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Type IIA Pati-Salam Flux Vacua

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Abstract

We show that for supersymmetric AdS vacua on Type IIA orientifolds with flux compactifications, the RR tadpole cancellation conditions can be completely relaxed, and then the four-dimensional $N = 1$ supersymmetry conditions are the main constraints on consistent intersecting D6-brane model building. We construct two kinds of three-family Pati-Salam models. In the first kind of models, the suitable three-family SM fermion masses and mixings can be generated at the stringy tree level, and then the rank one problem for the SM fermion Yukawa matrices can be solved. In the second kind of models, only the third family of the SM fermions can obtain masses at tree level. In these models, the complex structure parameters can be determined by supersymmetric D6-brane configurations, and all the moduli may be stabilized. The initial gauge symmetries $U(4)_C \times U(2)_L \times U(2)_R$ and $U(4)_C \times USp(2)_L \times U(2)_R$ can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ due to the Green-Schwarz mechanism and D6-brane splittings, and further down to the SM gauge symmetry around the string scale via the supersymmetry preserving Higgs mechanism. Comparing to the previous model building, we have less bidoublet Higgs fields. However, there generically exist some exotic particles.

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I. INTRODUCTION

The constructions of realistic Standard-like string models with moduli stabilization is the major challenge and lasting problem in string phenomenology. In the beginning string model building was mainly concentrated on the weakly coupled heterotic string theory, and rather successful models like flipped $SU(5)$ [1] in its stringy form were constructed [2]. After the second string revolution we can construct consistent four-dimensional chiral models with non-Abelian gauge symmetry on Type II orientifolds because of the advent of D-branes [3].

During last a few years, Type II orientifolds with intersecting D-branes has been particular interesting in string model building where the chiral fermions arise from the intersections of D-branes in the internal space [4] with T-dual description in terms of magnetized D-branes [5]. On Type IIA orientifolds with intersecting D6-branes, a large number of non-supersymmetric three-family Standard-like models and grand unified models, which satisfy the Ramond-Ramond (RR) tadpole cancellation conditions, were constructed [6, 7, 8]. However, there generically exist the uncancelled Neveu-Schwarz-Neveu-Schwarz (NSNS) tadpoles and the gauge hierarchy problem. To solve these two problems, the first quasi-realistic supersymmetric models [9, 10] have been constructed in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with intersecting D6-branes. Then, the supersymmetric Standard-like models, Pati-Salam models, $SU(5)$ models as well as flipped $SU(5)$ models have been constructed systematically [11, 12, 13, 14, 15, 16], and their phenomenological consequences have been studied [17, 18]. Moreover, the supersymmetric constructions on other orientifolds were also discussed [19]. There are two main constraints on supersymmetric model building: RR tadpole cancellation conditions and four-dimensional $N = 1$ supersymmetry conditions.

Even though some of the complex structure parameters (in the Type IIA picture) and dilaton field may be stabilized due to gaugino condensation in the hidden sector in some models (for example, see [18]), the moduli stabilization in open string and closed string sectors is still one of the most difficult problems. With supergravity fluxes which introduce a supergravity potential, we can stabilize the compactification moduli fields by lifting the continuous moduli space of the string vacua in the four-dimensional effective theory (for example, see [20]). On Type IIB orientifolds, the supergravity fluxes contribute large positive D3-brane charges due to the Dirac quantization conditions. Then, they modify the global RR tadpole cancellation conditions significantly and imposes strong constraints on consistent model building [21, 22]. By introducing magnetized D9-branes with large negative D3-brane charges in the hidden and observable sectors, we can construct the three-family and four-family Standard-like models [16, 23, 24, 25, 26, 27].

However, in above model building with or without fluxes, it is very difficult to explain the three-family Standard Model (SM) fermion masses and mixings. In the $SU(5)$ models, flipped $SU(5)$ models, and trinification models, the up-type quark Yukawa couplings, down-type quark Yukawa couplings, and lepton Yukawa couplings are forbidden by the anomalous $U(1)$ symmetries, respectively. And for the Pati-Salam models, although all the SM fermion Yukawa couplings can in principle be allowed by the anomalous $U(1)$ symmetries, we may not give masses to the three families of the SM fermions at the stringy ($M_S \sim 10^{17}$ GeV) tree level. The point is that the left-handed SM fermions, right-handed SM fermions and bidoublet Higgs fields in general arise from the intersections on different two-tori, and then the SM fermion Yukawa matrices are generically rank one, *i. e.*, only the third family of the SM fermions can obtain masses at the stringy tree level [28]. This is so called "rank one problem" in intersecting D-brane model building (for a possible solution, see [29]). In addition, if supersymmetry is broken by supergravity fluxes, the massless SM fermions may not obtain masses from quantum corrections [28] because the supersymmetry breaking trilinear soft terms are universal and the supersymmetry breaking soft masses for the left/right-chiral squarks and sleptons are universal [30, 31, 32] since the Kähler potential for the SM fermions depends only on the intersection angles [33, 34, 35, 36]. Thus, how to construct the Standard-like models, which can generate the suitable three-family SM fermion masses and mixings at the stringy tree level, is a very interesting open question.

Recently, the techniques for consistent chiral flux compactifications on Type IIA orientifolds with intersecting D6-branes were developed [37, 38, 39, 40]. We will show that in supersymmetric AdS vacua, the metric, NSNS and RR fluxes can contribute negative D6-brane charges to all the RR tadpole cancellation conditions, *i. e.*, the RR tadpole cancellation conditions give no constraints on consistent model building. Thus, the supersymmetric flux models on Type IIA orientifolds are mainly constrained by four-dimensional $N = 1$ supersymmetry conditions, and then it is possible to construct the rather realistic intersecting D6-brane models.

In this paper, we construct two kinds of three-family Pati-Salam models on Type IIA orientifolds with flux compactifications in supersymmetric AdS vacua. In the first kind of models, three families of the SM fermions can obtain suitable masses at the stringy tree level, while in the second kind of models, the third family of the SM fermions can obtain masses at tree level, and we assume that the masses for the first two families of the SM fermions may be generated from quantum corrections. In these models, we can determine the complex structure parameters via supersymmetric D6-brane configurations, and may stabilize all the moduli. The initial gauge symmetries are $U(4)_C \times U(2)_L \times U(2)_R$ and $U(4)_C \times USp(2)_L \times U(2)_R$ (note that $USp(2)_L \equiv SU(2)_L$). These initial gauge symmetries can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ due to the generalized Green-Schwarz mechanism and D6-brane splittings, and further down to the SM gauge symmetry at about the string scale via the supersymmetry preserving Higgs mechanism, where the Higgs fields come from the massless open string states in a $N = 2$ subsector. Moreover, if we want to use supergravity fluxes to relax the RR tadpole cancellation conditions in model building in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold, the fluxes will contribute large negative D6-brane charges due to the Dirac quantization conditions. Then we have to introduce many D6-branes in the hidden sector to completely cancel the RR tadpoles, and thus may introduce a lot of exotic particles simultaneously. Therefore, we mainly consider the Pati-Salam models in Type IIA theory on $\mathbf{T^6}$ orientifold with flux compactifications. We emphasize that all our Pati-Salam models on Type IIA $\mathbf{T^6}$ orientifold can be similarly constructed on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold by introducing more stacks of D6-branes in the hidden sector.

For the first kind of Pati-Salam models with $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry, we present five models (Models TI-U-i with i=1, ..., 5) on Type IIA $\mathbf{T^6}$ orientifold and two models (Models TI-U-6 and TI-U-7) on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold. Also, there are six bidoublet Higgs fields in Models TI-U-i with i=1, ..., 4. There are twelve pairs of vector-like bidoublet Higgs fields from the massless open string states in a $N = 2$ subsector in Model TI-U-5, and six pairs of vector-like bidoublet Higgs fields in Models TI-U-6 and TI-U-7. In particular, Model TI-U-6 is the only model in this paper that the supergravity fluxes do not contribute D6-brane charges, and then do not affect the RR tadpole cancellation conditions. Because Model TI-U-7 is on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold, its supergravity fluxes do contribute large negative D6-brane RR tadpoles due to the Dirac quantization conditions, and then there are many exotic particles from extra gauge groups due to the RR tadpole cancellation conditions. In addition, for the first kind of Pati-Salam models with

$U(4)_C \times USp(2)_L \times U(2)_R$ gauge symmetry, we give four models (TI-Sp-j with j=1, ..., 4) on Type IIA $\mathbf{T^6}$ orientifold. We have three bidoublet Higgs fields in Models TI-Sp-1, TI-Sp-2 and TI-Sp-3, and three pairs of vector-like bidoublet Higgs fields in Model TI-Sp-4.

For the second kind of Pati-Salam models with $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry, we present six models (Models TII-U-i with i=1, ..., 6) on Type IIA $\mathbf{T^6}$ orientifold. There are two bidoublet Higgs fields in Model TII-U-1, three in Model TII-U-2, four in Models TII-U-3 and TII-U-4. And there are two and four pairs of vector-like bidoublet Higgs fields in Models TII-U-5 and TII-U-6, respectively. For the second kind of Pati-Salam models with $U(4)_C \times USp(2)_L \times U(2)_R$ gauge symmetry, we give four models (Models TII-Sp-i with i=1, ..., 4) on Type IIA $\mathbf{T^6}$ orientifold, and one model (Model TII-Sp-5) on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold. Also, we have one bidoublet Higgs field in Model TII-Sp-1, three in Models TII-Sp-2, TII-Sp-3, and TII-Sp-5, and four pairs of vector-like bidoublet Higgs fields in Model TII-Sp-4. Similar to the Model TI-U-7, we have quite a few exotic particles in Model TII-Sp-5. Comparing to the previous Pati-Salam model building in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold without supergravity fluxes, we have less bidoublet Higgs fields.

Furthermore, the generic feature of our models is that there exist exotic particles. The phenomenological consequences, for example, the SM fermion masses and mixings, the moduli stabilization, and how to give masses to exotic particles, will be discussed elsewhere [41].

