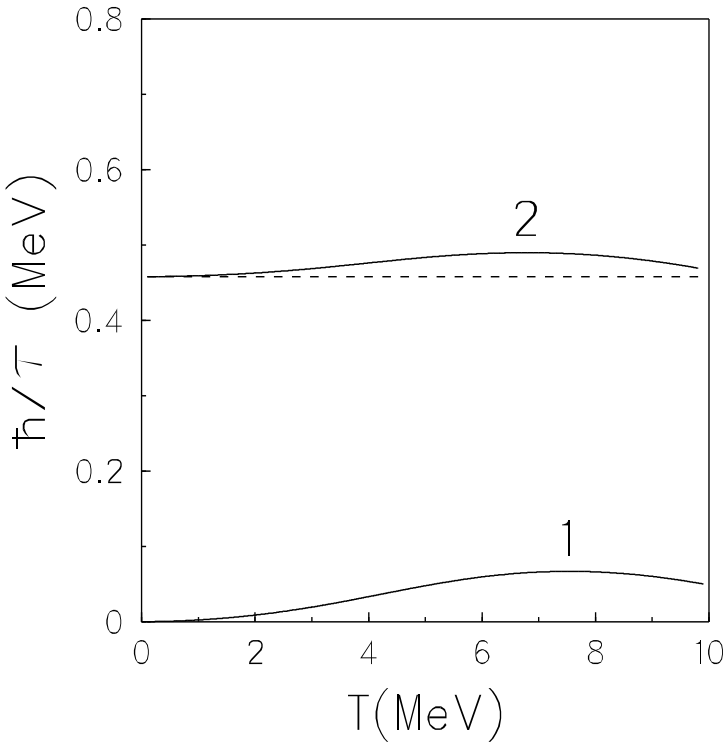

FIG. 1: Temperature dependence of $\hbar/\tau'$ (solid curve 1) and $\hbar/\tau''$ (solid curve 2) for the case of the temperature-dependent Fermi distribution function (30). The dashed line is the calculation of $\hbar/\tau''$ with the sharp Thomas-Fermi distribution function $\Theta(\epsilon_F - \epsilon)$.

ζ and ξ depend on the nuclear mean field potential V due to the space integrals r_{ij} and r_{ijk}, see Eqs. (A10) and (A21). This dependence appears after the Abrikosov-Khalatnikov transformation (A7) in the collision integral $\delta\mathrm{St}[f]$. However, due to the presence of the strongly picked functions $\partial f_{\mathrm{eq},i}/\partial\epsilon_i$, at $\epsilon = \epsilon_F$, in Eqs. (A10) and (A21) the final results for ζ and ξ are not sensitive to the specific choice of the mean field potential V at $T \ll \epsilon_F$. In Fig. 1, we have plotted the results of calculations of the quantities $\hbar/\tau'$ (solid curve 1) and $\hbar/\tau''$ (solid curve 2) as functions of temperature, T, for the nucleus with $A = 224$.

Both quantities $\hbar/\tau'$ and $\hbar/\tau''$ show a very broad and weak maximum. The magnitude of the maximum does not exceed the value of 0.07 MeV for $\hbar/\tau'$ and 0.49 MeV for $\hbar/\tau''$. The dashed line in Fig. 1 corresponds to the value of $\hbar/\tau''$ from Eq. (44). We can see from Fig. 1 and Eq. (44) that the simplest Thomas-Fermi distribution function $\Theta(\epsilon_F - \epsilon)$, with $\hbar/\tau'$ and $\hbar/\tau''$ from Eq. (44), provides a good description of both quantities $\hbar/\tau'$ and $\hbar/\tau''$. Fig. 2 shows the ratio of the collisional relaxation time τ_2 to both relaxation parameters τ'

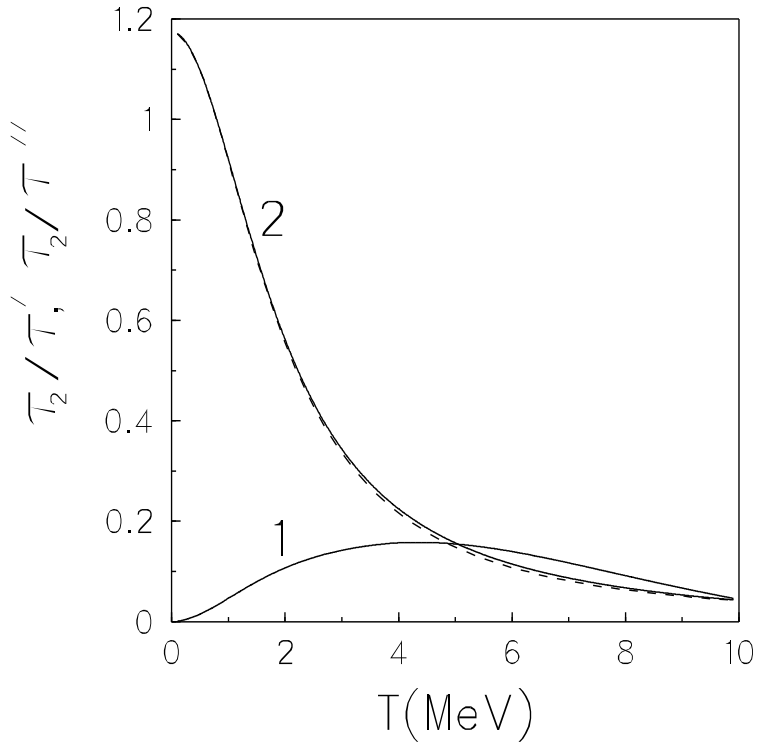

FIG. 2: Same as in Fig. 1 but for the ratio τ_2/τ' multiplied by the factor 10 (solid curve 1) and the ratio τ_2/τ'' (dashed and solid curves 2).

and τ''. For the relaxation time τ_2, we have used the expression from Ref. [14] which takes into account the memory effects. Namely,

$$\tau_2 = \frac{4\pi^2 \alpha \hbar}{(\hbar\omega_0)^2 + 4\pi^2 T^2}. \tag{45}$$

As seen from Fig. 2, the value of τ_2/τ' is relatively small over the entire range of the temperature. The value of τ_2/τ'' decreases with the temperature monotonically starting from $\tau_2/\tau'' = 1.16$ at zero temperature.

Let us now carry out a numerical study of the additional contribution to the friction coefficient, $\Delta\gamma$, caused by the nonlinear dissipativity, see Eqs. (40) and (41) and the corresponding contribution to the width Γ of the isoscalar giant quadrupole resonance (GQR). To apply Eqs. (38) and (41), we have to derive the spectral correlation function $\left(\delta\dot{\beta}^2\right)_\omega$. Using the correlation properties of the random force [30]

$$(\widetilde{y}^2)_\omega = \frac{2\gamma_0 T}{B}, \tag{46}$$

we obtain according to the fluctuation-dissipation theorem the following result [32]

$$\left(\delta\dot{\beta}^2\right)_\omega = \frac{2\mathcal{D}\,\omega^2}{(\omega_0^2-\omega^2)^2+\gamma_0^2\omega^2}, \tag{47}$$

where $\mathcal{D}$ is the diffusion coefficient

$$\mathcal{D} = \frac{\gamma_0 T}{B}. \tag{48}$$

To evaluate the relative contribution to the collisional width Γ from $\Delta\gamma$, we will start from the usual case with $\Delta\gamma = 0$. In this case, the width Γ of the GQR can be obtained from the solution in the form $\omega = \mathrm{Re}\,\omega + i\Gamma/2\hbar$ to the following secular equation, see Eq. (43),

$$(\omega_0^2-\omega^2)^2+\gamma_0^2\omega^2 = 0. \tag{49}$$

For the numerical solution of Eq. (49), we have used in Eq. (37) the liquid drop stiffness coefficient C_{LDM} in the form [33]

$$C_{LDM} = \frac{1}{4\pi}(L-1)(L+2)b_S A^{2/3} - \frac{5}{2\pi}\frac{L-1}{2L+1}b_C\frac{Z^2}{A^{1/3}}, \tag{50}$$

where $b_S = 17.2$ MeV and $b_C = 0.7$ MeV are respectively, the surface and Coulomb energy coefficients appearing in the nuclear mass formula.