This paper is organized as follows. In Section II, we briefly review the intersecting D6-brane model building on Type IIA orientifolds with flux compactifications. We study the general conditions for Pati-Salam model building in Section III. We discuss the first and second kinds of Pati-Salam models in Section IV. Discussion and conclusions are given in Section V. In Appendices A and B, we present the D6-brane configurations and intersection numbers for the first and second kinds of Pati-Salam models, respectively.

II. FLUX MODEL BUILDING ON TYPE IIA ORIENTIFOLDS

We briefly review the rules for the intersecting D6-brane model building in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold with flux compactifications [38, 39]. Because the model building rules in Type IIA theory on $\mathbf{T^6}$ orientifold with flux compactifications are quite similar, we only explain the differences for simplicity.

A. Type IIA Theory on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifold

We consider $\mathbf{T^6}$ to be a six-torus factorized as $\mathbf{T^6} = \mathbf{T^2} \times \mathbf{T^2} \times \mathbf{T^2}$ whose complex coordinates are z_i, $i = 1,\ 2,\ 3$ for the i-th two-torus, respectively. The θ and ω generators for the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ act on the complex coordinates as following

$$
\begin{aligned}
\theta &: \ (z_1, z_2, z_3) \to (-z_1, -z_2, z_3) \ , \\
\omega &: \ (z_1, z_2, z_3) \to (z_1, -z_2, -z_3) \ .
\end{aligned} \tag{1}
$$

We implement an orientifold projection ΩR, where Ω is the world-sheet parity, and R acts on the complex coordinates as

$$
R : (z_1, z_2, z_3) \to (\overline{z}_1, \overline{z}_2, \overline{z}_3) \ . \tag{2}
$$

Thus, we have four kinds of orientifold 6-planes (O6-planes) under the actions of ΩR, $\Omega R\theta$, $\Omega R\omega$, and $\Omega R\theta\omega$, respectively. In addition, we introduce some stacks of D6-branes which wrap on the factorized three-cycles. There are two kinds of complex structures consistent with orientifold projection for a two-torus – rectangular and tilted [10, 42]. If we denote the homology classes of the three cycles wrapped by a stack of N_a D6-branes as $n_a^i[a_i] + m_a^i[b_i]$ and $n_a^i[a_i'] + m_a^i[b_i]$ with $[a_i'] = [a_i] + \frac{1}{2}[b_i]$ for the rectangular and tilted two-tori respectively, we can label a generic one cycle by (n_a^i, l_a^i) in which $l_a^i \equiv m_a^i$ for a rectangular two-torus while $l_a^i \equiv 2\tilde{m}_a^i = 2m_a^i + n_a^i$ for a tilted two-torus [12]. For a stack of N_a D6-branes along the cycle (n_a^i, l_a^i), we also need to include their ΩR images $N_{a'}$ with wrapping numbers $(n_a^i, -l_a^i)$. For the D6-branes on the top of O6-planes, we count them and their ΩR images independently. So, the homology three-cycles for a stack of N_a D6-branes and its orientifold image a' are

$$
[\Pi_a] = \prod_{i=1}^{3} \left(n_a^i[a_i] + 2^{-\beta_i} l_a^i [b_i] \right), \quad [\Pi_{a'}] = \prod_{i=1}^{3} \left(n_a^i[a_i] - 2^{-\beta_i} l_a^i [b_i] \right) \ , \tag{3}
$$

where $\beta_i = 0$ if the i-th two-torus is rectangular and $\beta_i = 1$ if it is tilted. And the homology three-cycles wrapped by the four O6-planes are

$$
\Omega R : [\Pi_{\Omega R}] = 2^3 [a_1] \times [a_2] \times [a_3] \ , \tag{4}
$$

$$
\Omega R\omega : [\Pi_{\Omega R\omega}] = -2^{3-\beta_2-\beta_3} [a_1] \times [b_2] \times [b_3] \ , \tag{5}
$$

$$
\Omega R\theta\omega : [\Pi_{\Omega R\theta\omega}] = -2^{3-\beta_1-\beta_3} [b_1] \times [a_2] \times [b_3] \ , \tag{6}
$$

$$\Omega R\theta : [\Pi_{\Omega R}] = -2^{3-\beta_1-\beta_2}[b_1] \times [b_2] \times [a_3] \ . \tag{7}$$

Therefore, the intersection numbers are

$$I_{ab} = [\Pi_a][\Pi_b] = 2^{-k} \prod_{i=1}^{3}(n_a^i l_b^i - n_b^i l_a^i) \ , \tag{8}$$

$$I_{ab'} = [\Pi_a]\,[\Pi_{b'}] = -2^{-k} \prod_{i=1}^{3}(n_a^i l_b^i + n_b^i l_a^i) \ , \tag{9}$$

$$I_{aa'} = [\Pi_a]\,[\Pi_{a'}] = -2^{3-k} \prod_{i=1}^{3}(n_a^i l_a^i) \ , \tag{10}$$

$$I_{aO6} = [\Pi_a][\Pi_{O6}] = 2^{3-k}(-l_a^1 l_a^2 l_a^3 + l_a^1 n_a^2 n_a^3 + n_a^1 l_a^2 n_a^3 + n_a^1 n_a^2 l_a^3) \ , \tag{11}$$

where $[\Pi_{O6}] = [\Pi_{\Omega R}] + [\Pi_{\Omega R\omega}] + [\Pi_{\Omega R\theta\omega}] + [\Pi_{\Omega R\theta}]$ is the sum of O6-plane homology three-cycles wrapped by the four O6-planes, and $k = \beta_1 + \beta_2 + \beta_3$ is the total number of tilted two-tori.

Sector	Representation
aa	$U(N_a/2)$ vector multiplet and 3 adjoint chiral multiplets
$ab + ba$	I_{ab} $(\frac{N_a}{2}, \frac{\overline{N_b}}{2})$ chiral multiplets
$ab' + b'a$	$I_{ab'}$ $(\frac{N_a}{2}, \frac{N_b}{2})$ chiral multiplets
$aa' + a'a$	$\frac{1}{2}(I_{aa'} + \frac{1}{2}I_{aO6})$ anti-symmetric chiral multiplets
	$\frac{1}{2}(I_{aa'} - \frac{1}{2}I_{aO6})$ symmetric chiral multiplets

TABLE I: The general spectrum for the intersecting D6-brane model building in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold with flux compactifications.

For a stack of N_a D6-branes and its ΩR image, we have $U(N_a/2)$ gauge symmetry, while for a stack of N_a D6-branes and its ΩR image on the top of O6-plane, we obtain $USp(N_a)$ gauge symmetry. The general spectrum of D6-branes' intersecting at generic angles, which is valid for both rectangular and tilted two-tori, is given in Table I. The four-dimensional $N = 1$ supersymmetric models on Type IIA orientifolds with intersecting D6-branes are mainly constrained in two aspects: four-dimensional $N = 1$ supersymmetry conditions, and RR tadpole cancellation conditions.

To simplify the notation, we define the following products of wrapping numbers

$$\begin{array}{rl} A_a \equiv -n_a^1 n_a^2 n_a^3, \quad B_a \equiv n_a^1 l_a^2 l_a^3, \quad C_a \equiv l_a^1 n_a^2 l_a^3, \quad D_a \equiv l_a^1 l_a^2 n_a^3, \\ \tilde{A}_a \equiv -l_a^1 l_a^2 l_a^3, \; \tilde{B}_a \equiv l_a^1 n_a^2 n_a^3, \; \tilde{C}_a \equiv n_a^1 l_a^2 n_a^3, \; \tilde{D}_a \equiv n_a^1 n_a^2 l_a^3. \end{array} \tag{12}$$

(1) *Four-Dimensional $N = 1$ Supersymmetry Conditions*

The four-dimensional $N = 1$ supersymmetry can be preserved by the orientation projection (ΩR) if and only if the rotation angle of any D6-brane with respect to any O6-plane is an element of $SU(3)$ [4], *i. e.*, $\theta_1 + \theta_2 + \theta_3 = 0 \bmod 2\pi$, where θ_i is the angle between the $D6$-brane and the O6-plane on the i-th two-torus. Then the supersymmetry conditions can be rewritten as [12]

$$x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a \;=\; 0 \;, \tag{13}$$

$$A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D \;<\; 0 \;, \tag{14}$$

where $x_A = \lambda,\ x_B = \lambda 2^{\beta_2+\beta 3}/\chi_2\chi_3,\ x_C = \lambda 2^{\beta_1+\beta 3}/\chi_1\chi_3$, and $x_D = \lambda 2^{\beta_1+\beta 2}/\chi_1\chi_2$ in which $\chi_i = R_i^2/R_i^1$ are the complex structure parameters and λ is a positive real number.

(2) *RR Tadpole Cancellation Conditions*

The total RR charges from the D6-branes and O6-planes and from the metric, NSNS, and RR fluxes must vanish since the RR field flux lines are conserved. With the filler branes on the top of the four O6-planes, we obtain the RR tadpole cancellation conditions [38, 39]:

$$2^k N^{(1)} - \sum_a N_a A_a + \frac{1}{2}(m h_0 + q_1 a_1 + q_2 a_2 + q_3 a_3) \;=\; 16 \;, \tag{15}$$

$$-\, 2^{\beta_1} N^{(2)} + \sum_a 2^{-\beta_2-\beta 3} N_a B_a + \frac{1}{2}(m h_1 - q_1 b_{11} - q_2 b_{21} - q_3 b_{31}) \;=\; -2^{4-\beta_2-\beta_3} \;, \tag{16}$$

$$-\, 2^{\beta_2} N^{(3)} + \sum_a 2^{-\beta_1-\beta_3} N_a C_a + \frac{1}{2}(m h_2 - q_1 b_{12} - q_2 b_{22} - q_3 b_{32}) \;=\; -2^{4-\beta_1-\beta_3} \;, \tag{17}$$

TABLE II: Wrapping numbers of the four O6-planes.