Fig. 3 shows the results of the numerical solution of Eq. (49) for the nucleus with $A = 224$ and $Z = A(1-6\cdot 10^{-3}A^{2/3})/2 = 87$, which corresponds to the valley of beta-stability [33]. The energy of the collective excitation $E = \hbar\,\mathrm{Re}\,\omega$ decreases with temperature and approaches the hydrodynamic (liquid drop model) limit $E_{LDM} = \hbar\sqrt{C_{LDM}/B}$ at high temperatures. In Fig. 4 we have plotted the temperature dependence of the parameter $E\tau_2/\hbar$ which determines the sound regime: $E\tau_2/\hbar \gg 1$ for the zero sound (rare collision regime) and $E\tau_2/\hbar \ll 1$ for the first sound (frequent collision regime).

The solid curve in Fig. 4 corresponds to the calculation with the temperature dependence of E given by Fig. 3. For the dashed line, the phenomenological parametrization of the GQR energy $E = E_R = 60\cdot A^{-1/3}$ MeV was used. Using Eqs. (38), (41), (47) and (48), one can evaluate the contribution $\Delta\gamma$ to the friction coefficient due to the nonlinear dissipativity. In Fig. 5, the value of $\Delta\gamma/\gamma_0$ is shown as a function of temperature. The ratio $\Delta\gamma/\gamma_0$ equals to zero at $T = 0$ because $\Delta\gamma$ appears due to the thermodynamical fluctuations of the collective variable β. The ratio $\Delta\gamma/\gamma_0$ increases with temperature and reaches a maximum value, which does not exceed ≈ 0.2.

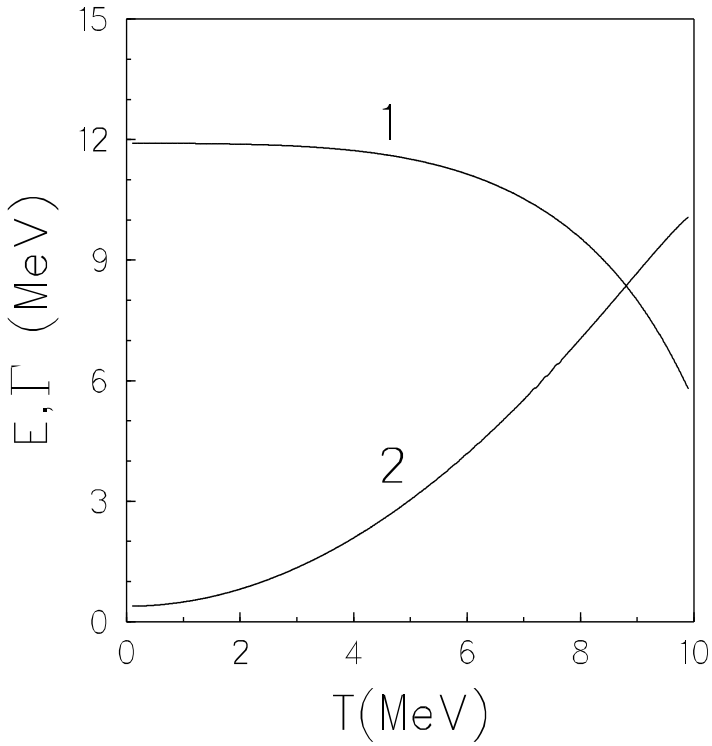

FIG. 3: Energy $E = \hbar\,\mathrm{Re}\,\omega$ of the isoscalar giant quadrupole resonance (GQR) and the corresponding collisional width Γ for the nucleus with $A = 224$ as obtained from Eq. (49).

In the high-temperature region the ratio $\Delta\gamma/\gamma_0$ decreases because the temperature dependence of $\gamma_0 \sim T^2$ is stronger than that of $\Delta\gamma \sim T$. For comparison, we have also performed the calculation of the ratio $\Delta\gamma/\gamma_0$ using the phenomenological parametrization for the GQR energy $E_R = 60 \cdot A^{-1/3}$ MeV (see dashed line in Fig. 5). In this case, the variation of $\Delta\gamma/\gamma_0$ with temperature is somewhat stronger. Taking into account the nonlinear dissipativity effects, the collisional width Γ' of the GQR is obtained from the solution, in the form $\omega = \mathrm{Re}\,\omega + i\Gamma'/2\hbar$, to the secular equation, see Eq. (43),

$$(\omega_0^2 + \Delta\omega_0^2 - \omega^2)^2 + (\gamma_0 + \Delta\gamma)^2\omega^2 = 0. \tag{51}$$

In Fig. 6 we have plotted the temperature dependence of the widths Γ (dashed lines) and Γ' (solid lines) for two choices of the resonance energy: $E = \hbar\,\mathrm{Re}\,\omega$ using Eq. (49) (curves 1) and $E_R = 60 \cdot A^{-1/3}$ MeV (curves 2).

We point out that an increase of the width is more apparent for curves 1 in Fig. 6 because of the temperature dependence of E. The comparison of the solid and dashed lines in Fig. 6 shows that the contribution of the nonlinear dissipative effects to the width Γ does not

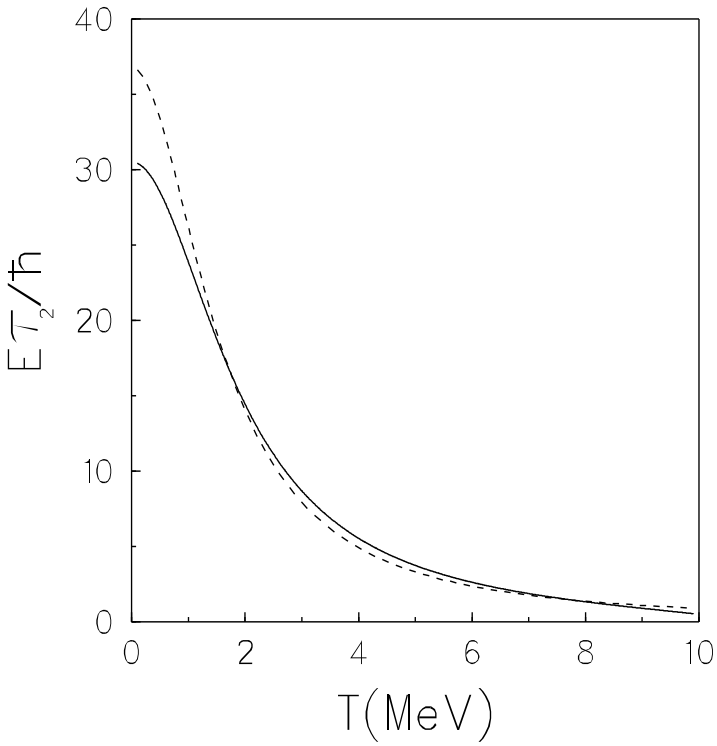

FIG. 4: Dependence of the dimensionless parameter $E\tau_2/\hbar$ on the temperature T for the GQR in the nucleus with $A = 224$ with τ_2 from Eq. (45). The solid curve was obtained using $E = \hbar\,\mathrm{Re}\,\omega$ from Eq. (49). The dashed curve was obtained with $E = E_R$, where $E_R = 60 \cdot A^{-1/3}$ MeV is the experimental value of the GQR energy.

exceed $\sim 20\%$.