Orientifold Action	O6-Plane	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$
ΩR	1	$(2^{\beta_1}, 0) \times (2^{\beta_2}, 0) \times (2^{\beta_3}, 0)$
$\Omega R\omega$	2	$(2^{\beta_1}, 0) \times (0, -2^{\beta_2}) \times (0, 2^{\beta_3})$
$\Omega R\theta\omega$	3	$(0, -2^{\beta_1}) \times (2^{\beta_2}, 0) \times (0, 2^{\beta_3})$
$\Omega R\theta$	4	$(0, -2^{\beta_1}) \times (0, 2^{\beta_2}) \times (2^{\beta_3}, 0)$

$$- 2^{\beta_3} N^{(4)} + \sum_a 2^{-\beta_1-\beta_2} N_a D_a + \frac{1}{2}(m h_3 - q_1 b_{13} - q_2 b_{23} - q_3 b_{33}) \; = \; -2^{4-\beta_1-\beta_2} \; , \quad (18)$$

where $2N^{(i)}$ are the number of filler branes wrapping along the i-th O6-plane which is defined in Table II. In addition, a_i and b_{ij} arise from the metric fluxes, h_0 and h_i arise from the NSNS fluxes, and m and q_i arise from the RR fluxes. We consider these fluxes (a_i, b_{ij}, h_0, h_i, m and q_i) quantized in units of 8 so that we can avoid the problems with flux Dirac quantization conditions.

In this paper, we concentrate on the supersymmetric AdS vacua with metric, NSNS and RR fluxes [39]. For simplicity, we assume that the Kähler moduli T_i satisfy $T_1 = T_2 = T_3$, then we obtain $q_1 = q_2 = q_3 \equiv q$ from the superpotential in [39]. To satisfy the Jacobi identities for metric fluxes, we consider the solution $a_i = a$, $b_{ii} = -b_i$, and $b_{ji} = b_i$ in which $j \neq i$ [39].

To have supersymmetric minima [39], we obtain that

$$3a \text{Re} S \; = \; b_i \text{Re} U_i \; , \quad \text{for} \quad i = 1, \; 2, \; 3 \; , \quad (19)$$

where

$$\text{Re} S \; \equiv \; \frac{e^{-\phi}}{\sqrt{\chi_1 \chi_2 \chi_3}} \; , \; \text{Re} U_i \; \equiv \; e^{-\phi} \sqrt{\frac{\chi_j \chi_k}{\chi_i}} \; , \quad (20)$$

where S and U_i are respectively dilaton and complex structure moduli, ϕ is the four-dimensional T-duality invariant dilaton, and $i \neq j \neq k \neq i$. And then we have

$$b_1 = \frac{3a}{\chi_2 \chi_3} \; , \; b_2 = \frac{3a}{\chi_1 \chi_3} \; , \; b_3 = \frac{3a}{\chi_1 \chi_2} \; . \quad (21)$$

Moreover, there are consistency conditions

$$3h_i a + h_0 b_i = 0 \ , \quad \text{for} \quad i = 1,\ 2,\ 3 \ . \tag{22}$$

So we have

$$h_1 = -\frac{h_0}{\chi_2\chi_3} \ , \ h_2 = -\frac{h_0}{\chi_1\chi_3} \ , \ h_3 = -\frac{h_0}{\chi_1\chi_2} \ . \tag{23}$$

Thus, the RR tadpole cancellation conditions can be rewritten as following

$$2^k N^{(1)} - \sum_a N_a A_a + \frac{1}{2}(h_0 m + 3aq) \ = \ 16 \ , \tag{24}$$

$$-\, 2^{\beta_1} N^{(2)} + \sum_a 2^{-\beta_2-\beta_3} N_a B_a - \frac{1}{2\chi_2\chi_3}(h_0 m + 3aq) \ = \ -2^{4-\beta_2-\beta_3} \ , \tag{25}$$

$$-\, 2^{\beta_2} N^{(3)} + \sum_a 2^{-\beta_1-\beta_3} N_a C_a - \frac{1}{2\chi_1\chi_3}(h_0 m + 3aq) \ = \ -2^{4-\beta_1-\beta_3} \ , \tag{26}$$

$$-\, 2^{\beta_3} N^{(4)} + \sum_a 2^{-\beta_1-\beta_2} N_a D_a - \frac{1}{2\chi_1\chi_2}(h_0 m + 3aq) \ = \ -2^{4-\beta_1-\beta_2} \ . \tag{27}$$

Therefore, if $(h_0 m + 3aq) < 0$, the supergravity fluxes contribute negative D6-brane charges to all the RR tadpole cancellation conditions, and then, the RR tadpole cancellation conditions give no constraints on the consistent model building because we can always introduce suitable supergravity fluxes and some stacks of D6-branes in the hidden sector to cancel the RR tadpoles. Also, if $(h_0 m + 3aq) = 0$, the supergravity fluxes do not contribute to any D6-brane charges, and then do not affect the RR tadpole cancellation conditions.

In addition, the Freed-Witten anomaly cancellation condition is [39]

$$-\, 2^{-k} h_0 \tilde{A}_a + 2^{-\beta_1} h_1 \tilde{B}_a + 2^{-\beta_2} h_2 \tilde{C}_a + 2^{-\beta_3} h_3 \tilde{D}_a = 0 \ . \tag{28}$$

It can be shown that if Eqs. (13), (19), and (22) are satisfied, the Freed-Witten anomaly is automatically cancelled. So, we will not consider the Freed-Witten anomaly in our model building.

Furthermore, in addition to the above RR tadpole cancellation conditions, the discrete D-brane RR charges classified by $\mathbb{Z}_2$ K-theory groups in the presence of orientifolds, which are subtle and invisible by the ordinary homology [23, 43], should also be taken into account [21]. The K-theory conditions for a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold are

$$\sum_a 2^{-k} \tilde{A}_a = \sum_a 2^{-\beta_1} \tilde{B}_a = \sum_a 2^{-\beta_2} \tilde{C}_a = \sum_a 2^{-\beta_3} \tilde{D}_a = 0 \text{ mod } 4 \ . \tag{29}$$

B. Type IIA Theory on $\mathbf{T^6}$ Orientifold

The intersecting D6-brane model building in Type IIA theory on $\mathbf{T^6}$ orientifold with flux compactifications is similar to that on $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. For the model building rules in the previous subsection, we only need to make the following changes: (1) For a stack of N_a D6-branes and its ΩR image, we have $U(N_a)$ gauge symmetry, while for a stack of N_a D6-branes and its ΩR image on the top of O6-plane, we obtain $USp(2N_a)$ gauge symmetry. Also, we present the general spectrum of D6-branes' intersecting at generic angles in Type IIA theory on $\mathbf{T^6}$ orientifold in Table III. (2) We only have the ΩR O6-planes, so, $[\Pi_{O6}] = [\Pi_{\Omega R}]$ in Eq. (11), and the right-hand sides of Eqs. (16), (17) and (18) are zero. (3) The metric, NSNS and RR fluxes (a_i, b_{ij}, h_0, h_i, m and q_i) are quantized in units of 2. (4) To have three families of the SM fermions, we obtain that at least one of the three two-tori is tilted. Thus, the right-hand side of Eq. (29) is 0 mod 2 for the K-theory conditions in our model building [44].

Sector	Representation
aa	$U(N_a)$ vector multiplet and 3 adjoint chiral multiplets
$ab+ba$	I_{ab} $(N_a, \overline{N_b})$ chiral multiplets
$ab'+b'a$	$I_{ab'}$ (N_a, N_b) chiral multiplets
$aa'+a'a$	$\frac{1}{2}(I_{aa'} + I_{aO6})$ anti-symmetric chiral multiplets
	$\frac{1}{2}(I_{aa'} - I_{aO6})$ symmetric chiral multiplets

TABLE III: The general spectrum for the intersecting D6-brane model building in Type IIA theory on $\mathbf{T^6}$ orientifold with flux compactifications, in particular, $I_{aO6} = [\Pi_a][\Pi_{O6}] = -2^{3-k} l_a^1 l_a^2 l_a^3$.

III. GENERAL CONDITIONS FOR PATI-SALAM MODEL BUILDING

In the $SU(5)$ models [12], flipped $SU(5)$ models [15], and trinification models [16], the up-type quark Yukawa couplings, down-type quark Yukawa couplings, and lepton Yukawa couplings are forbidden by the anomalous $U(1)$ symmetries, respectively. In the Pati-Salam models, all the SM fermions and Higgs fields belong to the bi-fundamental representations, which can naturally arise from the intersecting D6-brane model building. In particular, all the SM fermion Yukawa couplings can in principle be allowed by the anomalous $U(1)$ symmetries. Therefore, the Pati-Salam models are very interesting in the intersecting D6-brane model building.

In this paper, we construct the Pati-Salam models with following properties:

- Three families of the SM fermions.
- The Pati-Salam gauge symmetry can be broken down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings, and further down to the SM gauge symmetry around the string scale via supersymmetry preserving Higgs mechanism.
- The SM fermion Yukawa couplings are allowed by the anomalous $U(1)$ symmetries. We consider two kinds of Pati-Salam models: in the first kind of models, we can give suitable masses to three families of the SM fermions at the stringy tree level; in the second kind of models, we can only give masses to the third family of the SM fermions at tree level while we assume that the masses for the first two families of the SM fermions may be generated by quantum corrections.