In Figs. 7 and 8 we have plotted the strength function $S(\omega)$ of Eq. (43). The comparison between the solid and the dashed lines in Fig. 7 shows the accuracy of the derivation of the value of $\Delta\gamma$ directly from the strength function $S(\omega)$ of Eq. (43) and through the solution of the secular equation (51). The comparison of the solid lines with the dashed lines in Fig. 8 demonstrates the effect of the nonlinear dissipativity on the strength function.

V. SUMMARY AND CONCLUSIONS

Starting from the collisional kinetic equation with a random force and using the **p**-moments techniques, we have derived the equations of motion of the viscous fluid dynamic for the local values of particle density, velocity field and pressure tensor. The obtained

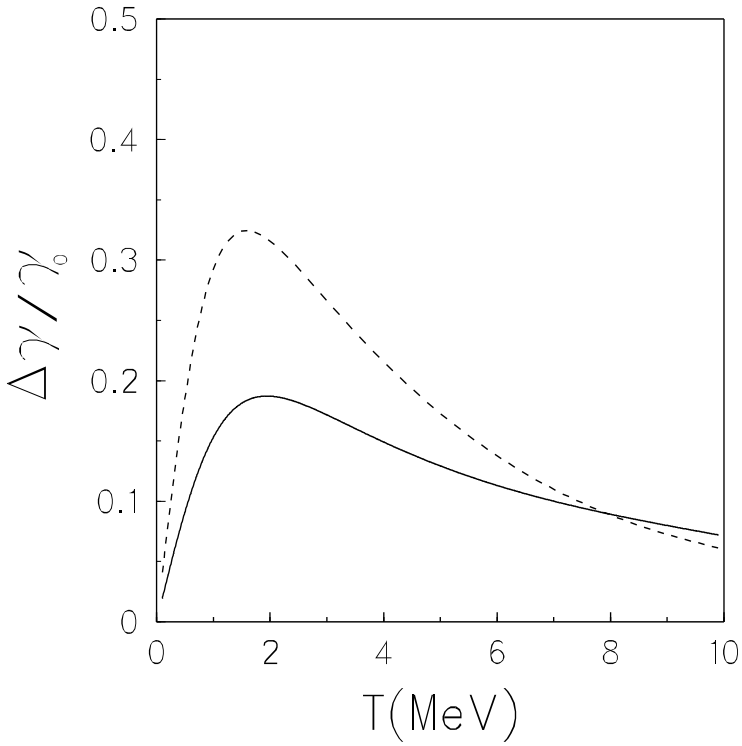

FIG. 5: Temperature dependence of the ratio $\Delta\gamma/\gamma_0$ for the nucleus with $A = 224$ for the GQR. The solid curve was obtained using Eq. (41) with ω from Eq. (49) (see also Fig. 3); the dashed line was obtained using Eq. (41) with γ_0 from Eq. (37) and $\omega = \omega_R = E_R/\hbar$ with the phenomenological parametrization $E_R = 60 \cdot A^{-1/3}$ MeV.

equations are closed due to the restriction imposed on the multipolarity l of the Fermi surface distortion, up to $l = 2$. The important features of these equations of motion are due to the non-Markovian form of the pressure tensor $P_{\alpha\beta}$. In contrast to the commonly used τ-approximation, we take into account the higher orders of the variation of the collision integral with respect to the variation of the phase-space distribution function. Using the Abrikosov-Khalatnikov transformation we have then obtained the collision integral in the form of the extended τ-approximation. Assuming a separable form for the displacement field, we have introduced the macroscopic collective variable $\beta(t)$ and reduced the problem to a macroscopic equation of motion for $\beta(t)$. Note that we do not assume the velocity $\dot{\beta}(t)$ to be small. The final macroscopic equation of motion (C1) includes both the memory effects and the nonlinear dissipativity terms $\sim \dot{\beta}^3$. We have separated the description of the collective motion into two parts. The first (slow) one is related to the driving force $\mathcal{F}_{\rm ext}(t)$

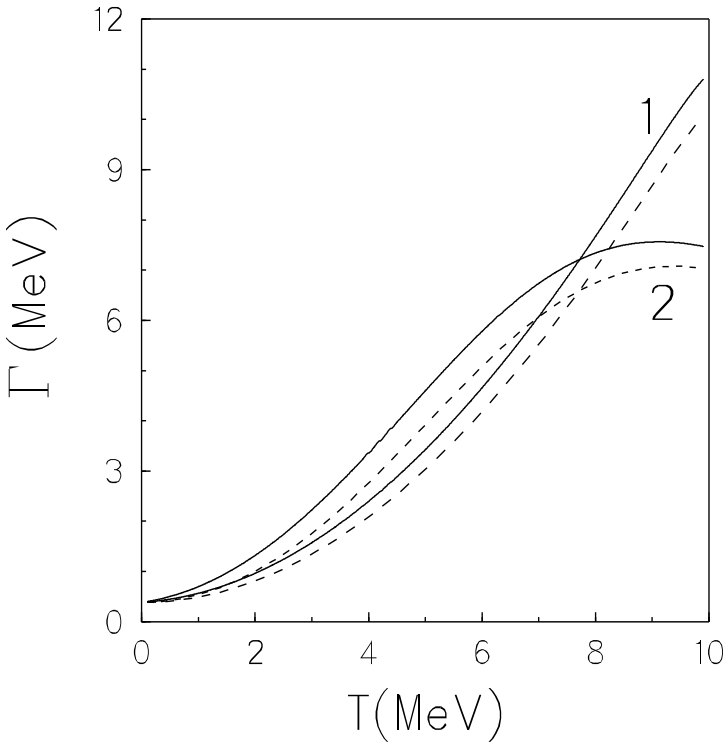

FIG. 6: Collisional width Γ as a function of temperature T for the nucleus with $A = 224$ for the GQR. The solid lines are for $\Gamma = 2\hbar\,\mathrm{Im}\,\omega$ from Eq. (49) and the dashed lines are for Γ' from Eq. (51). The curves 1 were obtained using the temperature dependent resonance frequency $\omega = \omega_R = \mathrm{Re}\,\omega$ from Eq. (49). The curves 2 were obtained using $\omega = \omega_R = E_R/\hbar$ with the phenomenological parametrization $E_R = 60 \cdot A^{-1/3}$ MeV (see also Fig. 5).

and it is associated with a slow motion having the velocity $\dot{\tilde{\beta}}$. The second (fast) one is due to the random force $y(t)$ with the velocity $\delta\dot{\beta} \gg \dot{\tilde{\beta}}$. Using the correlation properties of the random force, we have performed the averaging of the macroscopic equation of motion over the fast fluctuations $\sim \delta\dot{\beta}$, reducing the nonlinear dissipativity terms to the form $\sim \dot{\tilde{\beta}}(t)\ \left\langle \delta\dot{\beta}(t')\delta\dot{\beta}(t'') \right\rangle$, which is linear with respect to the slow collective motion $\sim \dot{\tilde{\beta}}$. Finally, assuming a periodic driving force $\mathcal{F}_{\mathrm{ext}}(t) \sim \exp(i\omega t)$, we have derived the macroscopic strength function $S(\omega)$. As seen from Eq. (43), the nonlinear dissipativity effect leads to the additional spreading $\Delta\gamma$ and the resonance shift $\Delta\omega_0$ in the strength function $S(\omega)$. The contribution $\Delta\gamma$ appears due to the thermodynamical fluctuations of the collective variable β. In contrast to the Fermi-liquid friction parameter γ_0 with $\gamma_0 \sim T^2$ (at $T \ll \epsilon_F$), the spreading $\Delta\gamma$ is a linear function of the temperature T. This fact provides a non-monotonic

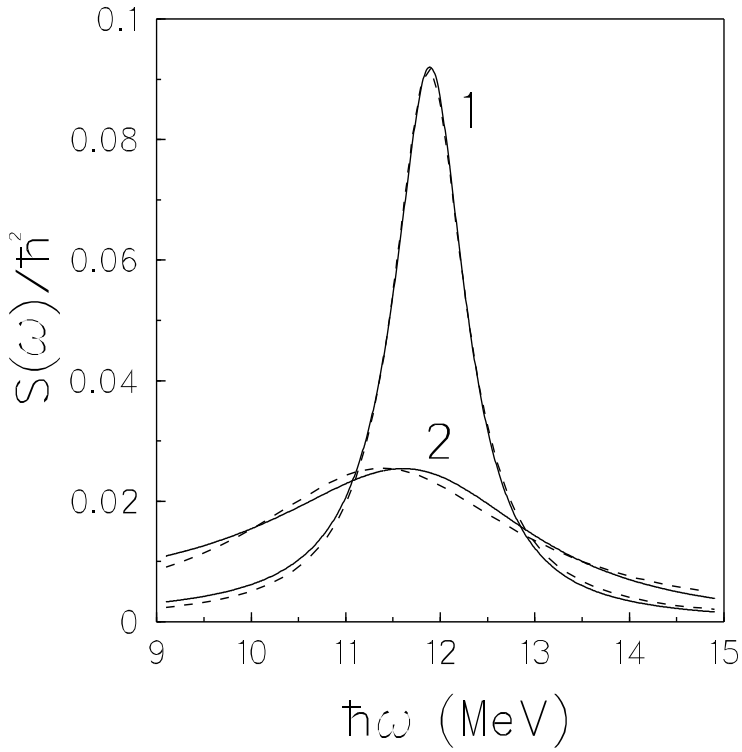

FIG. 7: The strength function $S(\omega)$ in $\hbar^2$-units for two temperatures: $T = 1.9$ MeV (curves 1) and $T = 5$ MeV (curves 2). The solid curves 1 and 2 were obtained from Eqs. (41), (42) and (43). The dashed curves 1 and 2 were obtained using Eq. (43) with $\gamma_0 + \Delta\gamma = \Gamma/\hbar$ and $\omega_0^2 + \Delta\omega_0^2 = (E/\hbar)^2$, where E and Γ are obtained from the solution, in the form $\omega = E/\hbar + i\Gamma/2\hbar$, to the secular equation (51).

behavior of the ratio $\Delta\gamma/\gamma_0$, see Fig. 5. As seen from Fig. 5, the nonlinear dissipativity effects are enhanced at the moderate temperatures $T \approx 2$ MeV and do not exceed $\approx 20\%$. The nonlinear dissipativity effect increases the collisional width of the GMR. Usually the total collisional width of the isoscalar GQR in cold nuclei does not exceed 30-40% of the experimental value and the main contribution to the width is due to the Landau damping. One can expect that the nonlinear dissipativity effect on the collisional width can lead to a deviation of the temperature dependent width $\Gamma(T)$ from the usual Fermi liquid prediction $\Gamma(T) \sim T^2$. Unfortunately, at present time, experimental data on the temperature behavior of $\Gamma(T)$ of the isoscalar GQR are not available. In this respect, it is more instructive to study the isovector giant dipole resonance where the temperature dependence of $\Gamma(T)$ was studied for some heavy nuclei [36, 37]. However our final results for the viscous tensor $Q_{\alpha\beta}$

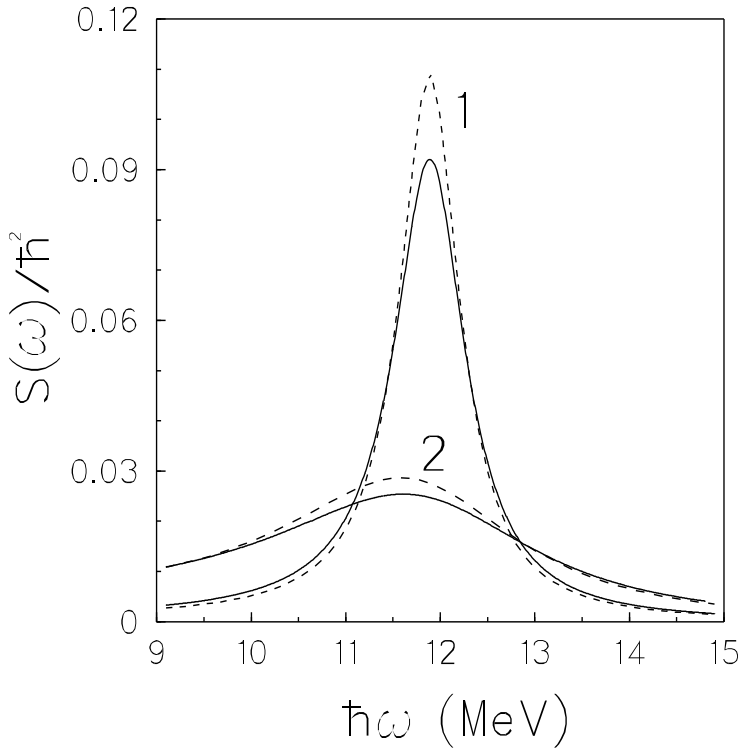

FIG. 8: The strength function $S(\omega)$ in $\hbar^2$-units for two temperatures: $T = 1.9$ MeV (curves 1) and $T = 5$ MeV (curves 2). The solid curves 1 and 2 are the same as in Fig. 7. The dashed curves 1 and 2 were obtained from Eqs. (43), but with $\Delta\gamma = 0$.

and the relaxation parameters ζ and ξ can not be applied directly to the isovector mode because the dipole distortion of the Fermi surface must be taken into account in the collision integral (13), in contrast to our case of the isoscalar GMR, see Sect. II. The generalization of our approach to the case isovector modes is now in progress.

VI. ACKNOWLEDGMENTS

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APPENDIX A

As a basic expression for the collision integral $\delta\mathrm{St}[f]$ we use Eq. (13). The second and third variations of Eq. (13) with respect to δf take the following form

$$\delta\mathrm{St}_2 = \int \frac{g^2 d\vec{p}_2 d\vec{p}_3 d\vec{p}_4}{(2\pi\hbar)^6} w(\{\vec{p}_j\}) \overline{\sum} \left. \frac{\delta^2 Q}{\delta f(i)\delta f(j)} \right|_{\mathrm{eq}} \delta f(i)\delta f(j)\ \delta(\Delta\epsilon)\delta(\Delta\vec{p}), \tag{A1}$$

$$\delta\mathrm{St}_3 = \int \frac{g^2 d\vec{p}_2 d\vec{p}_3 d\vec{p}_4}{(2\pi\hbar)^6} w(\{\vec{p}_j\}) \overline{\sum} \left. \frac{\delta^3 Q}{\delta f(i)\delta f(j)\delta f(k)} \right|_{\mathrm{eq}} \delta f(i)\delta f(j)\delta f(k)\ \delta(\Delta\epsilon)\delta(\Delta\vec{p}), \tag{A2}$$

where $\delta f(i) \equiv \delta f(\vec{p}_i)$ and the symbol $\overline{\sum}$ means a summation over indices $i, j, k = 1 \div 4$ with $i \neq j$, $j \neq k$, $k \neq i$. We will follow the fluid dynamic approach and represent the variation of the distribution function δf in the following form:

$$\delta f(i) = -\frac{\partial f_{\mathrm{eq},i}}{\partial \epsilon_i} \nu(i), \qquad \nu(i) = \sum_{l,m_l}^{l=2} \nu_{2m_l}(\vec{r},t) Y_{2m_l}(\Omega_i). \tag{A3}$$