To break the Pati-Salam gauge symmetry down to the SM gauge symmetry via D6-brane splittings and supersymmetry preserving Higgs mechanism, the $SU(4)_C$ and $SU(2)_R$ gauge symmetries must come from $U(4)_C$ and $U(2)_R$ gauge symmetries (see the following discussions). Thus, we introduce three stacks of D6-branes, a, b, and c with the numbers of D6-branes 8, 4 (or 2), and 4 in Type IIA theory on $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold, or with the numbers of D6-branes 4, 2 (or 1), and 2 in Type IIA theory on $\mathbf{T^6}$ orientifold. So, a, b, and c stacks of D6-branes give us the gauge symmetries $U(4)_C$, $U(2)_L$ (or $USp(2)_L$) and $U(2)_R$, respectively. The anomalies from global $U(1)$s are cancelled by the generalized Green-Schwarz mechanism, and the gauge fields of these $U(1)$s obtain masses via the linear $B \wedge F$ couplings. So, the effective gauge symmetry is $SU(4)_C \times SU(2)_L \times SU(2)_R$. In addition, we require that the intersection numbers satisfy

$$I_{ab} \;=\; 3 \;,\; I_{ab'} \;=\; 0 \text{ if } \mathrm{SU(2)_L} \text{ from } \mathrm{U(2)_L} \;, \tag{30}$$

$$I_{ac} \;=\; -3 \;,\; I_{ac'} \;=\; 0 \;. \tag{31}$$

The conditions $I_{ab} = 3$ and $I_{ac} = -3$ give us three families of the SM fermions with quantum numbers $(\mathbf{4}, \mathbf{2}, \mathbf{1})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2})$ under $SU(4)_C \times SU(2)_L \times SU(2)_R$ gauge symmetry. $I_{ac'} = 0$ implies that a stack of D6-branes is parallel to the ΩR image c' of the c stack of D6-branes along at least one two-torus, for example, the third two-torus. So, if a and c' stacks of D6-branes are on the top of each other on the third two-torus, we obtain $I_{ac'}^{(1,2)}$ pairs of the vector-like chiral multiplets with quantum numbers $(\mathbf{4}, \mathbf{1}, \mathbf{2})$ and $(\bar{\mathbf{4}}, \mathbf{1}, \bar{\mathbf{2}})$, where $I_{ac'}^{(1,2)}$ is the product of intersection numbers for a and c' stacks of D6-branes on the first two two-tori. These particles are the Higgs fields which can break the Pati-Salam gauge symmetry down to the SM gauge symmetry, and preserve the D- and F-flatness, *i. e.*, preserve supersymmetry. Also, the conditions in Eq. (31) imply that the $SU(4)_C$ and $SU(2)_R$ gauge symmetries must come from $U(4)_C$ and $U(2)_R$ gauge symmetries, respectively.

In addition, for the first kind of Pati-Salam models, we require that

$$I_{bc} \geq 2 , \tag{32}$$

and all the SM fermions and at least two bidoublet Higgs fields arise from the intersections on the same two-torus so that the suitable three-family SM fermion masses and mixings can be generated at the stringy tree level. And for the second kind of Pati-Salam models, we require that

$$I_{bc} \geq 1 , \tag{33}$$

and the left-handed SM fermions, right-handed SM fermions and bidoublet Higgs fields do not arise from the intersections on the same two-torus. Then we can only give masses to the third family of the SM fermions at tree level, and we assume that the first two families of the SM fermions can obtain masses from quantum corrections.

In order to break the Pati-Salam gauge symmetry, we split the a stack of D6-branes into a_1 and a_2 stacks with respectively 6 (3) and 2 (1) D6-branes, split the c stack of D6-branes into c_1 and c_2 stacks with 2 (1) D6-branes for each one on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold (Type IIA $\mathbf{T^6}$ orientifold). And then, the Pati-Salam gauge symmetry is broken down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$. To break this gauge symmetry down to the SM gauge symmetry, we assume that the a_2 and c_1' (ΩR image of c_1) stacks of D6-branes are parallel and on the top of each other on the third two-torus as an example, and then we obtain $I_{a_2 c_1'}^{(1,2)}$ pairs of vector-like chiral multiplets with quantum numbers $(\mathbf{1}, \mathbf{1}, -\mathbf{1}, \mathbf{1/2})$ and

$(\mathbf{1}, \mathbf{1}, \mathbf{1}, -\mathbf{1/2})$ under $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ gauge symmetry, where $I^{(1,2)}_{a_2 c'_1}$ is the product of intersection numbers for a_2 and c'_1 stacks on the first two two-tori. These particles can break the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ gauge symmetry down to the SM gauge symmetry and preserve supersymmetry in the mean time because their quantum numbers are the same as those of the right-handed neutrino and its Hermitian conjugate. In summary, the complete gauge symmetry breaking chains are

$$
\begin{aligned}
SU(4)_C \times SU(2)_L \times SU(2)_R \quad & \xrightarrow[a \to a_1 + a_2]{} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\
& \xrightarrow[c \to c_1 + c_2]{} SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L} \\
& \xrightarrow[\text{Higgs Mechanism}]{} SU(3)_C \times SU(2)_L \times U(1)_Y \ . \qquad (34)
\end{aligned}
$$

IV. TWO KINDS OF PATI-SALAM MODELS

In this Section, we present the first and second kinds of Pati-Salam models where all the SM fermion Yukawa couplings are allowed by the anomalous $U(1)$ symmetries. Because the supergravity fluxes on Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold contribute large negative D6-brane charges due to the Dirac quantization conditions if we want to use them to relax the RR tadpole cancellation conditions, many D6-branes in the hidden sector need to be introduced so that the RR tadpoles can be completely cancelled. Then there may exist a lot of exotic particles. Therefore, we mainly consider the Pati-Salam models on Type IIA $\mathbf{T^6}$ orientifold with flux compactifications. Also, we emphasize that the Pati-Salam models on Type IIA $\mathbf{T^6}$ orientifold can be similarly constructed on Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold by introducing more stacks of D6-branes in the hidden sector. In addition, we determine the complex structure parameters via supersymmetric D6-brane configurations in our model building. Similar to [39], all the moduli may be stabilized in our models.

A. The First Kind of Pati-Salam Models

We present the D6-brane configurations and intersection numbers for the first kind of Pati-Salam models, *i. e.*, the Models TI-U-i with i=1, ..., 7, and the Models TI-Sp-j with j=1, ..., 4, in Tables VII-XVII. In these models, all the SM fermions and at least three bidoublet Higgs fields arise from the intersections on the same two-torus. So, the suitable

Representation	Multiplicity	$U(1)_a$	$U(1)_b$	$U(1)_c$
$(4_a, \bar{2}_b)$	3	1	-1	0
$(\bar{4}_a, 2_c)$	3	-1	0	1
$(2_b, \bar{2}_c)$	6	0	1	-1
$(4_a, 2_c)$	3	1	0	1
$(\bar{4}_a, \bar{2}_c)$	3	-1	0	-1
6_a	1	2	0	0
$\overline{10}_a$	1	-2	0	0
1_c	2	0	0	-2
3_c	2	0	0	2

TABLE IV: The particle spectrum in observable sector in Model TI-U-4 with gauge symmetry $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times USp(2) \times USp(10)]_{hidden}$. Here, a, b and c denote the gauge groups $U(4)_C$, $U(2)_L$ and $U(2)_R$, respectively.

three-family SM fermion masses and mixings can be generated at the stringy tree level, and then the rank one problem for the SM fermion Yukawa matrices can be solved.

The observable gauge symmetry in Models TI-U-i is $U(4)_C \times U(2)_L \times U(2)_R$. The Models TI-U-i with i=1, ..., 5 are on Type IIA $\mathbf{T^6}$ orientifold while the Models TI-U-6 and TI-U-7 are on Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. Only Model TI-U-4 has a $U(4)_C$ symmetric representation. Moreover, there are six bidoublet Higgs fields in Models TI-U-1, TI-U-2, TI-U-3 and TI-U-4. There are twelve pairs of vector-like bidoublet Higgs fields from the massless open string states in a $N = 2$ subsector in Model TI-U-5, and six pairs of vector-like bidoublet Higgs fields in Models TI-U-6 and TI-U-7. Especially, the D6-brane configurations in Model TI-U-6 are the same as those in Model I-Z-10 in Ref. [13], and Model TI-U-6 is the only model that the supergravity fluxes do not contribute to the D6-brane RR tadpoles. Also, the D6-brane configurations in the observable sector in Model TI-U-7 are the same as

Representation	Multiplicity	$U(1)_a$	$U(1)_b$	$U(1)_c$	$U(1)_d$
$(4_a, \bar{2}_d)$	2	1	0	0	-1
$(\bar{4}_a, 2_e)$	3	-1	0	0	0
$(4_a, 10_{O6})$	1	1	0	0	0
$(\bar{2}_b, 2_d)$	1	0	-1	0	1
$(\bar{2}_b, \bar{2}_d)$	5	0	-1	0	-1
$(2_b, 2_e)$	6	0	1	0	0
$(2_c, 2_d)$	2	0	0	1	1
$(\bar{2}_c, 10_{O6})$	2	0	0	-1	0
$(2_d, 2_e)$	6	0	0	0	1

TABLE V: The exotic particle spectrum in Model TI-U-4 with gauge symmetry $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times USp(2) \times USp(10)]_{hidden}$. Here, a, b, c, d, e and $O6$ denote the gauge groups $U(4)_C$, $U(2)_L$, $U(2)_R$, $U(2)$, $USp(2)$ and $USp(10)$, respectively.

those in Model I-Z-10 in Ref. [13], and there are a lot of exotic particles from extra gauge groups due to the large supergravity fluxes and the RR tadpole cancellation conditions.

The observable gauge symmetry in Models TI-Sp-j is $U(4)_C \times USp(2)_L \times U(2)_R$, and all these models are on Type IIA $\mathbf{T^6}$ orientifold. There are $U(4)_C$ symmetric representations in Models TI-Sp-3 and TI-Sp-4. Also, there are three bidoublet Higgs fields in Models TI-Sp-1, TI-Sp-2 and TI-Sp-3, and three pairs of vector-like bidoublet Higgs fields in Model TI-Sp-4.