We point out that the $l = 0$ and 1 components of the expansion (A3) do not contribute to the collision integral (13), reflecting the conservation of particle number and momentum in a collision. The expansion coefficients $\nu_{2m}(\vec{r},t)$ in Eq. (A3) are related to the pressure tensor $P'_{\alpha\beta}$ of Eq. (9). Using Eqs. (9) and (A3), we obtain

$$mP'_{\alpha\beta} = -\frac{gI}{(2\pi\hbar)^3} \sum_{m_l=-2}^{2} \nu_{2m_l} \int d\Omega\ \hat{p}_\alpha\ \hat{p}_\beta Y_{2m_l}(\Omega), \tag{A4}$$

where

$$I = \int_0^\infty dp p^4 \frac{\partial f_{\mathrm{eq}}}{\partial \epsilon}, \tag{A5}$$

and $\hat{\vec{p}} = \vec{p}/p$ is the unit vector. In particular, performing the angle integration in Eq. (A4), we obtain

$$\nu_{20} = \frac{3}{4}\sqrt{\frac{5}{\pi}} \frac{(2\pi\hbar)^3 m}{gI} P'_0, \tag{A6}$$

where P'_0 is given by Eq. (18). To evaluate the collision integral $\delta\mathrm{St}_2$, we will substitute Eq. (A3) into (A1) and make use of the Abrikosov-Khalatnikov transformation in the following form [20]

$$\int d\vec{p}_2 d\vec{p}_3 d\vec{p}_4 \,(...)\, \delta(\Delta\vec{p}) \Rightarrow \frac{m^3}{2} \int_V^\infty d\epsilon_2 d\epsilon_3 d\epsilon_4 \int \frac{d\Omega d\phi_2}{\cos(\theta/2)} \,(...)\,, \tag{A7}$$

where $d\Omega = \sin\theta d\theta d\phi$, θ is the angle between $\vec{p}_1$ and $\vec{p}_2$, ϕ is the angle between the planes formed by $(\vec{p}_1, \vec{p}_2)$ and $(\vec{p}_3, \vec{p}_4)$, and ϕ_2 is the azimuthal angle of the momentum $\vec{p}_2$ in the co-ordinate system with z-axes along $\vec{p}_1$. We point out that the angle ϕ varies only from

0 to π because the particles are indistinguishable. Using the transformation (A7) and the relation (see Appendix B)

$$\int_0^{2\pi} \frac{d\phi_2}{2\pi} Y_{nm}(\Omega_i) Y_{n'm'}(\Omega_j) = Y_{nm}(\Omega_1) Y_{n'm'}(\Omega_1) P_n(\cos\theta_i) P_{n'}(\cos\theta_j), \tag{A8}$$

we obtain

$$\delta \mathrm{St}_2 = g^2 \frac{(2\pi)^2 m^3}{(2\pi\hbar)^6} \, [\nu(1)]^2 \, \overline{\sum} \langle w P_2(\cos\theta_i) P_2(\cos\theta_j) \rangle r_{ij}. \tag{A9}$$

Here, r_{ij} is given by

$$r_{ij} = \int_V^\infty d\epsilon_2 d\epsilon_3 d\epsilon_4 \, \frac{\delta^2 Q}{\delta f(i) \delta f(j)} \bigg|_{\mathrm{eq}} \frac{\partial f_{\mathrm{eq},i}}{\partial \epsilon_i} \frac{\partial f_{\mathrm{eq},j}}{\partial \epsilon_j} \, \delta(\Delta\epsilon), \tag{A10}$$

and the symbol $\langle ... \rangle$ denotes the following average

$$\langle w(\theta,\phi) P_2(\cos\theta_i) P_2(\cos\theta_j) \rangle = 2 \int_0^\pi d\theta \sin(\theta/2) \int_0^\pi \frac{d\phi}{2\pi} w(\theta,\phi) P_2(\cos\theta_i) P_2(\cos\theta_j),$$

where $\cos\theta_j \equiv (\hat{\vec{p}}_j \cdot \ \hat{\vec{p}}_1)$, i.e. $\theta_2 = \theta$, and

$$\cos\theta_3 = \cos^2(\theta/2) + \sin^2(\theta/2)\cos\phi, \cos\theta_4 = \cos^2(\theta/2) - \sin^2(\theta/2)\cos\phi, \tag{A11}$$

and $P_l(\cos\theta)$ is a Legendre polynomial. Using Eqs. (6), (14), (15) and (A9), we obtain

$$Q^{(2)}_{\alpha\beta} = \frac{1}{m} \int \frac{g d\vec{p}}{(2\pi\hbar)^3} (p_\alpha - m u_\alpha)(p_\beta - m u_\beta) \delta \mathrm{St}_2$$

$$= \frac{g^3}{m} \frac{(2\pi)^2 m^3}{(2\pi\hbar)^9} \overline{\sum} \langle w P_2(\cos\theta_i) P_2(\cos\theta_j) \rangle R_{ij} \int d\Omega_1 \ \hat{p}_{1,\alpha} \ \hat{p}_{1,\beta} [\nu(1)]^2, \tag{A12}$$

where

$$R_{ij} = \int_0^\infty dp_1 p_1^4 r_{ij}.$$

To exclude the unknown amplitude $\nu(1)$ from Eq. (A12), we will calculate the arbitrary partial contribution to the tensor $Q^{(2)}_{\alpha\beta}$. Using Eq. (A1), we will consider the partial contribution $Q^{(2)}_{\alpha\beta,12}$ to the tensor $Q^{(2)}_{\alpha\beta}$ given by

$$Q^{(2)}_{\alpha\beta,12} = \frac{g^3}{m(2\pi\hbar)^9} \int d\vec{p}_1 p_{1,\alpha} p_{1,\beta} \delta f(1) \int d\vec{p}_2 \delta f(2) \int d\vec{p}_3 d\vec{p}_4$$

$$\times \ w(\theta,\phi) \, \frac{\delta^2 Q}{\delta f(1) \delta f(2)} \bigg|_{\mathrm{eq}} \delta(\Delta\epsilon) \delta(\Delta\vec{p}). \tag{A13}$$

We will assume the isotropic probability scattering $w(\theta,\phi)=w_0$, and apply the Abrikosov-Khalatnikov transformation in the following form

$$\int d\vec{p}_3 d\vec{p}_4\,(...)\,\delta(\Delta\vec{p}) \Rightarrow \frac{m^2}{2p_F\cos(\theta/2)}\int d\epsilon_3 d\epsilon_4 d\phi\,(...)\,, \tag{A14}$$

where p_F is the Fermi momentum. Using Eqs. (A3) and (A14), we transform Eq. (A13) as

$$Q^{(2)}_{\alpha\beta,12} = \frac{g^3}{m}\frac{2\pi m^3 w_0}{2(2\pi\hbar)^9}R_{12}\sum_{m=-2}^{2}\nu_{2m}\int d\Omega_2\frac{Y_{2m}(\Omega_2)}{\cos(\theta/2)}\int d\Omega_1\;\hat{p}_{1,\alpha}\;\hat{p}_{1,\beta}\;\nu(1). \tag{A15}$$

Integrating over $d\Omega_2$ in Eq. (A15) and using Eq. (A6), we obtain

$$Q^{(2)}_{\alpha\beta,12} = g^2\frac{3\pi m^3 w_0}{(2\pi\hbar)^6}\frac{R_{12}}{I}P_0'\int d\Omega_1\;\hat{p}_{1,\alpha}\;\hat{p}_{1,\beta}\;\;\nu(1). \tag{A16}$$

Comparing Eq. (A16) with the partial $i=1$, $j=2$ term of Eq. (A12), we find the relation

$$\int d\Omega_1\;\hat{p}_{1,\alpha}\;\hat{p}_{1,\beta}\;[\nu(1)]^2 = \frac{3}{4\pi g\langle P_2(\cos\theta)\rangle}\frac{(2\pi\hbar)^3}{I}mP_0'\int d\Omega_1\;\hat{p}_{1,\alpha}\;\hat{p}_{1,\beta}\;\nu(1). \tag{A17}$$

Finally, from Eqs. (A12), (A4) and (A17) we obtain

$$Q^{(2)}_{\alpha\beta} = \frac{mP_0'}{\zeta}P'_{\alpha\beta}, \tag{A18}$$

where

$$\frac{1}{\zeta} = \frac{3\pi g m^3 w_0}{(2\pi\hbar)^3}\;\frac{\overline{\sum}\langle P_2(\cos\theta_i)P_2(\cos\theta_j)\rangle R_{ij}}{\langle P_2(\cos\theta)\rangle I^2}. \tag{A19}$$

Let us go now to the third order variation of the collision integral $\delta\mathrm{St}_3$ of Eq. (A2). Applying Eqs. (A3) and the transformation (A7) to Eq. (A2) and using the relation (see Appendix B)

$$\int\frac{d\phi_2}{2\pi}Y_{nm}(\Omega_i)Y_{n'm'}(\Omega_j)Y_{n''m''}(\Omega_k)$$
$$= Y_{nm}(\Omega_1)Y_{n'm'}(\Omega_1)Y_{n''m''}(\Omega_1)P_n(\cos\theta_i)P_{n'}(\cos\theta_j)P_{n''}(\cos\theta_k),$$

we will reduce the collision integral $\delta\mathrm{St}_3$ to the following form

$$\delta\mathrm{St}_3 = -g^2\frac{(2\pi)^2 m^3}{(2\pi\hbar)^6}\;[\nu(1)]^3\;\overline{\sum}\langle wP_2(\cos\theta_i)P_2(\cos\theta_j)P_2(\cos\theta_k)\rangle r_{ijk}, \tag{A20}$$

where

$$r_{ijk} = \int_V^\infty d\epsilon_2 d\epsilon_3 d\epsilon_4\;\frac{\delta^3 Q}{\delta f(i)\delta f(j)\delta f(k)}\bigg|_{eq}\frac{\partial f_{\mathrm{eq},i}}{\partial\epsilon_i}\frac{\partial f_{\mathrm{eq},j}}{\partial\epsilon_j}\frac{\partial f_{\mathrm{eq},k}}{\partial\epsilon_k}\;\delta(\Delta\epsilon). \tag{A21}$$

Using Eqs. (6), (14), (15) and (A20), we obtain

$$Q^{(3)}_{\alpha\beta} = \frac{1}{m}\int\frac{gd\vec{p}}{(2\pi\hbar)^3}(p_\alpha - mu_\alpha)(p_\beta - mu_\beta)\delta\mathrm{St}_3$$

$$= -\frac{g^3}{m}\frac{(2\pi)^2 m^3}{(2\pi\hbar)^9}\overline{\sum}\langle w P_2(\cos\theta_i)P_2(\cos\theta_j)P_2(\cos\theta_k)\rangle R_{ijk}\int d\Omega_1\ \hat{p}_{1,\alpha}\ \hat{p}_{1,\beta}\ \ [\nu(1)]^3, \quad \text{(A22)}$$

where

$$R_{ijk} = \int_0^\infty dp p^4 r_{ijk}. \quad \text{(A23)}$$

Similar to the previous evaluation of the tensor $Q^{(2)}_{\alpha\beta}$, we will consider the partial term of Eq. (A20) with $i = 1,\ j = 2,\ k = 3$ and the corresponding partial tensor $Q^{(3)}_{\alpha\beta,123}$ which is given by

$$Q^{(3)}_{\alpha\beta,123} = -\frac{g^3}{m(2\pi\hbar)^9}\int d\vec{p}_1\ \hat{p}_{1,\alpha}\hat{p}_{1,\beta}\ \delta f(1)$$
$$\times\int d\vec{p}_2\delta f(2)\int d\vec{p}_3\delta f(3)\int d\vec{p}_4 w(\theta,\phi)\delta(\Delta\epsilon)\delta(\Delta\vec{p}), \quad \text{(A24)}$$

where we have used the following relation

$$\left.\frac{\delta^3 Q}{\delta f(1)\delta f(2)\delta f(3)}\right|_{\text{eq}} = -1.$$

We will again assume the isotropic scattering probability: $w(\theta,\phi) = w_0$ and apply the transformation (A14) to Eq. (A24). The angle integrals over $d\Omega_2 d\phi$, appearing in Eq. (A24), can be transformed as

$$\int d\Omega_2 d\phi\frac{Y_{2m'}(\Omega_2)}{\cos(\theta/2)}Y_{2m''}(\Omega_3) = Y_{2m''}(\Omega_1)\int d\Omega_2 d\phi\frac{P_2(\cos\theta_3)}{\cos(\theta/2)}Y_{2m'}(\Omega_2), \quad \text{(A25)}$$

where we have used the relation [34]

$$\int_0^\pi \frac{d\phi_2}{2\pi}Y_{nm}(\Omega_i) = Y_{nm}(\Omega_1)P_n(\cos\theta_i). \quad \text{(A26)}$$

The result reads

$$Q^{(3)}_{\alpha\beta,123} = -\frac{g^3}{m}\frac{m^3 w_0}{2(2\pi\hbar)^9}R_{123}\sum_{m=-2}^{2}\nu_{2m}$$
$$\times\int d\Omega_2 d\phi\frac{P_2(\cos\theta_3)}{\cos(\theta/2)}Y_{2m}(\Omega_2)\int d\Omega_1\ \hat{p}_{1,\alpha}\ \hat{p}_{1,\beta}\ [\nu(1)]^2. \quad \text{(A27)}$$

Performing the integration over $d\Omega_2 d\phi$ in Eq. (A27) and using Eqs. (A17) and (A6), we obtain

$$Q^{(3)}_{\alpha\beta,123} = -\frac{27}{28}\frac{gm^3}{(2\pi\hbar)^3}\frac{w_0}{\langle P_2(\cos\theta)\rangle}\frac{R_{123}}{I^2}mP_0'^2\int d\Omega_1\ \hat{p}_{1,\alpha}\ \hat{p}_{1,\beta}\ \nu(1). \quad \text{(A28)}$$

Comparing Eq. (A28) with the partial $i = 1,\ j = 2,\ k = 3$ term of Eq. (A22) we obtain the following relation

$$\int d\Omega_1\ \hat{p}_{1,\alpha}\ \hat{p}_{1,\beta}\ \nu^3(1) = \frac{27}{28}\frac{(2\pi\hbar)^6}{(2\pi)^2 g^2 I^2}\frac{m^2 P_0'^2}{\langle P_2(\cos\theta)\rangle\langle P_2(\cos\theta)P_2(\cos\theta_3)\rangle}$$

$$\times \int d\Omega_1 \; \hat{p}_{1,\alpha} \; \hat{p}_{1,\beta} \; \nu(1). \tag{A29}$$

Finally, substituting Eq. (A29) into Eq. (A22) and using Eq. (A4), we obtain

$$Q^{(3)}_{\alpha\beta} = \frac{m^2 P_0'^2}{\xi} P'_{\alpha\beta}, \tag{A30}$$

where

$$\frac{1}{\xi} = \frac{27 m^3 w_0}{28} \; \frac{\overline{\sum} \langle P_2(\cos\theta_i) P_2(\cos\theta_j) P_2(\cos\theta_k)\rangle R_{ijk}}{\langle P_2(\cos\theta)\rangle \langle P_2(\cos\theta) P_2(\cos\theta_3)\rangle I^3}. \tag{A31}$$