We present the complete particle spectrum in Model TI-U-4 with six bidoublet Higgs fields in Tables IV and V, and the particle spectrum in the observable sector in Model TI-Sp-1 with three bidoublet Higgs fields in Table VI. The vector-like particles with quantum numbers $(4_a, 2_c)$ and $(\bar{4}_a, \bar{2}_c)$ are the Higgs fields which can break the Pati-Salam gauge symmetry down to the SM gauge symmetry. After suitable D6-brane splittings, only the vector-like particles with quantum numbers $(\mathbf{1}, \mathbf{1}, \mathbf{-1}, \mathbf{1/2})$ and $(\mathbf{1}, \mathbf{1}, \mathbf{1}, \mathbf{-1/2})$ under $SU(3)_C \times$

$SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ gauge symmetry from $(4_a, 2_c)$ and $(\bar{4}_a, \bar{2}_c)$ are massless, and they can break the $U(1)_{B-L} \times U(1)_{I_{3R}}$ gauge symmetry down to the $U(1)_Y$ gauge symmetry by supersymmetry preserving Higgs mechanism. It is apparent from Tables IV and VI that all the SM fermion Yukawa couplings are allowed by the anomalous $U(1)_{a,b,c}$ symmetries.

Representation	Multiplicity	$U(1)_a$	$U(1)_b$	$U(1)_c$
$(4_a, \bar{2}_b)$	3	1	-1	0
$(\bar{4}_a, 2_c)$	3	-1	0	1
$(2_b, \bar{2}_c)$	3	0	1	-1
$(4_a, 2_c)$	1	1	0	1
$(\bar{4}_a, \bar{2}_c)$	1	-1	0	-1
Exotic Particles and Hidden Sector Matter				

TABLE VI: The particle spectrum in the observable sector in Model TI-Sp-1 with gauge symmetry $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^4 \times USp(2)]_{hidden}$. Here, a, b and c denote the gauge groups $U(4)_C$, $USp(2)_L$ and $U(2)_R$, respectively.

B. The Second Kind of Pati-Salam Models

We present the D6-brane configurations and intersection numbers for the second kind of Pati-Salam models, *i. e.*, the Models TII-U-i with i=1, ..., 6, and the Models TII-Sp-j with j=1, ..., 5, in Tables XVIII-XXVII. Because the left-handed SM fermions, right-handed SM fermions and bidoublet Higgs fields do not arise from the intersections on the same two-torus in these models, only the SM fermion masses for the third family can be generated at the stringy tree level. We assume that the first two families of the SM fermions may obtain masses from quantum corrections.

Similar to the above subsection, the observable gauge symmetry in Models TII-U-i is $U(4)_C \times U(2)_L \times U(2)_R$. All these models are on Type IIA $\mathbf{T^6}$ orientifold. Moreover, there are two bidoublet Higgs fields in Model TII-U-1, three in Model TII-U-2, and four in Models

TII-U-3 and TII-U-4. There are two and four pairs of vector-like bidoublet Higgs fields in Models TII-U-5 and TII-U-6, respectively.

The observable gauge symmetry in Models TII-Sp-j is $U(4)_C \times USp(2)_L \times U(2)_R$. The Models TII-Sp-i with i=1, ..., 4 are on Type IIA $\mathbf{T^6}$ orientifold while the Model TII-Sp-5 is on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold. Also, there are $U(4)_C$ symmetric representations in Models TII-Sp-1, TII-Sp-3 and TII-Sp-4. Moreover, there are one bidoublet Higgs field in Model TII-Sp-1, three in Models TII-Sp-2, TII-Sp-3 and TII-Sp-5, and four pairs of vector-like bidoublet Higgs fields in Model TII-Sp-4.

V. DISCUSSION AND CONCLUSIONS

We considered the Pati-Salam model building on Type IIA orientifolds with flux compactifications in supersymmetric AdS vacua. We showed that the metric, NSNS and RR fluxes can contribute negative D6-brane charges to all the RR tadpole cancellation conditions, *i. e.*, the RR tadpole cancellation conditions can be completely relaxed. So, the major constraints on consistent model building on Type IIA orientifolds are the four-dimensional $N = 1$ supersymmetry conditions.

We constructed two kinds of three-family Pati-Salam models. In the first kind of models, the suitable three-family SM fermion masses and mixings can be generated at the stringy tree level, and then the rank one problem for the SM fermion Yukawa matrices can be solved. While in the second kind of models, only the third-family SM fermions can have masses at tree level, and we assume that the first two families of the SM fermions may obtain masses from quantum corrections. In these models, the complex structure parameters can be determined by supersymmetric D6-brane configurations, and all the moduli may be stabilized. The initial gauge symmetries $U(4)_C \times U(2)_L \times U(2)_R$ and $U(4)_C \times USp(2)_L \times U(2)_R$ can be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ due to the generalized Green-Schwarz mechanism and D6-brane splittings, and further down to the SM gauge symmetry at about the string scale via the supersymmetry preserving Higgs mechanism. Moreover, most of our models are on Type IIA $\mathbf{T^6}$ orientifold with flux compactifications because the supergravity fluxes on Type IIA $\mathbf{T^6}/(\mathbb{Z}_\mathbf{2} \times \mathbb{Z}_\mathbf{2})$ orientifold contribute large negative D6-brane RR tadpoles due to the Dirac quantization conditions if we use them to relax the RR tadpole cancellation conditions. Of course, our Pati-Salam models on Type IIA $\mathbf{T^6}$ orientifold can be similarly constructed on Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold by introducing more stacks of D6-branes in the hidden sector.

For the first kind of Pati-Salam models with $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry, we presented five models on Type IIA $\mathbf{T^6}$ orientifold and two models on Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. We have six bidoublet Higgs fields in the Models TI-U-i with i=1, ..., 4, twelve pairs of vector-like bidoublet Higgs fields in Model TI-U-5, and six pairs of vector-like bidoublet Higgs fields in Models TI-U-6 and TI-U-7. Especially, Model TI-U-6 is the only model that its supergravity fluxes do not contribute to the D6-brane RR tadpoles. Moreover, for the first kind of Pati-Salam models with $U(4)_C \times USp(2)_L \times U(2)_R$ gauge symmetry, we gave four models on Type IIA $\mathbf{T^6}$ orientifold. Models TI-Sp-1, TI-Sp-2 and TI-Sp-3 have three bidoublet Higgs fields, and Model TI-Sp-4 has three pairs of vector-like bidoublet Higgs fields.

For the second kind of Pati-Salam models with $U(4)_C \times U(2)_L \times U(2)_R$ gauge symmetry, we constructed six models on Type IIA $\mathbf{T^6}$ orientifold. There are two bidoublet Higgs fields in Model TII-U-1, three in Model TII-U-2, and four in Models TII-U-3 and TII-U-4. We also have two and four pairs of vector-like bidoublet Higgs fields in Models TII-U-5 and TII-U-6, respectively. For the second kind of Pati-Salam models with $U(4)_C \times USp(2)_L \times U(2)_R$ gauge symmetry, we presented four models on Type IIA $\mathbf{T^6}$ orientifold, and one model on Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold. There are one bidoublet Higgs field in Model TII-Sp-1, three in Models TII-Sp-2, TII-Sp-3 and TII-Sp-5, and four pairs of vector-like bidoublet Higgs fields in Model TII-Sp-4. Comparing to the previous Pati-Salam model building on Type IIA $\mathbf{T^6}/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orientifold without supergravity fluxes, we have less bidoublet Higgs fields.

Furthermore, the generic feature of our models is that there exist some exotic particles, in particular, in Models TI-U-7 and TII-Sp-5 because their supergravity fluxes contribute large negative D6-brane charges and we have to introduce quite a few stacks of D6-branes in the hidden sector to cancel the RR tadpoles. The phenomenological consequences of our models, for instance, the SM fermion masses and mixings, the moduli stabilization, and how to give masses to exotic particles, will be presented in detail elsewhere [41].

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APPENDIX A: THE D6-BRANE CONFIGURATIONS AND INTERSECTION NUMBERS FOR THE FIRST KIND OF PATI-SALAM MODELS

In Appendix A, we present the D6-brane configurations and intersection numbers for the first kind of Pati-Salam models. Let us explain the convention. Suppose b and c stacks of D6-branes are parallel on a two-torus and the product of intersection numbers on the other two two-tori is i, we denote their intersection number as $0(i)$.

1. $U(4)_C \times U(2)_L \times U(2)_R$ Models

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	$O6$
a	4	(1, 0) (-1,-1) (-1, 1)	0	0	3	0(3)	-3	0(3)	0(3)	-3	0(3)	3	0(1)	-	0(1)
b	2	(1,-3) (1,-1) (0, 2)	-6	6	-	-	6	0(1)	-6	0(18)	-9	-3	0(3)	-	-6
c	2	(-1,-3) (0, 2) (1, 1)	6	-6	-	-	-	-	9	3	6	0(18)	0(3)	-	6
d	2	(2,-3) (1, 1) (2, 0)	0	0	-	-	-	-	-	-	12	0(1)	-6	-	0(3)
e	2	(2, 3) (2, 0) (1,-1)	0	0	-	-	-	-	-	-	-	-	6	-	0(3)
f	1	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	0(4)
$O6$	1	(1, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE VII: D6-brane configurations and intersection numbers for Model TI-U-1 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2)^2 \times USp(2)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the first two-torus, and the complex structure parameters are $3\chi_1 = \chi_2 = \chi_3 = \sqrt{2}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -2(3q+4)$, $a = 4$, and $m = 2$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'
a	4	(1, 0) (-1,-1) (-1, 1)	0	0	3	0(3)	-3	0(3)	-1	2	1	-2	3	0(3)	-3	0(3)	1	-	-1	-
b	2	(-1, 3) (-2, 0) (1, 1)	0	0	-	-	6	0(1)	0(8)	0(2)	-2	4	0(1)	-6	0(12)	0(2)	0(1)	-	2	-
c	2	(1, 3) (-1, 1) (-2, 0)	0	0	-	-	-	-	2	-4	0(8)	0(2)	0(12)	0(2)	0(1)	6	-2	-	0(1)	-
d	1	(1, 1) (2, 0) (3,-1)	0	0	-	-	-	-	-	-	-2	0(1)	12	-6	-8	8	0(3)	-	-2	-
e	1	(1,-1) (3, 1) (2, 0)	0	0	-	-	-	-	-	-	-	-	8	-8	-12	6	2	-	0(3)	-
f	1	(1,-3) (1,-1) (0, 2)	-6	6	-	-	-	-	-	-	-	-	-	-	6	0(1)	0(1)	-	2	-
g	1	(1, 3) (0,-2) (1, 1)	6	-6	-	-	-	-	-	-	-	-	-	-	-	-	-2	-	0(1)	-
h	1	(0,-1) (2, 0) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0(4)	-
i	1	(0,-1) (0, 2) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE VIII: D6-brane configurations and intersection numbers for Model TI-U-2 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(2)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the first two-torus, and the complex structure parameters are $6\chi_1 = \chi_2 = \chi_3 = 2$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'
a	4	(1, 1) (1,-3) (1, 0)	0	0	3	0(1)	-3	0(3)	0(3)	0(1)	0(4)	0(1)	0(1)	-3	-1	2	1	-	0(1)	-
b	2	(2, 0) (1, 3) (1,-1)	0	0	-	-	6	0(3)	0(1)	3	1	-2	0(6)	0(4)	0(8)	0(2)	0(1)	-	-2	-
c	2	(1,-1) (2, 0) (1, 1)	0	0	-	-	-	-	-3	0(3)	-1	2	6	0(3)	2	-4	-2	-	2	-
d	2	(1, 1) (-1,-3) (0, 1)	3	-3	-	-	-	-	-	-	2	2	-3	0(1)	6	3	0(1)	-	-1	-
e	2	(3,-1) (1, 1) (1, 0)	0	0	-	-	-	-	-	-	-	-	6	-3	-1	0(1)	-1	-	0(3)	-
f	1	(0, 2) (-1, 3) (-1, 1)	-6	6	-	-	-	-	-	-	-	-	-	-	-8	-8	2	-	0(1)	-
g	1	(2, 0) (1,-1) (3, 1)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	0(3)	-	2	-
h	1	(2, 0) (0,-2) (0, 1)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0(2)	-
i	1	(0,-2) (0, 2) (1, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE IX: D6-brane configurations and intersection numbers for Model TI-U-3 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2)^2 \times U(1)^2 \times USp(2)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the second two-torus, and the complex structure parameters are $\chi_1 = 3\chi_2 = 2\chi_3 = 2$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	$O6$
a	4	(0,-1) (1, 1) (3, 1)	1	-1	3	0(1)	-3	0(3)	2	0(2)	-3	-	1
b	2	(-1,-1) (2, 0) (-3, 1)	0	0	-	-	6	0(3)	-1	-5	6	-	0(1)
c	2	(1,-1) (-1, 1) (0,-2)	-2	2	-	-	-	-	0(10)	2	0(1)	-	-2
d	2	(2, 3) (1,-1) (2, 0)	0	0	-	-	-	-	-	-	6	-	0(3)
e	1	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-	-	-	-	-	0(4)
$O6$	5	(1, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-

TABLE X: D6-brane configurations and intersection numbers for Model TI-U-4 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times USp(2) \times USp(10)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the third two-torus, and the complex structure parameters are $6\chi_1 = 2\chi_2 = \chi_3 = 2\sqrt{6}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -12(q+2)$, $m = 2$, and $a = 8$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	$O6$
a	4	(1, 0) (-1,-1) (-1, 1)	0	0	3	0(3)	-3	0(3)	0(1)	0(1)	2	2	-2	-2	0(1)
b	2	(1,-3) (1,-1) (0, 2)	-6	6	-	-	0(12)	0(2)	0(1)	1	2	-1	6	3	-6
c	2	(1, 3) (-1, 1) (-2, 0)	0	0	-	-	-	-	0(1)	-1	-6	-3	-2	1	0(3)
d	2	(0, 1) (1,-1) (1,-1)	-1	1	-	-	-	-	-	-	0(4)	0(1)	0(4)	0(1)	-1
e	1	(0,-1) (3, 1) (1, 3)	3	-3	-	-	-	-	-	-	-	-	0(16)	0(25)	3
f	1	(0,-1) (1, 3) (3, 1)	3	-3	-	-	-	-	-	-	-	-	-	-	3
$O6$	4	(1, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XI: D6-brane configurations and intersection numbers for Model TI-U-5 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^2 \times USp(8)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the first two-torus, and the complex structure parameters are $6\chi_1 = \chi_2 = \chi_3 = 2$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	$O6^1$	$O6^2$	$O6^3$	$O6^4$
a	8	(0,-1) (1, 1) (1, 1)	0	0	3	0(3)	-3	0(3)	1	-1	0	0
b	4	(3, 1) (1, 0) (1,-1)	-2	2	-	-	0(6)	0(1)	0	1	0	-3
c	4	(3,-1) (0, 1) (1,-1)	2	-2	-	-	-	-	-1	0	3	0
$O6^1$	2	(1, 0) (1, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-
$O6^2$	2	(1, 0) (0,-1) (0, 2)	-	-	-	-	-	-	-	-	-	-
$O6^3$	2	(0, -1) (1, 0) (0, 2)	-	-	-	-	-	-	-	-	-	-
$O6^4$	2	(0, -1) (0, 1) (2, 0)	-	-	-	-	-	-	-	-	-	-

TABLE XII: D6-brane configurations and intersection numbers for Model TI-U-6 on Type IIA $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [USp(2)^4]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the first two-torus, and the complex structure parameters are $2\chi_1 = 6\chi_2 = 3\chi_3 = 6$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = \pm 3a$, and $m = \mp q$ so that the supergravity fluxes do not contribute to the D6-brane RR tadpoles.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	$O6^1$	$O6^2$
a	8	(0,-1) (1, 1) (1, 1)	0	0	3	0(3)	-3	0(3)	18	36	15	-9	-18	-36	-15	9	8	8	-8	-8	1	-1
b	4	(3, 1) (1, 0) (1,-1)	-2	2	-	-	0(6)	0(1)	-12	6	-36	-60	18	36	15	-9	-3	12	20	5	0	1
c	4	(3,-1) (0, 1) (1,-1)	2	-2	-	-	-	-	-18	-36	-15	9	12	-6	36	60	-5	-20	-12	3	-1	0
d	2	(9,-1) (-3,-1) (-2, 0)	12	-12	-	-	-	-	-	-	0	0	-288	0	462	78	-12	-42	-90	-60	0	-6
e	2	(3, 4) (1,-4) (2, 0)	-16	16	-	-	-	-	-	-	-	-	462	-78	-720	0	276	-204	-140	340	0	8
f	2	(9, 1) (-1,-3) (0, 2)	-12	12	-	-	-	-	-	-	-	-	-	-	0	0	60	90	42	12	6	0
g	2	(3,-4) (-4, 1) (0,-2)	16	-16	-	-	-	-	-	-	-	-	-	-	-	-	-340	140	204	-276	-8	0
h	2	(1, 0) (5, 3) (5,-3)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	0	0
i	2	(1, 0) (3,-5) (3, 5)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0
$O6^1$	4	(1, 0) (1, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$O6^2$	4	(1, 0) (0,-1) (0, 2)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XIII: D6-brane configurations and intersection numbers for Model TI-U-7 on Type IIA $\mathbf{T}^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^6 \times USp(4)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the first two-torus, and the complex structure parameters are $2\chi_1 = 6\chi_2 = 3\chi_3 = 6$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -6(q+8)$, $m = 8$, and $a = 16$. A lot of exotic particles arise from extra gauge groups due to the large supergravity fluxes and the RR tadpole cancellation conditions.

2. $U(4)_C \times USp(2)_L \times U(2)_R$ Models

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'	$O6$
a	4	(1, 0) (-1,-1) (-3, 1)	0	0	3	-	-3	0(1)	-2	0(1)	1	-2	2	0(2)	2	1	0(2)	0(8)	0(1)
b	1	(0,-1) (1, 0) (0, 2)	0	0	-	-	3	-	6	-	-3	-	0(1)	-	0(1)	-	-3	-	0(2)
c	2	(-1,-1) (0, 1) (3, 1)	2	-2	-	-	-	-	2	4	0(8)	0(2)	-2	-1	-2	0(2)	2	-1	1
d	2	(-3,-1) (-1, 1) (2, 0)	0	0	-	-	-	-	-	-	6	0(1)	0(3)	-6	-4	-2	4	2	0(1)
e	1	(3,-1) (0, 1) (1,-1)	-2	2	-	-	-	-	-	-	-	-	0(3)	3	2	0(4)	0(1)	-1	-1
f	1	(0,-1) (1,-1) (-1, 1)	-2	2	-	-	-	-	-	-	-	-	-	-	-1	0(1)	0(4)	2	-1
g	1	(1,-1) (1, 0) (1, 1)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-3	0(3)	0(1)
h	1	(1, 0) (-1,-3) (-1, 1)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0(3)
$O6$	1	(1, 0) (1, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XIV: D6-brane configurations and intersection numbers for Model TI-Sp-1 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^4 \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the third two-torus, and the complex structure parameters are $2\chi_1 = 6\chi_2 = \chi_3 = 2\sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -6(q+2)$, $m = 2$, and $a = 4$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'
a	4	(1, 1) (-1,-1) (-1, 3)	12	0	3	-	-3	0(9)	6	0(3)	-36	-36	0(2)	0(3)	52	22	1	-	-3	-
b_{O6}	1	(1, 0) (2, 0) (1, 0)	-	-	-	-	3	-	-6	-	0(25)	-	0(1)	-	0(1)	-	0(1)	-	0(2)	-
c	2	(0,-1) (1,-1) (-2, 3)	-6	6	-	-	-	-	18	6	6	-9	-3	0(3)	-42	-30	0(2)	-	0(3)	-
d	1	(2, 1) (0, 2) (-1,-3)	12	-12	-	-	-	-	-	-	-27	33	9	-3	110	26	-4	-	0(6)	-
e	1	(1, 5) (1,-5) (1, 0)	0	0	-	-	-	-	-	-	-	-	0(18)	0(8)	-80	70	-5	-	0(1)	-
f	1	(1,-1) (1, 1) (1, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-2	4	1	-	0(1)	-
g	1	(3,-1) (2, 0) (4, 1)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	0(12)	-	6	-
h	2	(0,-1) (2, 0) (0, 1)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0(2)	-
i	2	(0,-1) (0, 2) (1, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XV: D6-brane configurations and intersection numbers for Model TI-Sp-2 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(4)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the third two-torus, and the complex structure parameters are $4\chi_1 = 2\chi_2 = 3\chi_3 = 2\sqrt{2}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+4)$, $m = 2$, and $a = 8$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'
a	4	(0,-1) (1, 3) (3, 1)	3	-3	3	-	-3	0(2)	2	1	15	6	-3	0(1)	0(1)	-
b_{O6}	1	(1, 0) (2, 0) (2, 0)	-	-	-	-	3	-	0(1)	-	0(1)	-	6	-	0(2)	-
c	2	(1,-1) (-1, 3) (-1,-1)	-6	0	-	-	-	-	-2	0(2)	-24	0(9)	4	4	1	-
d	2	(1, 1) (1,-1) (2, 0)	0	0	-	-	-	-	-	-	0(1)	-2	4	-2	0(1)	-
e	1	(1, 1) (2, 0) (7,-1)	0	0	-	-	-	-	-	-	-	-	16	-20	-2	-
f	1	(1,-3) (0, 2) (3,-1)	-6	6	-	-	-	-	-	-	-	-	-	-	0(1)	-
g	1	(0,-1) (0, 2) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XVI: D6-brane configurations and intersection numbers for Model TI-Sp-3 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^2 \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the second two-torus, and the complex structure parameters are $14\chi_1 = 7\chi_2 = \chi_3 = 2\sqrt{7}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'
a	4	(0,-1) (1, 3) (3, 1)	3	-3	3	-	-3	0(2)	12	15	3	-6	0(1)	-
b_{O6}	1	(1, 0) (2, 0) (2, 0)	-	-	-	-	0(3)	-	0(2)	-	-12	-	0(2)	-
c	2	(1, 0) (1,-3) (1, 1)	0	0	-	-	-	-	10	-8	-12	-6	1	-
d	1	(3,-2) (3, 1) (2, 0)	0	0	-	-	-	-	-	-	-40	64	0(9)	-
e	1	(1, 6) (1, 1) (0,-2)	12	-12	-	-	-	-	-	-	-	-	-2	-
f	2	(0,-1) (0, 2) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-