APPENDIX B

In this Appendix, we will consider some angle integrals which appear in the calculations of the collision integral and its variations. Let us start from the integral

$$M_{23} = \int \frac{d\Omega_2}{4\pi} P_n^m(\cos\Theta_2) e^{im\Phi_2} P_{n'}^{m'}(\cos\Theta_3) e^{im\Phi_3} P_l(\cos\theta), \tag{B1}$$

where (Θ_j, Φ_j) are the angle coordinates of the momentum vectors $\vec{p}_j$ in the arbitrary co-ordinate frame ($j = 1 \div 4$) and θ is the angle between the vectors $\vec{p}_1$ and $\vec{p}_2$. Using the addition theorem for spherical harmonics [35]

$$P_l(\cos\theta) = \sum_{r=-l}^{l} \frac{(l-|r|)!}{(l+|r|)!} P_l^{|r|}(\cos\Theta_1) P_l^{|r|}(\cos\Theta_2) e^{ir(\Phi_1-\Phi_2)}, \tag{B2}$$

we find

$$\begin{aligned} M_{23} &= \sum_{r=-l}^{l} \frac{(l-|r|)!}{(l+|r|)!} \int \frac{d\Phi_2}{4\pi} \int \sin\Theta_2 d\Theta_2 P_n^m(\cos\Theta_2) P_{n'}^{m'}(\cos\Theta_3) \\ &\quad \times P_l^{|r|}(\cos\Theta_2) P_l^{|r|}(\cos\Theta_1) e^{im\Phi_2} e^{im'\Phi_3} e^{ir(\Phi_1-\Phi_2)} \\ &= \frac{\delta_{nl}}{2n+1} P_n^m(\cos\Theta_1) e^{im\Phi_1} P_{n'}^{m'}(\cos\Theta_1) e^{im'\Phi_1} P_{n'}(\cos\Theta_3). \end{aligned} \tag{B3}$$

Here, θ_3 is the angle between $\vec{p}_1$ and $\vec{p}_3$. On the other hand, using the direction of $\vec{p}_1$ as a polar axis with $d\Omega_2 = \sin\theta d\theta d\phi_2$ where ϕ_2 is the azimuthal coordinate of $\vec{p}_2$ in the new co-ordinate frame, we will rewrite Eq. (B1) as

$$M_{23} = \int \frac{d\theta}{2} \sin\theta P_l(\cos\theta) \left\{ \int \frac{d\phi_2}{2\pi} P_n^m(\cos\Theta_2) e^{im\Phi_2} P_{n'}^{m'}(\cos\Theta_3) e^{im'\Phi_3} \right\}. \tag{B4}$$

Here and below the angles Θ_i and Φ_i are dependent on the angles θ and ϕ_2. Comparing Eqs. (B3) and (B4) and using the orthogonality condition for the Legendre polynomial $P_l(\cos\theta)$ in Eq. (B4), one obtains the following integral relation

$$\int \frac{d\phi_2}{2\pi} P_n^m(\cos\Theta_2) e^{im\Phi_2} P_{n'}^{m'}(\cos\Theta_3) e^{im'\Phi_3}$$

$$= P_n^m(\cos\Theta_1) e^{im\Phi_1} P_{n'}^{m'}(\cos\Theta_1) e^{im'\Phi_1} P_n(\cos\theta) P_{n'}(\cos\theta_3). \tag{B5}$$

Starting from the integral

$$M_{24} = \int \frac{d\Omega_2}{4\pi} P_n^m(\cos\Theta_2) e^{im\Phi_2} P_{n'}^{m'}(\cos\Theta_4) e^{im\Phi_4} P_l(\cos\theta),$$

we will also obtain an integral relation analogous to Eq. (B5) but with the replacement $3 \to 4$. Let us consider now the integral

$$M_{34} = \int \frac{d\Omega_3}{4\pi} P_n^m(\cos\Theta_3) e^{im\Phi_3} P_{n'}^{m'}(\cos\Theta_4) e^{im'\Phi_4} P_l(\cos\theta_3). \tag{B6}$$

Using the addition theorem for $P_l(\cos\theta_3)$ (see Eq. (B2)), we reduce Eq. (B6) as

$$M_{34} = \frac{\delta_{nl}}{2n+1} P_n^m(\cos\Theta_1) e^{im\Phi_1} \int \frac{d\Phi_3}{2\pi} P_{n'}^{m'}(\cos\Theta_4) e^{im'\Phi_4}$$

$$= \frac{\delta_{nl}}{2n+1} P_n^m(\cos\Theta_1) e^{im\Phi_1} P_{n'}^{m'}(\cos\Theta_1) e^{im'\Phi_1} P_n(\cos\Theta_4). \tag{B7}$$

Replacing in Eq. (B6) the integration over Φ_3 to integration over Φ_2 and using the direction of $\vec{p}_1$ as a polar axis, we will rewrite Eq. (B6) as

$$M_{34} = \int \frac{d\theta_3}{2} \sin\theta_3 P_l(\cos\theta_3) \left\{ \int \frac{d\phi_2}{2\pi} P_n^m(\cos\Theta_3) e^{im\Phi_3} P_{n'}^{m'}(\cos\Theta_4) e^{im'\Phi_4} \right\}. \tag{B8}$$

Comparing Eqs. (B7) and (B8) and using the orthogonality conditions for the Legendre polynomials, we obtain

$$\int \frac{d\phi_2}{2\pi} P_n^m(\cos\Theta_3) e^{im\Phi_3} P_{n'}^{m'}(\cos\Theta_4) e^{im'\Phi_4}$$

$$= P_n^m(\cos\Theta_1) e^{im\Phi_1} P_{n'}^{m'}(\cos\Theta_1) e^{im'\Phi_1} P_n(\cos\theta_3) P_{n'}(\cos\theta_4). \tag{B9}$$

Using the representation of the spherical function $Y_{nm}(\Omega)$ via the Legendre polynomials $P_n^m(\cos\theta)$ [35]

$$Y_{nm}(\Omega) = \sqrt{\frac{2n+1}{4\pi} \frac{(n-m)!}{(n+m)!}} P_n^m(\cos\theta) e^{im\phi}, \tag{B10}$$

and collecting Eqs. (B5) and (B9) we obtain the following integral relation

$$\int \frac{d\phi_2}{2\pi} Y_{nm}(\Omega_i) Y_{n'm'}(\Omega_j) = Y_{nm}(\Omega_1) Y_{n'm'}(\Omega_1) P_n(\cos\theta_i) P_{n'}(\cos\theta_j), \tag{B11}$$

where $i, j = 1 \div 4$. Let us consider finally the integral

$$M_{234} = \int \frac{d\Omega_2}{4\pi} P_n^m(\cos\Theta_2) e^{im\Phi_2} P_{n'}^{m'}(\cos\Theta_3) e^{im\Phi_3} P_{n''}^{m''}(\cos\Theta_4) e^{im\Phi_4} P_l(\cos\theta). \tag{B12}$$

Similar to the previous consideration, we will transform Eq. (B12) as

$$\begin{aligned} M_{234} &= \int \frac{d\theta}{2} \sin\theta P_l(\cos\theta) \\ &\times \left\{ \int \frac{d\phi_2}{2\pi} P_n^m(\cos\Theta_2) e^{im\Phi_2} P_{n'}^{m'}(\cos\Theta_3) e^{im'\Phi_3} P_{n''}^{m''}(\cos\Theta_4) e^{im'\Phi_4} \right\} \\ &= \frac{\delta_{nl}}{2n+1} P_n^m(\cos\Theta_1) e^{im\Phi_1} P_{n'}^{m'}(\cos\Theta_1) e^{im'\Phi_1} \\ &\times P_{n''}^{m''}(\cos\Theta_1) e^{im''\Phi_1} P_{n'}(\cos\Theta_3) P_{n''}(\cos\Theta_4). \end{aligned} \tag{B13}$$

Using the orthogonality conditions for the Legendre polynomials, we obtain

$$\begin{aligned} &\int \frac{d\phi_2}{2\pi} P_n^m(\cos\Theta_2) e^{im\Phi_2} P_{n'}^{m'}(\cos\Theta_3) e^{im'\Phi_3} P_{n''}^{m''}(\cos\Theta_4) e^{im''\Phi_4} \\ &= P_n^m(\cos\Theta_1) e^{im\Phi_1} P_{n'}^{m'}(\cos\Theta_1) e^{im'\Phi_1} P_{n''}^{m''}(\cos\Theta_1) e^{im''\Phi_1} \\ &\times P_n(\cos\theta) P_{n'}(\cos\theta_3) P_{n''}(\cos\theta_4). \end{aligned} \tag{B14}$$