TABLE XVII: D6-brane configurations and intersection numbers for Model TI-Sp-4 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(1)^2 \times USp(4)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on the second two-torus, and the complex structure parameters are $12\chi_1 = 3\chi_2 = \chi_3 = 2\sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

APPENDIX B: THE D6-BRANE CONFIGURATIONS AND INTERSECTION NUMBERS FOR THE SECOND KIND OF PATI-SALAM MODELS

In Appendix B, we present the D6-brane configurations and intersection numbers for the second kind of Pati-Salam models.

1. $U(4)_C \times U(2)_L \times U(2)_R$ Models

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'
a	4	(1, 0) (-1,-3) (-1, 3)	0	0	3	0(1)	-3	0(1)	0(1)	3	6	3	0(3)	-9	-6	3	0(1)	-
b	2	(1,-1) (1,-3) (0, 2)	-6	6	-	-	2	0(1)	0(6)	0(0)	0(2)	0(8)	-6	0(6)	-14	-20	0(1)	-
c	2	(-1,-1) (0, 2) (1, 3)	6	-6	-	-	-	-	0(1)	-2	-4	-2	9	3	15	21	0(1)	-
d	1	(1, 1) (1, 3) (0,-2)	6	-6	-	-	-	-	-	-	0(8)	0(2)	0(6)	6	20	14	0(1)	-
e	1	(1,-3) (-1, 1) (0,-2)	-6	6	-	-	-	-	-	-	-	-	-20	14	0(19)	-34	0(3)	-
f	1	(2,-1) (1, 3) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	0(16)	0(4)	-2	-
g	1	(6, 1) (1,-1) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	2	-
h	1	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XVIII: D6-brane configurations and intersection numbers for Model TII-U-1 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $\chi_1 = 3\chi_2 = 3\chi_3 = \sqrt{2}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -2(3q+2)$, $m = 2$, and $a = 4$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	$O6$
a	4	(2, 0) (3, 1) (3,-1)	0	0	3	0(1)	-3	0(1)	2	1	0(1)	0(4)	0(6)	0(6)	-9	0(3)	0(1)
b	2	(3,-1) (2, 0) (3, 1)	0	0	-	-	3	0(1)	0(1)	0(4)	-2	-1	9	0(3)	0(6)	0(6)	0(1)
c	2	(3, 1) (3,-1) (2, 0)	0	0	-	-	-	-	-2	-1	2	1	0(3)	-9	0(3)	9	0(1)
d	3	(1,-1) (2, 0) (1, 1)	0	0	-	-	-	-	-	-	-1	0(1)	2	1	4	-4	0(1)
e	1	(2, 0) (1, 1) (1,-1)	0	0	-	-	-	-	-	-	-	-	-4	4	-2	-1	0(1)
f	1	(0, 2) (3,-1) (3,-1)	-1	1	-	-	-	-	-	-	-	-	-	-	27	0(9)	-2
g	1	(3, 1) (0,-2) (3, 1)	1	-1	-	-	-	-	-	-	-	-	-	-	-	-	2
$O6$	1	(2, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XIX: D6-brane configurations and intersection numbers for Model TII-U-2 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(3) \times U(1)^3 \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $\chi_1 = \chi_2 = \chi_3 = 6$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -36(q+4)$, $m = 2$, and $a = 24$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	$O6$
a	4	(1, 0) (-1,-3) (-1, 3)	0	0	3	0(1)	-3	0(1)	3	6	6	3	0(9)	-27	-6	3	0(1)	-	3	-	0(9)
b	2	(2,-1) (1,-3) (0, 2)	-6	6	-	-	4	0(1)	0(16)	0(4)	0(4)	0(16)	12	0(20)	-12	-8	0(1)	-	0(6)	-	-6
c	2	(-2,-1) (0, 2) (1, 3)	6	-6	-	-	-	-	-4	-8	-8	-4	30	-6	6	18	0(1)	-	-4	-	6
d	1	(2, 3) (1, 1) (0,-2)	6	-6	-	-	-	-	-	-	0(12)	0(0)	-36	24	20	0(28)	0(3)	-	0(2)	-	6
e	1	(2,-3) (1,-1) (0, 2)	-6	6	-	-	-	-	-	-	-	-	-24	36	0(28)	-20	0(3)	-	0(2)	-	-6
f	1	(4,-3) (1, 3) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	0(32)	0(8)	-6	-	24	-	0(9)
g	1	(4, 1) (1,-1) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	2	-	-8	-	0(1)
h	1	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0(2)	-	0(4)
i	2	(0,-1) (2, 0) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0(2)
$O6$	3	(1, 0) (2, 0) (2, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XX: D6-brane configurations and intersection numbers for Model TII-U-3 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^4 \times USp(2) \times USp(4) \times USp(6)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $\chi_1 = 2\chi_2 = 2\chi_3 = 2\sqrt{6}/3$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+4)$, $m = 2$, and $a = 8$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'
a	4	(1, 0) (-1,-1) (-1, 3)	0	0	3	0(3)	-3	0(2)	-1	2	0(1)	3	0(3)	-9	2	-1	0(1)	-
b	2	(2,-3) (1,-1) (0, 2)	-6	6	-	-	4	2	0(16)	0(4)	-8	-4	0(1)	12	-16	-20	0(3)	-
c	2	(0,-1) (1, 3) (1, 3)	9	-9	-	-	-	-	0(2)	-6	18	0(6)	-6	0(2)	18	36	-1	-
d	2	(2, 1) (1, 3) (0,-2)	6	-6	-	-	-	-	-	-	0(3)	12	-8	-4	24	12	0(1)	-
e	2	(-2,-1) (2, 0) (-1, 3)	0	0	-	-	-	-	-	-	-	-	0(16)	-24	0(16)	0(4)	2	-
f	1	(2,-3) (0,-2) (-1, 3)	-18	18	-	-	-	-	-	-	-	-	-	-	-64	-40	0(3)	-
g	1	(6,-1) (2, 0) (1, 1)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-2	-
h	4	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XXI: D6-brane configurations and intersection numbers for Model TII-U-4 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2)^2 \times U(1)^2 \times USp(8)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $\chi_1 = \chi_2 = 3\chi_3 = 2/\sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -4(3q+2)$, $m = 2$, and $a = 8$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'
a	4	(1, 0) (-1,-3) (-1, 1)	0	0	3	0(1)	-3	0(1)	-12	-6	-6	12	0(1)	-	3	-
b	2	(1,-1) (1,-3) (0, 1)	-6	6	-	-	0(2)	0(3)	12	6	-36	-18	0(1)	-	0(3)	-
c	2	(1, 1) (-1, 3) (-1, 0)	0	0	-	-	-	-	-2	4	6	-12	1	-	-3	-
d	1	(1, 3) (0,-2) (3, 1)	12	-12	-	-	-	-	-	-	96	0(20)	0(9)	-	-6	-
e	1	(3, 1) (2, 0) (3,-1)	0	0	-	-	-	-	-	-	-	-	6	-	0(9)	-
f	2	(1, 0) (0,-2) (0, 1)	0	0	-	-	-	-	-	-	-	-	-	-	0(2)	-
g	2	(0,-1) (2, 0) (0, 1)	0	0	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XXII: D6-brane configurations and intersection numbers for Model TII-U-5 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(1)^2 \times USp(4)^2]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $2\chi_1 = 3\chi_2 = 2\chi_3 = 2$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -2(3q+4)$, $m = 2$, and $a = 4$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'
a	4	(1, 0) (3, 1) (3,-1)	0	0	3	0(2)	-3	0(2)	-2	-4	0(9)	-	3	-
b	2	(0, 1) (3,-1) (1,-1)	-1	1	-	-	0(4)	0(1)	-2	1	3	-	0(1)	-
c	2	(0,-1) (1, 1) (3, 1)	1	-1	-	-	-	-	0(1)	1	-3	-	0(3)	-
d	2	(1,-2) (1, 1) (2, 0)	0	0	-	-	-	-	-	-	-4	-	2	-
e	1	(1, 0) (0,-2) (0, 2)	0	0	-	-	-	-	-	-	-	-	0(2)	-
f	2	(0,-1) (2, 0) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-

TABLE XXIII: D6-brane configurations and intersection numbers for Model TII-U-6 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times U(2)_L \times U(2)_R]_{observable} \times [U(2) \times USp(2) \times USp(4)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $4\chi_1 = \chi_2 = \chi_3 = 2\sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -12(q+2)$, $m = 2$, and $a = 8$.