Finally, taking into account Eq. (B10) we will generalize Eq. (B14) as

$$\begin{aligned} &\int \frac{d\phi_2}{2\pi} Y_{nm}(\Omega_i) Y_{n'm'}(\Omega_j) Y_{n''m''}(\Omega_k) \\ &= Y_{nm}(\Omega_1) Y_{n'm'}(\Omega_1) Y_{n''m''}(\Omega_1) P_n(\cos\theta_i) P_{n'}(\cos\theta_j) P_{n''}(\cos\theta_k). \end{aligned} \tag{B15}$$

APPENDIX C

In this Appendix we give a proof of the macroscopic equation of motion for the nuclear shape variable $\beta(t)$ derived by the displacement field as $\vec{\chi}(\vec{r}, t) = \beta(t)\vec{v}(\vec{r})$, see Sect. III. Substituting this separable form in Eq. (27) and multiplying by v_α, summing over α, and

integrating over $\vec{r}$ space, we obtain the equation of motion for the collective variable $\beta(t)$. Namely,

$$
\begin{aligned}
&B\ddot{\beta} + D_0\dot{\beta}^2 + A_0\int_{-\infty}^{t} dt' \exp\left(\frac{t'-t}{\tau_2}\right)\ \dot{\beta}(t') + C_{LDM}\beta \\
&-D_1\int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1'\ \exp\left(\frac{t_1'-t}{\tau_2}\right)\ \dot{\beta}(t')\dot{\beta}(t_1') \\
&-D_2\int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1' \int_{-\infty}^{t'} dt_2' \exp\left(\frac{t_1'-t}{\tau_2}\right)\exp\left(\frac{t_2'-t'}{\tau_2}\right)\ \dot{\beta}(t_1')\dot{\beta}(t_2') \\
&+A_1\int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1' \int_{-\infty}^{t'} dt_2' \int_{-\infty}^{t'} dt_3' \exp\left(\frac{t_1'-t}{\tau_2}\right)\exp\left(\frac{t_2'-t'}{\tau_2}\right) \\
&\qquad\times\exp\left(\frac{t_3'-t'}{\tau_2}\right)\ \dot{\beta}(t_1')\dot{\beta}(t_2')\dot{\beta}(t_3') \\
&+A_2\int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1' \int_{-\infty}^{t_1'} dt_2' \int_{-\infty}^{t'} dt_3' \exp\left(\frac{t_2'-t}{\tau_2}\right)\exp\left(\frac{t_3'-t'}{\tau_2}\right)\ \dot{\beta}(t_1')\dot{\beta}(t_2')\dot{\beta}(t_3') \\
&+A_3\int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1' \int_{-\infty}^{t_1'} dt_2' \int_{-\infty}^{t_1'} dt_3' \int_{-\infty}^{t'} dt_4' \exp\left(\frac{t_2'-t}{\tau_2}\right)\exp\left(\frac{t_3'-t_1'}{\tau_2}\right) \\
&\qquad\times\exp\left(\frac{t_4'-t'}{\tau_2}\right)\dot{\beta}(t_2')\dot{\beta}(t_3')\dot{\beta}(t_4') \\
&+A_4\int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1' \int_{-\infty}^{t_1'} dt_2' \exp\left(\frac{t_2'-t}{\tau_2}\right)\ \dot{\beta}(t')\dot{\beta}(t_1')\dot{\beta}(t_2') \\
&+A_5\int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt_1' \int_{-\infty}^{t_1'} dt_2' \int_{-\infty}^{t_1'} dt_3'\ \exp\left(\frac{t_2'-t}{\tau_2}\right)\exp\left(\frac{t_3'-t_1'}{\tau_2}\right)\ \dot{\beta}(t')\dot{\beta}(t_2')\dot{\beta}(t_3') \\
&\qquad = B\ \mathcal{F}_{\rm ext}(t) + B\ \widetilde{y}(t),
\end{aligned}
\tag{C1}
$$

where $B\mathcal{F}_{\rm ext}$ and $B\ \widetilde{y}(t)$ are, respectively, the external and random forces in the collective space of the variable β (we have separated the mass coefficient B from the external and random forces for technical convenience). The transport coefficients in Eq. (C1) are given by

$$
B = m\int d\vec{r}\rho_{\rm eq}v^2, \qquad C_{LDM} = \int d\vec{r}\ \left(\frac{\delta^2\varepsilon}{\delta\rho^2}\right)_{\rm eq}\left[\frac{\partial}{\partial r_\nu}(\rho_{\rm eq}v_\nu)\right]^2, \tag{C2}
$$

$$
D_0 = m\int d\vec{r}\rho_{\rm eq}v_\alpha v_\nu\frac{\partial v_\alpha}{\partial r_\nu}, \qquad D_1 = \int d\vec{r}\ \hat{\overline{L}}\left(P_{\rm eq}\overline{\Lambda}_{\alpha\nu}\right)\frac{\partial v_\alpha}{\partial r_\nu}, \tag{C3}
$$

$$
D_2 = \int d\vec{r}\ \frac{P_{\rm eq}^2}{\zeta}\overline{\Lambda}_0\overline{\Lambda}_{\alpha\nu}\frac{\partial v_\alpha}{\partial r_\nu}, \tag{C4}
$$

$$
A_0 = \int d\vec{r}P_{\rm eq}\overline{\Lambda}_{\alpha\nu}\frac{\partial v_\alpha}{\partial r_\nu}, \qquad A_1 = m^2\int d\vec{r}\ \frac{P_{\rm eq}^3}{\xi}\overline{\Lambda}_0^2\overline{\Lambda}_{\alpha\nu}\frac{\partial v_\alpha}{\partial r_\nu}, \tag{C5}
$$

$$A_2 = m \int d\vec{r}\, \frac{P_{\rm eq}}{\zeta} \left(\overline{\Lambda}_{\alpha\nu} \hat{\overline{L}}(P_{\rm eq}\overline{\Lambda}_0) + \overline{\Lambda}_0 \hat{\overline{L}}(P_{\rm eq}\overline{\Lambda}_{\alpha\nu}) \right) \frac{\partial v_\alpha}{\partial r_\nu}, \tag{C6}$$

$$A_3 = 2m^2 \int d\vec{r}\, \frac{P_{\rm eq}^3}{\zeta^2} \overline{\Lambda}_0^2 \overline{\Lambda}_{\alpha\nu} \frac{\partial v_\alpha}{\partial r_\nu}, \qquad A_4 = \int d\vec{r}\, \hat{\overline{L}}^2 \left(P_{\rm eq}\overline{\Lambda}_{\alpha\nu} \right) \frac{\partial v_\alpha}{\partial r_\nu}, \tag{C7}$$

$$A_5 = m \int d\vec{r}\, \hat{\overline{L}} \left(\frac{P_{\rm eq}^2}{\zeta} \overline{\Lambda}_0 \overline{\Lambda}_{\alpha\nu} \right) \frac{\partial v_\alpha}{\partial r_\nu}, \tag{C8}$$

with

$$\hat{\overline{L}} P'_{\alpha\beta} = v_\nu \frac{\partial P'_{\alpha\beta}}{\partial r_\nu} + P'_{\alpha\beta} \frac{\partial v_\nu}{\partial r_\nu} + P'_{\alpha\nu} \frac{\partial v_\beta}{\partial r_\nu} + P'_{\beta\nu} \frac{\partial v_\alpha}{\partial r_\nu},$$

$$\overline{\Lambda}_{\alpha\beta} = \frac{\partial v_\alpha}{\partial r_\beta} + \frac{\partial v_\beta}{\partial r_\alpha} - \frac{2}{3}\delta_{\alpha\beta} \frac{\partial v_\nu}{\partial r_\nu},$$

and

$$\overline{\Lambda}_0 = \frac{1}{2}(\overline{\Lambda}_{xx} + \overline{\Lambda}_{yy} - \overline{\Lambda}_{zz}).$$

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