2. $U(4)_C \times USp(2)_L \times U(2)_R$ Models

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	j	j'
a	4	(0,-1) (1, 3) (3, 1)	6	-6	3	-	-3	0(1)	-6	-9	3	0(5)	0(2)	0(8)	42	30	12	6	-3	-	0(9)	-
b_{O6}	1	(1, 0) (1, 0) (2, 0)	-	-	-	-	1	-	0(6)	-	0(2)	-	-1	-	0(1)	-	0(1)	-	0(2)	-	0(2)	-
c	2	(1,-1) (0, 1) (3,-1)	-2	2	-	-	-	-	1	-2	0(1)	-3	2	1	-14	-10	-4	-2	0(3)	-	3	-
d	2	(1, 0) (1, 6) (1,-1)	0	0	-	-	-	-	-	-	0(4)	0(16)	5	0(7)	-13	11	-7	5	0(1)	-	6	-
e	2	(1, 0) (1, 2) (3,-1)	0	0	-	-	-	-	-	-	-	-	2	3	-5	3	-3	1	0(3)	-	6	-
f	2	(0,-1) (1, 1) (1, 1)	2	-2	-	-	-	-	-	-	-	-	-	-	18	6	6	0(3)	-1	-	0(1)	-
g	1	(6, 1) (2,-1) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	0(3)	0(27)	4	-	-12	-
h	1	(3, 1) (1,-1) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	-	-6	-
i	3	(1, 0) (0,-1) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
j	1	(0, -1) (1, 0) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XXIV: D6-brane configurations and intersection numbers for Model TII-Sp-1 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2)^3 \times U(1)^2 \times USp(6) \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $\chi_1 = 3\chi_2 = \chi_3/4 = \sqrt{3/2}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -6(q+4)$, $m = 2$, and $a = 4$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	$O6$
a	4	(1, 0) (3, 1) (3,-1)	0	0	3	-	-3	0(1)	0(2)	0(8)	0(3)	9	3	0(5)	-3	-	0(1)
b	1	(0,-1) (2, 0) (0, 1)	0	0	-	-	3	-	-1	-	0(2)	-	0(2)	-	0(2)	-	0(1)
c	2	(3, 1) (3,-1) (1, 0)	0	0	-	-	-	-	2	1	-3	0(5)	0(3)	-9	0(9)	-	0(1)
d	2	(1, 0) (1, 1) (1,-1)	0	0	-	-	-	-	-	-	1	2	-2	3	-1	-	0(1)
e	1	(2, 1) (3, 1) (0,-1)	2	-2	-	-	-	-	-	-	-	-	-12	0(4)	-6	-	1
f	1	(0, 1) (3,-1) (2,-1)	-2	2	-	-	-	-	-	-	-	-	-	-	0(3)	-	-1
g	2	(0,-1) (0, 2) (1, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	0(2)
$O6$	4	(1, 0) (2, 0) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XXV: D6-brane configurations and intersection numbers for Model TII-Sp-2 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^2 \times USp(4) \times USp(8)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $2\chi_1 = \chi_2 = 2\chi_3 = 2\sqrt{6}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -12(3q+4)$, $m = 2$, and $a = 24$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	j	j'
a	4	(0,-1) (1, 3) (3, 1)	6	-6	3	-	-3	0(1)	-2	-1	-12	-15	33	21	5	1	0(3)	9	9	0(3)	0(9)	-
b_{O6}	1	(1, 0) (1, 0) (2, 0)	-	-	-	-	3	-	1	-	-3	-	0(4)	-	0(4)	-	0(2)	-	0(2)	-	0(2)	-
c	2	(1,-3) (0, 1) (3,-1)	-6	6	-	-	-	-	0(2)	0(8)	0(50)	0(32)	-33	-21	-5	-1	-3	0(5)	0(5)	-3	3	-
d	1	(1,-1) (0, 1) (1,-1)	-2	2	-	-	-	-	-	-	0(8)	0(2)	-15	-3	-3	1	2	-3	-3	2	1	-
e	1	(3, 1) (0, 1) (-1,-3)	6	-6	-	-	-	-	-	-	-	-	-27	81	-15	21	28	-25	-25	28	-3	-
f	1	(3, 2) (3,-2) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	0(16)	0(64)	-16	-8	-8	-16	-12	-
g	1	(1, 2) (1,-2) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-8	0(2)	0(2)	-8	-4	-
h	1	(1,-2) (-1, 0) (-3,-1)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0(12)	0(0)	0(3)	-
i	1	(1, 2) (1, 0) (3,-1)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0(3)	-
j	2	(0,-1) (1, 0) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XXVI: D6-brane configurations and intersection numbers for Model TII-Sp-3 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(1)^6 \times USp(4)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $12\chi_1 = 12\chi_2 = \chi_3 = 2\sqrt{6}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -6(q+2)$, $m = 2$, and $a = 4$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'
a	4	(0, 1) (-1,-3) (3, 1)	6	-6	3	-	-3	0(7)	5	1	-18	0(3)	16	-4	28	20	-3	-
b_{O6}	1	(1, 0) (1, 0) (2, 0)	-	-	-	-	0(4)	-	0(2)	-	12	-	0(25)	-	0(1)	-	0(2)	-
c	2	(-1, 0) (1, 4) (-3, 1)	0	0	-	-	-	-	-6	2	42	6	-45	-5	-9	7	0(3)	-
d	2	(1, 1) (1,-2) (2, 0)	0	0	-	-	-	-	-	-	6	-10	0(9)	0(49)	0(9)	0(25)	2	-
e	1	(-1, 2) (-1, 3) (0, 2)	-24	24	-	-	-	-	-	-	-	-	36	16	-90	-98	0(2)	-
f	1	(2, 5) (-1, 5) (-2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	0(162)	0(242)	10	-
g	1	(4, 1) (2,-1) (2, 0)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	4	-
h	1	(1, 0) (0,-1) (0, 2)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XXVII: D6-brane configurations and intersection numbers for Model TII-Sp-4 on Type IIA $\mathbf{T^6}$ orientifold. The complete gauge symmetry is $[U(4)_C \times USp(2)_L \times U(2)_R]_{observable} \times [U(2) \times U(1)^3 \times USp(2)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $12\chi_1 = 24\chi_2 = \chi_3 = 4\sqrt{3}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0 = -8(3q+2)$, $m = 2$, and $a = 16$.

stack	N	$(n_1,l_1)(n_2,l_2)(n_3,l_3)$	A	S	b	b'	c	c'	d	d'	e	e'	f	f'	g	g'	h	h'	i	i'	j	j'	k	k'	$O6^1$	$O6^4$
a	8	(1, 0) (3, 1) (3,-1)	0	0	3	-	-3	0	-1	-2	-15	-12	33	21	30	24	15	-21	21	-33	6	-2	-36	-15	0	-3
b_{O6^3}	2	(0,-1) (2, 0) (0, 1)	-	-	-	-	3	-	-1	-	3	-	0	-	0	-	-8	-	-12	-	0	-	0	-	0	0
c	4	(3, 1) (3,-1) (1, 0)	0	0	-	-	-	-	0	0	0	0	-33	-21	-30	-24	42	6	54	0	5	-2	-15	12	0	0
d	2	(1,-1) (-1,-1) (-1, 0)	0	0	-	-	-	-	-	-	0	0	3	15	6	12	-6	-10	-6	-12	-1	0	-1	2	0	0
e	2	(1, 3) (1,-3) (1, 0)	0	0	-	-	-	-	-	-	-	-	-81	27	-54	0	26	22	30	24	-7	-16	-3	0	0	0
f	2	(2,-3) (-2, 0) (-2,-3)	0	0	-	-	-	-	-	-	-	-	-	-	0	0	-140	-28	-168	0	-8	-16	96	120	0	12
g	2	(1,-3) (-2, 0) (-1,-3)	0	0	-	-	-	-	-	-	-	-	-	-	-	-	-110	26	-144	60	-5	-7	51	57	0	6
h	2	(4,-1) (0, 2) (1,-2)	4	-4	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0	0	2	6	52	-44	-4	0
i	2	(3,-1) (0, 2) (2,-3)	6	-6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	2	10	60	-48	-6	0
j	2	(2,-1) (1,-1) (0, 1)	1	-1	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	24	12	-1	2
k	2	(0, 1) (-1,-3) (6, 1)	-3	3	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	3	0
$O6^1$	6	(1, 0) (2, 0) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
$O6^4$	4	(0,-1) (0, 2) (1, 0)	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

TABLE XXVIII: D6-brane configurations and intersection numbers for Model TII-Sp-5 on Type IIA $\mathbf{T}^6/\mathbb{Z}_2\times\mathbb{Z}_2$ orientifold. The complete gauge symmetry is $[U(4)_C\times USp(2)_L\times U(2)_R]_{observable}\times[U(1)^8\times USp(6)\times USp(4)]_{hidden}$, the SM fermions and Higgs fields arise from the intersections on different two-tori, and the complex structure parameters are $\chi_1=\chi_2/2=\chi_3=\sqrt{2}$. To satisfy the RR tadpole cancellation conditions, we choose $h_0=-4(3q+8)$, $m=8$, and $a=32$. A lot of exotic particles arise from extra gauge groups due to the large supergravity fluxes and the RR tadpole cancellation conditions.